Magnetic impurities and universality in AdS/CMT

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MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

1. Kondo models from holography
   - Model — J.E., Hoyos, O'Bannon, Wu 1310.3271
   - Quantum quenches — J.E., Flory, Newrzella, Wu in progress
   - Entanglement entropy — J.E., Flory, Newrzella 1410.7811; J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu 1511.03666
   - Two-point functions — J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu in progress

2. S-Wave Superconductivity in Anisotropic Holographic Insulators
   J.E., Herwerth, Klug, Meyer, Schalm 1501.07615
   - Scalar condenses in helical Bianchi VII background
   - Homes' Law
     (See also talk by Keun-Young Kim on Thursday; Kim, Kim, Park 1604.06205)

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

Kondo models from gauge/gravity duality

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Motivation for study within gauge/gravity duality:

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1. Kondo model: Simple model for a RG flow with dynamical scale generation

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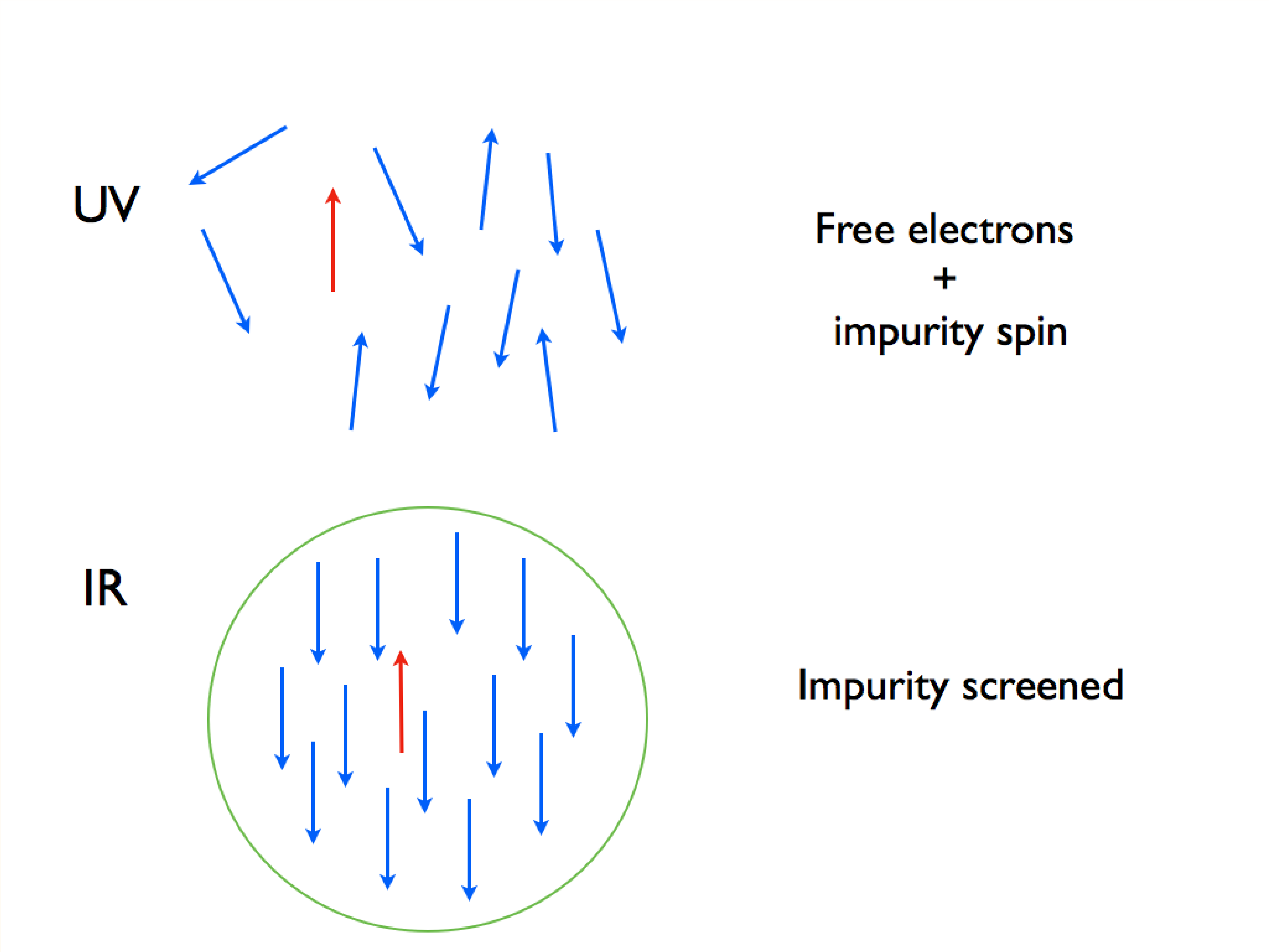
2. New applications of gauge/gravity duality to condensed matter physics:

   - Magnetic impurity coupled to strongly correlated electron system
   - Entanglement entropy
   - Quantum quench
   - Relation to Sachdev-Ye-Kitaev model

Kondo effect



Kondo model

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Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

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Hamiltonian:

$$H = \frac{v_F}{2\pi}\psi^\dagger i\partial_x\psi + \lambda_K v_F \delta(x)\vec{S}\cdot\vec{J}\,, \quad \vec{J} = \psi^\dagger \frac{1}{2}\vec{T}\psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Gauge/gravity requires large N: Spin group $SU(N)$

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In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

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Screened phase has condensate $\langle \mathcal{O} \rangle$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192

Senthil, Sachdev, Vojta cond-mat/0209144

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation
- Holographic superconductor: Condensate forms in AdS_2
- Power-law scaling of conductivity in IR with real exponent
- Screening, phase shift

Field-operator map

From top-down construction involving D3, D5 and D7 branes:

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $Q = \chi^\dagger \chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger \chi$	$\Leftrightarrow$	2d complex scalar Φ

Bottom-up gravity dual for Kondo model

Action:

$$S = S_{\text{Einstein-Hilbert}} + S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi}\int_{AdS_3} \text{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right),$$

$$S_{AdS_2} = -N \int d^3x \, \delta(x) \sqrt{-g}\left[\frac{1}{4}\text{Tr} f^{mn} f_{mn} + g^{mn} \left(D_m \Phi\right)^\dagger D_n \Phi + V(\Phi^\dagger \Phi)\right]$$

$$V(\Phi) = M^2 \Phi^\dagger \Phi$$

Metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2}\left(\frac{dz^2}{h(z)} - h(z)\, dt^2 + dx^2\right),$$

$$h(z) = 1 - z^2/z_H^2 \,, \qquad T = 1/(2\pi z_H)$$

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

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Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

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Witten hep-th/0112258

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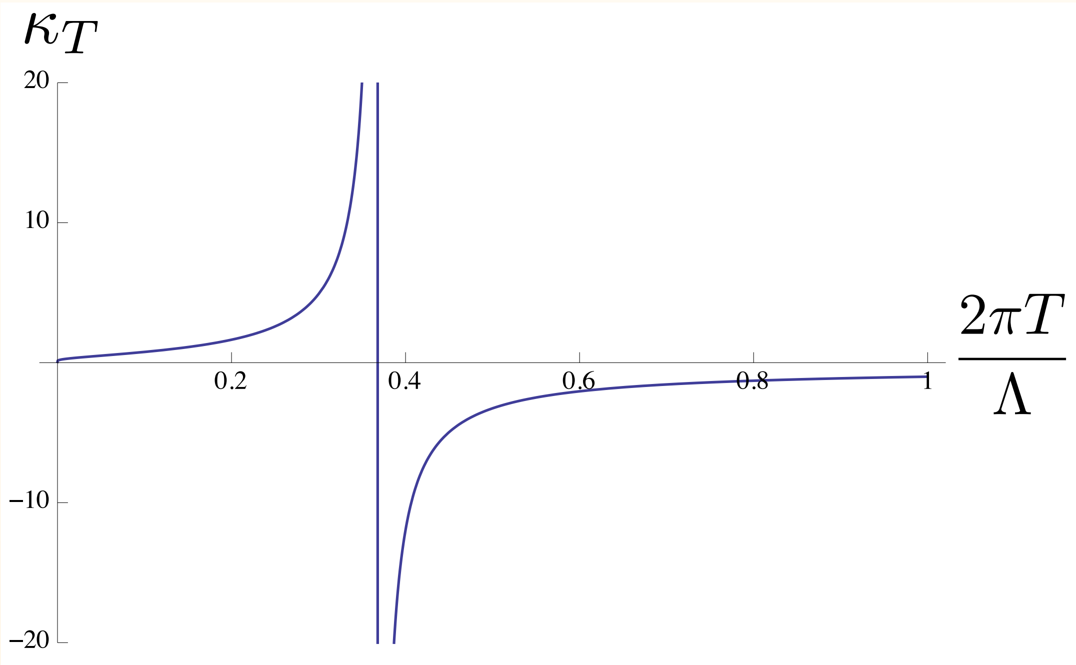
Dynamical scale generation

