Particle Production at Strong Coupling

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Based on:

M.R., Mukund Rangamani, Mark van Raamsdonk, JHEP 1509 (2015) 213, arXiv:1505.03901

Outline:

1. Introduction and Motivation
   - QFT in curved spacetime
   - Non-Equilibrium many-Body Physics
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3. Results in Free Field Theory
4. Holographic Results
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   - Comparison to Hydrodynamics
   - Saturation for Large Amplitudes
5. Conclusions and Outlook

QFT in curved spacetime

- Consistent decoupling of Planck scale physics by sending $l_p \to \infty$. Get "QFT in curved spacetime".
- The theory can be strongly fluctuating and non-linear, with all the complexities of QFT.
- Much of what is known about black hole physics and the early universe is an application of this formalism.

However:

- known problems and mysteries, e.g. black hole information paradox. What is precisely the limit of reliability?
- Almost all results are at the free-field, or at best perturbative level.
- Many of the celebrated concepts (e.g. vacuum ambiguity and particle production) are inherently perturbative.

Motivation to study the subject non-perturbatively, ask for concepts and results more general that the free-field limit.

Non-Equilibrium Many-Body Physics

In this talk we explore the simplest possible question, first out of many: how does particle production in a cosmological (time-dependent) background get modified in strongly coupled holographic QFT?

Immediate sub-question: What does one mean by particle production in strongly coupled QFT?

Our answer: We can focus on entropy production, which is a standard concept in non-equilibrium physics.

Similarly, many other questions about QFT is curved spacetime can (should?) be viewed as questions about driven (yet closed!) many-body quantum systems.

More generally, it is useful to frame the discussion in the language of NEQ QFT. It is then interesting to see if and how results of NEQ QFT get modified at strong coupling. Here we will see a simple example of this.

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Basic Setup

We'd like to calculate particle production in an expanding FRW universe, sticking with 3+1 D. Since FRW is conformally flat, we need a mass scale to get non-trivial results, e.g. the classic results in Birrell and Davies apply to a massive scalar field.

We go a different route: Consider the $4+1$ dimensional spacetime

$$ds^2 = -dt^2 + dx_c^2 + a^2(t)\, d\vec{x}^2\,,$$

where x_c parameterizes a $\mathbf{S}^1$ of fixed size l_c, and anti-periodic boundary conditions are imposed for fermions. We choose to stay in the deconfined phase throughout the evolution. In that phase local quantities do not depend on the scale l_c ("Large N Volume Independence").

We choose the scale factor $a(t)$ to evolve from $a = 1$ in the infinite past to $a = \alpha$ at the infinite future, e.g.

$$a(t) = \frac{\alpha + 1}{2} + \frac{\alpha - 1}{2} \tanh(v\, t)\,.$$

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Generalizations

This resembles a quench lasting for a duration v^{-1}. The parameters are then α and $\mathfrak{v} = v/T$, the duration v measured in units of the initial temperature. We discuss our results as function of these two dimensionless parameters.

Our initial and finals states are chosen to be equilibrium states where thermodynamic quantities are well-defined. We quantify "particle production" in terms of the change in these quantities before and after the quench. We parametrize the results in two functions F_ϵ and G_T, defined below.

One can generalize in many ways, e.g. have spatial curvature or different expansion profiles. The boundary metric does not even have to satisfy Einstein equations, let alone any energy conditions...

We also specialize to observables which are well-understood, by restricting attention to initial and final states. Maybe more interesting is tracking local quantities during the strong deviation from equilibrium.

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Observables

We compare the equilibrium states before and after the expansion using two (equivalent) observables:

Using scale invariance, the final energy density can be written as $C\,\ell_c\,v^5\,f_\epsilon(\alpha,\mathfrak{v})$. We also know that in the adiabatic limit $\mathfrak{v}\to 0$ there is no particle (or entropy) production, so we define

$$F_\epsilon(\alpha,\mathfrak{v}) = f_\epsilon(\alpha,\mathfrak{v}) - f_\epsilon(\alpha,\mathfrak{v}\to 0)$$

Alternatively, we can consider the change in temperature in terms of a scaling function:

$$T_f = T_f(\alpha,\mathfrak{v}\to 0) + v\,G_T(\alpha,\mathfrak{v})$$

We calculate both those observables as function of $\alpha,\mathfrak{v}$ both for free theories and strongly coupled CFTs.

Sketch of the Free Field Calculation

First, we calculate F_ϵ and G_T in free field theory. We are interested in 4+1 dimensional CFTs compactified on a circle. At zero coupling an example of such theory is one free massless scalar field.

In 3+1 dimensional, after compactification, this corresponds to a tower of massive scalar fields. For each one we can calculate the number of quanta produced by the expansion, following the classic Birrell-Davies discussion.

As is well-known the "in" and "out" vacua are inequivalent and are related by the Bogoliubov transformation

$$\mathbf{a} = \alpha^* \mathbf{A} - \beta^* \mathbf{B}^\dagger, \qquad \mathbf{A} = \alpha\, \mathbf{a} + \beta^* \mathbf{b}^\dagger$$
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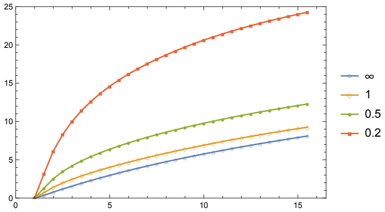
Free Field Result

When the dust settles we get the following answers for the total particle production, quantified by the functions we defined above.
First compare the energy density before and after the expansion:

F_ϵ as function of the amplitude α, for varying $\mathfrak{v}$ (quench duration or inverse initial temperature). Bottom (blue) line is zero initial T, and the other lines are ordered for increasing initial T.



