Large N limits and chaos in quantum mechanics and AdS_2

Kristan Jensen (SFSU) - Current Themes in Holography - NBI 26/04/2016

Based on WIP with Chris Herzog

1605.XXXXX

AdS/CFT @ ~ 20 years

- d=6: N=(2,0) ADE, N=(1,0) zoo

  e.g. [del Zotto, Heckman, Tomasiello, Vafa], [Gaiotto, Tomasiello]
- d=5: superconformal quivers; massive IIA

  e.g. [Bergman, Rodriguez-Gomez]
- d=4: need it be said?
- d=3: ABJM, CSM
- d=2: D1/D5, symmetric orbifolds, ``pure'' gravity
- d=1: ??

  [D0s, BFSS [Anagnostopoulos, Hanada, et al] not AdS]

What about AdS_2/CFT_1 ?

Never satisfactorily developed relative to its higher-dimensional cousins

At least two reasons:

1. No explicit* candidate dualities
2. Thorny issues on both sides of a putative correspondence

*I'm ruling out implicit dualities, like the DLCQ of 2d CFT with a gravity dual

Thorns in the side of AdS_2

Let's pick two:

1. AdS_2 does not support finite-energy excitations

Anti-de Sitter Fragmentation *

Juan Maldacena[1], Jeremy Michelson[2,1] and Andrew Strominger[1]

Equivalent ways to say this:

- dual stress tensor identically vanishes
- `ADM' mass = 0 for any $AAdS_2$ spacetime
- AdS_2 throats do not admit a decoupling limit

*Fragmentation is a property of higher-dim geometries with AdS_2 near-horizon

Thorns in the side of AdS$_2$

2. Two-dimensional gravity is topological; need a dilaton to have AdS$_2$ vacua

$$S = \frac{1}{2\kappa^2}\int d^2x\sqrt{-g}\Big(\varphi R + U[\varphi]\Big)$$
$$-\frac{1}{2}\int d^2x\sqrt{-g}\Big(Z_0[\varphi](\partial\chi)^2 + Z_1[\varphi]m^2\chi^2\Big)$$

- Semi-universal
- AdS solutions at roots of U

$$\varphi = \varphi_0\,,$$
$$g = L^2\left(-r^2dt^2 + \frac{dr^2}{r^2}\right)$$

$$L^2 = -\frac{2}{U'[\varphi_0]}$$

Thorns in the side of CFT_1*

Summarized in Polchinski's paradox:

Scale invariance in 1d implies a density of states

$$\rho(E) = e^{S_0}\delta(E) + \frac{s}{E}$$

*There is an old question about whether CFT_1 is a conformal QM (CQM), or the chiral half of a CFT_2; for reasons I can explain, CFT_1 = CQM in this talk

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Log-divergent partition function, need Λ_{IR}

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Topological correlations; no dynamics

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim t^{-2\Delta}?$$

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Sachdev-Ye-Kitaev (SYK)

Enter a set of QM theories which Kitaev has conjectured to have a gravity dual:

2N Majorana fermions with quenched disorder

$$H = \sum_{a,b,c,d} \frac{J_{abcd}}{4!} \psi^a \psi^b \psi^c \psi^d$$

$$\overline{J_{abcd}} = 0\,, \qquad \overline{J_{abcd} J^{abcd}} = \frac{3J^2}{(2N)^3}\,,$$

Sachdev-Ye-Kitaev (SYK)

Some features:

1. Single dimensionless coupling J/T
2. Solvable large N limit
3. Emergent conformal symmetry as $J/T \to 0$

$$\langle \psi^a(t)\psi^b(0)\rangle = \left(\frac{1}{\sqrt{4\pi J^2}}\right)^{1/4} \frac{\mathrm{sgn}(t)\delta^{ab}}{|t|^{1/2}}\,, \qquad t \gg 1/J\,.$$

4. Generalization to theories labeled by (q,N)

$$H = \sum_{a_1,..,a_q} \frac{J_{a_1..a_q}}{q!}\psi^{a_1}..\psi^{a_q}$$

Sachdev-Ye-Kitaev (SYK)

Evidence for a gravity dual:

1. Low energy conformal symmetry
2. Large extremal entropy $(\Delta = 1/q)$

$$\frac{S}{N} = (1 - 2\Delta)\ln(2\cos(\pi\Delta)) - \frac{\mathrm{Li}_2\left(-e^{2\pi i\Delta}\right) - \mathrm{Li}_2\left(-e^{-2\pi i\Delta}\right)}{2\pi i}$$

3. Maximally chaotic [Kitaev]

$$\lambda_L = 2\pi T$$

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See also:
[Polchinski, Rosenhaus]
[Maldacena, Stanford]

Goals for today

1. Assess viability of SYK/AdS correspondence
2. Resolve old issues in AdS_2/CFT_1

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In particular, I want a framework for computing holographic n-point functions, as well as the gravitational Lyapunov exponent

$$\lambda_L = 2\pi T??$$

SYK/AdS

Two simple arguments why the SYK models **do not** have a conventional gravity dual:

SYK/AdS

Two simple arguments why the SYK models **do not** have a conventional gravity dual:

1. Too many fields: quenched disorder preserves SO(2N) flavor symmetry, with 2N ψ^a 's

$$G_N \sim \frac{1}{N} \Rightarrow \ln \mathcal{Z}_{\text{classical}} \sim \ln \mathcal{Z}_{\text{one-loop}}$$

SYK/AdS

Two simple arguments why the SYK models **do not** have a conventional gravity dual:

2. Large N factorization guarantees that there are ``multi-trace'' operators, e.g. $\psi^a \partial_t \psi_a$, with 1/N-suppressed anomalous dimension.

   Decomposing Witten diagrams in the conformal block expansion, these ``multi-trace'' operators have OPE coefficients comparable to ``single-traces''

Not seen in explicit computation of $\langle \psi^a \psi^b \psi^c \psi^d \rangle$

[Polchinski, Rosenhaus] [Maldacena, Stanford]

gauged SYK/AdS?