Scale generation

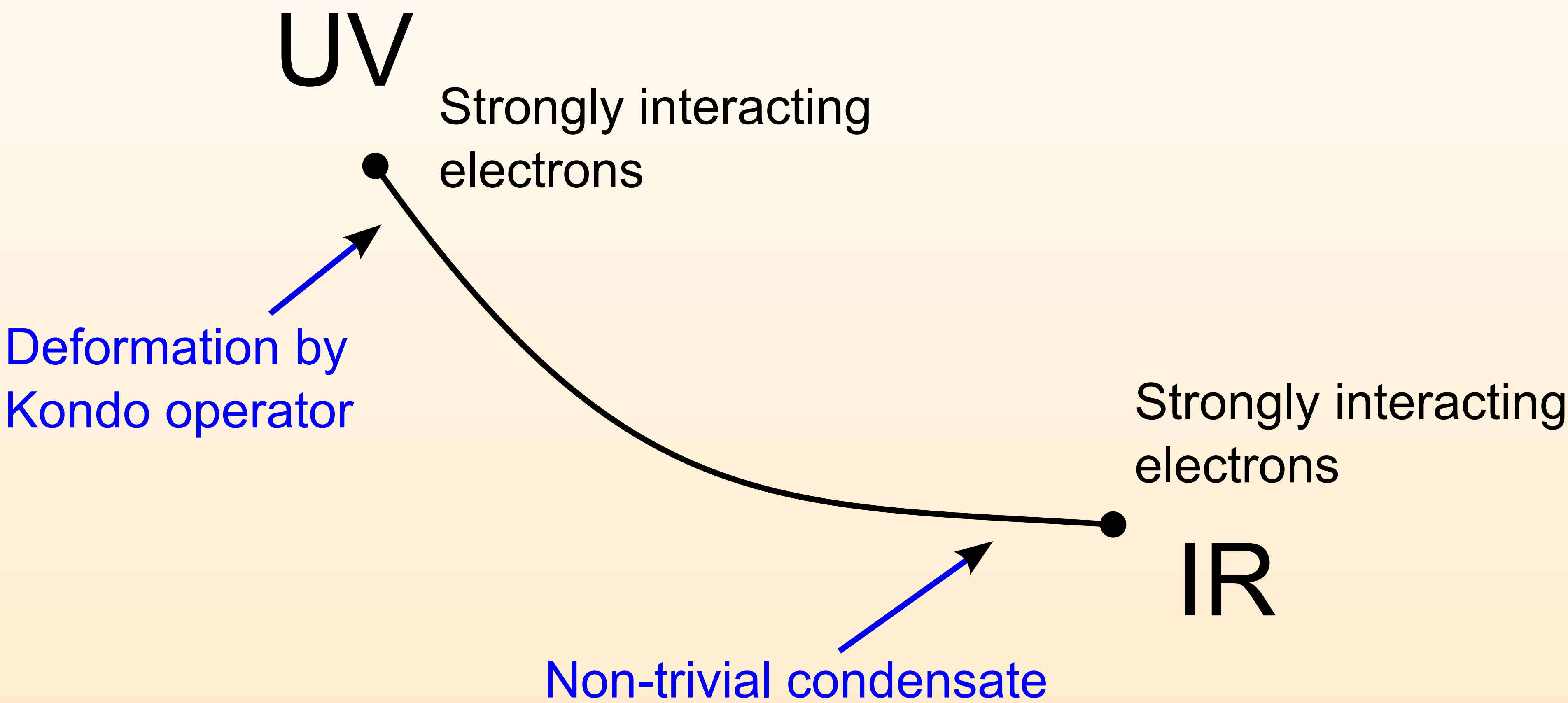


Divergence of Kondo coupling determines Kondo temperature T_K

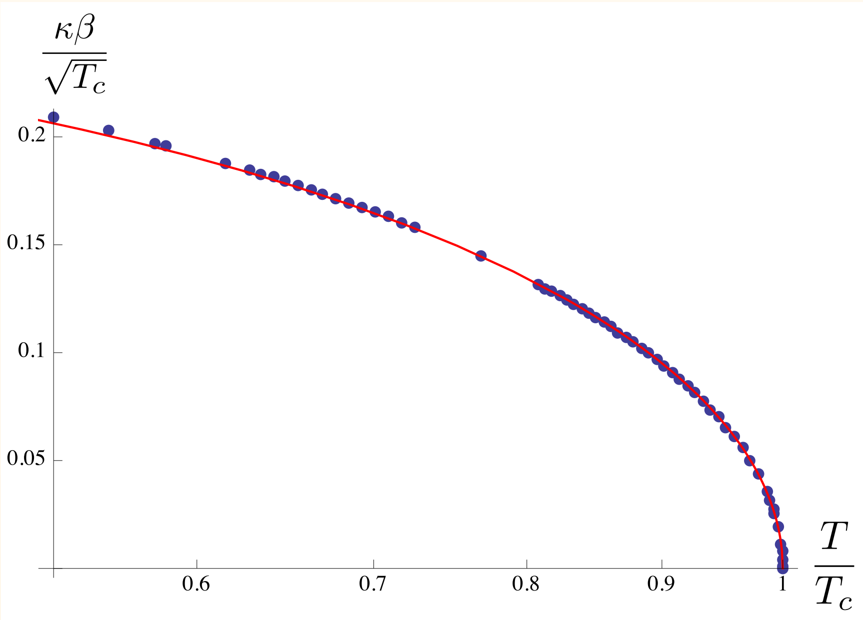
Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

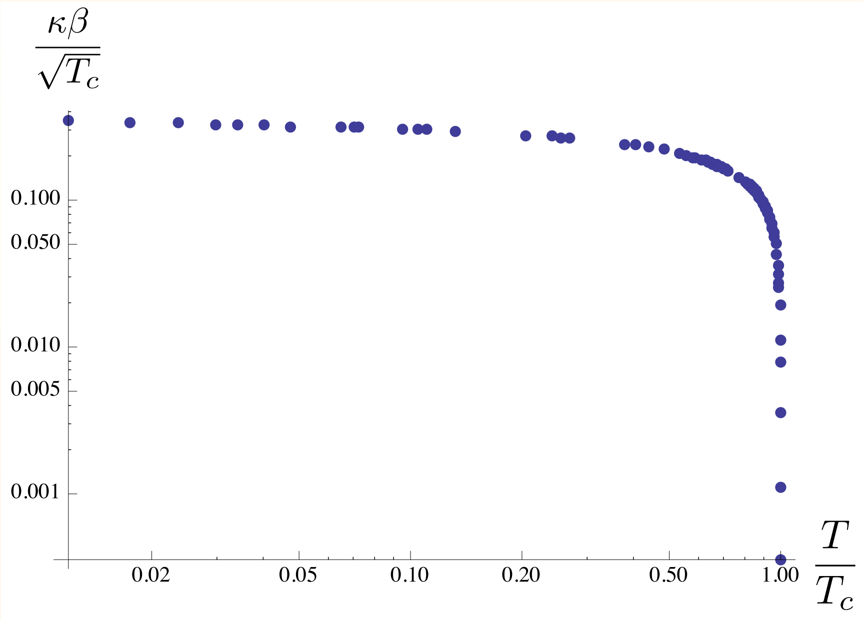
RG flow


UV
Strongly interacting electrons
Deformation by Kondo operator
Strongly interacting electrons
IR
Non-trivial condensate

Normalized condensate $\langle \mathcal{O} \rangle \equiv \kappa\beta$ as function of the temperature



(a)



(b)

Mean field transition

$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$

Time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

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Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

Time dependence

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Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

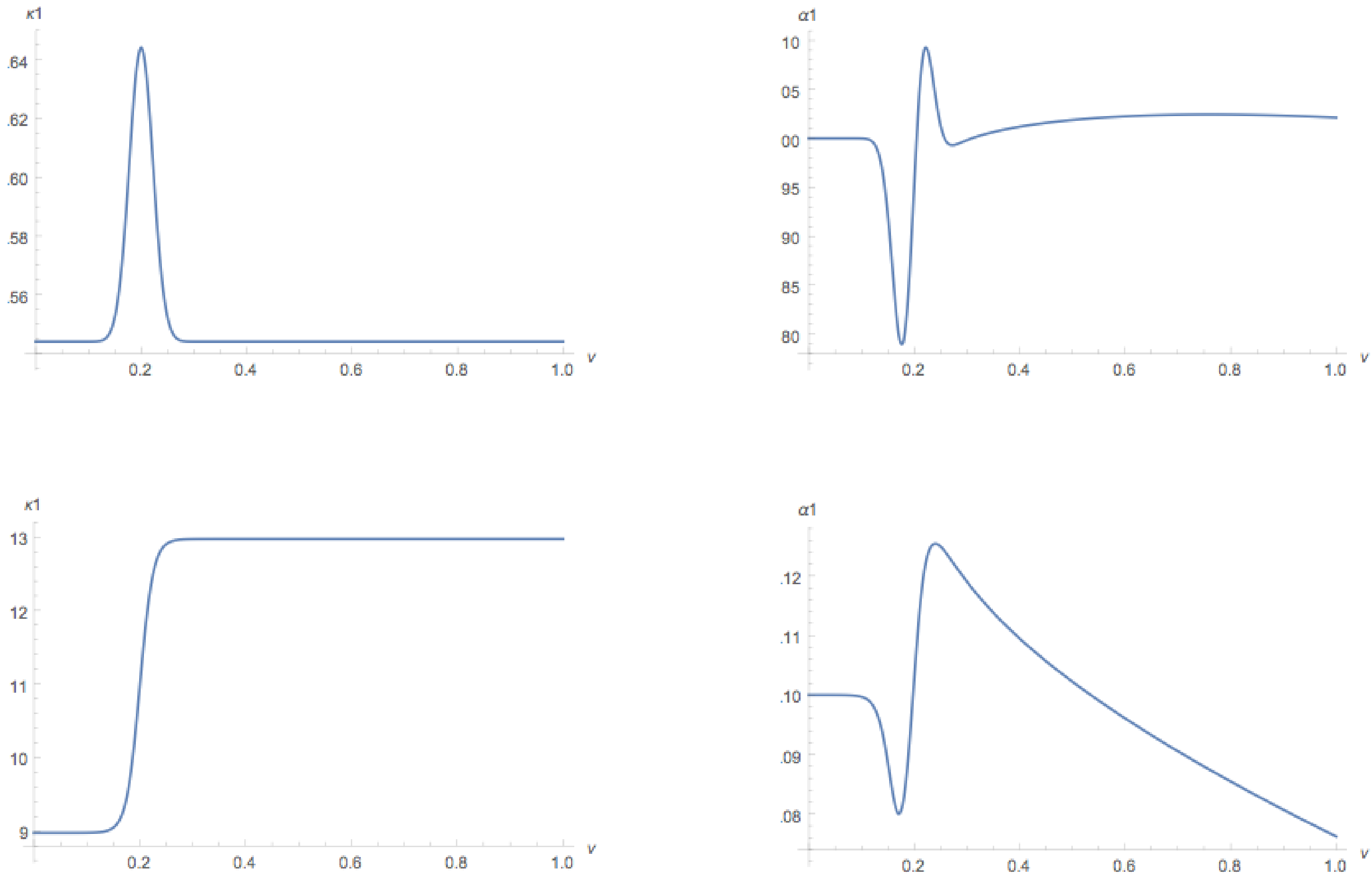
Observations:

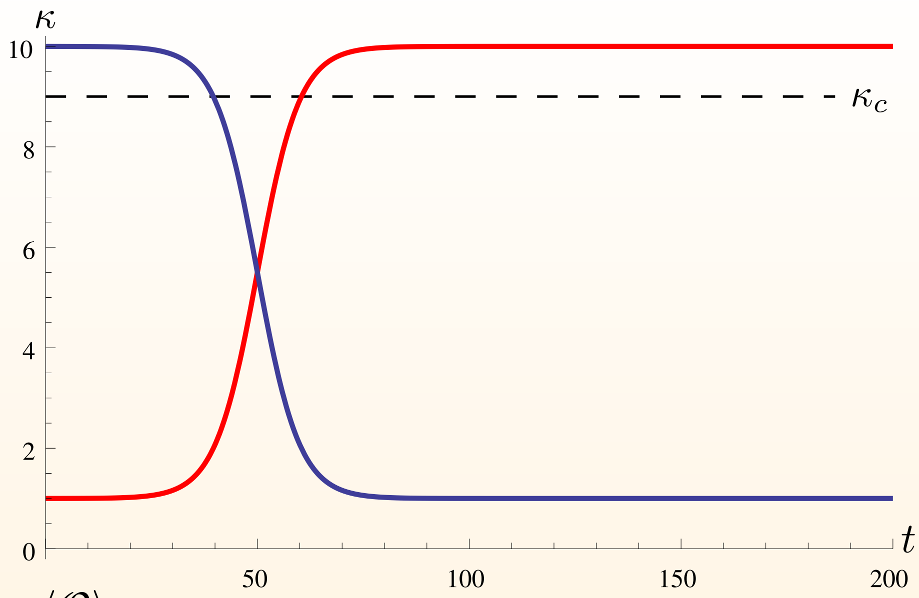
Different timescales depending on whether the condensate is asymptotically small or large

