Entanglement Holography

[Part 1]

Robert Myers

with de Boer, Haehl, Heller & Neiman

arXiv:1509.00113; arXiv:1605.nnnnn

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Quantum Entanglement

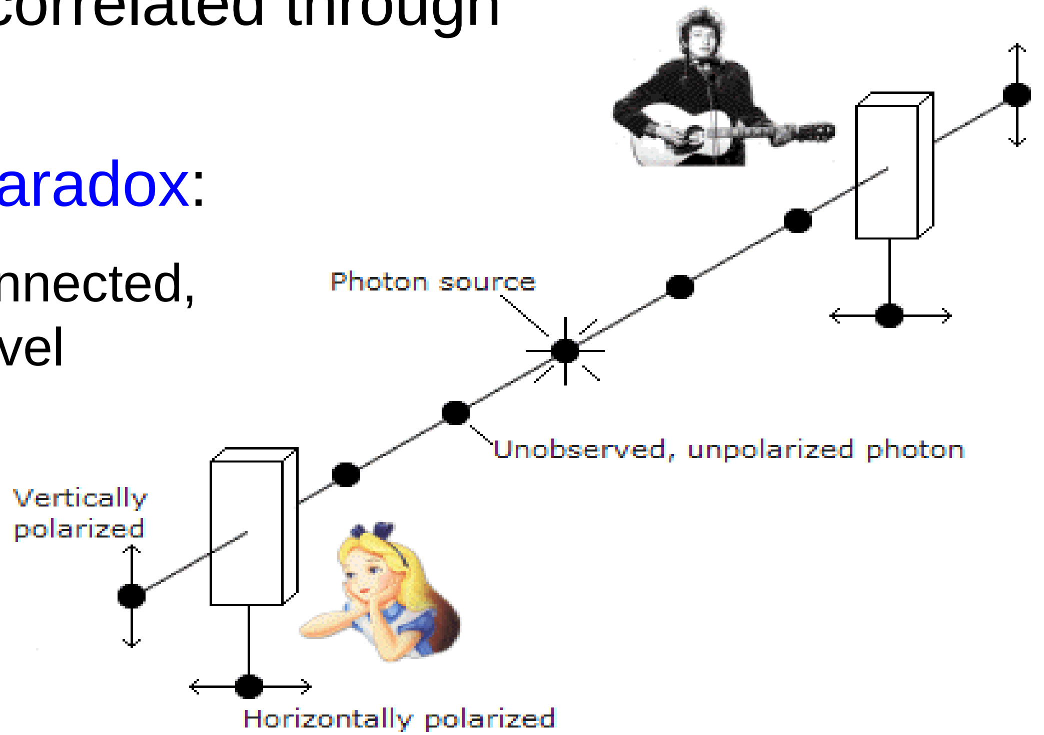
- different subsystems are correlated through global state of full system

Einstein-Podolsky-Rosen Paradox:

- properties of pair of photons connected, no matter how far apart they travel

"*spukhafte Fernwirkung*" = spooky action at a distance

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big)$$



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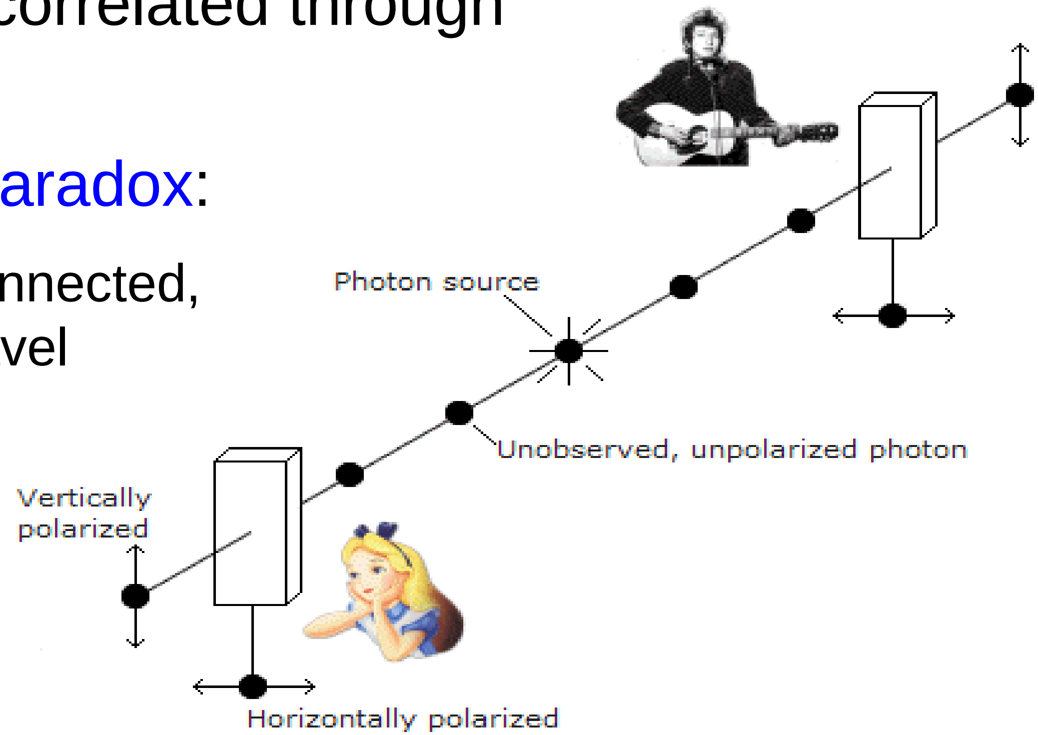

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$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big)$$

compare: $$|\psi'\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\Big)$$

$$|\psi''\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle - |\downarrow\downarrow\rangle\Big)$$

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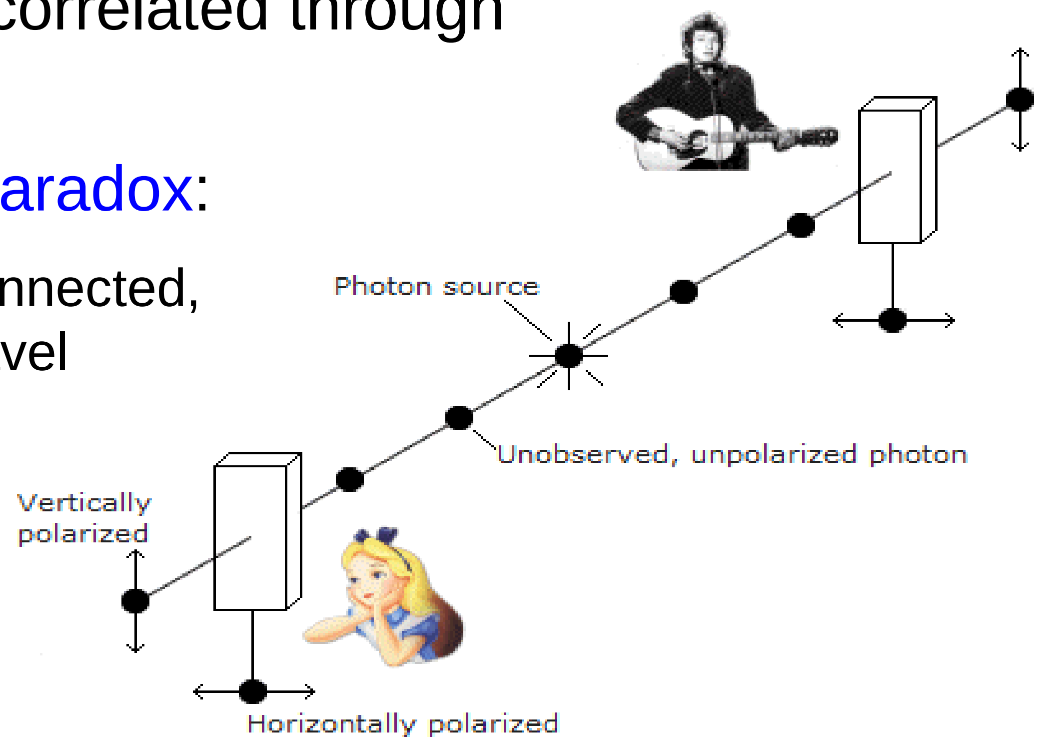


compare:

$$|\psi'\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\Big)$$

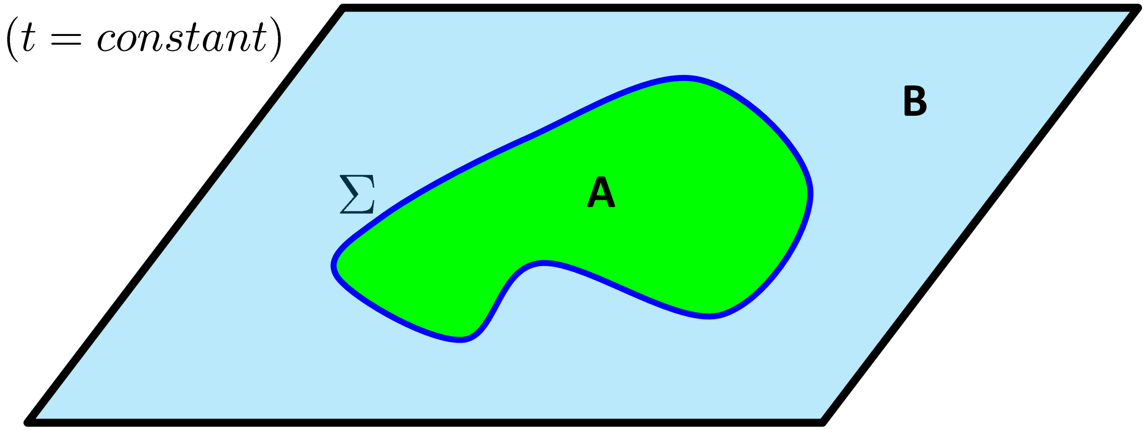
$$= \frac{1}{2}\Big(|\uparrow\rangle + |\downarrow\rangle\Big) \otimes \Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$ ➡ **No Entanglement!!**

$$|\psi''\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle - |\downarrow\downarrow\rangle\Big)$$ ➡ **Entangled!!**