Obvious route to salvage: gauge some flavor symmetry

1. Permutation orbifold $S_N \subset SO(2N)$

2. Continuous symmetry $SU(N) \subset SO(2n)$

WIP

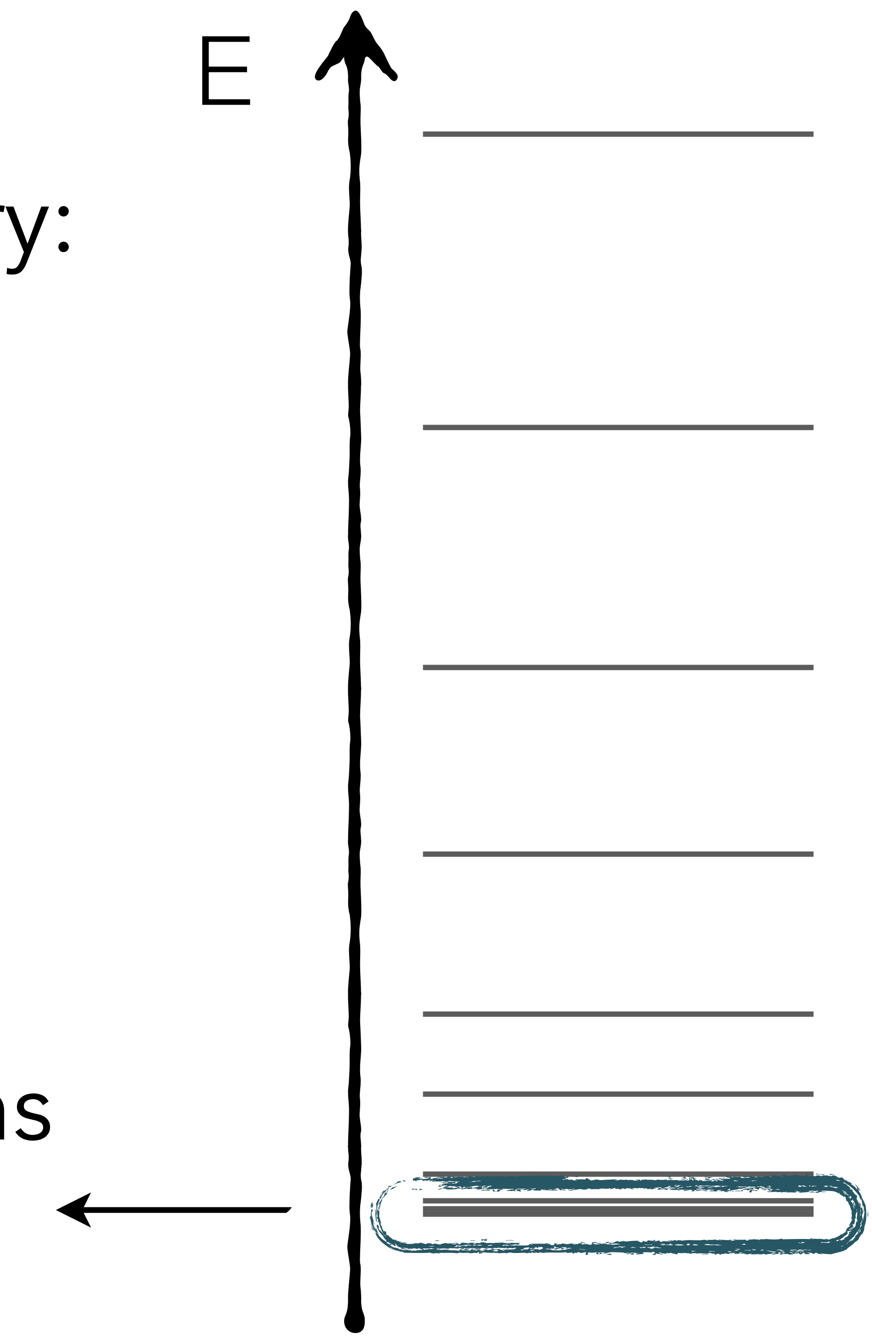
CQM

Conformal symmetry in 0+1

QM analogue of RG flow,

emergent conformal symmetry:

Long (Euclidean) time correlations
with power-law decay



Conformal symmetry in 0+1

Three observations:

1. $\rho(E) = e^{S_0}\delta(E) + \frac{s}{E} \;\Rightarrow\; S = S_0 + s\ln\left(\frac{T}{\Lambda_{IR}}\right)$

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(As we'll see shortly, AdS_2 black holes have constant S)

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   (But conformal symmetry got to break at 1/N!)

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3. Conformal symmetry => Virasoro with c=0
   Weyl Ward identity sets $T^{tt} = 0$

AdS_2

Fields and operators

For Einstein gravity in AdS_{d+1} with $d>1$, the metric is dual to the boundary stress tensor.

What about dilaton gravity in AdS_2?

Fields and operators

For Einstein gravity in AdS_{d+1} with $d>1$, the metric is dual to the boundary stress tensor.

What about dilaton gravity in AdS_2?

Neither the metric nor dilaton are dual to operators; however, they mimic a $\Delta = 2$ scalar

Near-extremal, near-horizon black holes

Turning on the dilaton destroys AdS_2

$$\varphi = \varphi_0 + \ell r\left(1+\frac{r_h^2}{r^2}\right) + \mathcal{O}(\ell^2 r^2)\,,$$

$$g = -r^2\left(1-\frac{r_h^2}{r^2}\right)^2 dt^2 + \frac{dr^2}{r^2} + r^2 h(r) dt^2 + \mathcal{O}(\ell^2 r^2)\,,$$

$$h = -\frac{\ell r U''[\varphi_0]}{3}\left(1-\frac{r_h^2}{r^2}\right)^2\left(1+\frac{r_h^2}{r^2}\right)\,.$$