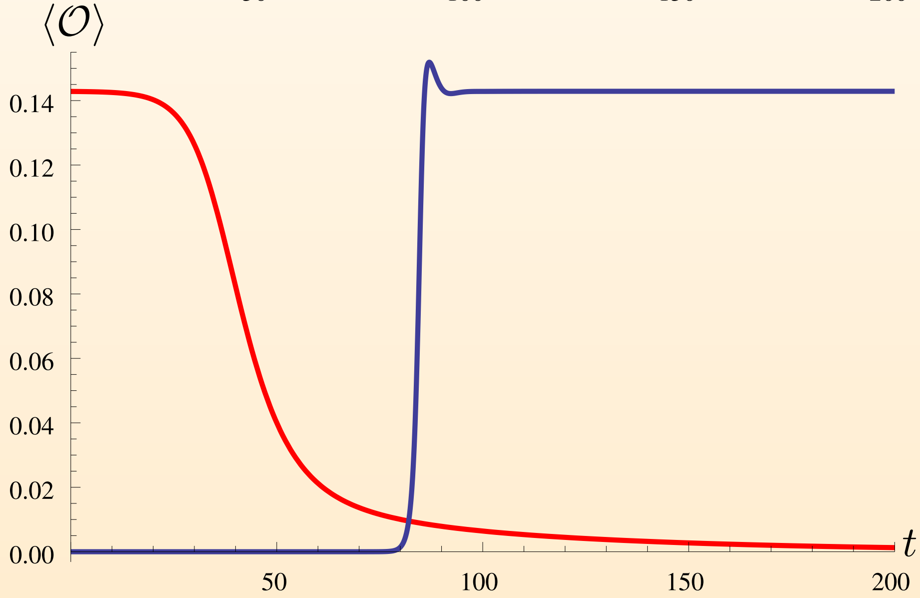
Timescales governed by quasinormal modes

Time dependence

Kondo coupling → Condensate







Quantum quenches in holographic Kondo model

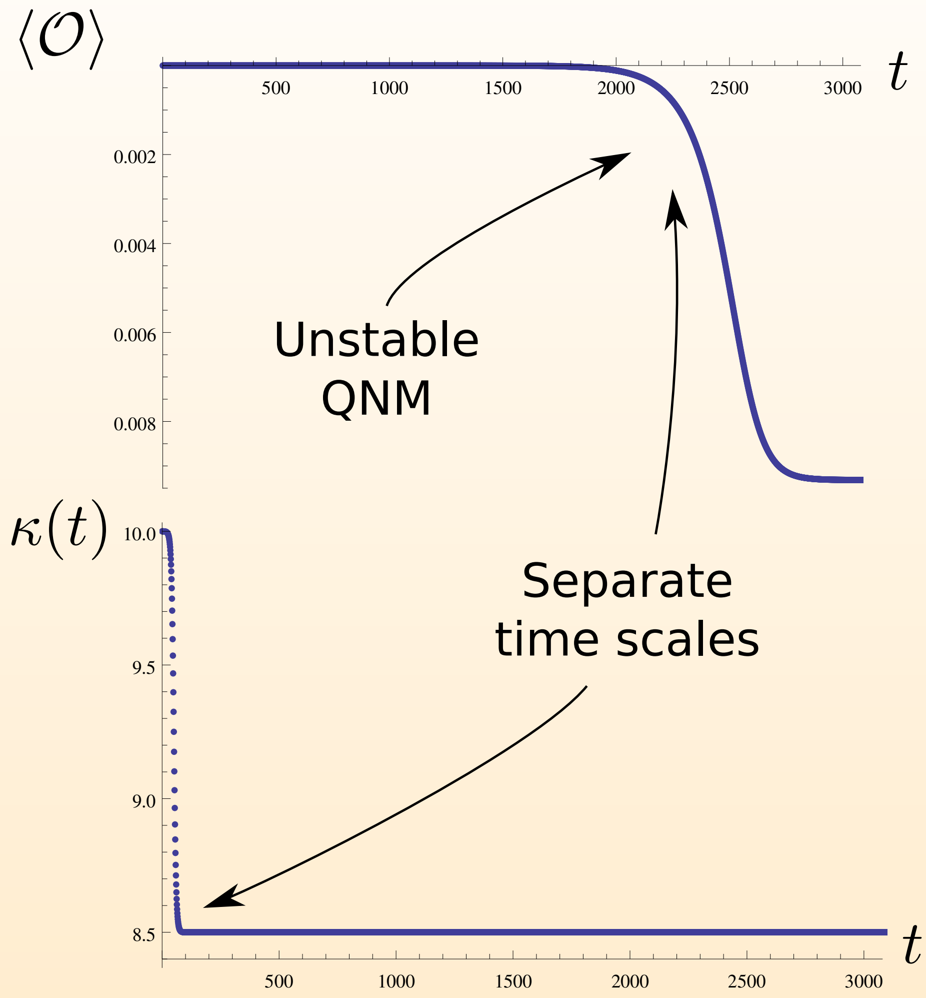
To and from condensed phase

Timescales determined by quasinormal modes

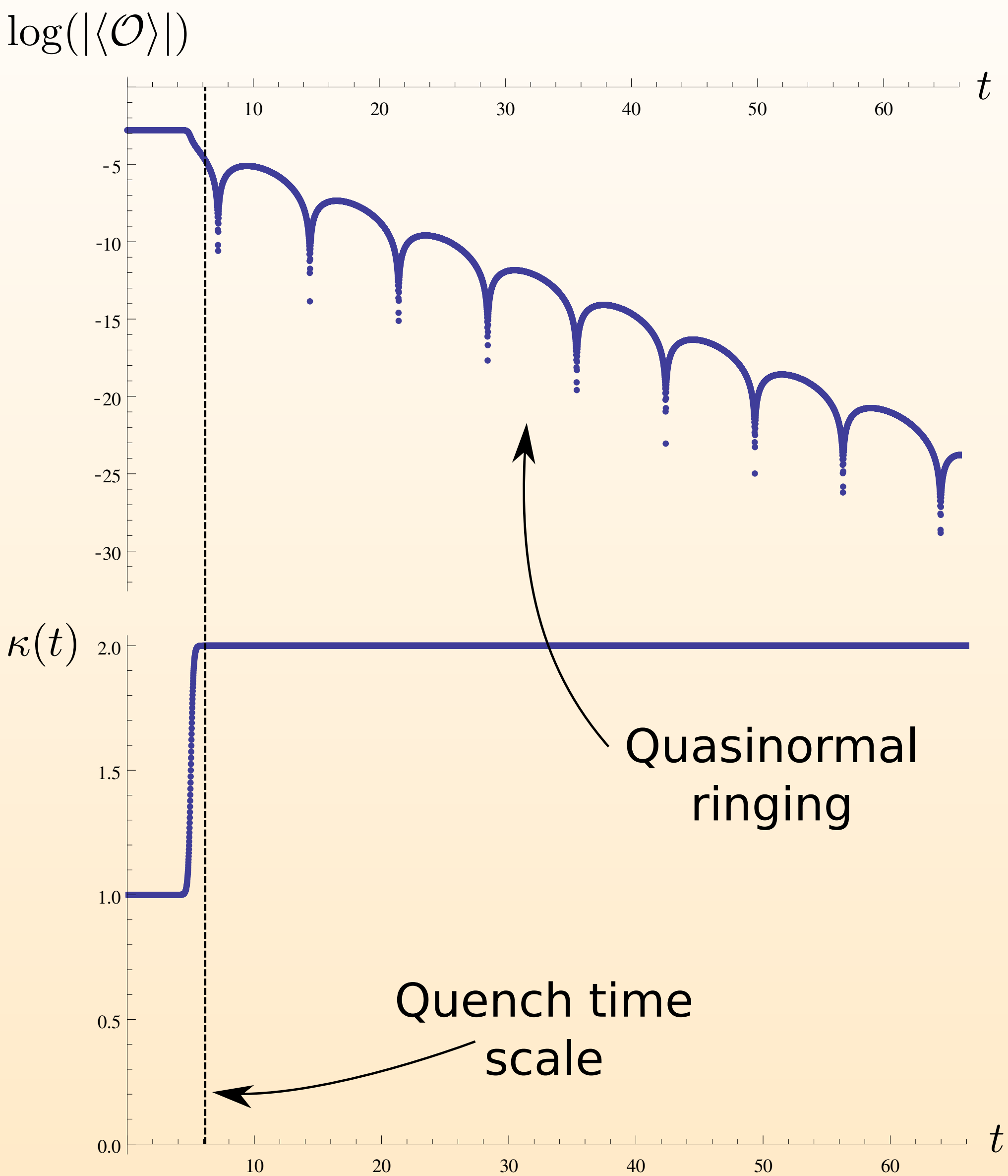
J.E., Flory, Newrzella, Strydom, Wu

cf. Quench in s-wave holographic superconductor,
Bhaseen, Gauntlett, Simons, Sonner, Wiseman PRL 2012