Entanglement Entropy

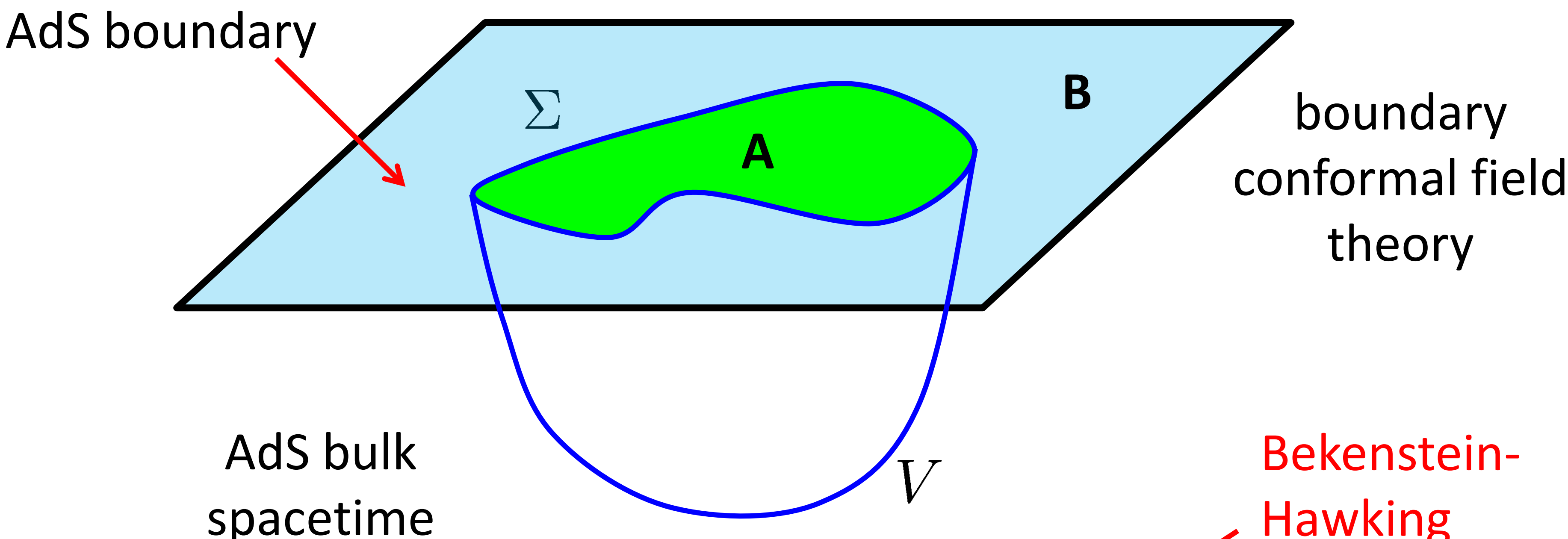
- general tool; divide quantum system into two parts and use entropy as measure of correlations between subsystems
- in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
- integrate out degrees of freedom in "outside" region
- remaining dof are described by a density matrix ρ_A

⟶ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \log \rho_A\right]$



Holographic Entanglement Entropy:

(Ryu & Takayanagi)



Bekenstein-Hawking formula

$$S(A) = \underset{V \sim A}{\text{ext}} \frac{A_V}{4G_N}$$

- 2006 conjecture ⟶ many detailed consistency tests

(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Faulkner, . . .)

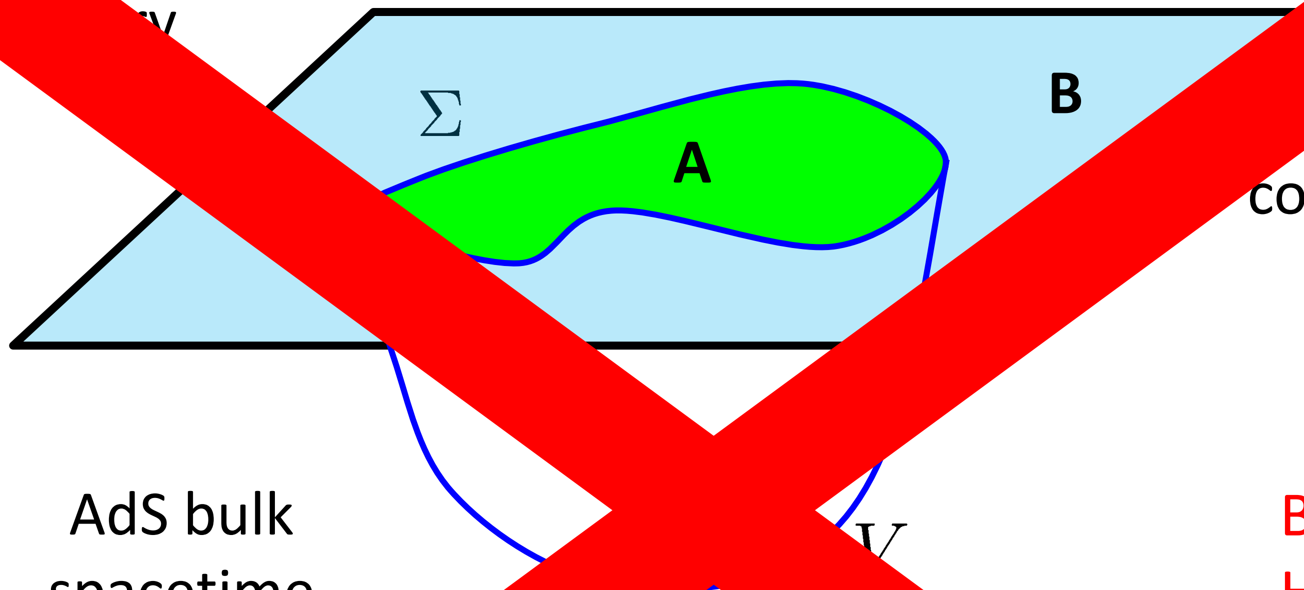
- 2013 proof (for static geometries)

(Maldacena & Lewkowycz)

H…graphic Entanglement Entropy:

(Ryu & Takaya…i)

AdS b…ry



…oundary conformal field theory

AdS bulk spacetime

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(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Fa… .)

- …3 proof (for static geometries) (Maldacena & Lewko…)

First Law of Entanglement

- entanglement entropy: $S(\rho_A) = -\mathrm{tr}(\rho_A \log \rho_A)$
- make a small perturbation of state: $\tilde{\rho} \;=\; \rho_A + \delta\rho$

$$\longrightarrow \quad \delta S = -\mathrm{tr}(\delta\rho \log \rho_A) - \underbrace{\mathrm{tr}(\rho_A\, \rho_A^{-1} \delta\rho)}_{= \mathrm{Tr}\,(\delta\rho) = 0} + O(\delta\rho^2)$$

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- modular (or entanglement) Hamiltonian: $\rho_A = \exp(-H_A)$

$$\delta S_A \;=\; \delta\langle H_A\rangle \;=\; \mathrm{Tr}\,(\delta\rho\, H_A)$$

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"1st law" of entanglement entropy

- this is **the** 1st law for thermal states: $\rho_A = \exp(-H/T)$

"1st law" of entanglement entropy: $\delta S_A \;=\; \delta\langle H_A\rangle$

- generally H_A is "**nonlocal mess**" and flow is nonlocal/**not geometric**

$$H_A = \int d^{d-1}x\, \gamma_1^{\mu\nu}(x)\, T_{\mu\nu} + \int d^{d-1}x \int d^{d-1}y\, \gamma_2^{\mu\nu;\rho\sigma}(x,y)\, T_{\mu\nu}T_{\rho\sigma} + \cdots$$

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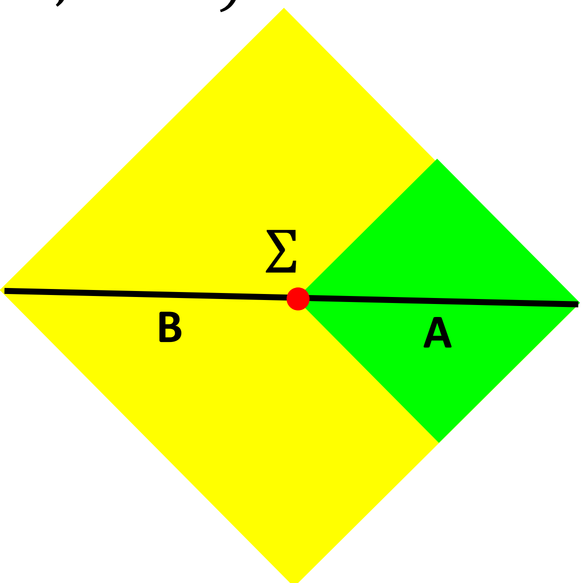
- famous exception: **Rindler wedge**
- any QFT in Minkowski vacuum; choose $\Sigma = (\mathrm{x}=0, \mathrm{t}=0)$

$$H_A = 2\pi K$$ ⟵ boost generator

$$= 2\pi \int_{A(x>0)} d^{d-2}y\,dx\,\left[\,x\,T_{tt}\,\right] + c'$$



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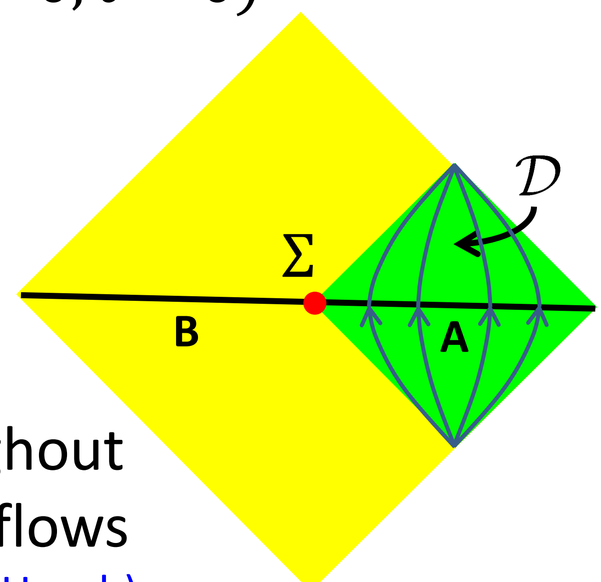


- by causality, ρ_A and H_A describe physics throughout domain of dependence $\mathcal{D}$; eg, generate boost flows

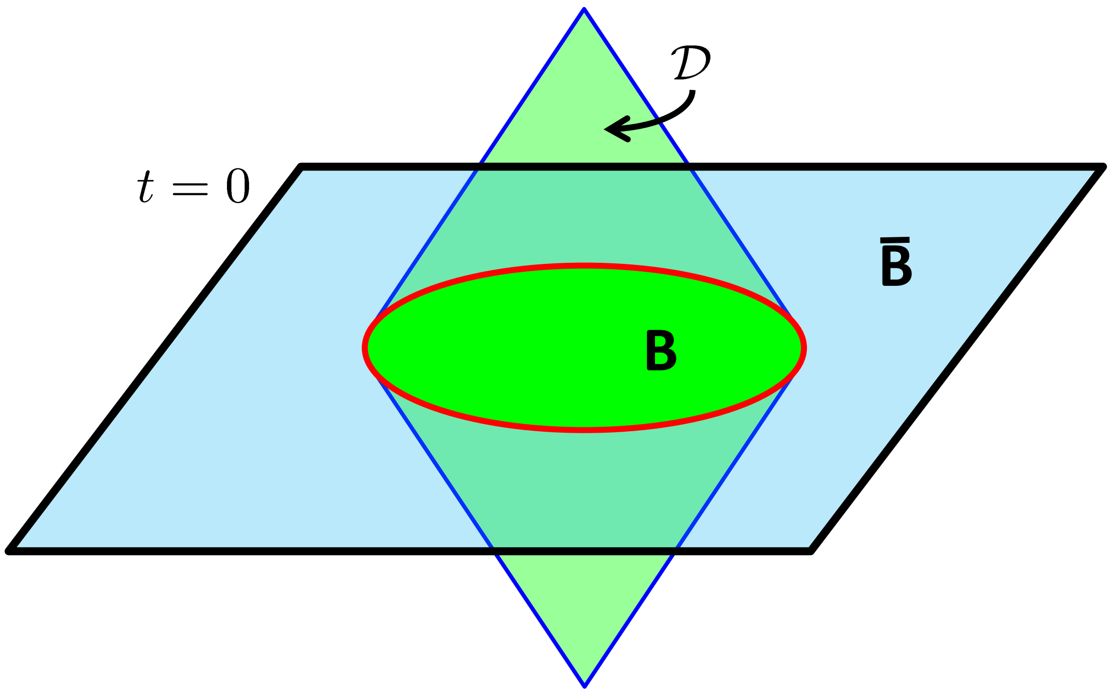
(Bisognano & Wichmann; Unruh)

