# String interactions and integrability

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## Outline

**Introduction and motivation**

**String Field Theory vertex**

The conventional approach for noninteracting worldsheet theory

What will change for an interacting worldsheet theory?

**Interlude: Form factors in an integrable quantum field theory**

**Functional equations for the string vertex**

**What happens in $AdS_5 \times S^5$?**

The kinematical $AdS_5 \times S^5$ Neumann coefficient

**Conclusions & outlook**

Focus on $\mathcal{N}=4$ Super-Yang-Mills theory — a 4D gauge theory which is a conformal theory...

**Key questions:**

- Find the spectrum of conformal weights
  $\equiv$ eigenvalues of the dilatation operator
  $\equiv$ (anomalous) dimensions of operators

$$\langle O(0)O(x)\rangle = \frac{1}{|x|^{2\Delta}}$$

The dimensions are complicated functions of the coupling:

$$\Delta = \underbrace{\Delta_0(\lambda)}_{planar} + \underbrace{\frac{1}{N_c^2}\Delta_1(\lambda) + \dots}_{nonplanar} \qquad \text{where } \lambda \equiv g_{YM}^2 N_c$$

- Find the OPE coefficients $C_{ijk}$ defined through

$$\langle O_i(x_1)O_j(x_2)O_k(x_3)\rangle = \frac{C_{ijk}}{|x_1-x_2|^{\Delta_i+\Delta_j-\Delta_k}|x_1-x_3|^{\Delta_i+\Delta_k-\Delta_j}|x_2-x_3|^{\Delta_j+\Delta_k-\Delta_i}}$$

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# The AdS/CFT correspondence

$\mathcal{N} = 4$ Super Yang-Mills theory $\equiv$ Superstrings on $AdS_5 \times S^5$

## The AdS/CFT dictionary

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| Operators in $\mathcal{N} = 4$ SYM | $\longleftrightarrow$ | (quantized) string states in $AdS_5 \times S^5$ |
| Single trace operators | $\longleftrightarrow$ | single string states |
| Multitrace operators | $\longleftrightarrow$ | multistring states |
| Large $N_c$ limit | $\longleftrightarrow$ | suffices to consider single string states |
| Operator dimension | $\longleftrightarrow$ | Energy of a string state in $AdS_5 \times S^5$ |
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1. **Anomalous dimensions in the planar limit:**

   $\equiv$ energy levels of a single string in $AdS_5 \times S^5$
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2. **Nonplanar corrections to the dilatation operator or OPE coefficients:**

   $\equiv$ string interactions
   $\equiv$ the specific 2D QFT on a string 'pants' topology:

This is the **string field theory vertex** $\leftarrow$ focus of this talk

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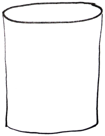

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   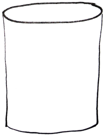

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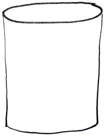

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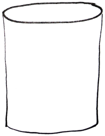

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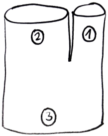


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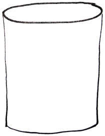

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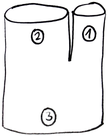


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- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory

**Key question:**

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
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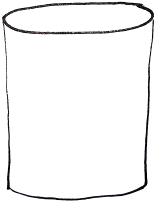

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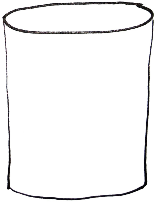

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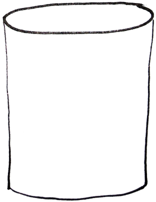

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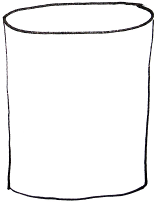

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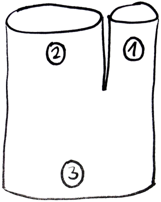

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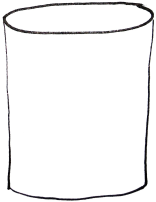

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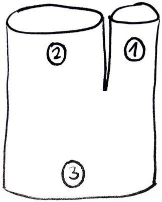


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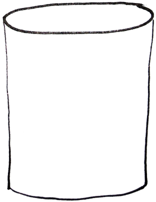