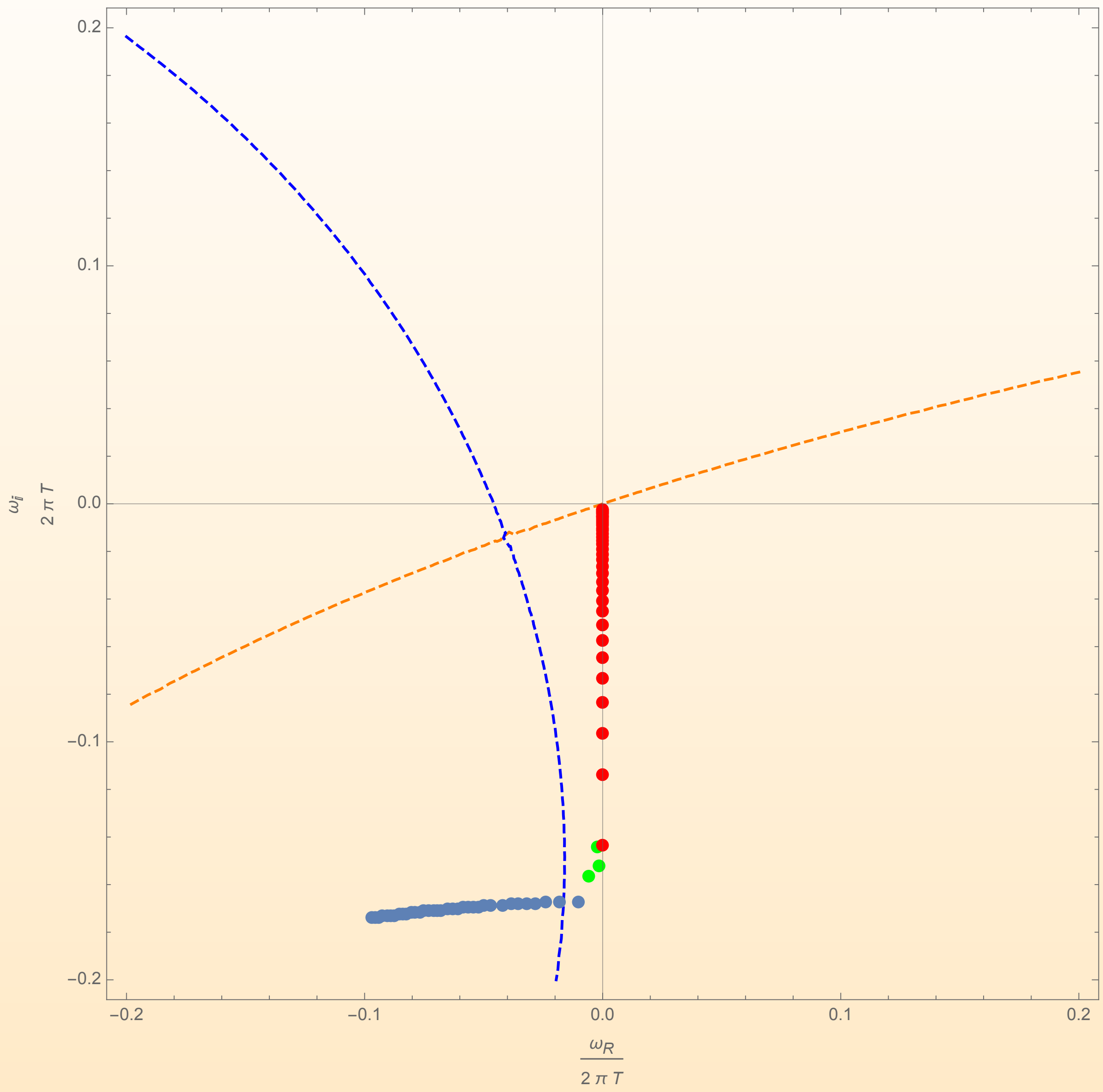
Timescales in quantum quench



Timescales in quantum quench



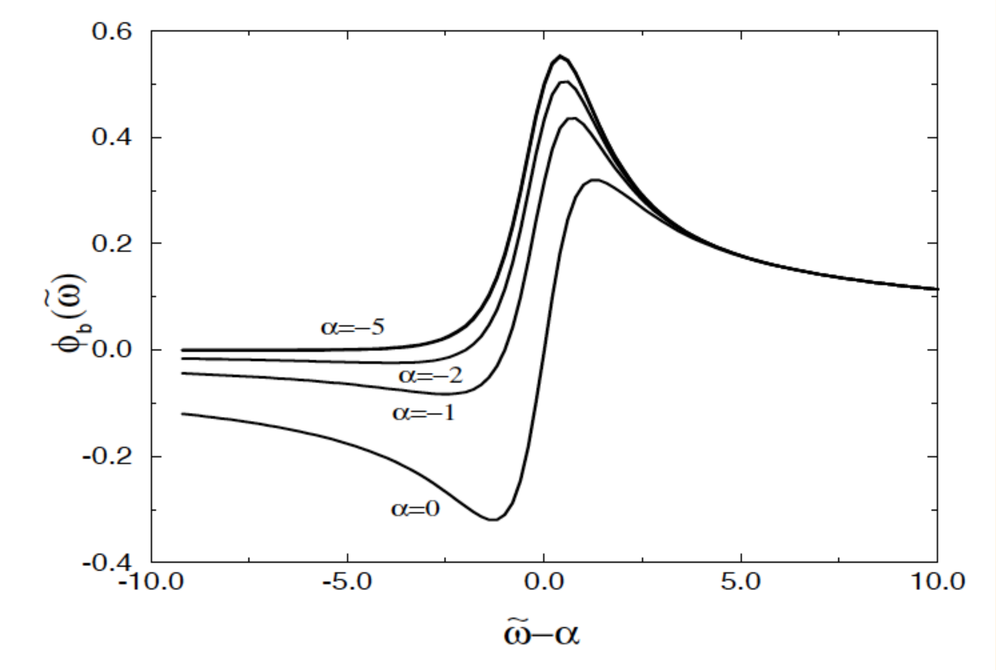
Quasinormal modes



Kondo model: Two-point functions

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192: Large N Kondo model

Spectral asymmetry ω_s: Particle-hole symmetry broken

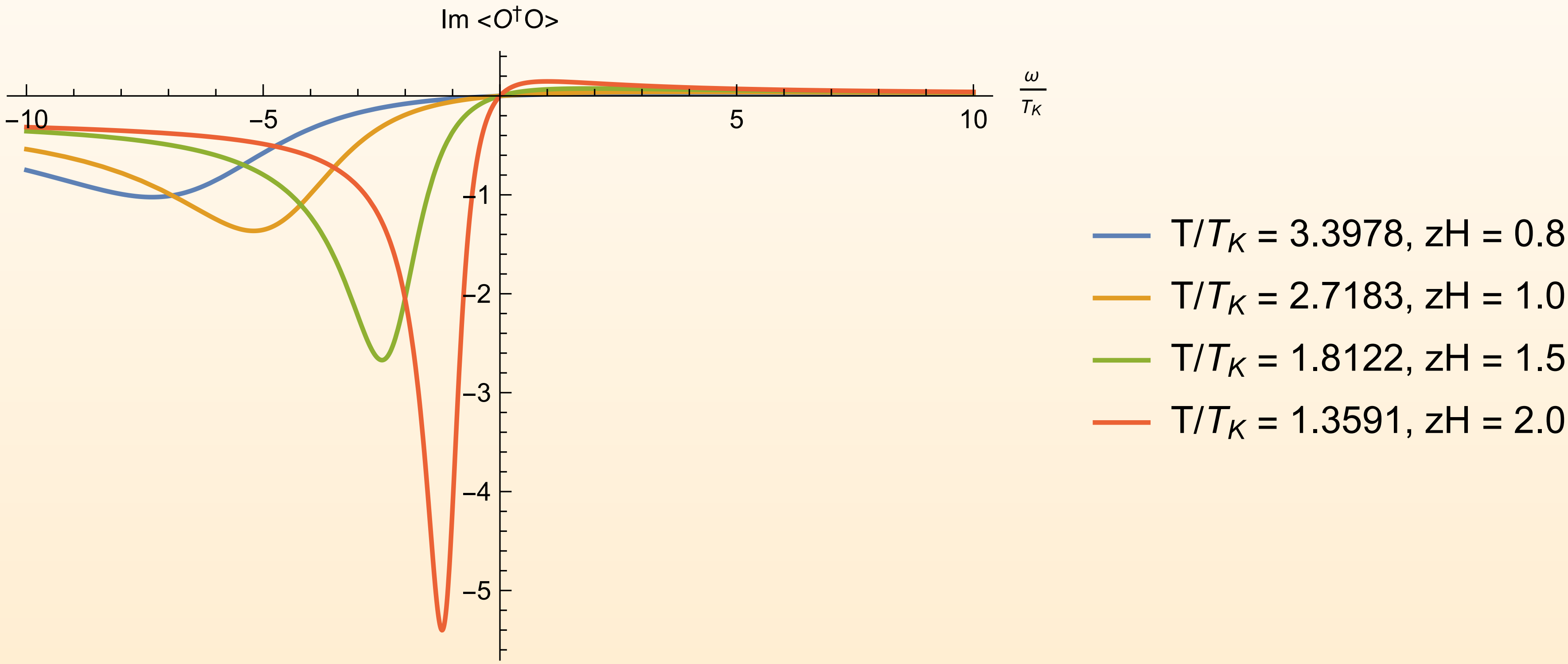


$-\mathrm{Im}G^R$ for bosonic $\langle \mathcal{O}\mathcal{O}^\dagger \rangle$

$\omega_s = \frac{qT}{\hbar}\frac{\partial S}{\partial \mathcal{Q}}$ see also Sachdev 1506.05111, AdS_2 black hole (fermions)

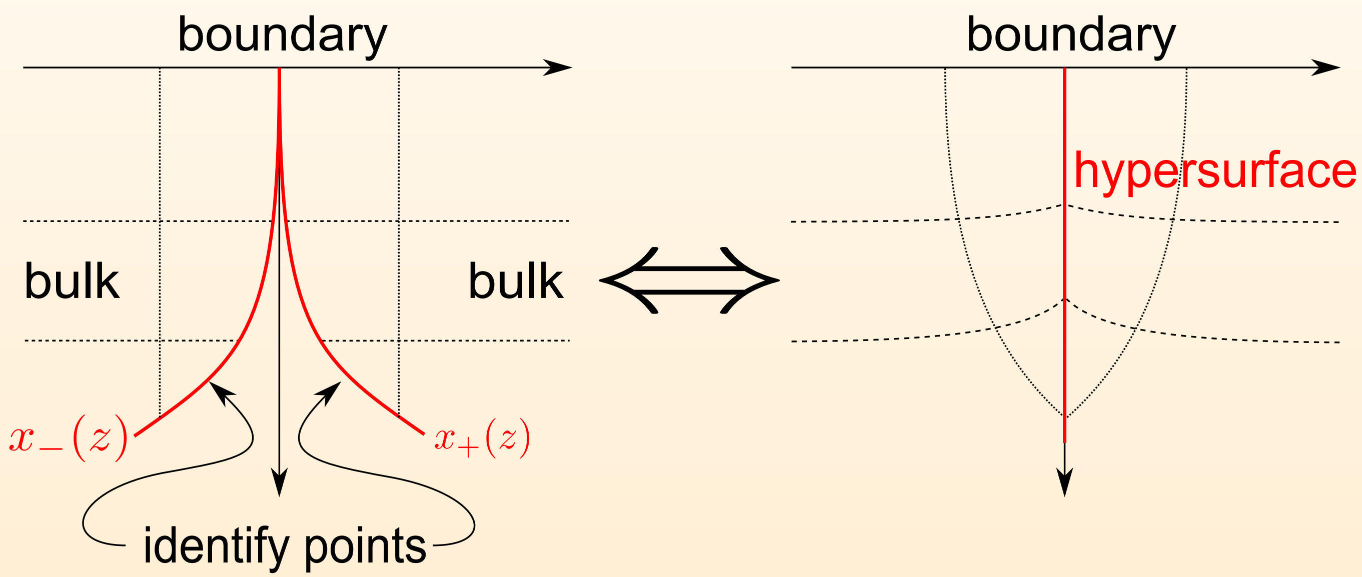
Two-point function in holographic Kondo model