“1st law” of entanglement entropy: $\delta S_A \;=\; \delta\langle H_A\rangle$

- **another exception**: CFT in vacuum of d-dim. flat space and entangling surface which is S^{d-2} with radius R

$$H_B = 2\pi \int_B d^{d-1}y \;\frac{R^2 - |\vec{y}|^2}{2R}\, T_{tt}(\vec{y}) + c'$$

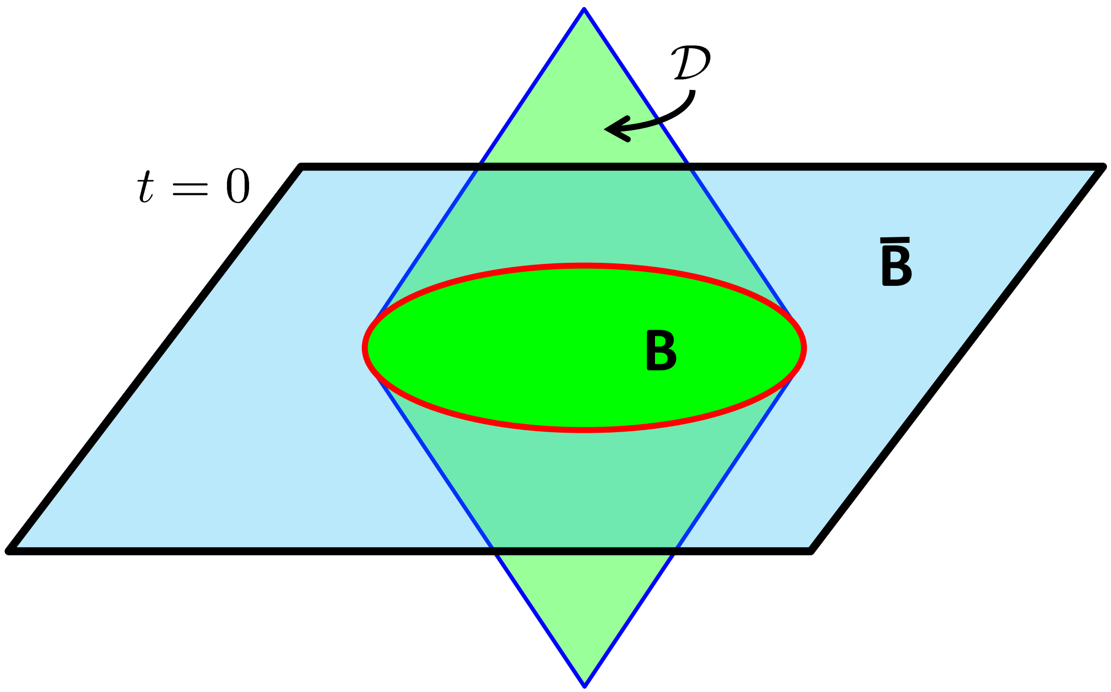
(Casini, Huerta & RM; Hislop & Longo)



"1st law" of entanglement entropy: $\delta S_A \;=\; \delta\langle H_A\rangle$

- small excitations of CFT vacuum in d-dim. flat space and entangling surface which is S^{d-2} with radius R:

$$\delta S = \delta\langle H_B\rangle = 2\pi \int_B d^{d-1}y \; \frac{R^2 - |\vec{y}|^2}{2R} \, \langle T_{tt}(\vec{y})\rangle$$



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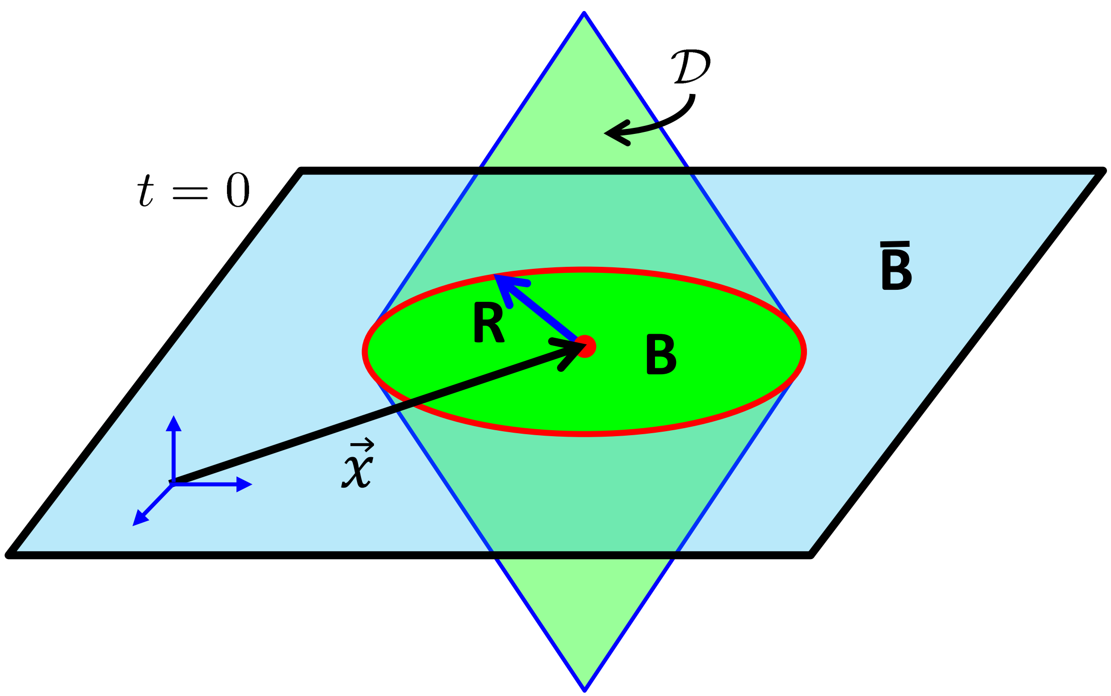

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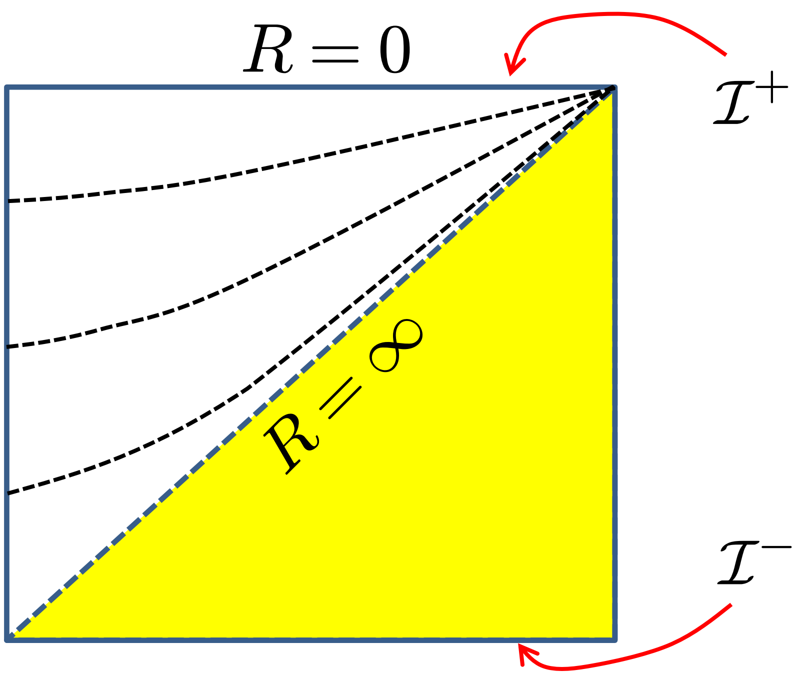
- **boundary-to-bulk propagator in d-dim de Sitter space!**

(eg, see: Xiao 1402.7080)



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- straightforward to show δS satisfies wave equation in dS_d

$$\left(\nabla^2_{dS} - m^2\right)\,\delta S = 0 \qquad \text{with} \qquad m^2\,L^2 = -d$$

Boundary data:

- de Sitter metric: $ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$
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⟶ 2 independent sol's: $$\delta S \overset{R\to 0}{=} F(\vec{x})/R + f(\vec{x})\,R^d + \cdots$$

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- $\langle T_{tt}\rangle$ sets δS at very small R and EE perturbations at larger scales determined by the **local Lorentzian propagation** into dS geometry
- $m^2 L^2 = -d$: mass tachyonic! → above precisely removes the "non-normalizable" or unstable modes

Example:

$$\delta S(R,\vec{x}) = 2\pi \int_B d^{d-1}y \; \frac{R^2 - |\vec{y}-\vec{x}|^2}{2R} \langle T_{tt}(\vec{y})\rangle$$

