GRAVITATIONAL CONSTRAINTS FROM ENTANGLEMENT INEQUALITIES

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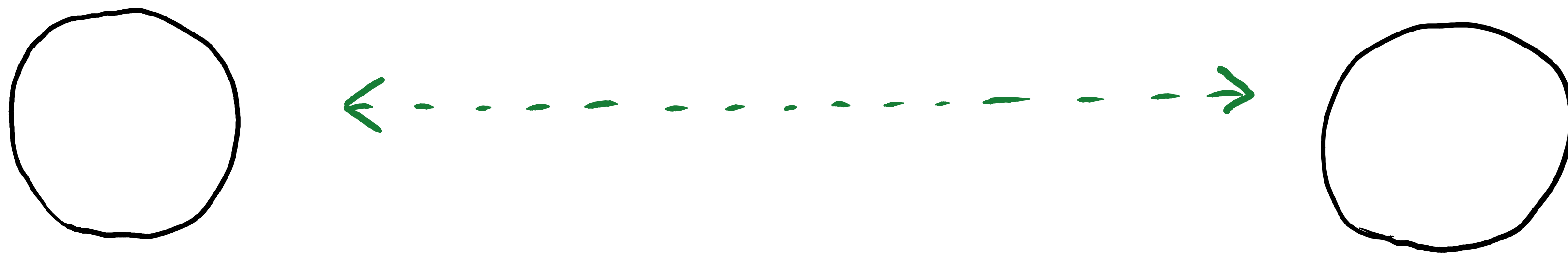
AdS/CFT: CFT on ∂M $\longleftrightarrow$ Quant. grav. for asympt. AdS w. boundary ∂M

(some) CFT states

(some) Asympt. AdS spacetimes boundary ∂M



which ones?

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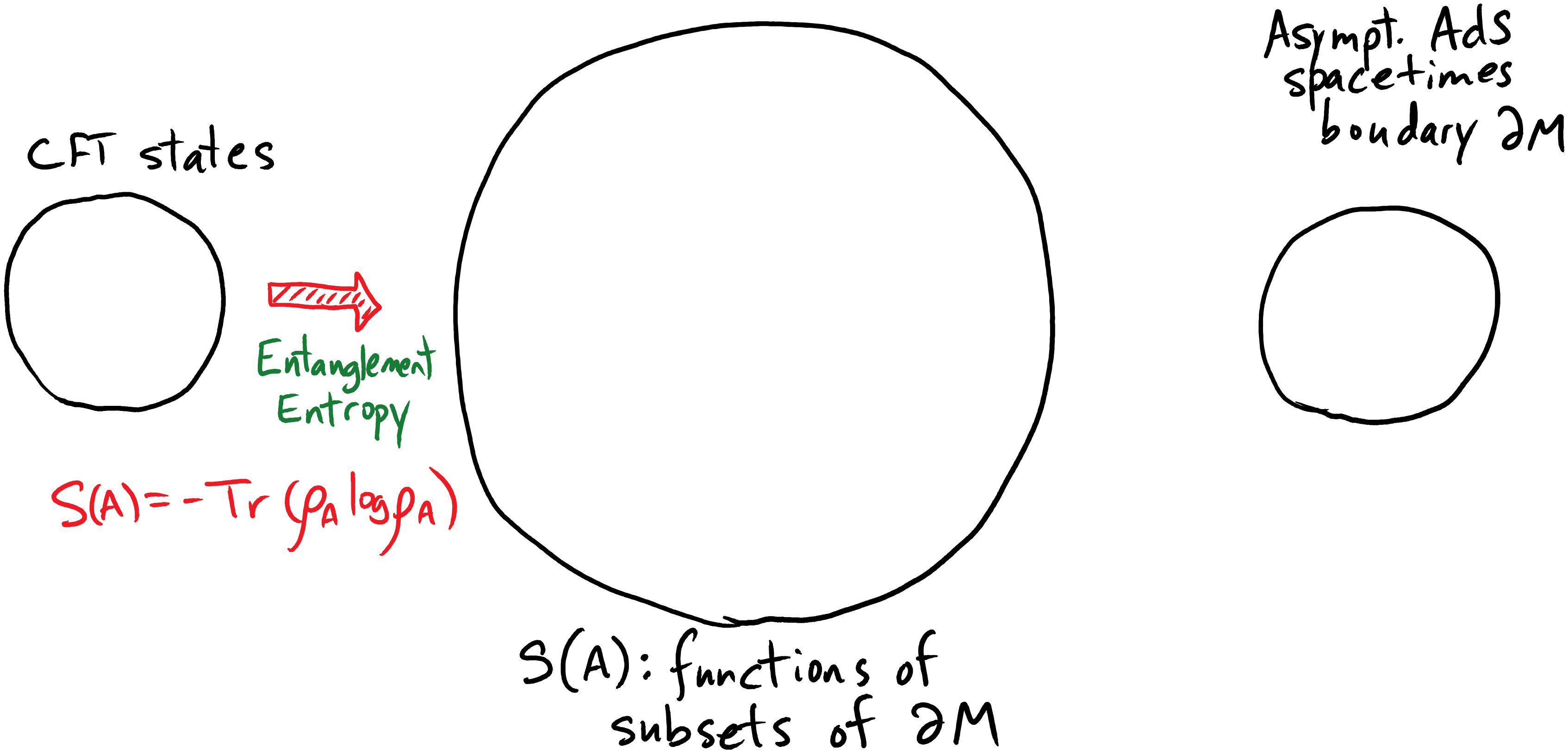

