A Goldstino at the bottom of the cascade

Matteo Bertolini

based on 1509.03594 (JHEP)
w/ D. Musso, I. Papadimitriou & H. Raj

[see also 1412.6499 (PRD) w/ Argurio, Musso, Porri & Redigolo
and 1310.6897 (JHEP) w/ Argurio, Di Pietro, Porri & Redigolo]

HOLOGRAV 2016 - NBI, Copenhagen

Motivations

- Understand (super)symmetry breaking in strongly coupled QFTs → Holography it's a powerful tool!
- Here we consider the breaking of supersymmetry. Within top-down models this can also say something on ~~SUSY~~ in String theory and existence of metastable vacua.
- In this talk I focus on 4-dim N=1 theories arising from D-branes at CY singularities (*i.e.* quiver gauge theories):
  - Improve understanding of holographic renormalization for quiver gauge theories.
  - Contribute to ongoing debate about antiD-branes in warped throats (from a complementary perspective).

Preliminaries

- A necessary condition for a SQFT to break SUSY is that conformal invariance is explicitly broken:

$E_{vac} = \langle T_{00} \rangle \sim \langle T^{\mu}_{\mu} \rangle$ at odds with operator identity $T^{\mu}_{\mu} = 0$

From Lorentz invariance

The SCFT must be deformed by (marginally) relevant, SUSY-preserving operators.

Note: This means that dual backgrounds cannot be AdS!

~~AdS~~

Should depart from AdS-ness...
and do it at enough pace!

In QGT there is a sharp departure from AdS-ness: log-divergent, not even AAdS!

Cascading backgrounds

Preliminaries

- ***Recall***: in AdS/CFT a QFT vacuum is described by a given solution of bulk EOM. [A ***necessary*** condition for different solutions to describe vacua of ***same*** QFT is to have same asymptotic.]
- Suppose to have a bulk solution which breaks SUSY. There are two basic questions one should answer:
  - Q1: Is the solution gravitationally (meta)stable?

    YES: then the solution describes an actual QFT vacuum.
  - Q2: Is the bulk mode dual to the goldstino present?

    YES: then SUSY is broken spontaneously in the FT dual.

Preliminaries

- It should be possible to answer these two questions independently:
  - The goldstino appears as a massless pole in supercurrent 2-point function (in IR $\mathbf{S}_\mu = \sigma_\mu \bar{\mathbf{G}}$)

$$\langle S_{\mu\alpha}\, \bar{S}_{\nu\dot{\beta}} \rangle$$

Complicated structure; it depends on the vacuum one is considering!

  - In fact, we don't need it all! Information fully encoded in (quasi local) contact term implied by SUSY Ward identity

$$\langle \partial^\mu S_{\mu\alpha}(x)\, \bar{S}_{\nu\dot{\beta}}(0) \rangle = -2\sigma^\mu_{\alpha\dot{\beta}}\, \langle T_{\mu\nu} \rangle\, \delta^4(x)$$

Upon integration, it relates pole residue to vacuum energy.

WIs depend on UV data. Vacuum stability is IR property.

The basic goal

- This disentanglement should hold also from a holographic dual perspective.
  - *Goal*: see how much can we learn on structure of QFT vacua ***without*** detailed knowledge of the deep interior.
  - More concretely, we would like to:

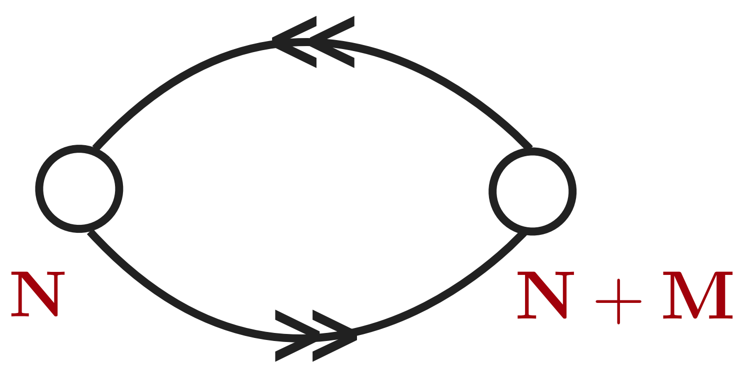
  1. Derive the SUSY Ward identities holographycally.
  2. See if and when a goldstino mode is present.

- I will focus on the conifold theory, a prototype for (a large class of) QGT which can accommodate ~~SUSY~~ vacua.
- Underlying ***question***: are cascading theories renormalizable?

~~SUSY~~ in String th. & Holography

- The conifold theory (*i.e.* the KS model) in a nutshell.