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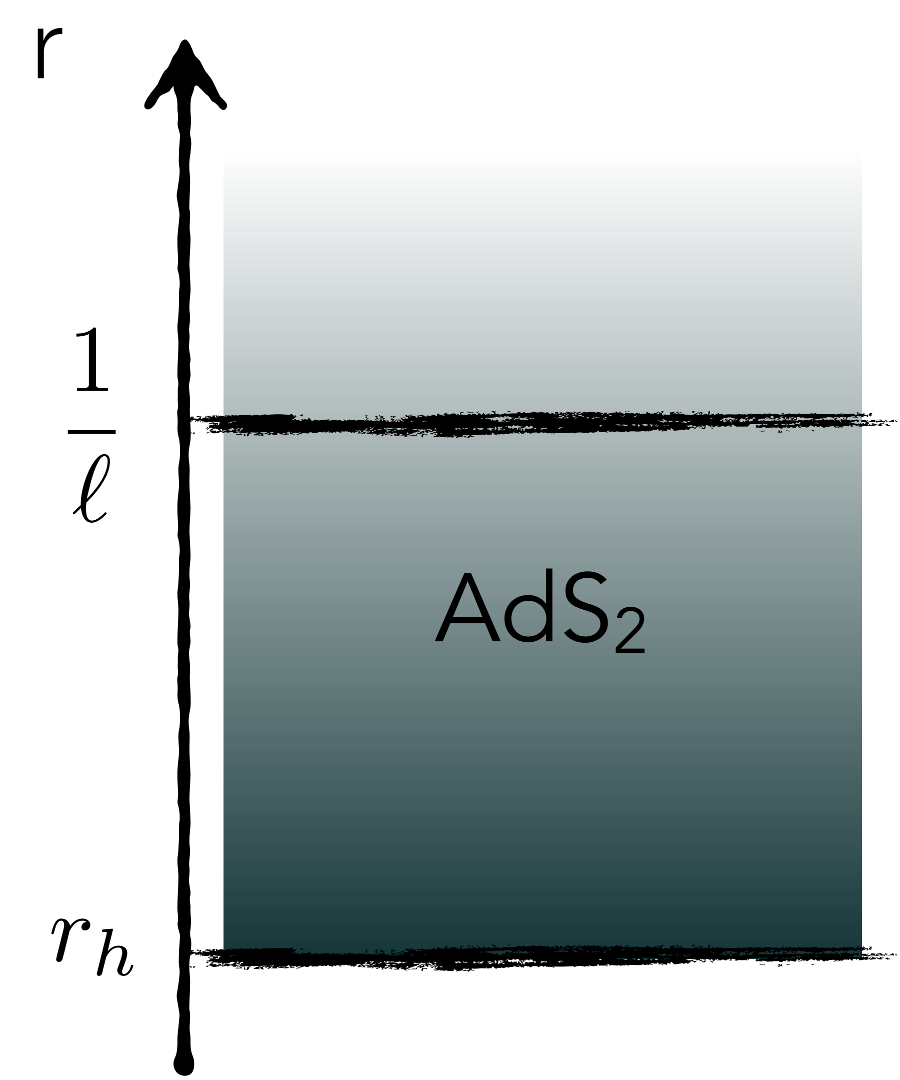


irrelevant deformation $\ell r_h \ll 1$

$$S = \frac{2\pi}{\kappa^2}\left(\varphi_0 + 2\pi \ell T + \mathcal{O}(\ell^2 T^2)\right)$$

Mimesis with $\Delta = 2$.

While the dilaton is not dual to an operator, the low-energy thermodynamics is that of a CQM deformed by a $\Delta = 2$ operator

$$\rho(E) = e^{S_0}\left(\delta(E) + \boxed{\frac{2\pi^2\ell}{\kappa^2}} + \mathcal{O}(E)\right)$$