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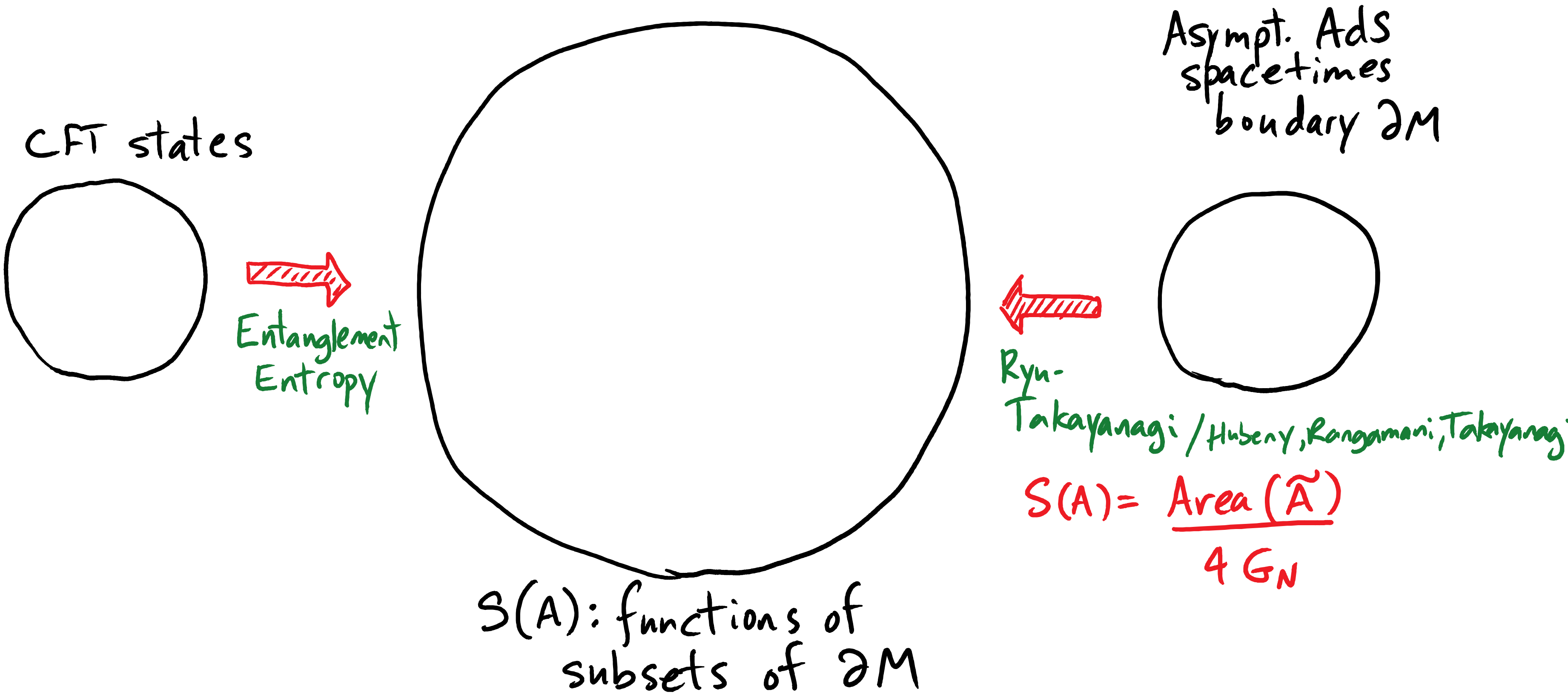