[Klebanov-Strassler '00]



Gauge Group	$SU(N+M)\times SU(N)$
Global Symmetries	$SU(2)\times SU(2)\times U(1)_B\times Z_{2M}$
Bifundamental Matter	$A_i, B_k (i,k=1,2)$
Superpotential	$W=\lambda Tr(A_iB_kA_jB_l)\epsilon^{ij}\epsilon^{kl}$

→ N regular and M fractional D3-branes at the tip of $\mathbf{C(T^{1,1})}$.

- For $\mathbf{M=0}$ the theory is (super)conformal. [Klebanov-Witten '98]

$$\mathcal{O}_\phi\sim\frac{1}{g_1^2}+\frac{1}{g_2^2}\longleftrightarrow e^{-\phi}\quad,\quad \mathcal{O}_{\tilde b}\sim\frac{1}{g_1^2}-\frac{1}{g_2^2}\longleftrightarrow \tilde b^\Phi=e^{-\phi}b^\Phi$$

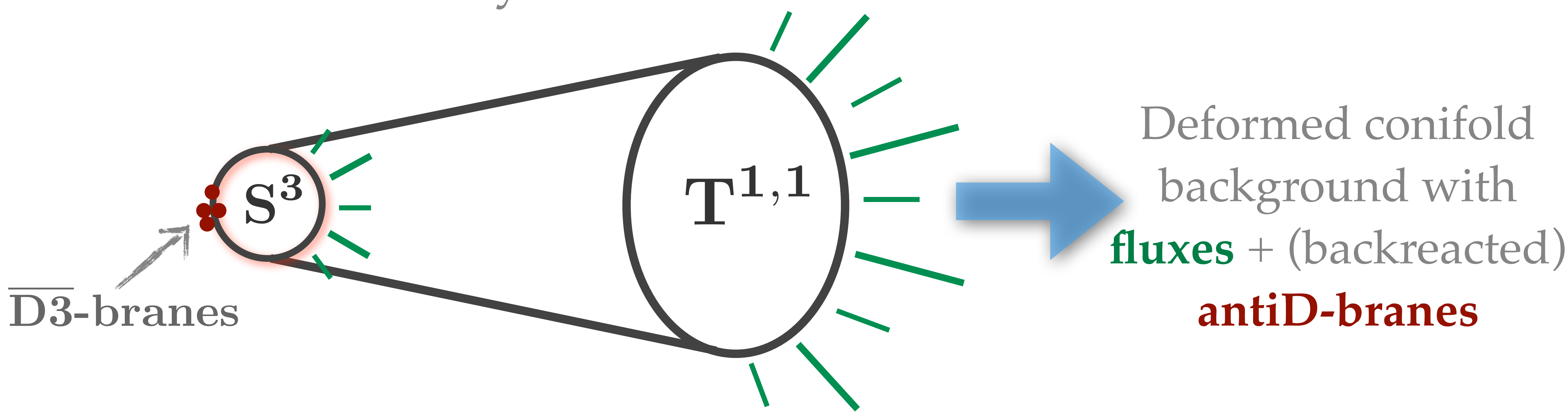
- For $\mathbf{M\neq 0}$ conformal invariance is broken: $\mathcal{O}_{\tilde b}$ becomes relevant and triggers an RG-flow → duality cascade.

~~SUSY~~ in String th. & Holography

- For $\mathbf{N = kM}$ there are both mesonic and baryonic branches of SUSY vacua.
- For $\mathbf{N = kM - p}$ with $\mathbf{p < M}$ the baryonic branch is lifted and only the mesonic branch survives.

[Dymarsky-Klebanov-Seiberg'05]

[Kachru-Pearson-Verlinde '01] argued that there exist ~~SUSY~~ vacua on the would-be baryonic branch!



- *Question*: is there a goldstino there?

Cascading theories from 5d SUGRA

- Holographic dictionary (and machinery) defined in terms of 5d effective d.o.f. → need to compactify type IIB on $\mathbf{T^{1,1}}$. The resulting effective theory is very complicated. However, several simplifications make our life simpler:
  - We need to look to UV asymptotic only, up to the order ~~SUSY~~ deformation appears.

    → need to look for solutions up to order $\mathbf{z^4}$ only.
  - We focus on $\mathbf{SU(2)} \times \mathbf{SU(2)}$ invariant sector (and restrict to fields invariant under an extra $\mathbf{U(1)}$ symmetry).