J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu in progress



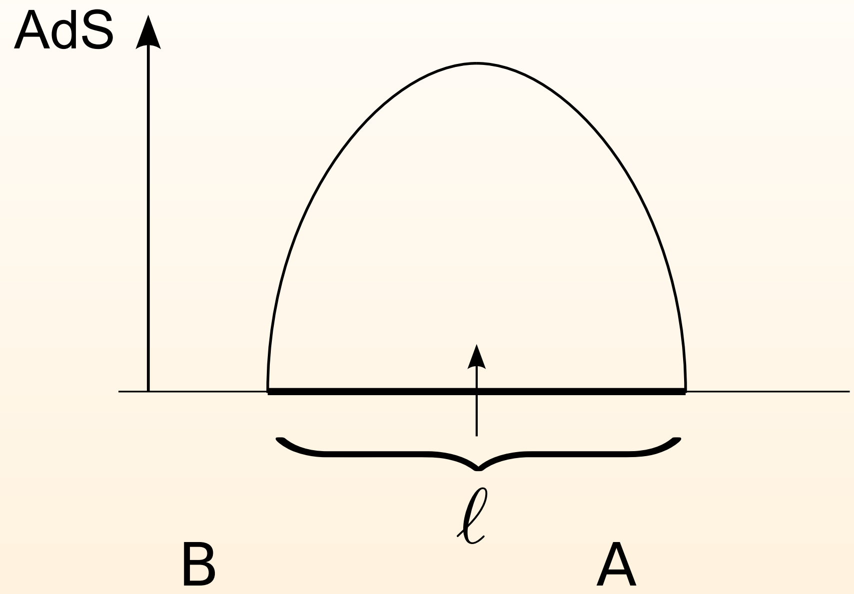
Including the backreaction using a thin brane and Israel junction conditions

Israel junction conditions $K_{\mu\nu} - \gamma_{\mu\nu} K = -\frac{\kappa}{2} T_{\mu\nu} \Leftrightarrow$ Energy conditions



J.E., Flory, Newrzella 1410.7811

Entanglement entropy for magnetic impurity



Impurity entropy:

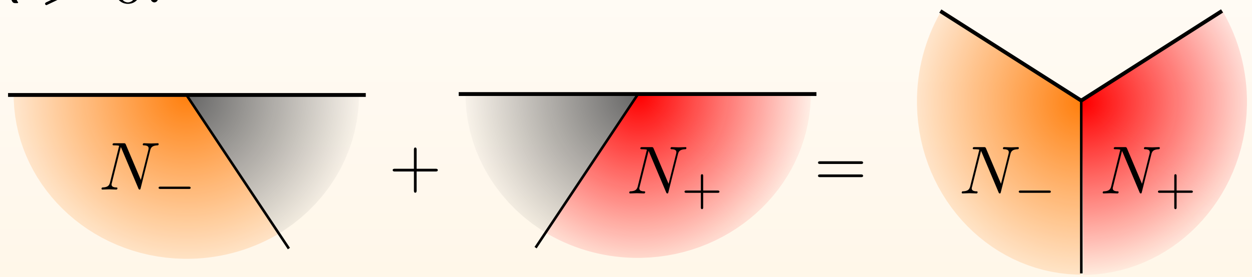
$$S_{\text{imp}} = S_{\text{condensed phase}} - S_{\text{normal phase}}$$

Subtraction also guarantees UV regularity

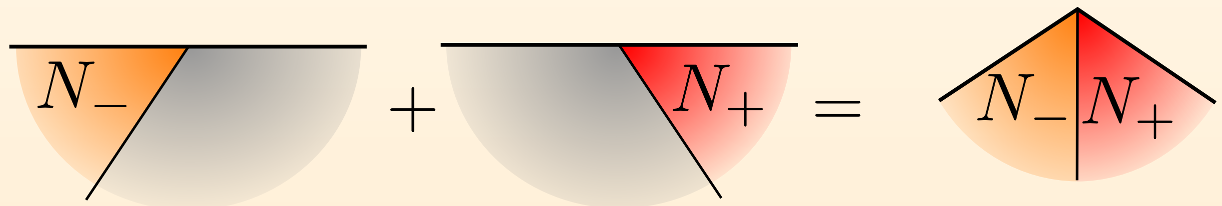
Entanglement entropy for magnetic impurity

J.E., Flory, Newrzella 1410.7811

$\lambda > 0$:



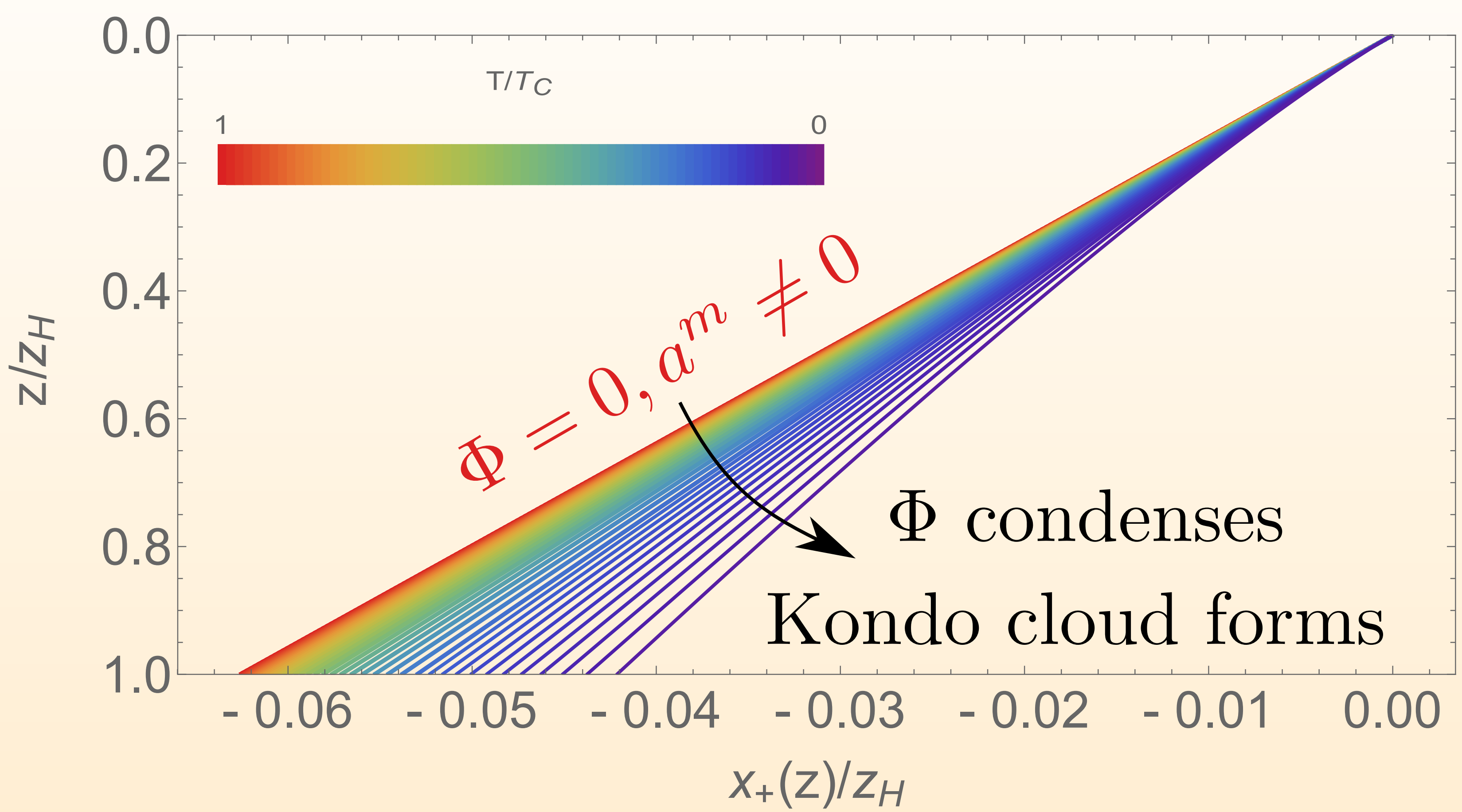
$\lambda < 0$:



Depending on the brane tension λ, the total space is enhanced or reduced

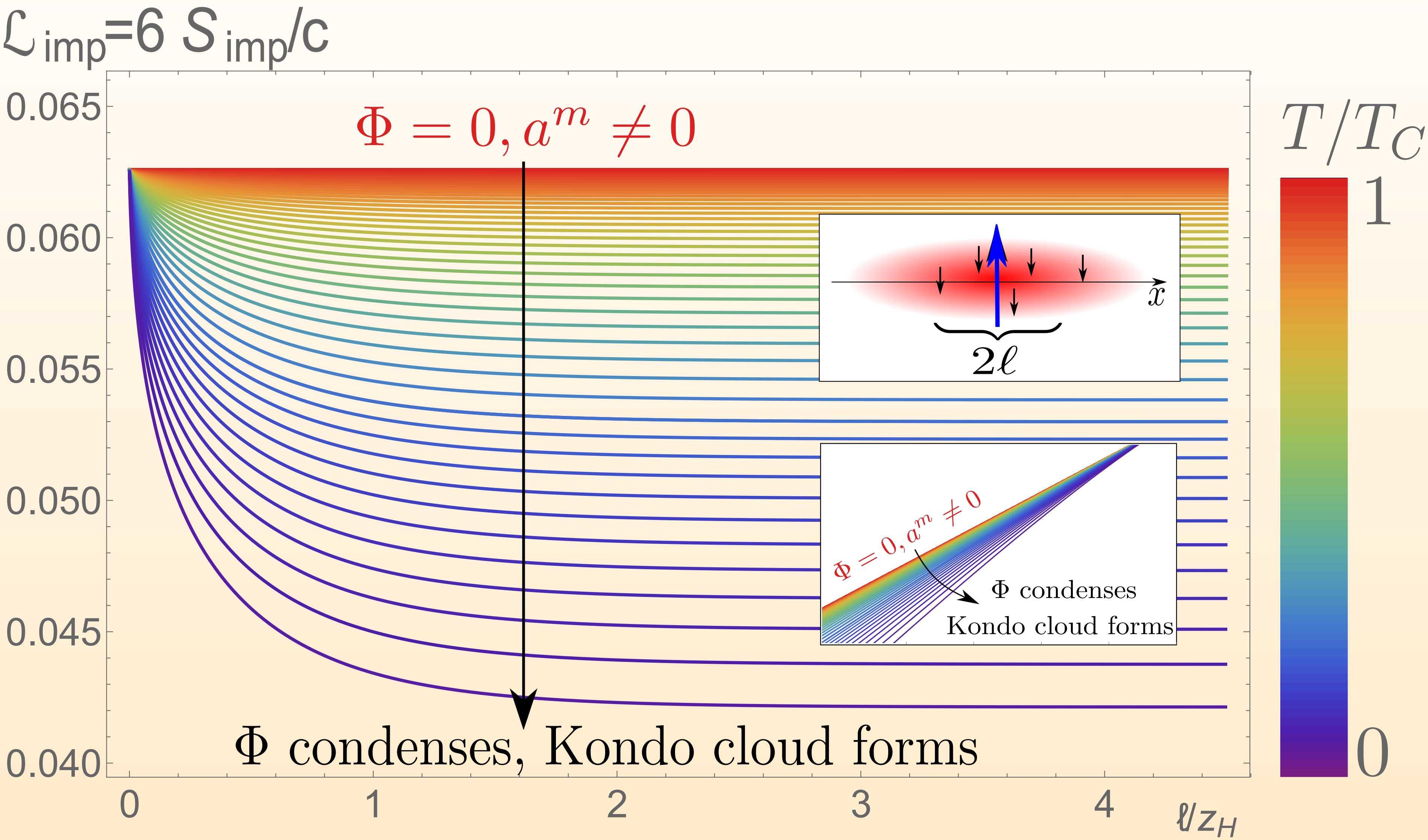
Entanglement entropy for magnetic impurity