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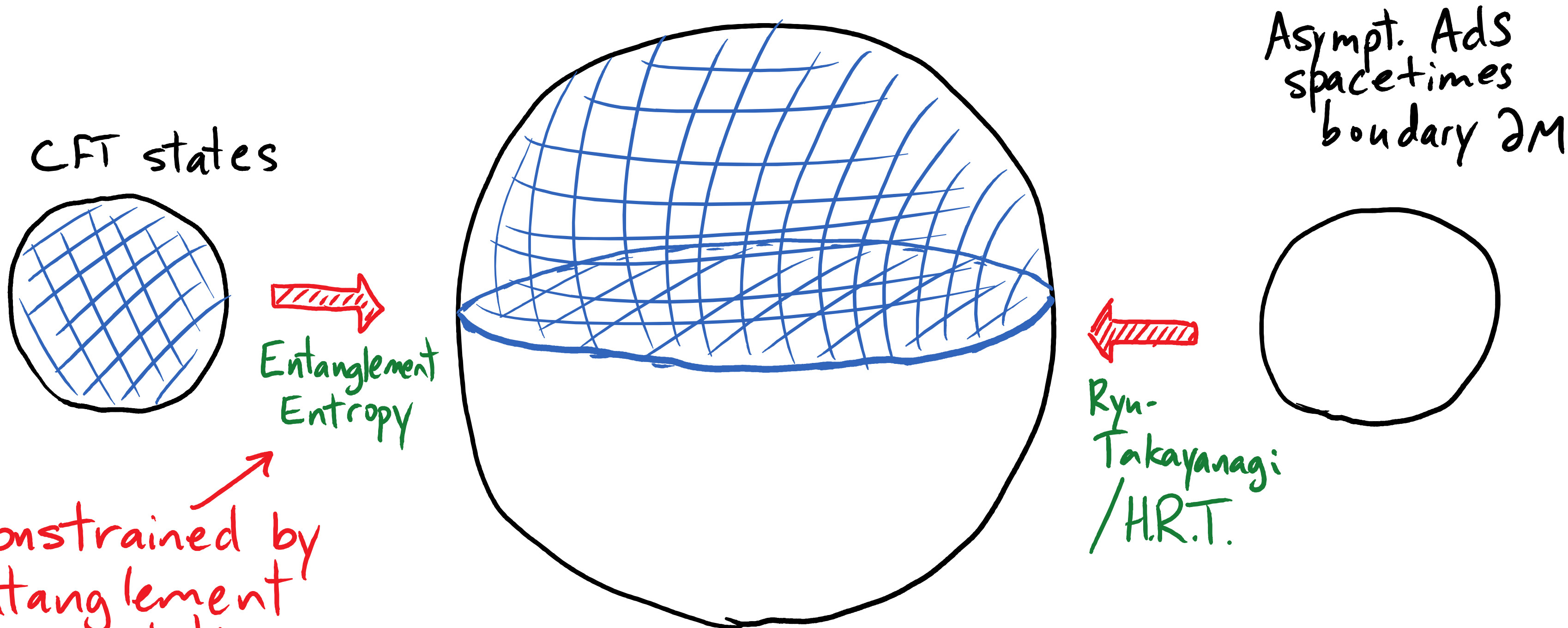

constrained by entanglement inequalities

$S(A)$: functions of subsets of ∂M

e.g.