universal feature
resolves Polchinski's paradox

Four-point functions

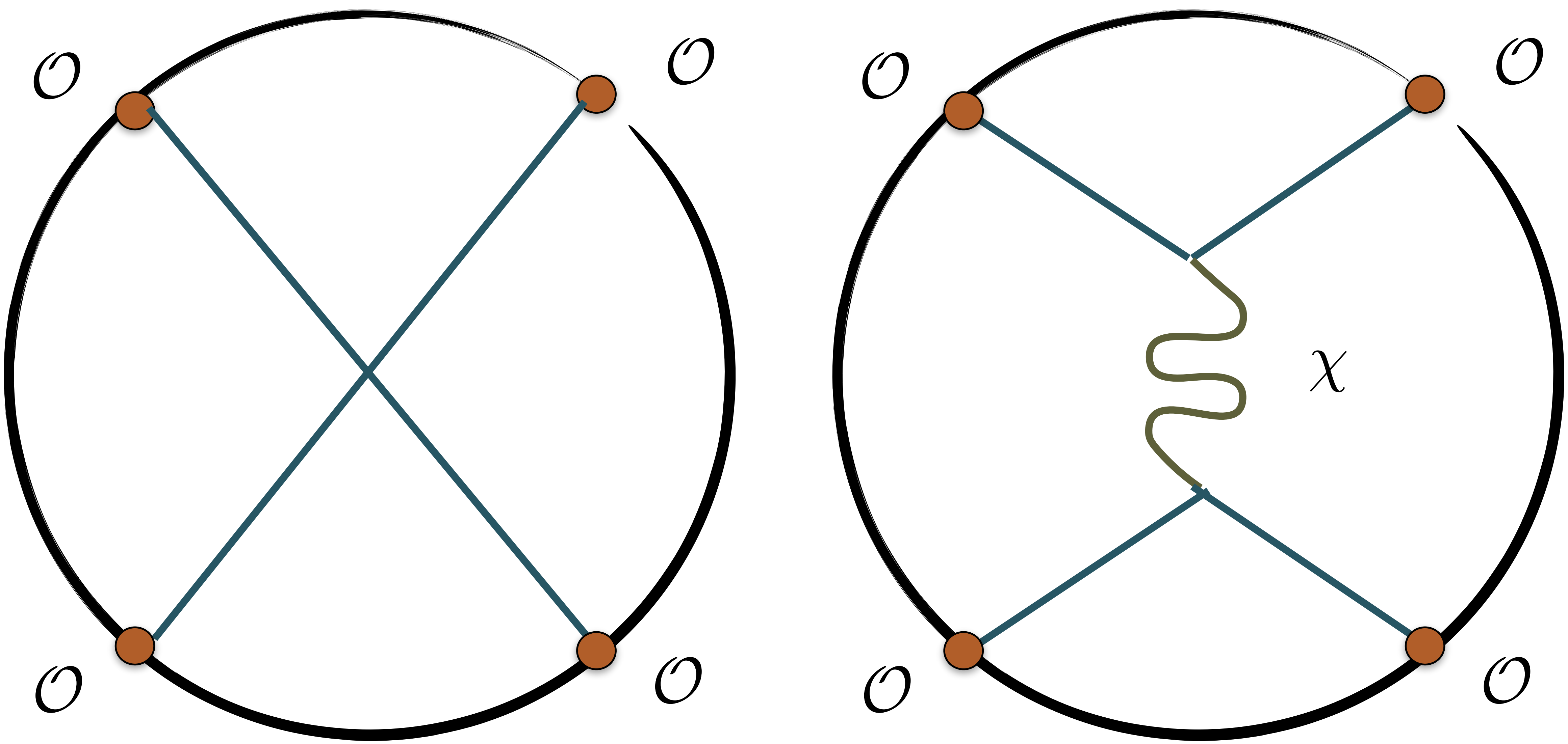
Goal:

Outline computation protocol for

$$\langle \mathcal{O}(t_1)\mathcal{O}(t_2)\mathcal{O}(t_3)\mathcal{O}(t_4)\rangle$$

Matter Witten diagrams

Proceeds as in higher-dimensional AdS_{d+1}



Equations of motion

What about the metric/dilaton?

$$S = \frac{1}{2\kappa^2}\int d^2x\sqrt{-g}\Big(\varphi R + U[\varphi]\Big)$$
$$-\frac{1}{2}\int d^2x\sqrt{-g}\Big(Z_0[\varphi](\partial\chi)^2 + Z_1[\varphi]m^2\chi^2\Big)$$

$$T_{\mu\nu} = -D_\mu D_\nu\varphi + g_{\mu\nu}\Box\varphi - \frac{U}{2}g_{\mu\nu}\,,$$
$$\Phi = R + U'$$
$$0 = D_\mu(Z_1 D^\mu\chi) - Z_2 m^2\chi$$

Perturbative expansion

Introduce ε , rescale $\ell \to \varepsilon^2 \ell$

Then black hole mass and matter stress tensor are perturbatively small and comparable

$$\chi = \varepsilon \chi_1 + \varepsilon^3 \chi_3 + \dots \, ,$$

$$\varphi = \varphi_0 + \varepsilon \varphi_2 + \dots \, ,$$

$$g = 4e^{2\omega} \left(1 + \varepsilon^2 \sigma_2 + \dots\right) dz d\bar{z} \, , \qquad e^{2\omega} = \frac{1}{1 - |z|^2} \, .$$