J.E., Flory, Newrzella 1410.7811



The larger the condensate, the shorter the geodesic

Impurity entropy from gauge/gravity duality



Entanglement entropy for magnetic impurity: Comparison to field theory

Field theory result

Sorensen, Chang, Laflorencie, Affleck 2007
(Eriksson, Johannesson 2011)

$$\Delta S_{\mathrm{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0$$

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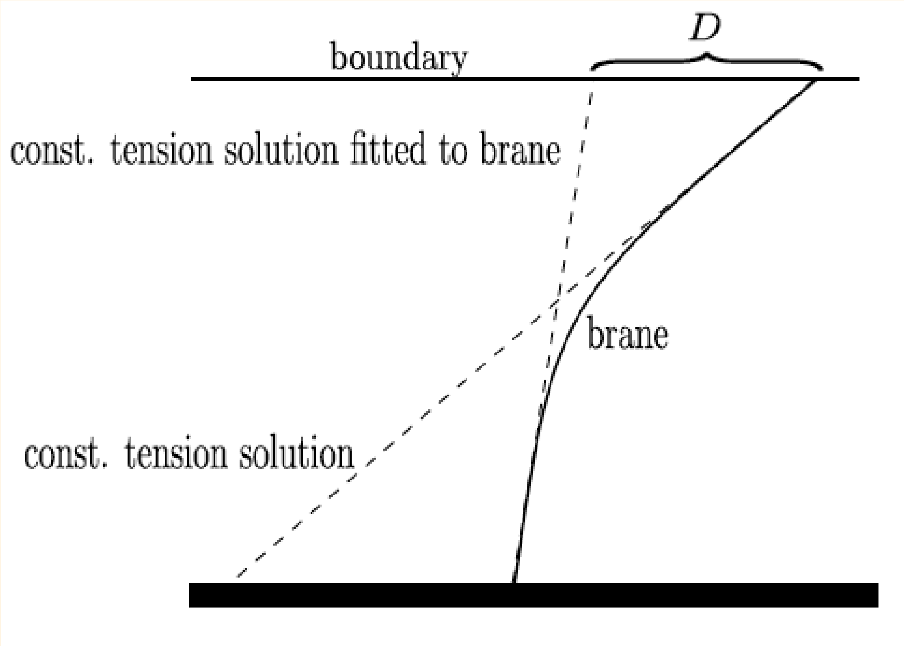
In our gravity approach:

J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu 1511.03666



Entanglement entropy for magnetic impurity: Comparison to field theory

On gravity side:

Impurity entropy from difference of entanglement entropies for constant tension branes

$$\Delta S_{\rm imp}(\ell) = c_0 + S_{BH}(\ell + D) - S_{BH}(\ell)$$

$$S_{BH}(\ell) = \frac{c}{3} \ln\left(\frac{1}{\pi\epsilon T}\sinh(2\pi\ell T)\right)$$

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For $D \ll \ell$:

$$\Delta S_{\text{imp}}(\ell) \sim c_0 + D \cdot \partial_\ell S_{BH}(\ell) = c_0 + \frac{2\pi D T}{3} \coth(2\pi \ell T)$$

Agrees with field theory result subject to identification $D \sim \xi_K$

Universal properties of superconductors

Universality: IR fixed point determines physical properties
Macroscopic properties do not depend on microscopic degrees of freedom

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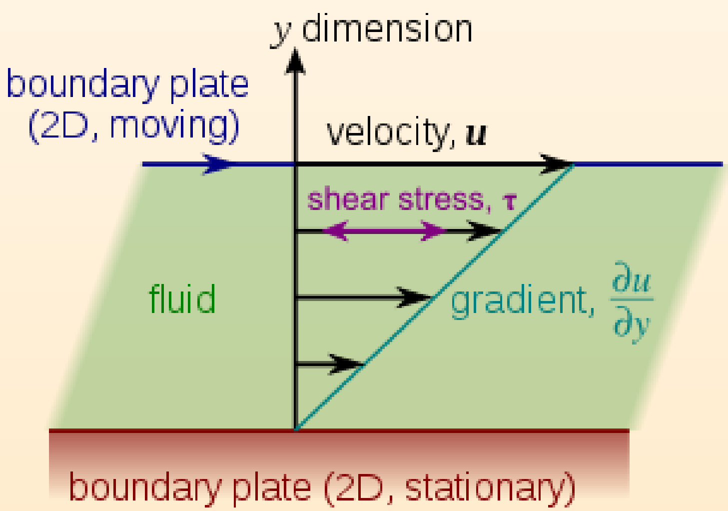
Example: Universal result from gauge/gravity duality:

Shear viscosity over entropy density:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$



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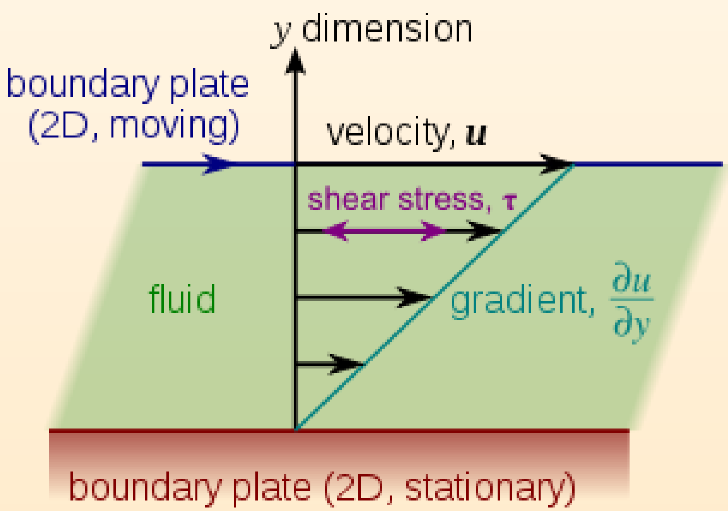


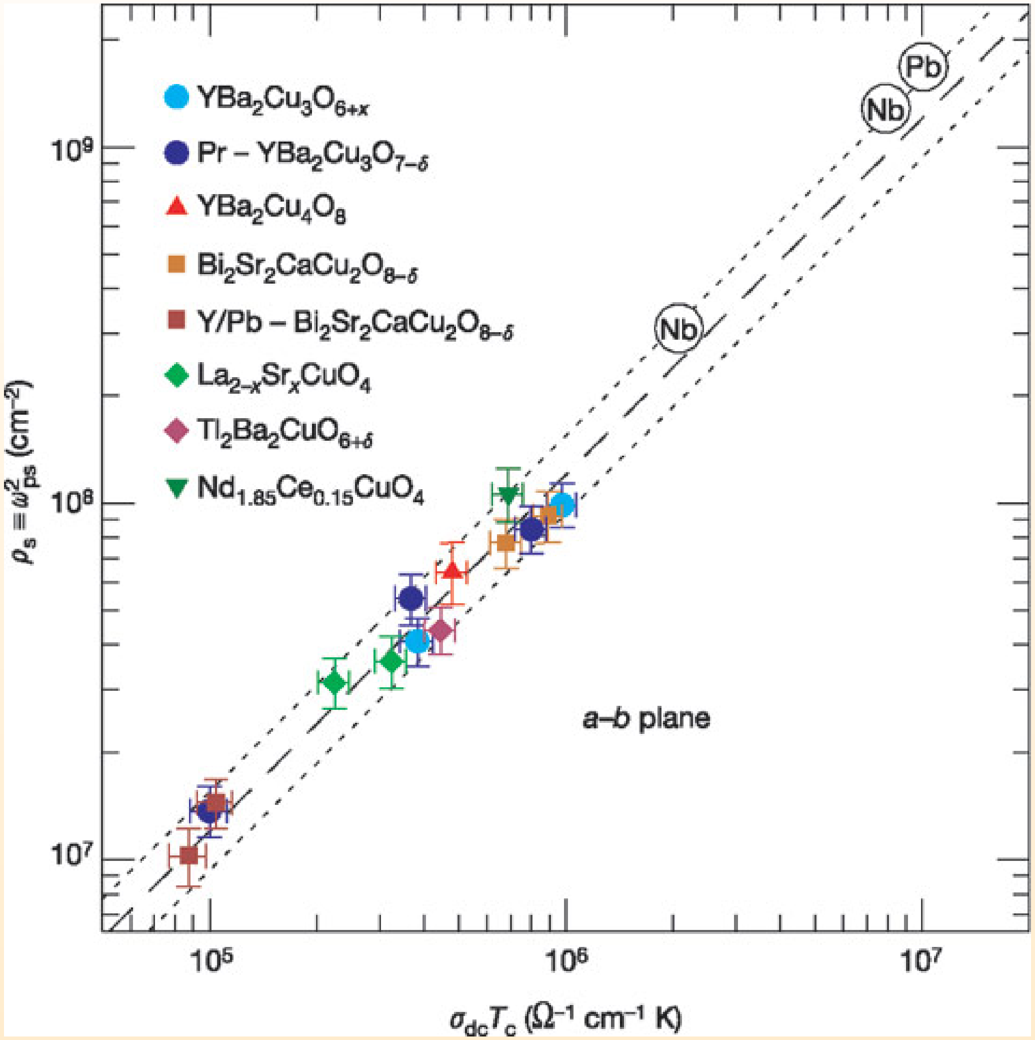
Planckian dissipator: relaxation time $\tau = \frac{\hbar}{k_B T}$

Damle, Sachdev 1997

Is there a similiar universal result for applications of the duality within condensed matter physics?