- consider state: $|\psi\rangle = |0\rangle \; + \; \epsilon \, T_{tt}(t_0 + i\tau, \vec{x}_0)|0\rangle$

small expansion parameter → ϵ

$\epsilon/\tau^d \ll 1$

regulate UV divergences → $i\tau$

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- expectation value is fixed by 2-pt function $\langle 0|T_{tt}(t,\vec{x})\, T_{tt}(0,\vec{0})|0\rangle$

$$\langle\psi|T_{tt}(t,x)|\psi\rangle = \epsilon\, C_T \left[\frac{1}{(\Delta x^2 - (\Delta t + i\,\tau)^2)^d}\left(\frac{(\Delta x^2 + (\Delta t + i\,\tau)^2)^2}{(\Delta x^2 - (\Delta t + i\,\tau)^2)^2} - \frac{1}{d}\right) + \text{c.c.}\right] + \mathcal{O}(\epsilon^2)$$

with $\Delta x^2 = |\vec{x} - \vec{x}_0|^2$ and $\Delta t^2 = |t - t_0|^2$

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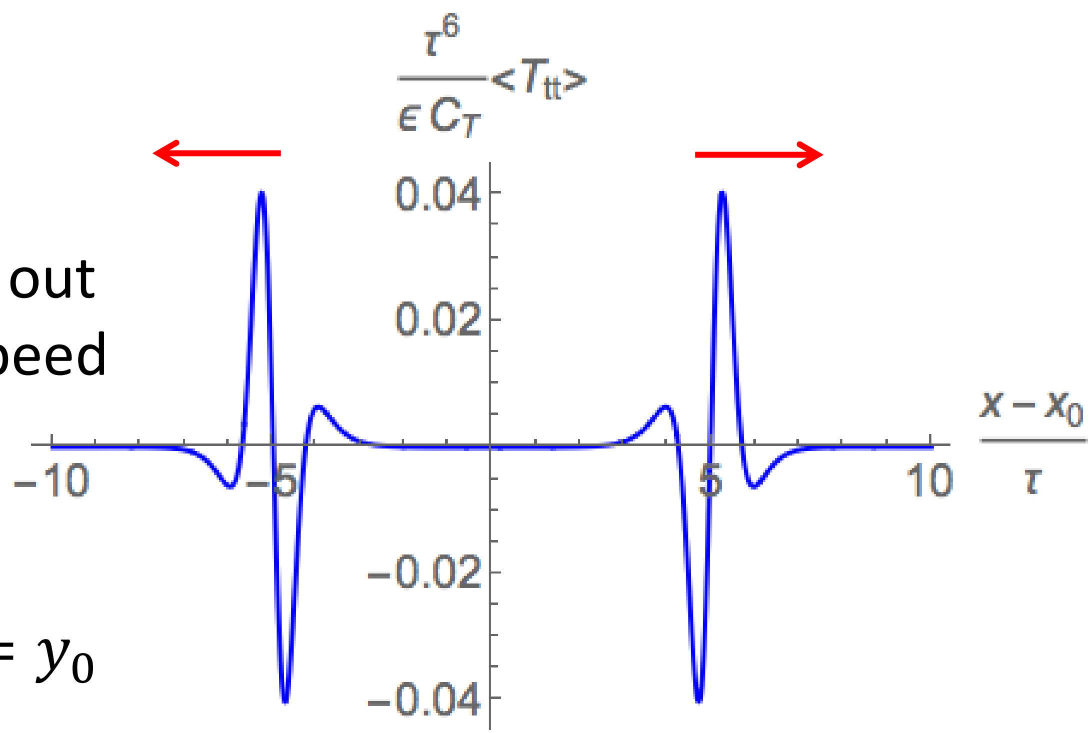
- sphere expanding out from $(t_0, \vec{x}_0)$ at speed of light



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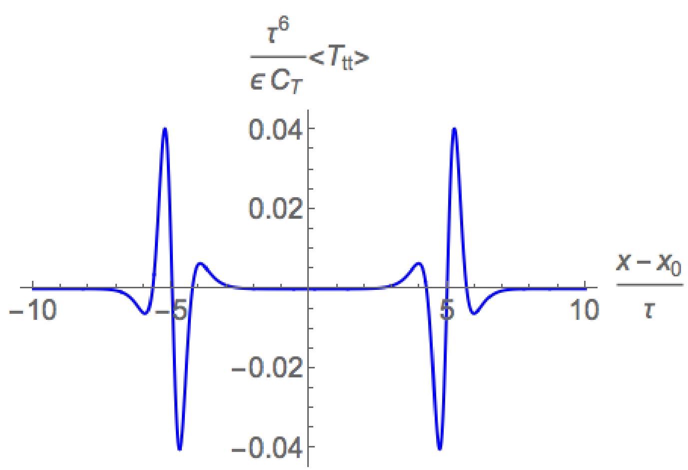

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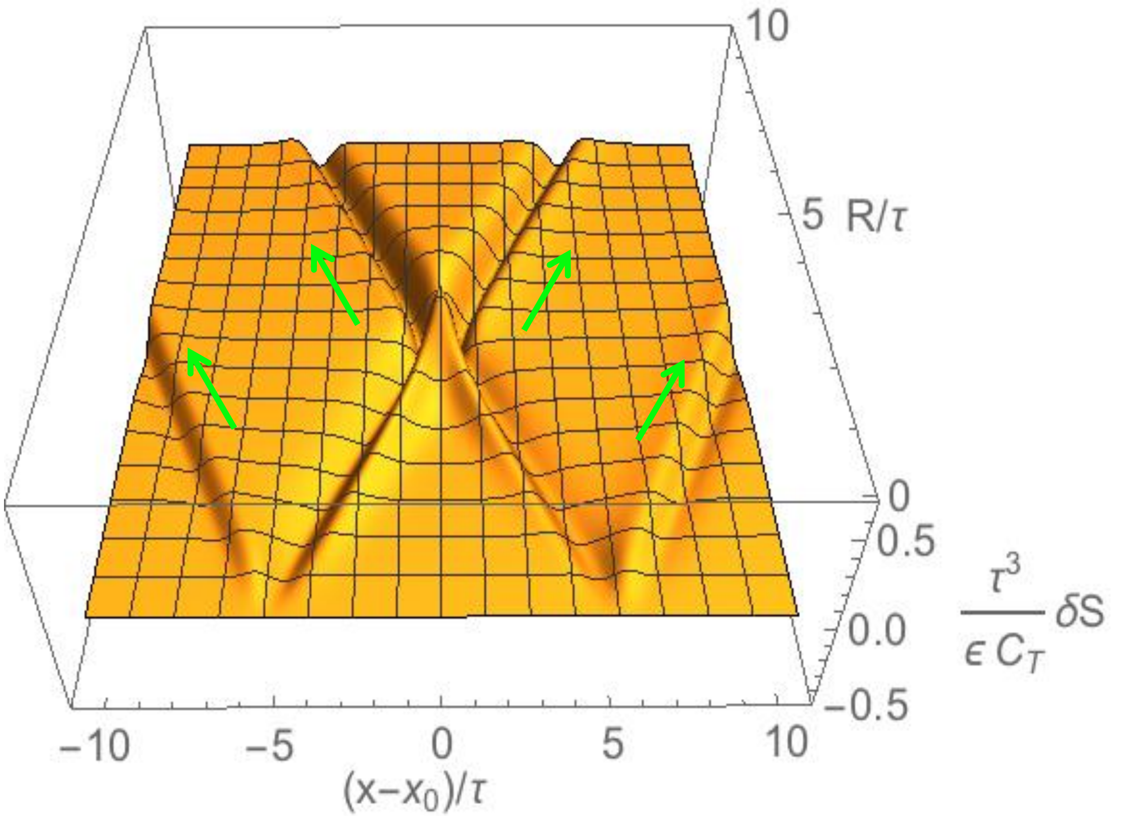

Alternate conformal frames:

- wave equation $\left(\nabla^2_{dS} - m^2\right)\, \delta S = 0$ is covariant

  $\longrightarrow$ can use any coordinate system on dS geometry

- coord transformation in bulk corresponds to conformal transformation in boundary theory $\longrightarrow$ new holographic construction extends to CFT in any conformally flat background

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- for example, consider same wave equation in global dS coord’s

$$ds^2 = L^2(-d\tau^2 + \cosh(\tau)^2 d\ \ {}_{d-1})$$

  → asymptotic boundary ($\tau \to 0$) is round S^{d-1}

  → CFT time slice is S^{d-1}, in cylindrical bkgd $R \times S^{d-1}$

Compare and Contrast: begin with d-dim. CFT

- **Entanglement Holography:**

$$ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$$

- **AdS/CFT correspondence:**

$$ds^2 = \frac{L^2}{z^2}\left(+dz^2 - dt^2 + d\vec{x}^2\right)$$

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  → holographic CFT requires strong coupling and large # of d.o.f.

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undetermined constant? → L (circled)

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spatial coordinates in (d–1)-dim. time slice

- two-derivative wave equation relies only on first law of entanglement
  → appropriate states in any CFT in any number of dimensions

- **AdS/CFT correspondence:**

fixed by ℓ_P → L^2 (circled)

Euclidean → $+dz^2$

$$ds^2 = \frac{L^2}{z^2}\left(+dz^2 - dt^2 + d\vec{x}^2\right)$$

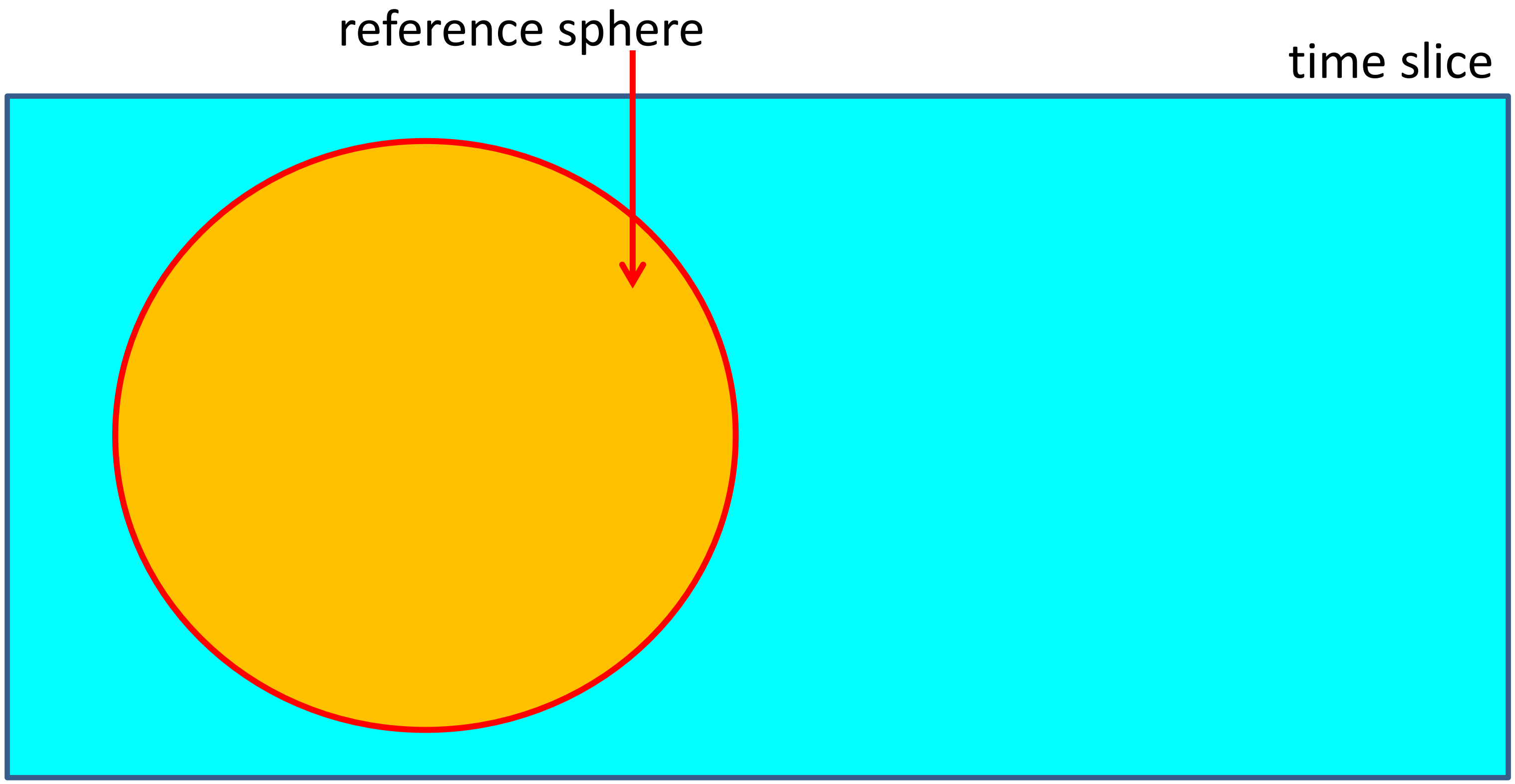
holo. coordinate = scale (roughly)

spacetime coordinates for d-dim. CFT

- two-derivative bulk theory relies on weak curvature and weak coupling
  → holographic CFT requires strong coupling and large # of d.o.f.