    [Cassani-Faedo '10, Liu-Szepietowski '11]

    see also [Buchel '05]
- *Note*: Solutions should be domain-wall like (metric+scalars) and have all same asymptotic (~KT like [Klebanov-Tseytlin '00])!

Cascading theories from 5D SUGRA

- The most general solution compatible with UV b.c. is

$$\begin{cases} ds^2 = \dfrac{1}{z^2}\left(e^{2Y(z)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2X(z)}dz^2\right) \\ e^{2Y} = h^{\frac{1}{3}}(z)\, h_2^{\frac{1}{2}}(z)\, h_3^{\frac{1}{2}}(z) \quad , \quad e^{2X} = h^{\frac{4}{3}}(z)\, h_2^{\frac{1}{2}}(z) \end{cases}$$

$$\begin{cases} e^{2U} = h^{\frac{5}{2}}(z)\, h_2^{\frac{3}{2}}(z) \quad , \quad e^{2V} = h_2^{-\frac{3}{2}}(z) \\ b^\Phi(z) = -\dfrac{9}{2}g_s M \ \log\left(z/z_0\right) \\ \qquad + z^4\left[\left(\dfrac{9\pi N}{4M} + \dfrac{99}{32}g_s M - \dfrac{27}{4}g_s M\log\left(z/z_0\right)\right)\mathcal{S} - \dfrac{9}{8}g_s M\varphi\right] + \mathcal{O}(z^8) \\ \phi(z) = \log g_s + z^4\left(3\,\mathcal{S}\log\left(z/z_0\right) + \varphi\right) + \mathcal{O}(z^8) \end{cases}$$

$$h(z) = \frac{27\pi}{4g_s}\left(g_s N + \frac{3}{8\pi}(g_s M)^2 - \frac{3}{2\pi}(g_s M)^2\log\left(z/z_0\right)\right)$$

$$+ \frac{z^4}{g_s}\left[\left(\frac{54\pi g_s N}{64} + \frac{81}{4}\frac{13}{64}\left(g_s M\right)^2 - \frac{81}{16}\left(g_s M\right)^2\log\left(z/z_0\right)\right)\mathcal{S} - \frac{81}{64}\left(g_s M\right)^2\varphi\right] + \mathcal{O}(z^8)$$

$$h_2(z) = 1 + \frac{2}{3}\,\mathcal{S}\, z^4 + \mathcal{O}(z^8) \quad , \quad h_3(z) = 1 + \mathcal{O}(z^8)$$

Two-parameter family of ~~SUSY~~ solutions.

SEE ALSO [DE WOLFE ET AL. '08]

Holographic Ward identities

- Ward identities are relations among correlators of local operators, descending from global symmetries.
- Turn on sources for local operators, gauge the global symmetries and require invariance of the generating functional under local gauge transformations.

→ Get relations between 1pt-functions (at finite source!) and in turn, upon differentiation, the WIs.

- ***Holography*** naturally adapted to this procedure:
  - global symmetries on the boundary ⟷ gauge symmetries in the bulk
  - bulk fields contain arbitrary sources for local operators, and transform under local symmetries in the bulk.

Holographic Ward identities

- *Note*: work at a finite cut-off, identify sources with induced fields at cut-off shell & remove the cut-off at the end, only.

- Renormalized 1-point functions in the presence of sources

$$\langle T^{\mu\nu}\rangle = \frac{2}{\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial \tilde{\gamma}_{\mu\nu}} \qquad \langle \overline{S}^{-\mu}\rangle = \frac{-2i}{\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial \tilde{\Psi}^{+}_{\mu}}$$

$$\langle \mathcal{O}_{\phi}\rangle = \frac{1}{2\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial \phi} \qquad \langle \overline{\mathcal{O}}^{+}_{\zeta_\phi}\rangle = \frac{1}{\sqrt{-\tilde{\gamma}}}\frac{i}{\sqrt{2}}\frac{\partial S_{ren}}{\partial \zeta^{-}_{\phi}}$$

$$\langle \mathcal{O}_{\tilde{b}}\rangle = \frac{1}{2\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial \tilde{b}^{\Phi}} \qquad \langle \overline{\mathcal{O}}^{+}_{\tilde{\zeta}_b}\rangle = \frac{1}{\sqrt{-\tilde{\gamma}}}\frac{i}{\sqrt{2}}\frac{\partial S_{ren}}{\partial \tilde{\zeta}^{-}_{b}}$$

$\mathcal{S}_{\mathbf{ren}} = \mathcal{S}_{\mathbf{reg}} + \mathcal{S}_{\mathbf{ct}}$: renormalized action (at finite cut-off!)

Explicit expression of counter-terms not needed to derive Ward identities!