Candidate: Homes' relation

$\rho_s(T=0) = C\,\sigma_{\rm DC}(T_c)\,T_c$



C. Homes et al, Nature 2004

Universal properties of superconductors

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general form may be deduced from Planckian dissipation Zaanen 2004

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general form may be deduced from Planckian dissipation Zaanen 2004

J.E., Herwerth, Klug, Meyer, Schalm arXiv:1501.07615:

Investigation of C in a family of gauge/gravity duality models

In particular region of parameter space:

$$C \approx 6.2$$

High-T_c in (ab)-plane and BCS superconductors in 'dirty limit': $C = 8.1$,
High-T_c superconductors in c-plane: $C = 4.4$

Holography:

J.E., Kerner Müller 2012

Conditions for identifying ρ_s:

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Translation symmetry broken $\Rightarrow$ Drude peak

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Applicability of sum rules:

All normal state degrees of freedom condense at $T = 0$

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Horowitz, Santos 2013

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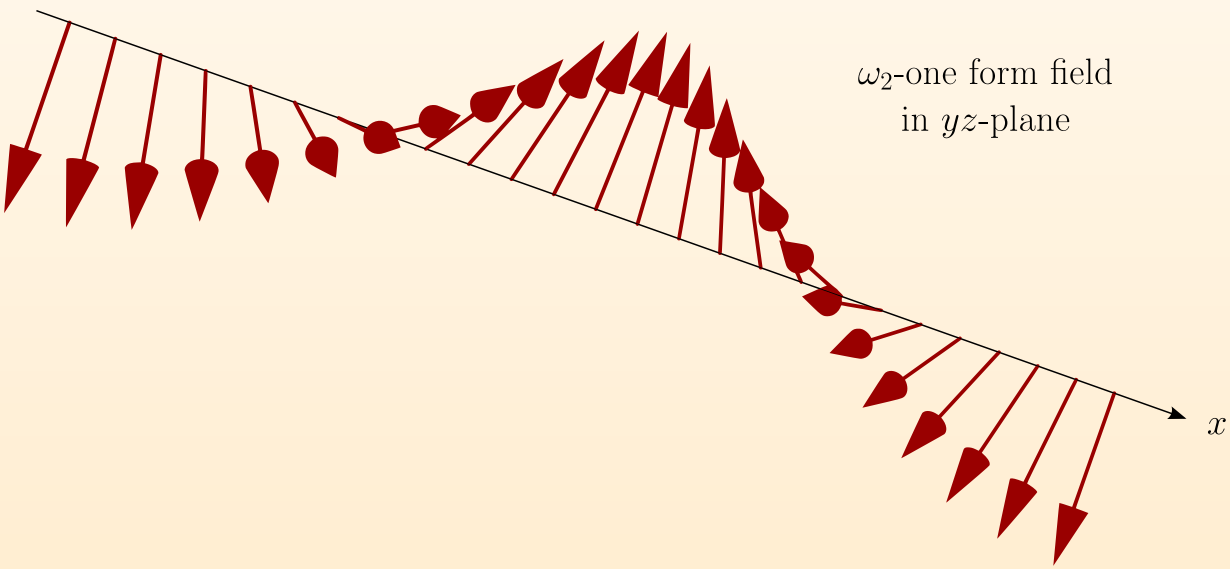
Horowitz, Santos 2013

Use background with helical symmetry

Background: Helical Bianchi VII symmetry

Donos, Gauntlett 2011; Donos, Hartnoll 2012

Model with broken translation symmetry:



Background: (Hartnoll, Donos)

$$S_{\rm helix} = \int {\rm d}^{4+1}x\,\sqrt{-g}\Big[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2 B_\mu B^\mu\Big] - \frac{\kappa}{2}\int B \wedge F \wedge W.$$

$$B = w(r)\omega_2, \qquad w(\infty) = \lambda$$

$$\omega_1 = {\rm d}x\,,$$
$$\omega_2 = \cos(px)\,{\rm d}y - \sin(px)\,{\rm d}z$$
$$\omega_3 = \sin(px)\,{\rm d}y + \cos(px)\,{\rm d}z$$

S-wave superconductivity in helical symmetry background

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Add charged scalar:

$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x\,\sqrt{-g}\Big[-|\partial\rho - \mathrm{i}qA\rho|^2 - m_\rho^2|\rho|^2\Big]$$

S-wave superconductivity in helical symmetry background

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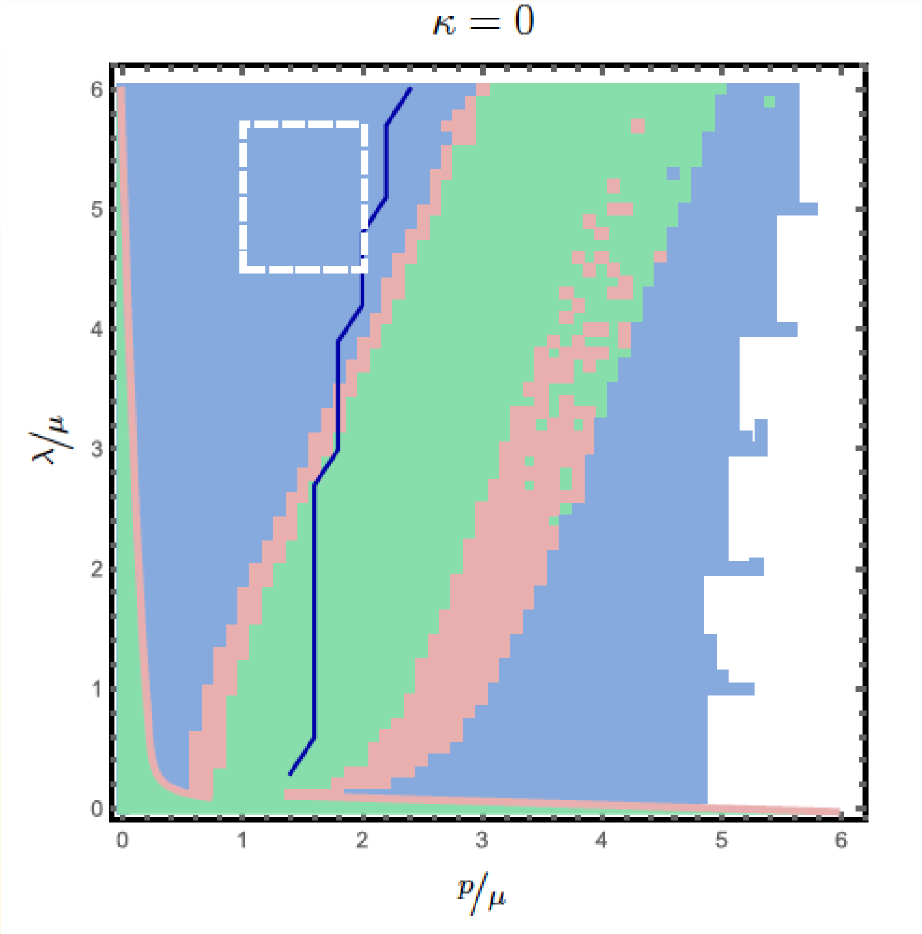
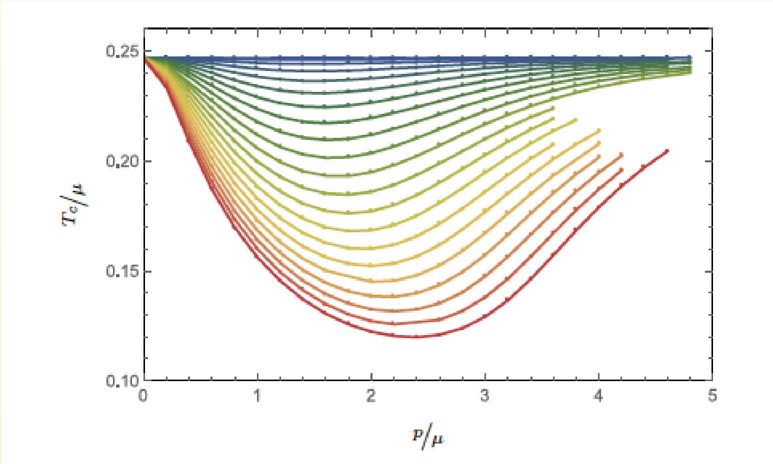
All charged degrees of freedom condense at $T = 0$