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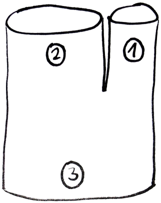

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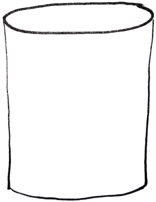

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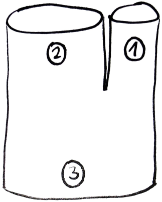

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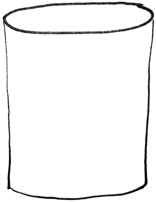

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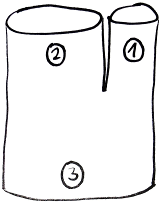


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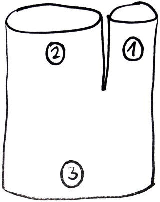
**Standard approach to the light cone String Field Theory vertex...**

## Light-cone String Field Theory Vertex




String Field Theory vertex describes the splitting/joining of 3 strings

**Comments:**

1. In light cone gauge, the lengths of the strings are directly proportional to some conserved charges of the theory
2. They always have to add up by charge conservation

$$L_3 = L_1 + L_2$$

3. In typical applications to $AdS_5 \times S^5$/pp-wave, the lengths are directly the R-charges w.r.t. $U(1) \subset SO(6)$

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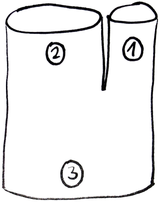


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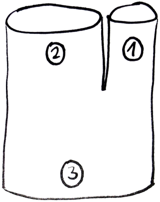


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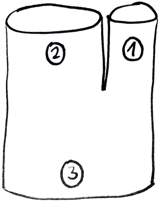


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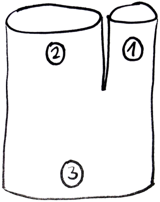


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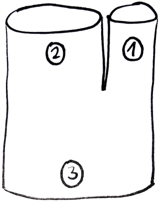


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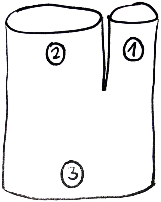


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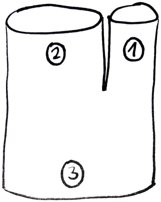


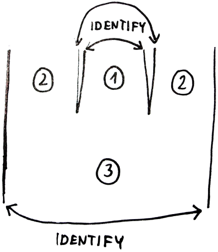


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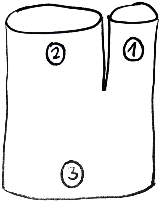


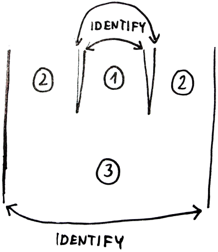


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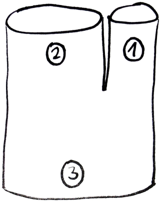


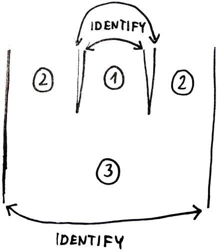


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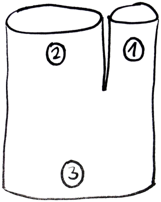


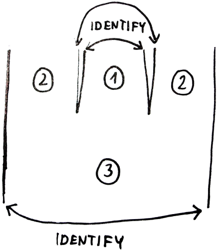


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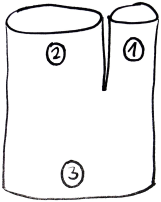


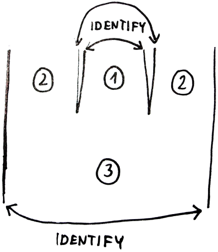


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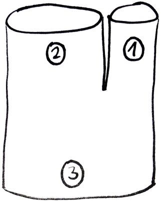


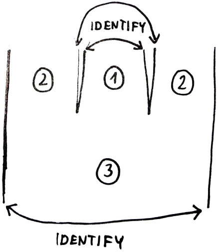


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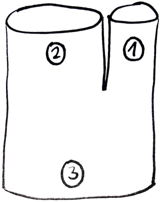