$$S(A\cup B)+S(B\cup C)\geq S(B)+S(A\cup B\cup C)$$

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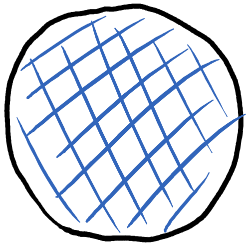
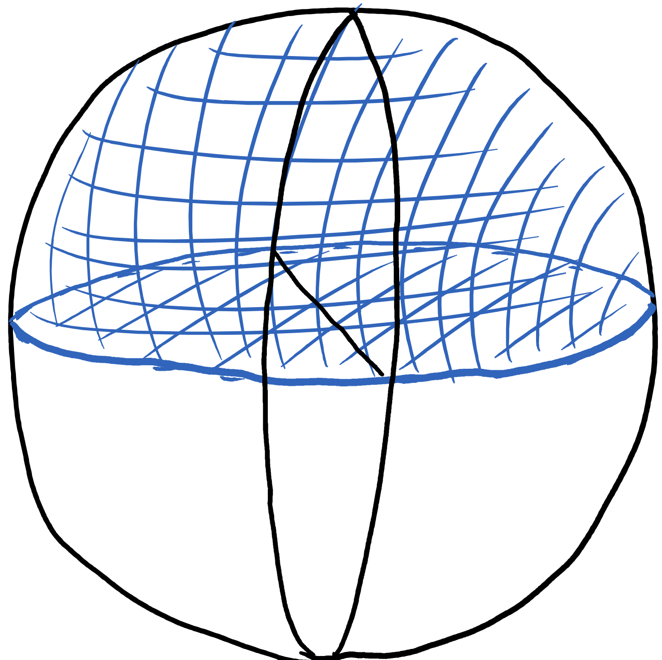
CFT states



Entanglement Entropy

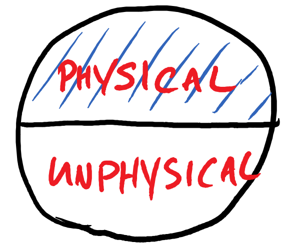
constrained by entanglement inequalities



$S(A)$: functions of subsets of ∂M

Ryu-Takayanagi /H.R.T.

Asympt. AdS spacetimes boundary ∂M



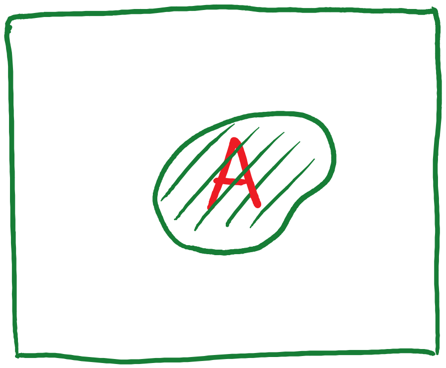
Which geometries M are allowed?

see: Lashkari
Rabideau
Sabella-Garnier
M.V.R.

Gives universal constraints:

Spacetime M with $S(A)$ violating constraints $\longrightarrow$ Cannot correspond to consistent state in ANY UV complete theory for which dual CFT obeys RT/HRT.