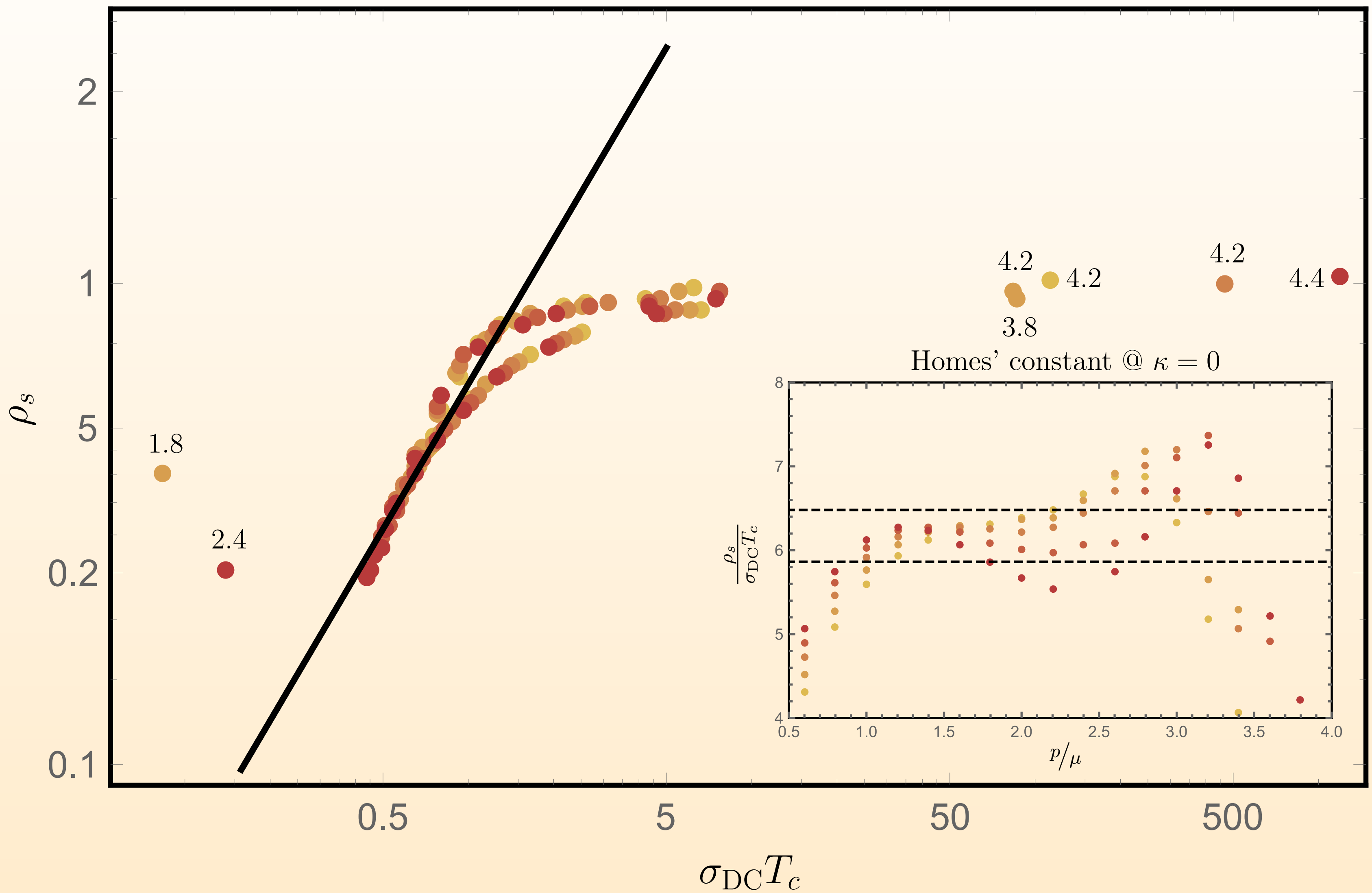
Phase diagram



T_c as function of helix frequency

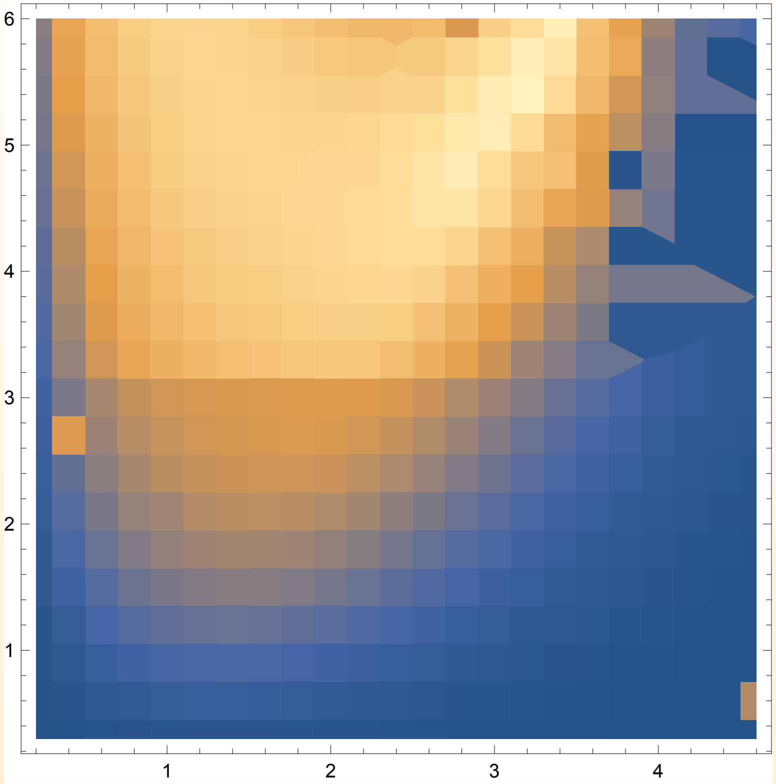


Homes' relation for $q = 6$ & $\kappa = 0$



J.E., Herwerth, Klug, Meyer, Schalm 1501.07615

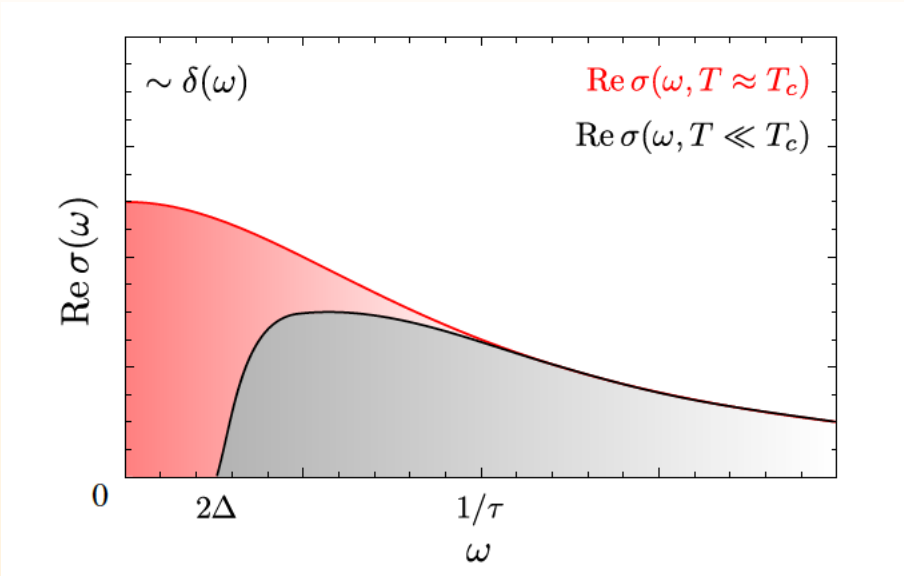
Homes' constant and comparison with phase diagram in normal phase



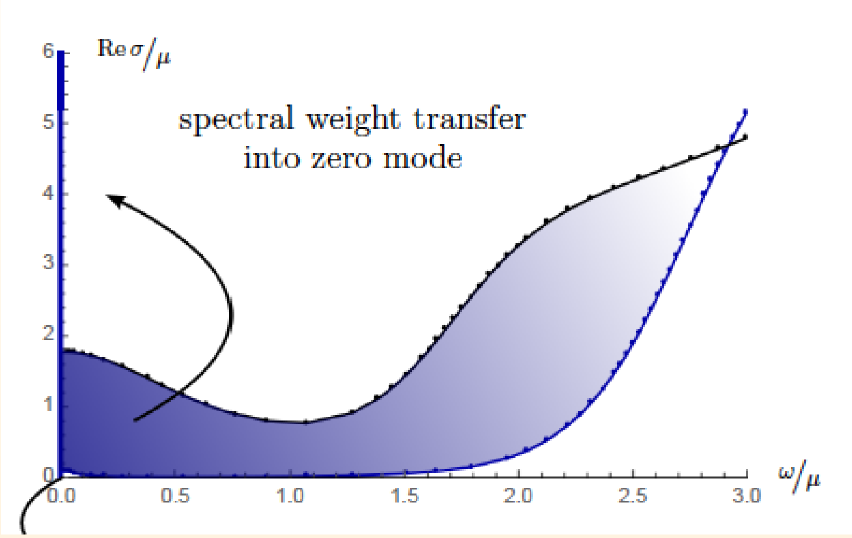
J.E., Meyer, Schalm, Shock in progress

Homes' relation holds in region of phase diagram which is insulating (incoherent metal) in normal phase

Sum rules

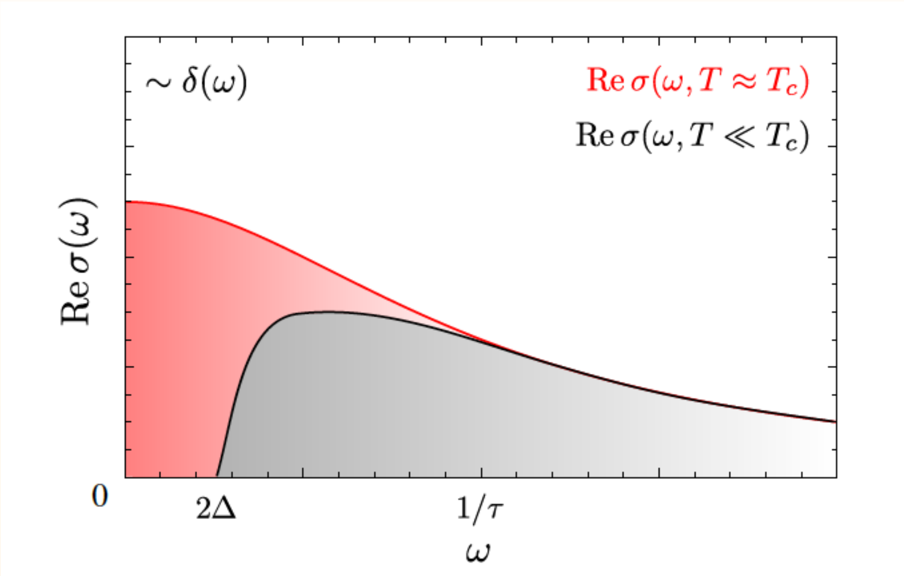


BCS



Holography for helical background

Sum rules



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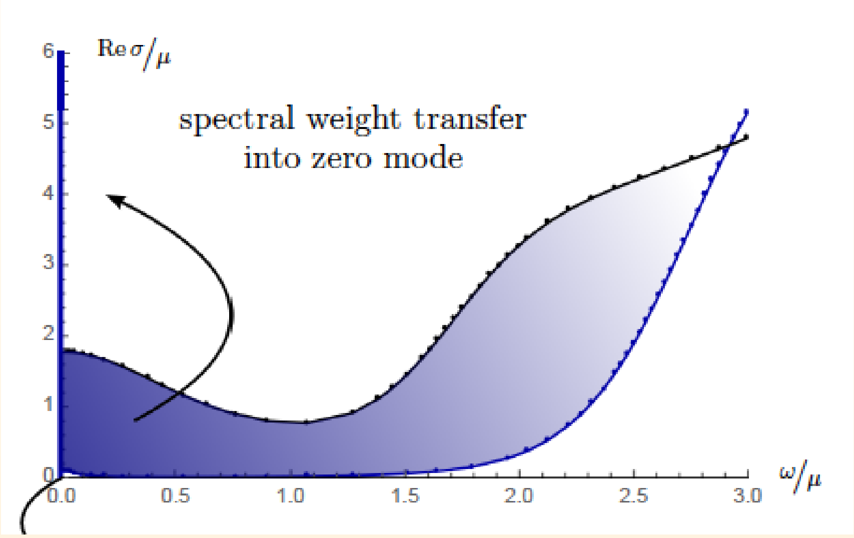

Holography for helical background

(related work by Kim, Kim, Park)

Conclusions and outlook

- Kondo model:
- Magnetic impurity coupled to strongly coupled system
- Quantum quench
- Entanglement entropy
- Two-point and spectral functions
- S-wave superconductor in Bianchi VII background:
- Homes' Relation

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