Perturbative expansion

$$\Box = e^{-2\omega}\partial\bar{\partial}$$

dilaton is a constrained field

$$\left(\Box - m^2\right)\chi_1 = 0\,,$$

$$e^{\omega}\partial^2\left(e^{-\omega}\varphi_2\right) = -T_2\,,$$

$$e^{\omega}\bar{\partial}^2\left(e^{-\omega}\varphi_2\right) = -\overline{T}_2\,,$$

$$\left(\Box - 2\right)\varphi_2 = \left(T^{\mu}_{\mu}\right)_2\,,$$

$$\left(\Box - 2\right)\sigma_2 = U''[\varphi_0]\varphi_2 - \Phi_2\,,$$

$$\left(\Box - m^2\right)\chi_3 = \mathcal{S}_3$$

Solution for massless scalar

This case was considered by [Almeihri, Polchinski]

$$\chi_1 = \frac{1}{2\pi}\int d\tau' \left(\frac{1}{e^{i\tau'} - z} + \frac{1}{e^{-i\tau'} - \bar{z}} - 1\right) j(\tau')\,,$$

$$\varphi_2 = \frac{a(1+|z|^2) + bz + \bar{b}\bar{z}}{1-|z|^2} + I + \bar{I}$$

$$I = \frac{1}{1-|z|^2}\int_0^z dw(1-w\bar{z})(w-z)T_2(w)$$

$$\boxed{\lim_{|z|^2\to 1}(1-|z|^2)\varphi_2 = 2a + b + \bar{b} + \text{(depends on matter)}}$$

Conformal transformation

BUT: we ought to fix dilaton, metric asymptotics as BC

In terms of CQM: fix ``source'' for $\Delta = 2$

Do so by combination of Weyl rescaling, diff
(depends on state!)

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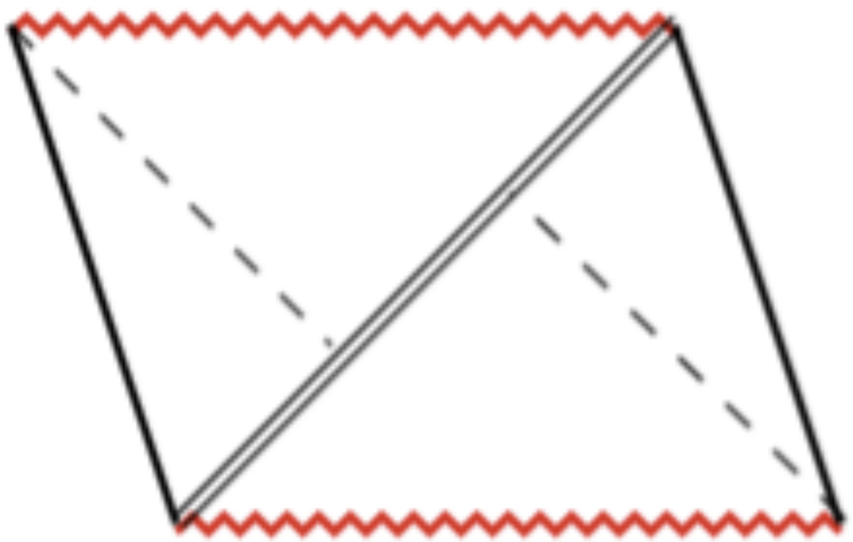
Plugging back into two-point function leads to non-conformal contribution to $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$

(there's also a conformal contribution which can be interpreted as a Witten diagram)

Shocks and chaos

What about the Lyapunov exponent?

Send in a shock to the left side of a two-sided BH a la [Shenker, Stanford]



$$T = E\delta(u)$$

$$\varphi = \varphi_0 + \varepsilon^2 \frac{2\ell r_h(1-uv) - Eu\Theta(u)}{1-uv}$$

Shocks and chaos

Horizon traversing geodesic approximates out-of-time-ordered four-point function

Usual formula for geodesic lengths in AdS_2, with the new definition of time gives

$$\sim \left(1+\frac{E}{\ell r_h}e^{2\pi T t}\right)^{-2m} \longrightarrow \boxed{\lambda_L = 2\pi T}$$

Chaos comes from the ``non-conformal block''

Summary

1. No SYK/AdS; perhaps gauged SYK/AdS?
2. Set up algorithm to compute four-point functions in near-AdS_2 region
3. Universal ``non-conformal blocks''
4. Dilaton gravity on AdS_2 is maximally chaotic

$$\rho = e^{S_0}\left(\delta(E) + \frac{2\pi^2\ell}{\kappa^2} + \mathcal{O}(E)\right)$$