Focus on RELATIVE ENTROPY inequalities:

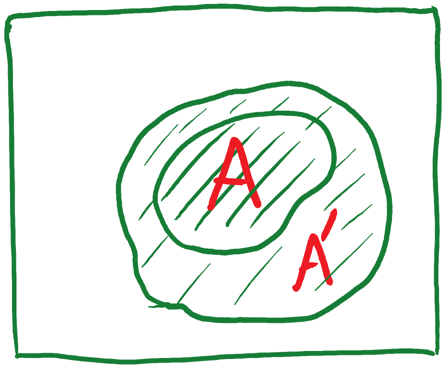


$$|\psi\rangle \longrightarrow \rho_A \qquad |vac\rangle \longrightarrow \sigma_A$$

$$S(\rho_A \| \sigma_A) \equiv \mathrm{Tr}(\rho_A \log \rho_A) - \mathrm{Tr}(\rho_A \log \sigma_A)$$

Measure of distinguishability: POSITIVE for $\rho_A \neq \sigma_A$

Focus on RELATIVE ENTROPY inequalities:



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Measure of distinguishability: POSITIVE for $\rho_A \neq \sigma_A$

Also MONATIONIC: $S(\rho_{A'} \| \sigma_{A'}) \geq S(\rho_A \| \sigma_A)$

for $A' \supset A$

What do relative entropy inequalities imply for $\mathcal{M}$?

Have: $S(\rho_A \| \sigma_A) = -\mathrm{tr}(\rho_A \log \sigma_A) + \mathrm{tr}(\sigma_A \log \sigma_A)$

$+\mathrm{tr}(\rho_A \log \rho_A) - \mathrm{tr}(\rho_A \log \rho_A)$

$= \Delta \langle H_A \rangle - \Delta S_A$

modular Hamiltonian

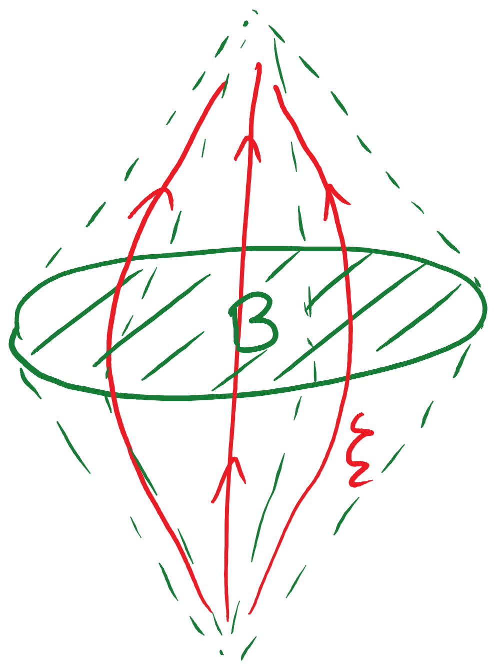
$H_A \equiv -\log \sigma_A$

entanglement entropy

Map to properties of dual spacetime for holographic $|\psi\rangle$.

Blanco, Casini, Hung, Myers

Focus on ball-shaped region B



$$S(\rho_B \| \sigma_B) = \Delta\langle H_B\rangle - \Delta S_B$$

Can write explicity as

$$H_B = 2\pi \int \xi^\mu T_{\mu\nu} \varepsilon^\nu$$

conformal Killing vector

CFT stress tensor

volume form

(Casini, Huerta, Myers)

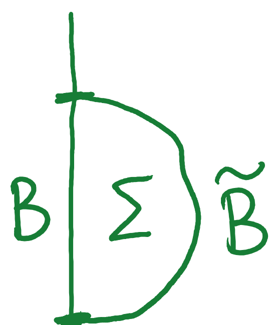
Any CFT: $S(\rho_B \| \sigma_B) = 2\pi \int_B \xi^\mu \Delta \langle T_{\mu\nu} \rangle \varepsilon^\nu - \Delta S_B$

Assume $|\psi\rangle$ has entanglement calculated from some $\mathcal{M}$ via RT/HRT

$$ds^2_{\mathcal{M}} = \frac{\ell_{AdS}^2}{z^2}\left[dz^2 + \Gamma_{\mu\nu}(z,x)\,dx^\mu dx^\nu\right]$$

Then:

$$S(\rho_B \| \sigma_B) = 2\pi \int_B \xi^\mu \, c \cdot \Gamma_{\mu\nu}^{(x)}\Big|_{z^d} \varepsilon^\nu - \frac{\Delta \operatorname{Area}(\tilde{B})}{4 G_N}$$