Why is scale time-like?

- geometry naturally gives partial ordering of spheres



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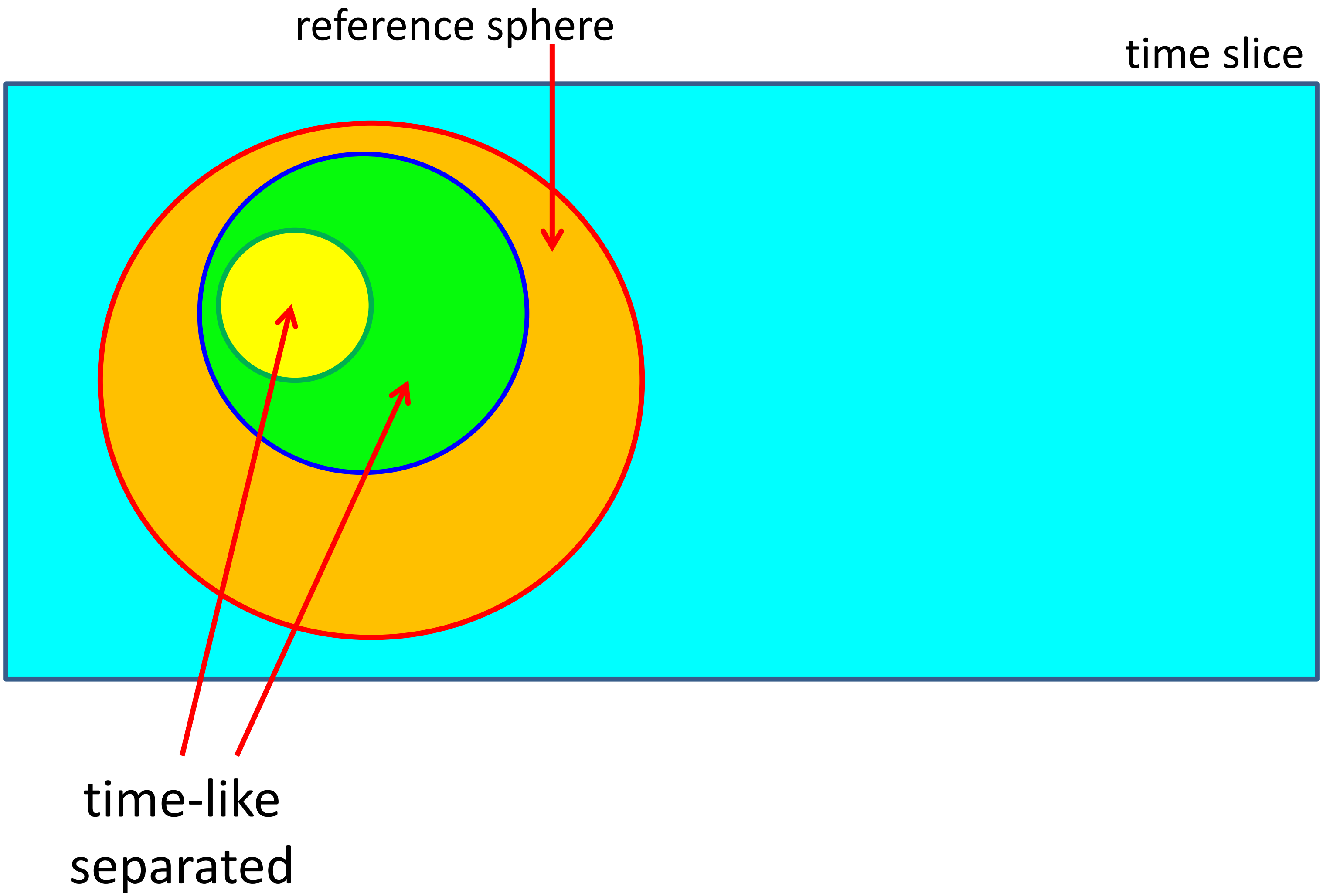

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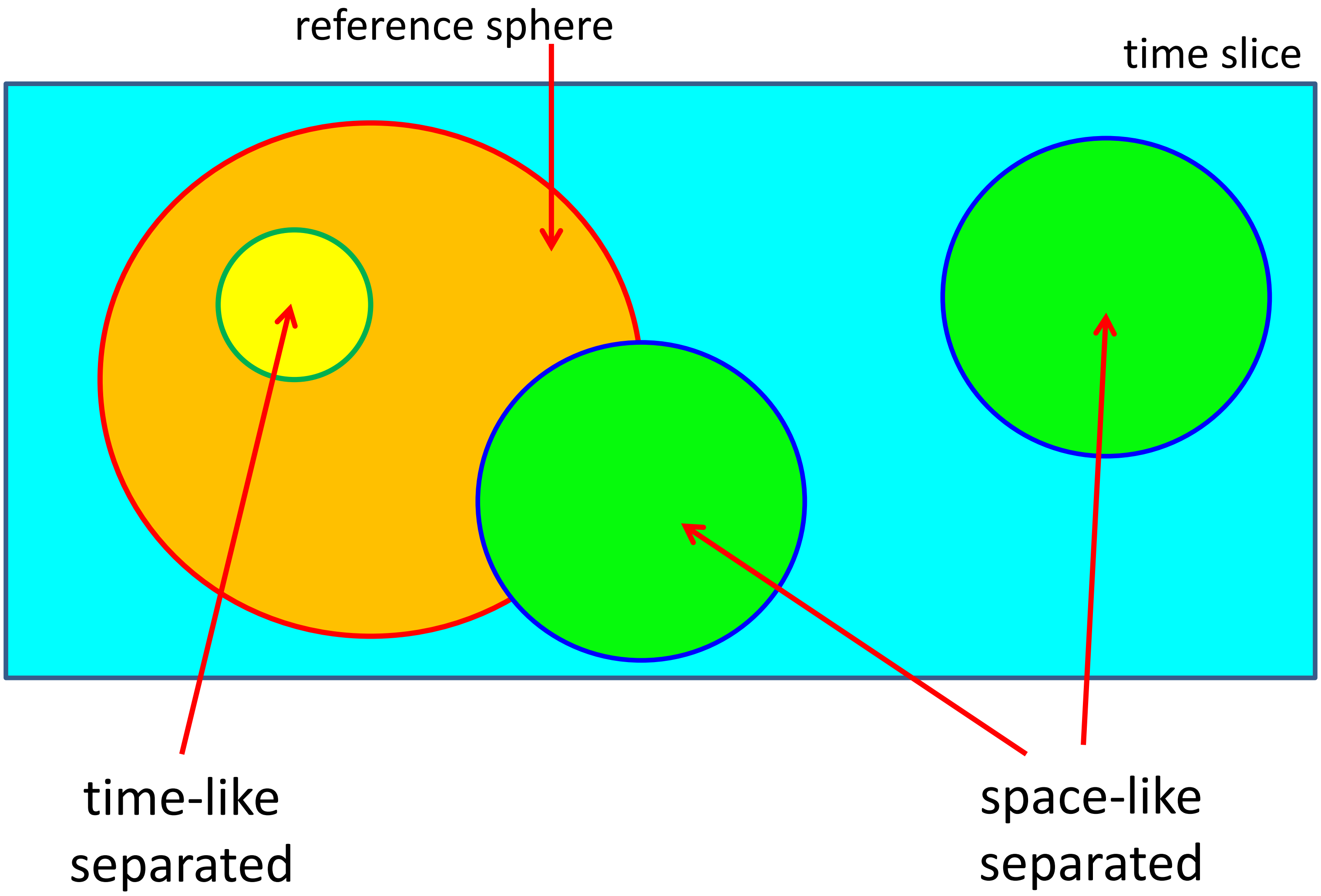