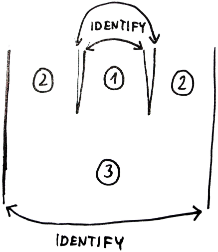


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- Implement these relations as operator equations acting on a state $|V\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ — the SFT vertex
- The state has the form

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He, Schwarz, Spradlin, Volovich

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**Our goal:** **Concentrate on defining (and constructing) the string field theory vertex for a generic integrable worldsheet theory**

## What will change?

- We no longer have any mode expansions at our disposal...
- Even if we had, inverting the infinite dimensional matrices would be hopeless...
- It is extremely difficult to work directly in **finite volume** — even the single string spectrum is given only implicitly in terms of Bethe Ansatz equations...
- We do **not** expect the exponential structure to hold...

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- We expect to obtain **separate** but **possibly related** amplitudes for various numbers of external particles

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- In particular no separation between the $(Prefactor)$ and the Neumann coefficient part...
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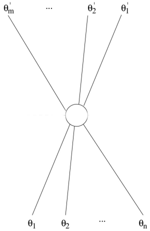
## Interlude: Form factors in an integrable quantum field theory

## Form factors




- Form factors are expectation values of a local operator sandwiched between specific multiparticle *in* and *out* states $p_k = m \sinh \theta$
- Form factors in infinite volume (on an infinite plane) satisfy a concrete set of functional equations

$$f(\theta_1, \theta_2) = S(\theta_1, \theta_2)\, f(\theta_2, \theta_1)$$
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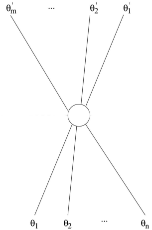


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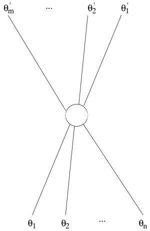


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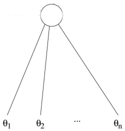


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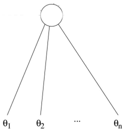


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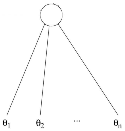


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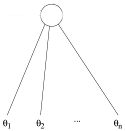


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- They have numerous solutions — for each local operator in the theory...

**Finite volume $\equiv$ form factors on a cylinder**

- Up to wrapping corrections ($\sim e^{-mL}$), very simple way to pass to finite volume (cylinder of circumference $L$): Pozsgay, Takacs

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1. How does this relate to the pp-wave SFT which seemed inherently tied to finite volume??
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  (these are exactly wrapping terms $e^{-ML_r}$!!)
- Going to an exponential basis (BMN basis) one got in this limit

$$N^{rs}_{mn} = \left[ \frac{\sqrt{(\omega^r_m + \mu\alpha_r)(\omega^s_n + \mu\alpha_s)}}{\omega^r_m + \omega^s_n} - \frac{\sqrt{(\omega^r_m - \mu\alpha_r)(\omega^s_n - \mu\alpha_s)}}{\omega^r_m + \omega^s_n} \right] \cdot (simple)$$

- Instead of integer mode numbers use rapidities... $p_k = M \sinh\theta_k$

$$N^{33}(\theta_1, \theta_2) = \frac{-1}{\cosh\frac{\theta_1-\theta_2}{2}} \cdot \sin\frac{p_1 L_1}{2} \sin\frac{p_2 L_1}{2}$$

- The integer mode numbers (characteristic of finite volume) are completely inessential – they only obscure a simple underlying structure
- Pole at $\theta_1 = \theta_2 + i\pi$ (position of kinematical singularity as for form factors!) $\longrightarrow$ there should be some underlying axioms...
- Still some surprising features — the $\sin\frac{p_k L_1}{2}$ factors

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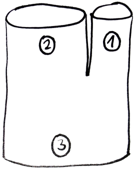
**Proceed to the generic string vertex...**

# The decompactified string vertex

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**Key new feature:** string $\#1$ 'eats up volume' $\longrightarrow$ the "operator" should have a $e^{-ipL}$ branch cut defect...