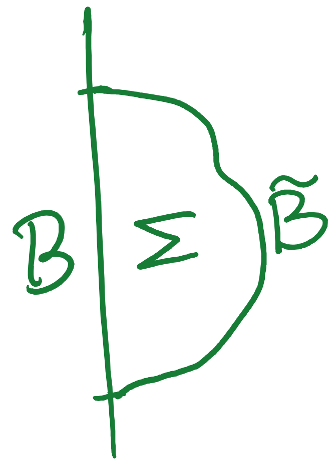


integral over B

integral over $\tilde{B}$

Use Wald technology to rewrite as integral over Σ

Basic idea:



Find $(d-1)$-form χ s.t.

$$\int_B \chi = \Delta E_B^{grav} \qquad \int_{\tilde{B}} \chi = \Delta S_B^{grav}$$

Then:

$$S(\rho_B \| \sigma_B) = \Delta E_B^{grav} - \Delta S_B^{grav} = \int_B \chi - \int_{\tilde{B}} \chi$$

$$= \int_\Sigma d\chi$$

Interpret this

Start w. perturbative constraints: $|\psi(x)\rangle$, $|\psi(0)\rangle = |vac\rangle$

1st order: $S(\rho_B \| \sigma_B)\Big|_{\mathcal{O}(\lambda)} = 0$

$$\Longrightarrow \quad \delta E_B^{grav} = \delta S_B^{grav}$$

$$\Longrightarrow \quad \int_\Sigma \left(\text{something} \propto \text{Einstein tensor}\right) = 0$$

w. McDermott, Lashkari
w. Faulkner, Guica, Hartman, Myers
w. Swingle

constaint = linearized Einstein eqns

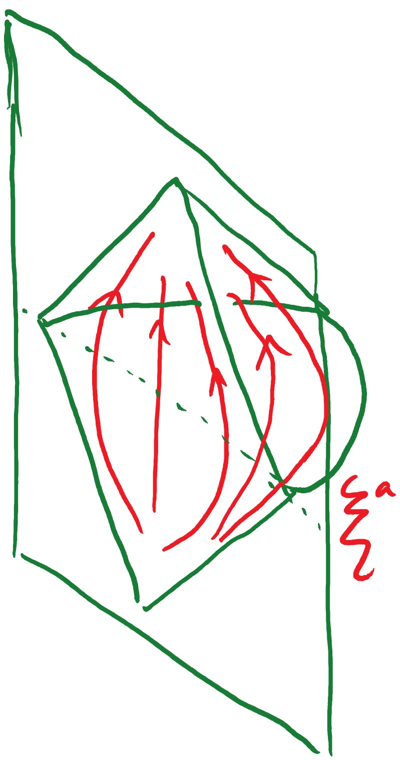
2nd order: $S(\rho_B(\lambda)\|\sigma_B)\Big|_{\mathcal{O}(\lambda^2)} \equiv \langle \delta\rho_B, \delta\rho_B \rangle$

(w. Lashkari)

QUANTUM FISHER INFORMATION

Maps to: $W_\Sigma(g, \delta g, \mathcal{L}_\xi \delta g) \equiv \mathcal{E}(\delta g, \delta g)$