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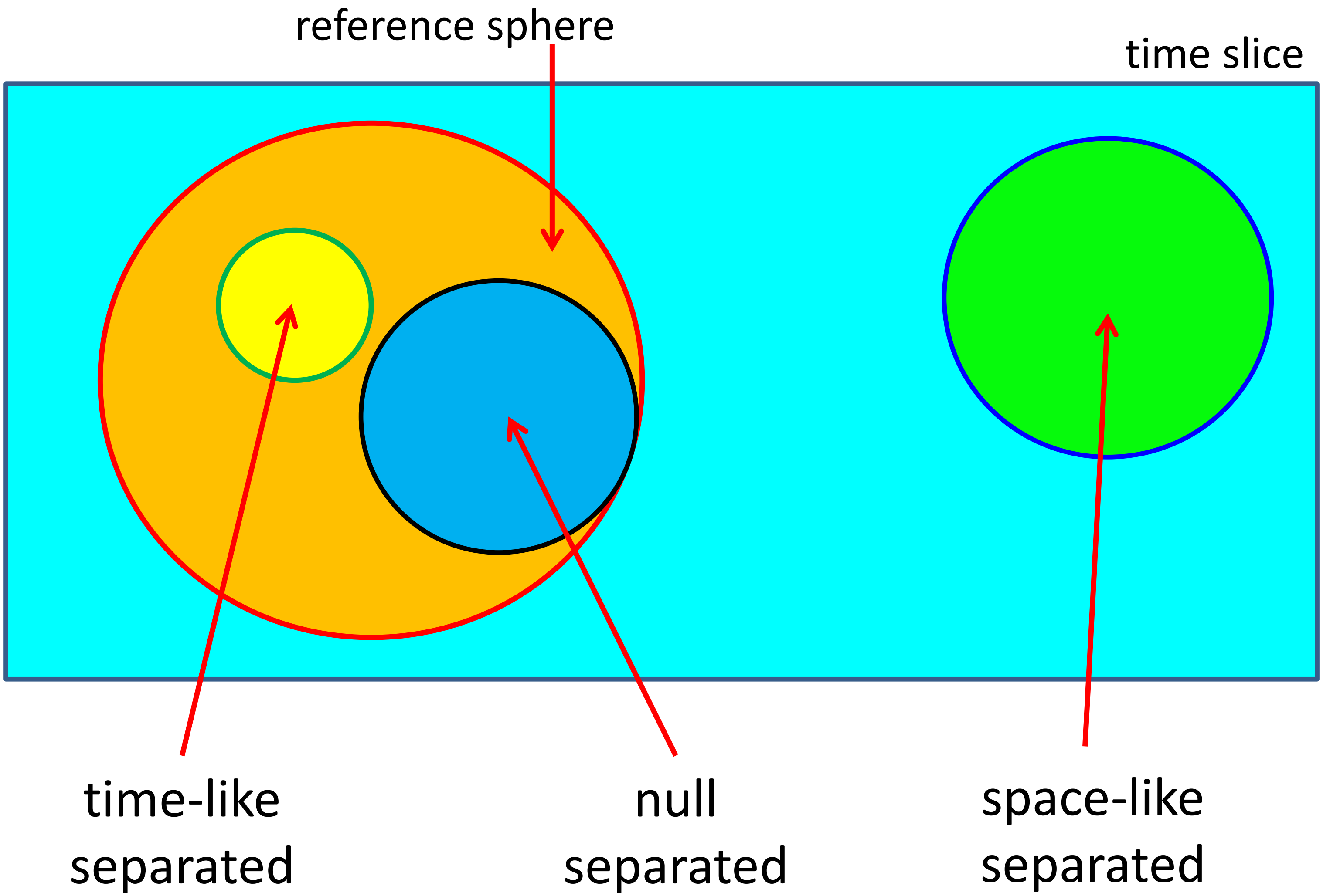
Why is scale time-like?

- geometry naturally gives partial ordering of spheres

→ suggests auxiliary/holographic geometry should be Lorentzian



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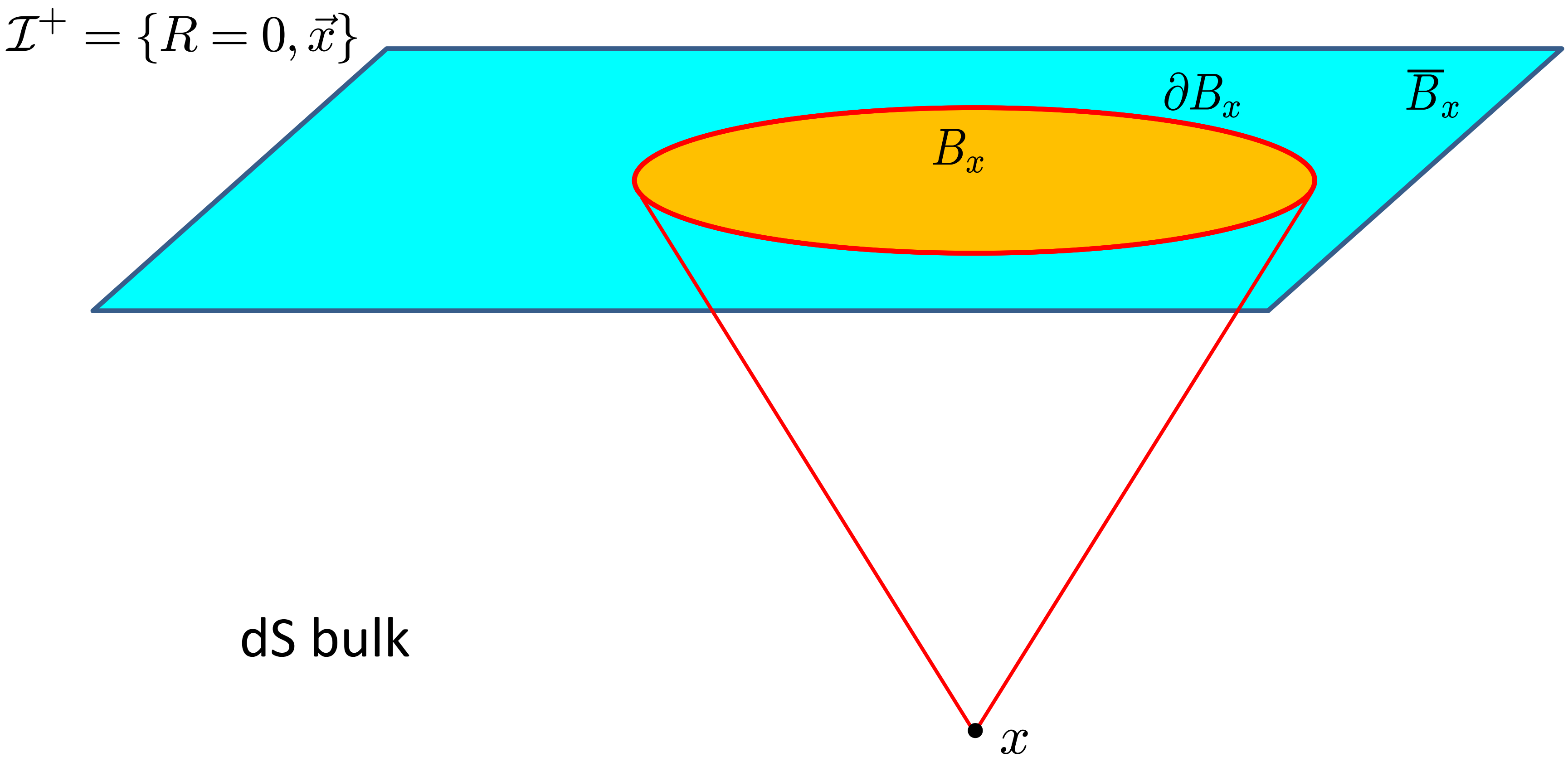
Mapping deSitter ↔ Balls?

- choose one of asymptotic boundaries of dS (eg, $\mathcal{I}^+$) ↔ time slice
- for any point x in bulk and send out future light cone to $\mathcal{I}^+$
- intersects $\mathcal{I}^+$ on a sphere and interior uniquely defines `dual' ball B_x



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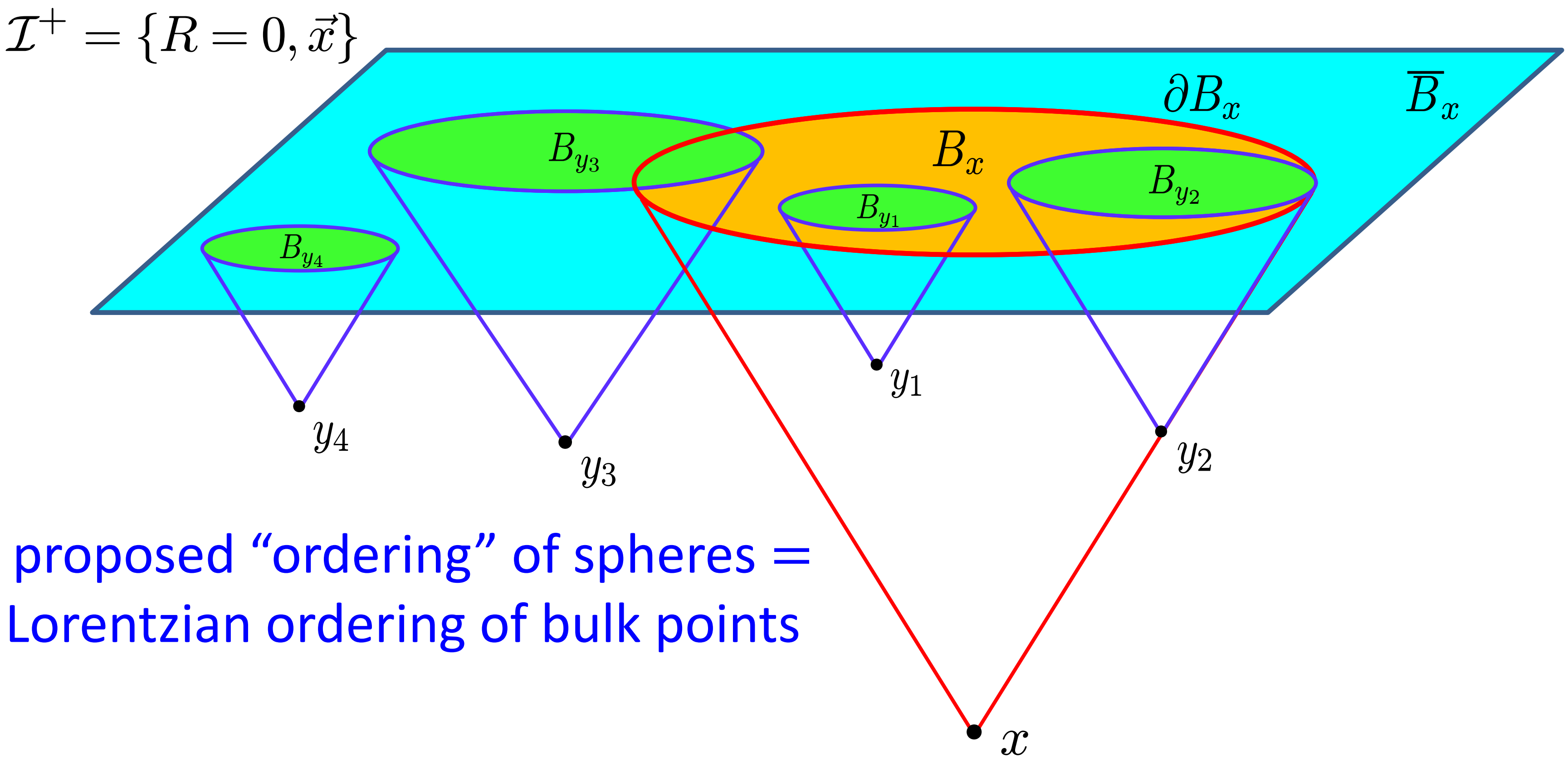