Holographic Ward identities

- ***Recipe***: 1. Fix bulk gauge redundancy by choosing a gauge, 2. Study transformations of the sources under residual local symmetries preserving the gauge.
- Gauge fixing condition (Fefferman-Graham gauge):

$$ds^2 = dr^2 + \gamma_{\mu\nu} dx^\mu dx^\nu \quad , \quad \Psi_r = 0 \qquad (dr = -e^{X(z)} dz/z\,)$$

- Bulk diff preserving this gauge:

$$\dot{\xi}^r = 0\ ,\ \dot{\xi}^\mu + \gamma^{\mu\nu}(r,x)\partial_\nu \xi^r = 0$$

Solution: $\xi^r = \sigma(x)\ ,\ \xi^\mu = \xi_0^\mu(x) - \int^r dr' \gamma^{\mu\nu}(r',x)\partial_\nu \sigma(x)$

- Bulk SUSY transformations preserving this gauge:

$$(\nabla_r + \frac{1}{6}\mathcal{W}\,\Gamma_r)\epsilon = 0$$

Solution: $\begin{cases} \epsilon^+(z,x) = z^{-1/2} h(z)^{1/12} \epsilon_0^+(x) + \mathcal{O}(z^4) \\ \epsilon^-(z,x) = z^{1/2} h(z)^{-1/12} \epsilon_0^-(x) + \mathcal{O}(z^4) \end{cases}$

$\epsilon = \epsilon^+ + \epsilon^-$

P-T superpotential

Holographic Ward identities

- $\sigma(\mathbf{x})$ parametrizes Weyl transformations: $\delta_\sigma \mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\langle T^\mu_\mu \rangle + 9M\langle \mathcal{O}_{\tilde{b}} \rangle + \left[\frac{i}{4} \langle \overline{S}^{-\mu} \rangle \tilde{\Psi}^+_\mu + \frac{i}{\sqrt{2}} \langle \overline{\mathcal{O}}^+_{\zeta^\phi} \rangle \zeta^-_\phi + \frac{i}{\sqrt{2}} \langle \overline{\mathcal{O}}^+_{\tilde{\zeta}^b} \rangle \tilde{\zeta}^-_b + \text{h.c.} \right] = 0$$

- $\epsilon^-(\mathbf{x})$ parametrizes superWeyl transformation: $\delta_{\epsilon^-} \mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\frac{i}{2} \langle \overline{S}^{-\mu} \tilde{\Gamma}_\mu \rangle = \frac{9M}{\sqrt{2}} \langle \overline{\mathcal{O}}^+_{\tilde{\zeta}_b} \rangle$$

- $\epsilon^+(\mathbf{x})$ parametrizes SUSY transformations: $\delta_{\epsilon^+} \mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\frac{i}{2} e^{-\frac{2}{15}U} \langle \partial_\mu \overline{S}^{-\mu} \rangle = -\frac{1}{2} \langle T^{\mu\nu} \rangle \overline{\tilde{\Psi}}^+_\mu \tilde{\Gamma}_\nu + i \langle \mathcal{O}_\phi \rangle \overline{\zeta}^-_\phi + i \langle \mathcal{O}_{\tilde{b}} \rangle \overline{\tilde{\zeta}}^-_b$$

- Ward identities can be obtained differentiating the above relations wrt covariant sources, then removing the cut-off (and set sources to 0, when needed).

at the cut-off w/ non-zero sources!

Holographic Ward identities

- From ϵ^+ identity, further differentiating wrt gravitino/ hyperini we get:

$$\langle \partial^\mu S_{\mu\alpha}(x)\ \bar{S}_{\nu\dot{\beta}}(0)\rangle = -2\,\sigma^\mu_{\alpha\dot{\beta}}\langle T_{\mu\nu}\rangle\ \delta^4(x)$$

$$\langle \partial^\mu S^\alpha_\mu(x)\ \mathcal{O}_{\zeta_\phi\alpha}(0)\rangle = -\sqrt{2}\ \langle \mathcal{O}_\phi\rangle\ \delta^4(x)$$

$$\langle \partial^\mu S^\alpha_\mu(x)\ \mathcal{O}_{\tilde{\zeta}_b\alpha}(0)\rangle = -\sqrt{2}\ \langle \mathcal{O}_{\tilde{b}}\rangle\ \delta^4(x)$$

Higher-components operators VEVs → break SUSY!