CANONICAL ENERGY



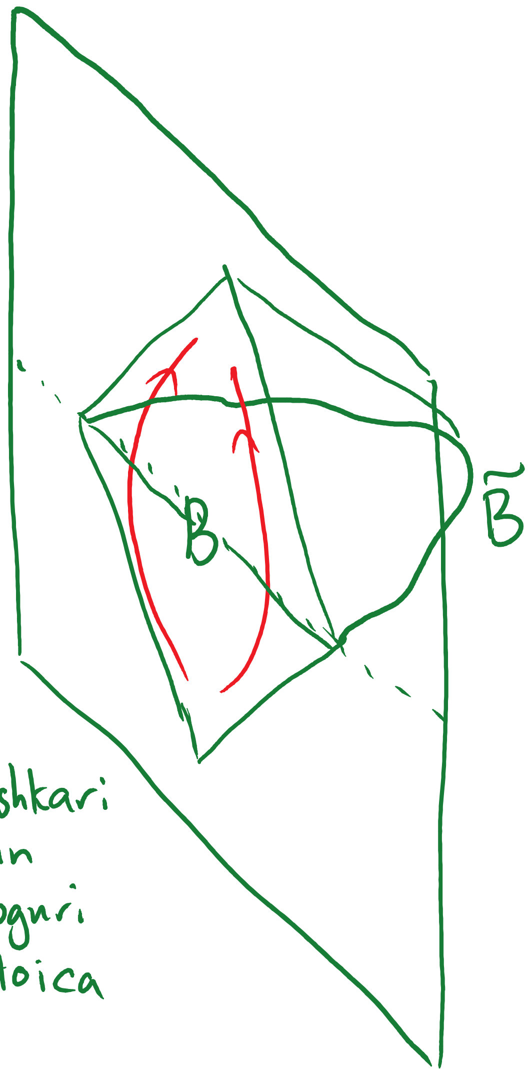
ζ_B extends to ξ_B: Killing vector of pure AdS = Rindler time

$$\mathcal{E}(\delta g, \delta g) \sim \int \xi^a T^{(2)}_{ab} \varepsilon_b + \mathcal{E}^{grav}$$

= gravitational + matter energy associated w. ξ

Positivity/monotonicity give energy conditions

Non-perturbative: $S(\rho_B \| \sigma_B)$ again maps to bulk integral.



w. Lashkari
Lin
Ooguri
Stoica

→ Can interpret as energy associated w. vector field X

Require: $X \to \xi$, $\nabla_{(a} X_{b)} \to 0$ at B

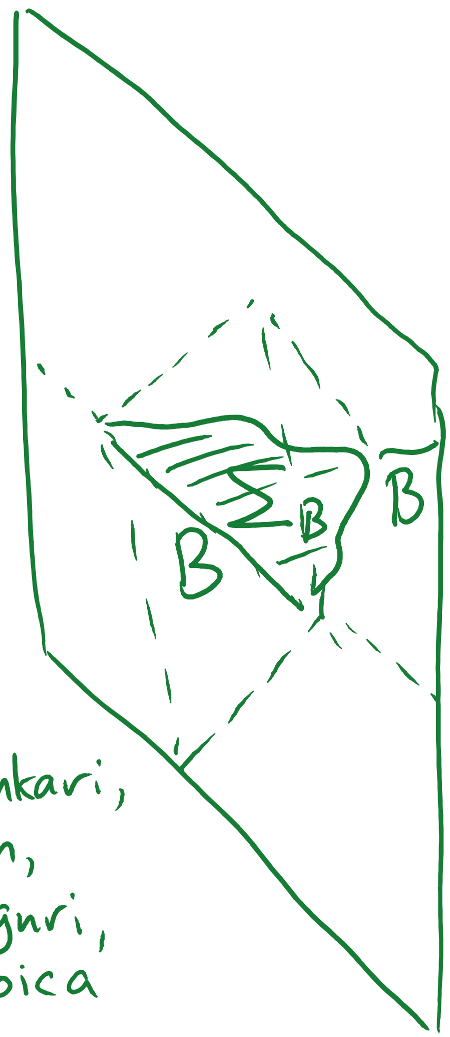
$X \to 0$, $\nabla^a X^b \to 2\pi n^{ab}$ at $\tilde{B}$

Then:

$$S(\rho_B \| \sigma_B) = \underbrace{\int_\Sigma J_X}^{\text{Noether charge}} = H_X$$

Independent details of X!

New positive energy theorem for subsystems.



w. Lashkari, Lin, Ooguri, Stoica

Given any asympt. AdS M in consistent UV completion of Einstein grav + matter

For any $B \subset \partial M$ can define energy H_X associated w. Σ_B.

☆ H_X must be positive + increase w. increasing B ☆