**Ready to formulate functional equations...**

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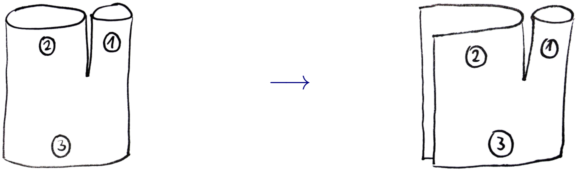


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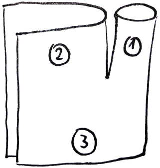


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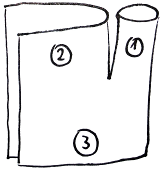


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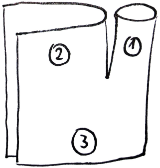


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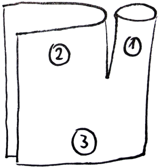


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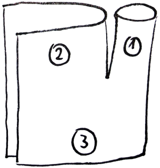


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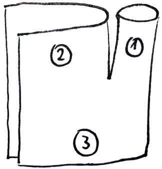


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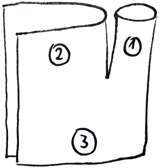


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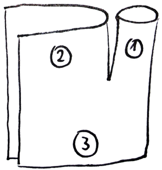


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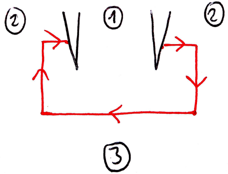


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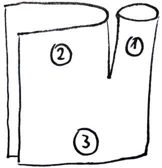


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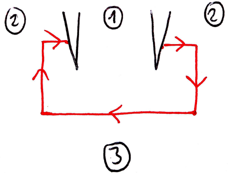


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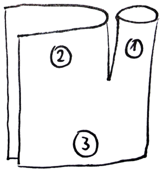


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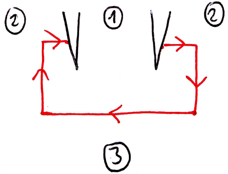


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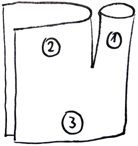
### Functional equations for the (decompactified) string vertex

written here for two incoming particles and, for the moment, free theory




- The exact pp-wave solution, involving the $\Gamma_\mu(\theta)$ special function solves these equations and can be reconstructed from them!

$$n(\theta)n(\theta+i\pi) = -\frac{1}{2\pi^2} ML \sinh\theta \sin\frac{p(\theta)L}{2}$$

- This includes **all exponential wrapping corrections** $e^{-\mu\alpha_1} = e^{-ML}$ for the $\#1$ string
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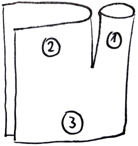


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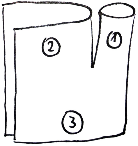


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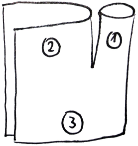


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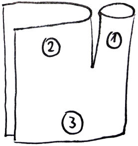
## The decompactified string vertex

### Functional equations for the (decompactified) string vertex

written here for two incoming particles and, for the moment, free theory

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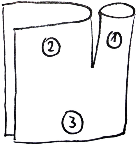


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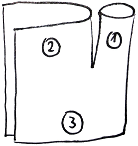


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## What happens in $AdS_5 \times S^5$?

**Novel kinematics**

- Complex rapidities $z$ are defined on a covering of an elliptic curve
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- The phase factors $e^{ip\mathbf{L}}$ make sense directly only for integer $\mathbf{L}$ which is nice from the point of view of $\mathcal{N} = 4$ SYM...