- From σ and ϵ^- we get the trace operator identities:

$$\langle T^\mu_\mu\rangle = -9M\langle \mathcal{O}_{\tilde{b}}\rangle \quad , \quad \langle \sigma^\mu_{\alpha\dot{\beta}}\bar{S}^{\dot{\beta}}_\mu\rangle = -9\sqrt{2}M\ \langle \mathcal{O}_{\tilde{\zeta}_b\,\alpha}\rangle$$

All relations are in perfect agreement with QFT expectations:

$$\Delta(\mathcal{O}_{\tilde{b}}) < 4\,,\ \Delta(\mathcal{O}_\phi) = 4\ \text{ and } T^\mu_\mu = -(1/2)\sum \beta_i\, O_i$$

→ Goldstino eigenstate: **G** $\sim \langle \mathcal{O}_{\tilde{b}}\rangle\, \mathcal{O}_{\tilde{\zeta}_b}$!

Goldstino & antiD-branes

- Finally, evaluating the bosonic 1pt-func on our solutions (explicit expression for (bosonic) $\mathcal{S}_{\mathbf{ren}}$ is known!) we get

$$\langle T^{\mu}_{\mu} \rangle = -12\,\mathcal{S} \quad , \quad \langle \mathcal{O}_{\phi} \rangle = \frac{(3\mathcal{S} + 4\varphi)}{2} \quad , \quad \langle \mathcal{O}_{\tilde{b}} \rangle = \frac{4}{3\,M}\mathcal{S}$$

SEE ALSO [AHARONY ET AL. '05, DE WOLFE ET AL. '08]

- SUSY case $\mathcal{S} = \varphi = \mathbf{0}$: 0 vacuum energy, no goldstino.
- Branch $\mathcal{S} = \mathbf{0}\,,\ \varphi \neq \mathbf{0}$: 0 vacuum energy, no goldstino but ~~SUSY~~!? Not a vacuum of KS theory: explicit breaking!
- Branch $\mathcal{S} \neq \mathbf{0}\,,\ \varphi = \mathbf{0}$: $\neq 0$ vacuum energy, goldstino mode, ~~SUSY~~ VEV, trace identity fulfilled. Spontaneous breaking!
- *Note*: these conclusions can be reached by looking just at the asymptotic solution. One does not need the full solution!

Goldstino & antiD-branes

- ***Note***: in the *asymptotic* solution $\mathcal{S}$ and φ are integration constants. In a full solution, they get fixed by IR b.c.

$$\mathcal{S} = \mathcal{S}(k, M, p, \epsilon, \dots)\,,\ \varphi = \varphi(k, M, p, \epsilon, \dots)$$

- Solution w/ $\mathcal{S} = \varphi = \mathbf{0}$ is known: KS geometry for $\mathbf{p = 0}$ or KS with $\mathbf{M - p}$ mobile D3s for $\mathbf{p \neq 0}$.

- Solution w/ $\mathcal{S} = \mathbf{0}\,,\ \varphi \neq \mathbf{0}$ also known [Kuperstein et al. '14] . Our analysis shows that no matter how φ depends on the IR data, it does not describe a KS theory vacuum.

- Solution w/ $\mathcal{S} \neq \mathbf{0}\,,\ \varphi = \mathbf{0}$ corresponds to ~~SUSY~~ vacua of KS theory. IR analysis [de Wolfe et al. '08, Bena et al. '11] shows that the mode $\mathcal{S}$ is *sourced* by antiD3, $\mathcal{S} \sim \mathbf{p\, e}^{-\frac{\mathbf{8\pi k}}{\mathbf{3 g_s M}}}$ → if stable antiD configuration exists, it is a ~~SUSY~~ KS-theory vacuum!

Summary

- Holographic ***derivation*** of Ward identities for cascading backgrounds (providing further evidence that such theories can be consistently renormalized [krasnitz '02, aharony et al. '05]).

Note: insensitive to IR, but give constraints on solutions that correspond to given vacua (e.g. ~~SUSY~~ vacua in KS theory).

- Provided a ***necessary*** consistency check for the existence of metastable vacua in the KS cascading gauge theory.

- Our results give further evidence for 1-1 ***correspondence*** between spontaneous ~~SUSY~~ in (a class of) quiver gauge theories and antiD-brane states in warped throats.

OUTLOOK

- On with the program of HR for the conifold theory (more generally, for cascading backgrounds) → do a systematic derivation of fermionic counter-terms.

- Extension of present analysis to the full background (including conifold deformation parameter)... but we don't expect any dramatic *qualitative* change.

- Working at finite cut-off in terms of induced fields looks promising for systematics of HR in generic set-ups. E.g., derive SUSY-preserving counter-terms for SQFT on curved manifolds.

Thank you!