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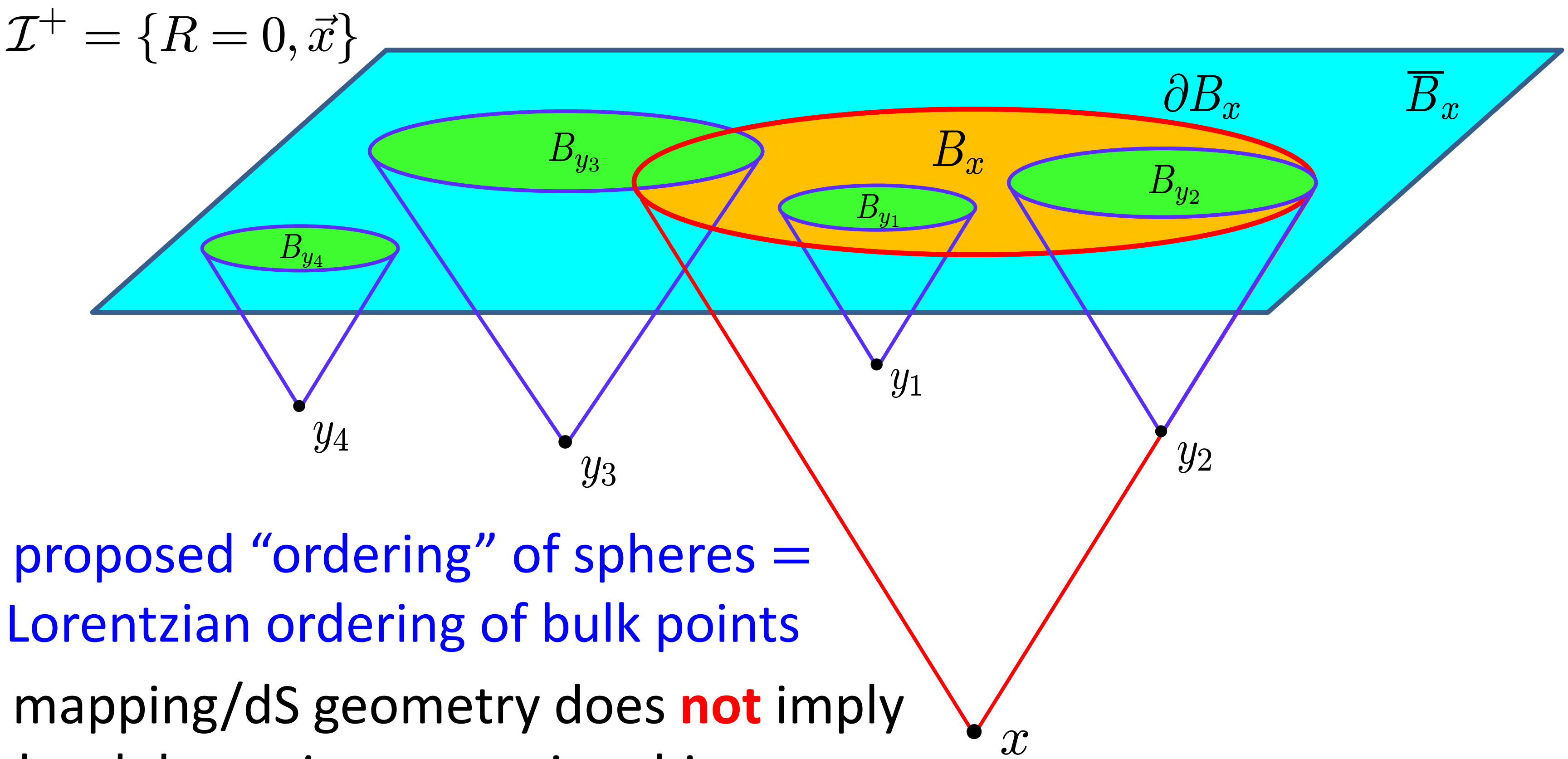


- proposed "ordering" of spheres = Lorentzian ordering of bulk points
- mapping/dS geometry does **not** imply local dynamics respecting this structure

Comments:

- same wave equation derived from AdS/CFT correspondence

Nozaki, Numasawa, Prudenziati& Takayanagi: arXiv:1304.7100
Bhattacharya, Takayanagi: arXiv:1308.3792

- Eg, linearized Einstein eqs in AdS$_4$ implied for holographic EE

$$\left[\frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial}{\partial R} - \frac{3}{R^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right] \delta S(t,x,y,R) = 0$$

- can be recast as d=3 deSitter wave equation:

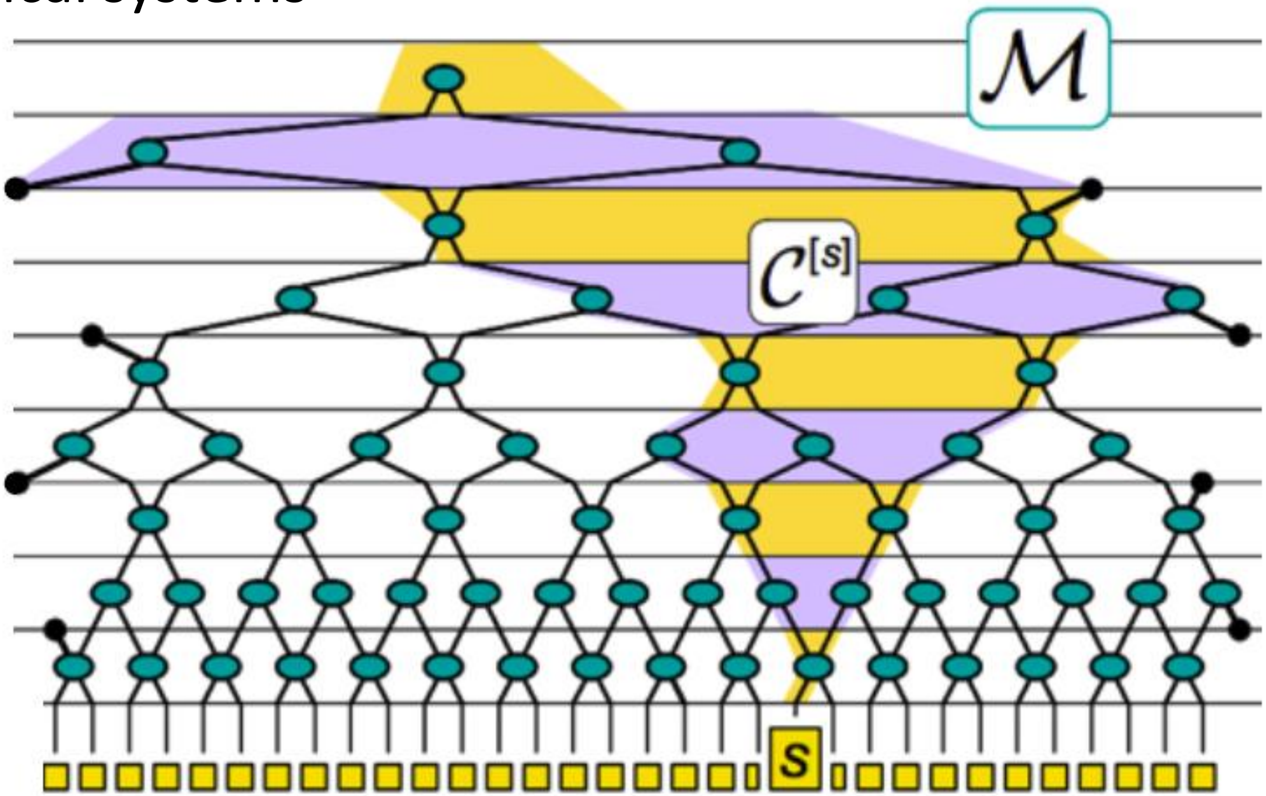
$$\left[\underbrace{-\frac{R^3}{L^2}\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{R^2}{L^2}\frac{\partial^2}{\partial x^2} + \frac{R^2}{L^2}\frac{\partial^2}{\partial y^2}}_{\text{d'Alembertian on dS}_3} + \underbrace{\frac{3}{L^2}}_{\text{mass term}}\right] \delta S(t,x,y,R) = 0$$

d'Alembertian on dS$_3$ mass term

- here, we see equation readily extends to any d and follows purely from underlying conformal symmetry

Comments:

- MERA (Multi-scale Entanglement Renormalization Ansatz) provides efficient tensor network representation of ground-state wave-function in d=2 critical systems (Vidal)



- has been argued that MERA has (Lorentzian) causal structure with coarse-graining direction being time-like! (Beny; Czech etal)

Comments:

- deSitter geometry appears in recent discussions of integral geometry and the interpretation of MERA in terms of AdS_3/CFT_2
  (Czech, Lamprou, McCandlish & Sully: arXiv:1505.05515; arXiv:1512.01548)

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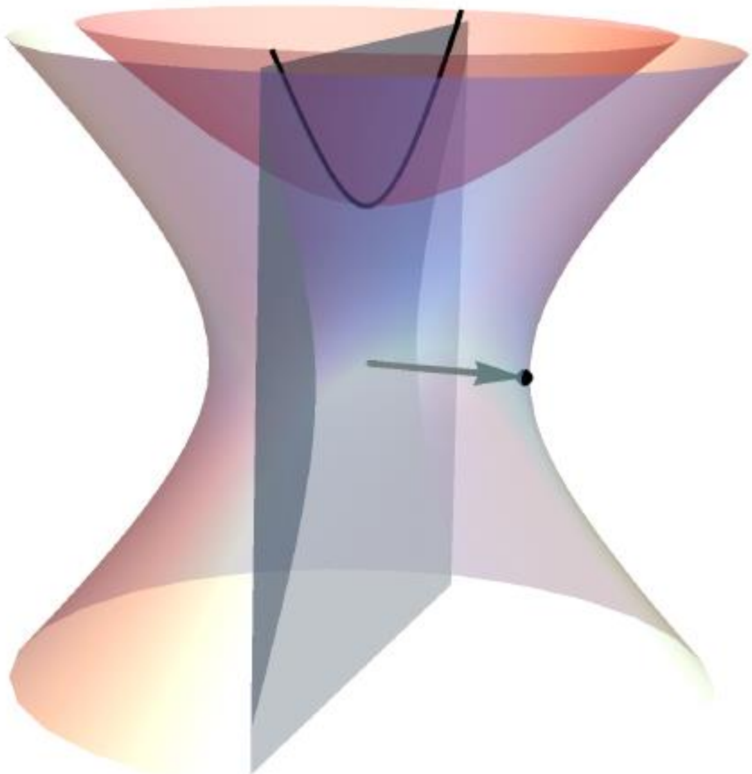
$\longleftrightarrow$ space of geodesics on 2d slice of AdS_3 $\longleftrightarrow$ pts in 2d de Sitter