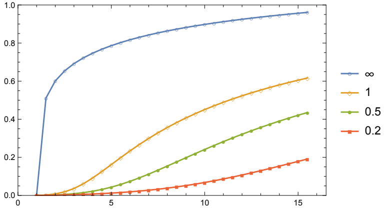
Results are pretty structureless. This will change at strong coupling.

Free Field Result

We can also quantify the particle production as change in temperature between initial and final states.

Importantly, the final state at zero coupling is not thermal. Assuming the particles produced ultimately thermalize by an independent mechanism, one can then discuss the final temperature. We have then the second function G_T:

G_T as function of the amplitude α, for varying $\mathfrak{v}$ (quench duration or inverse initial temperature). Top (blue) line is zero initial T, and the other lines are ordered for increasing initial T.



We can now perform the same calculation in the holographic context.

Holographic Calculation

We start with strongly coupled CFT in 4+1 dimensions, dual to asymptotically AdS_6 spacetime. We require the boundary metric to be of the specified form (FRW times a circle).

The bulk metric is taken in the Bondi-Sachs form:

$$ds^2 = -2A\, e^{2\chi}\, dt^2 + 2e^{2\chi}\, dt\, dr + \Sigma^2(e^B\, d\vec{x}^2 + e^{-3B}\, dx_c^2)$$

This results in a nested system of linear equations for the metric functions A, χ, Σ, depending on the radial coordinate r and time.

We solve the equations numerically, using pseudo-spectral discretization in the radial direction and RK45 (with adaptive step size) for time evolution. Some care has to be taken to extract the boundary energy-momentum correctly (e.g. one has to work with multiple precision).

Comparison to Hydrodynamics

We start by describing features of the gravity solution. One way to do it is compare to the fluid/gravity solution, as quantified by the function G_T.

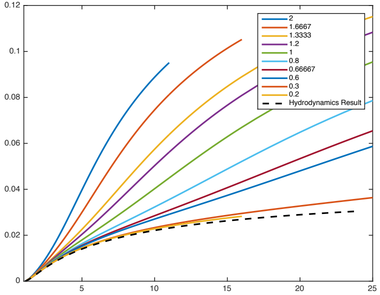
In hydro the expansion of the metric leads to entropy production through the shear viscosity. When parametrizing this as change in temperature through the function G_T we get:

$$G_T(\alpha, \mathfrak{v}) = \frac{1}{7\pi} \frac{\alpha^{\frac{7}{4}} + 7\,\alpha^{\frac{3}{4}} - 7\,\alpha - 1}{\alpha^{\frac{3}{4}}(\alpha - 1)}$$

We will find that for small enough amplitudes the results are universal (independent of $\mathfrak{v}$) and are well-captured by hydrodynamics .

Comparison to Hydrodynamics

Plotted here is the function G_T as function of the amplitude α, for varying values of the duration (or inverse initial temperature) $\mathfrak{v}$. Hydro corresponds to the dashed line.



For large amplitudes hydro fails and the numerical solution is not well-approximated by fluid-gravity. This confirms that we are indeed very far from equilibrium during the expansion.

Saturation at Strong Coupling

Next we look at the results presented in the function F_ϵ.

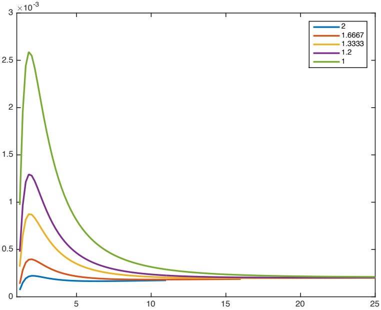
For adiabatic expansion between initial temperature T_i and final temperature T_f, the 4+1 dimensional energy density scales as T^5 whereas the total entropy stays fixed. This means that the temperature scales as $a^{-3/4}$ and the energy density as $a^{-15/4}$. This is intermediate between matter and radiation in 3+1 dimensions.

We use these results for the subtraction of the adiabatic quantities to get the function F_ϵ defined previously.

Saturation at Strong Coupling

Plotted is the function F_ϵ as function of the amplitude α for various values of $\mathfrak{v}$. As we increase $\mathfrak{v}$ the peak also increases.

Interestingly, the large amplitude results always saturate to a value independent of $\mathfrak{v}$ (representing the initial temperature). This saturation does not exist at weak coupling.



The intuitive reason of the difference is the separation of time scales existing at weak coupling between particle production and thermalization (entropy production). Famously holographic fluids experience fast thermalization, so there is no separation of scales between the two processes, they are intertwined.

Conclusions

In this work we explored particle (entropy) production at strong coupling:

- The results are most naturally phrased in the language of non-equilibrium QFT.
- The results are dramatically different at weak and strong coupling. In particular there is a new feature (saturation) of the strong coupling results, which does not exist at weak coupling.
- The mechanism of entropy production (fast thermalization) different at weak and strong coupling.

More along those lines of "strongly coupled cosmology"

- CMB fluctuations.
- Phenomenological applications.

Also a good laboratory to look at general issues in non-equilibrium physics (e.g. entropy production, work identities, fluctuations theorems).