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- The S-matrix does not depend on the difference of rapidities
- The S-matrix is nondiagonal which drastically complicates solving form factor axioms (which are a special case of our SFT axioms)

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## The $AdS_5 \times S^5$ Neumann coefficient

- Consider an amplitude with only two particles in the ingoing string #3 and vacuum on the two outgoing strings...
- The equations satisfied by $\mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1,i_2}$ are

$$\begin{aligned} \mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1,i_2} &= S^{kl}_{i_1 i_2}(z_1, z_2)\mathbf{N}^{33}_{\mathbf{L}}(z_2, z_1)_{l,k} \\ \mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1,i_2} &= e^{-ip(z_1)\mathbf{L}}\mathbf{N}^{33}_{\mathbf{L}}(z_2, z_1 - \tau)_{i_2,i_1} \\ \mathrm{res}_{z'=z}\, \mathbf{N}^{33}_{\mathbf{L}}(z' + \tau/2, z)_{\bar{i},i} &= \left(1 - e^{ip(z)\mathbf{L}}\right) \end{aligned}$$

- Suppose that we know a solution of 2-particle form factor equations in $AdS$ $\mathbf{F}(z_1, z_2)_{i_1,i_2}$ s.t.

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$$\begin{aligned} N^{33}_{\mathbf{L}}(z_1, z_2) &= N^{33}_{\mathbf{L}}(z_2, z_1) \\ N^{33}_{\mathbf{L}}(z_1, z_2) &= e^{-ip(z_1)\mathbf{L}} N^{33}_{\mathbf{L}}(z_2, z_1 - \tau) \\ \mathrm{res}_{z'=z}\, N^{33}_{\mathbf{L}}(z' + \tau/2, z) &= \left(1 - e^{ip(z)\mathbf{L}}\right) \end{aligned}$$

- $N^{33}_{\mathbf{L}}(z_2, z_1)$ incorporates all **L** dependence (all wrapping corrections w.r.t. string $\#1$) at any coupling
- Conversely, if we have *any* solution of the SFT axioms, then the ratio

$$\frac{\mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1 i_2}}{N^{33}_{\mathbf{L}}(z_1, z_2)}$$

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- We will solve the equations following the general structure of the pp-wave answer:

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- The denominator generalizes directly to the $AdS$ case – however it in addition to the kinematical singularity pole at $\theta_1 = \theta_2 + i\pi$, it has another pole at $\theta_1 = -\theta_2 + i\pi$
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- Such a $f(z)$ can be constructed using $q$-theta functions $\theta_0(z)$:

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- $n(z)$ satisfies in particular

$$n(z)n(z+\tau/2) \propto \sin\frac{pL}{2}$$

- This already implies a set of zeroes – we should distribute them on the real line    we consider $L = 2n$

$$n(z) \propto \sqrt{\frac{L}{2}} \prod_{k=1}^{n-1} \frac{\sqrt{1+16g^2\sin^2\frac{\pi k}{L}} - E(z)}{4g\sin\frac{\pi k}{L}}$$

- $n(z)$ also satisfies a monodromy property

$$n(z+\tau) = e^{-ip(z)L} n(z)$$

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We performed a number of checks:

1. We verified that our formula has the correct pp-wave limit
2. The $L$ dependence in the weak coupling limit agrees with spin chain calculations
3. We observe 'vanishing of monodromy' in the asymptotic large $L$ limit i.e. for any $L$ we have

$$\lim_{\varepsilon \to 0^+} \frac{n(z+\tau-i\varepsilon)}{n(z+i\varepsilon)} = e^{-ip(z)L}$$

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## Conclusions & outlook

- We propose a framework for formulating functional equations for string interactions (light cone string field theory vertex) when the worldsheet theory is **integrable**
- This approach should work in particular for strings in the full $AdS_5 \times S^5$ geometry
- A key step is the existence of an infinite volume setup, which allows for formulating functional equations incorporating e.g. crossing
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- Understand links with the subsequent 'hexagon' approach of Basso, Komatsu, Vieira

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- Understand links with the subsequent 'hexagon' approach of Basso, Komatsu, Vieira

## Conclusions & outlook

- We propose a framework for formulating functional equations for string interactions (light cone string field theory vertex) when the worldsheet theory is **integrable**
- This approach should work in particular for strings in the full $AdS_5 \times S^5$ geometry
- A key step is the existence of an infinite volume setup, which allows for formulating functional equations incorporating e.g. crossing
- We reproduced pp-wave string field theory formulas for the Neumann coefficients
- We solved for the 'kinematical' part of the $AdS_5 \times S^5$ Neumann coefficient describing exact volume dependence (for even $L$) at any coupling – may describe all order wrapping w.r.t. one string

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