AdS/CFT



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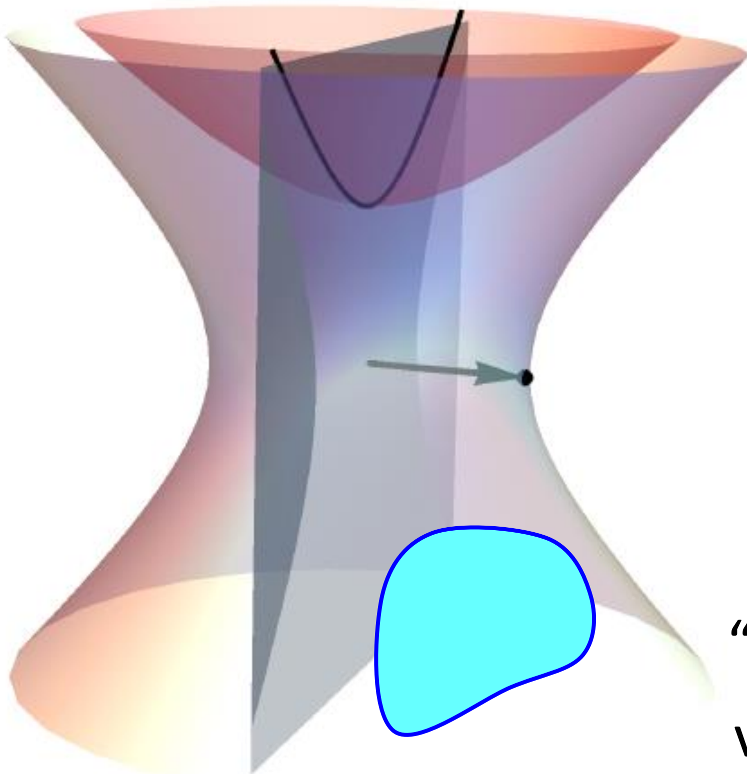

$$ds^2 = L^2 \, \frac{du\, dv}{(v-u)^2}$$

dS scale?

motivate the choice: $L^2 = \frac{c}{3}$

$\longrightarrow$ $ds^2 = \partial_u \partial_v S_0 \, du\, dv$

with $S_0 = \frac{c}{3} \log \frac{v-u}{\delta}$

"hole-ography":

volume in dS_2 = length in AdS_3 slice

Recap:

- EE of excitations of CFT vacuum arranged in novel holographic manner
- δS satisfies wave equation in dS_d where **scale** plays the role of time

$$\left(\nabla^2_{dS} - m^2\right)\,\delta S = 0 \qquad \text{with} \qquad m^2\,L^2 = -d$$

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Extension to Higher Spin Charges:

- CFT with conserved symmetric traceless currents $T_{\mu_1 \cdots \mu_s}$ with $s \geq 1$
- modular Hamiltonian is flux of $J^{(2)}_\mu = T_{\mu\nu} K^\nu$ through B where K^ν is conformal Killing vector that leaves ∂B invariant

$$\longrightarrow \quad H_B = \int d\Sigma^\mu \, J^{(2)}_\mu$$

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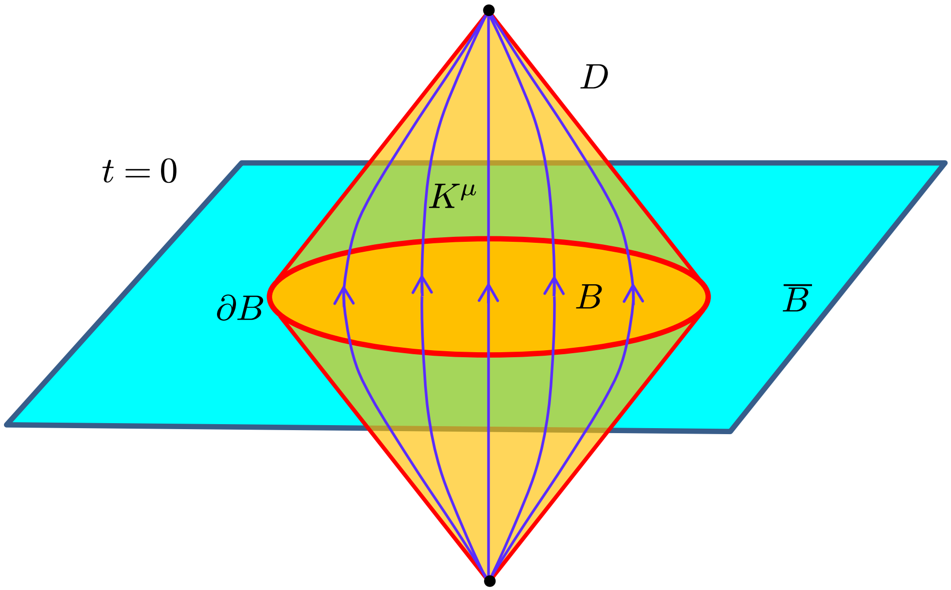
$$Q^{(s)} = \int d\Sigma^\mu \, J^{(s)}_\mu \quad \text{with} \quad J^{(s)}_\mu = T_{\mu\mu_2\cdots\mu_s} K^{\mu_2} \cdots K^{\mu_s}$$



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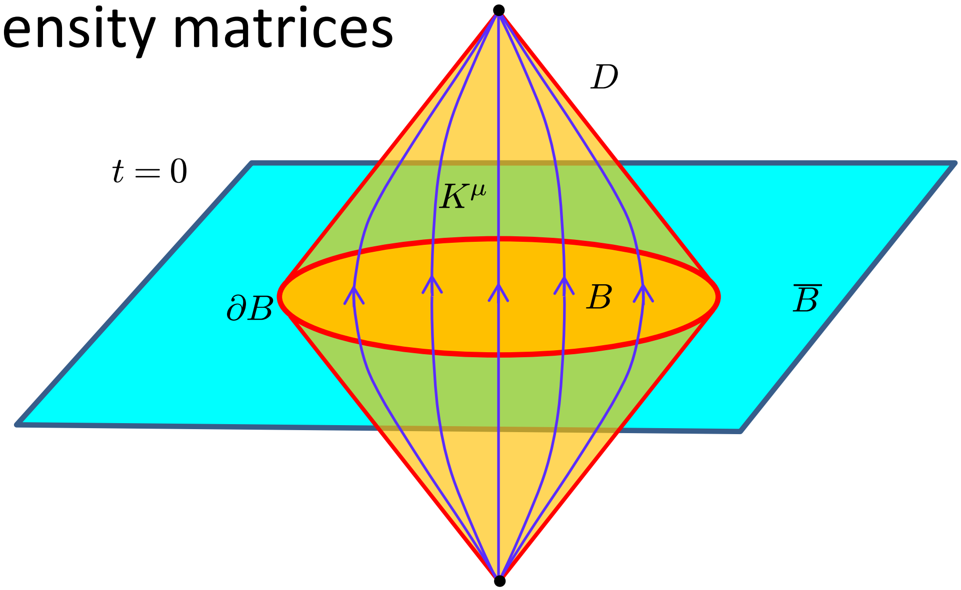
- appear in discussion of modified density matrices

$$\rho_B \sim \exp\left[-\sum \mu_s Q^{(s)}\right]$$

(s=1: Belin, Hung etal;
s≥3: Hijano & Kraus)



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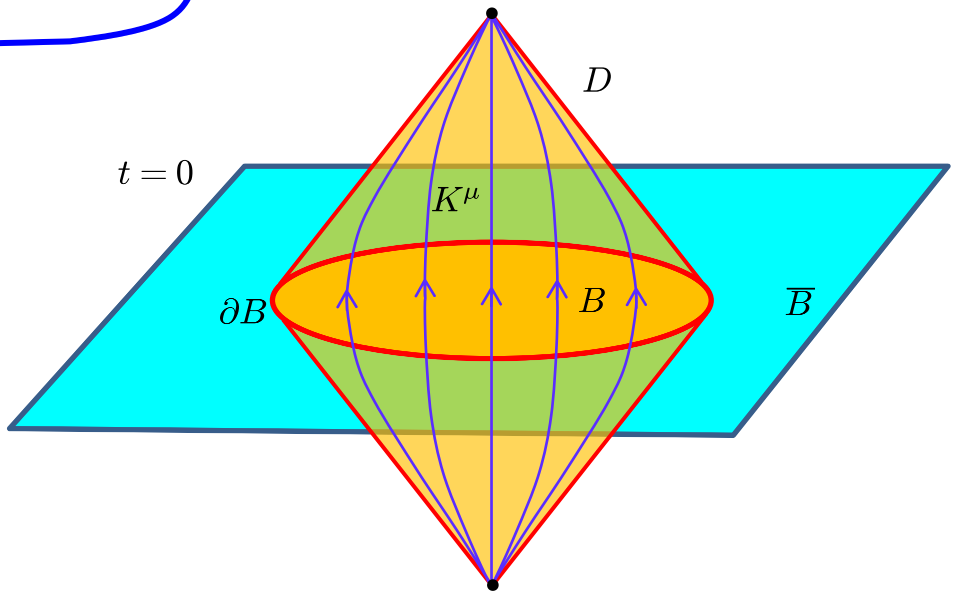
bdry-to-bulk propagator for deSitter



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with

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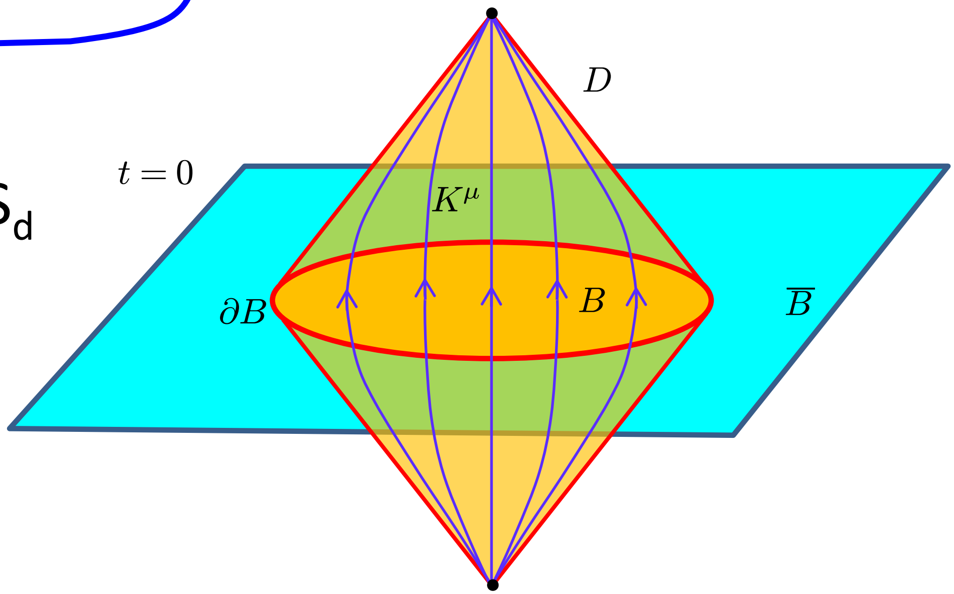

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Question: Interacting fields in dS spacetime?

- how describe finite excitations? ⟶ need to go beyond 1st law!!
- do we have nonlinear but **local** equation on dS geometry?

e.g., $$\left(\nabla^2_{dS} - m^2\right)\, \delta S = g\, \delta S^2 + \cdots$$ **???**

- can higher spin charges be included, as well as δS?

(stay tuned for part 2)

Question: What about time dependence in CFT?

- so far focused on single time slice; natural to consider perturbations of EE for all spheres throughout spacetime on any time slice & any frame

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Lots to explore!!

(Stay tuned for Part 2)