Current Themes in Holography, Copenhagen, April 27, 2016

Theta dependence in Holographic QCD

Francesco Bigazzi (INFN, Pisa)

- *FB, Aldo L. Cotrone, Roberto Sisca, JHEP 1508 (2015) 090*
- *Lorenzo Bartolini, FB, Stefano Bolognesi, Aldo L. Cotrone, Andrea Manenti (in progress)*

Plan

- The θ-angle in Yang-Mills and QCD
- Holographic Yang-Mills at finite θ-angle
- Holographic QCD at finite θ-angle

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Effects of the θ parameter interesting but challenging.

The θ-angle in Yang-Mills

- Euclidean Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{i}{32\pi^2} \theta \, \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma}$$

- θ term breaks P,T and hence CP
- θ multiplies topological charge density, whose 4d integral is integer.
- Hence θ is an angle. Physics periodic under $\theta \longrightarrow \theta + 2\pi$

- Effects: vacuum structure, CP violating effects in QGP [Kharzeev et al], mass and interactions of η' meson in QCD [Witten-Veneziano], cosmology (axions)

- Neutron electric dipole moment: $|\theta| < 10^{-10}$. Strong CP problem.

The θ-angle in Yang-Mills

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$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2}\theta\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

- θ-dependence due to instantons . Non-perturbative effect.

- Real θ-angle challenging for Lattice (sign problem: imaginary term)
- Go to imaginary θ, then analytically continue to real around θ=0
- Alternatively, compute n-point correlators of topological charge at θ=0
- Lattice results obtained in this way: first few terms in θ expansion

The θ-angle in Lattice Yang-Mills

- Ground state energy density [Vicari, Panagopoulos, 08; Bonati, D'Elia, Vicari, Panagopoulos, 13]

$$f(\theta) - f(0) = \frac{1}{2}\chi_g \theta^2 \left[1 + \bar{b}_2 \frac{\theta^2}{N_c^2} + \bar{b}_4 \frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right] \quad \chi_g = f''(\theta)|_{\theta=0} \neq 0 \text{ topological susceptibility}$$

$\bar{b}_2 \approx -0.2$ (from N_c=3,…,6 data) $|b_4| < 0.001$ (N_c=3)

- String tension [Del Debbio, Manca, Panagopoulos, Skouropathis, Vicari, 06]

$$T_s(\theta) = T_s(0)\left[1 + \bar{s}_2 \frac{\theta^2}{N_c^2} + \mathcal{O}(\theta^4)\right], \quad \bar{s}_2 \approx -0.9 \text{ (from } N_c\text{=3,…,6 data)}$$

- Lowest 0^{++} glueball mass [Del Debbio, Manca, Panagopoulos, Skouropathis, Vicari, 06]

$$M_{0^{++}}(\theta) = M_{0^{++}}(0)\left[1 + g_2\theta^2 + \mathcal{O}(\theta^4)\right], \quad g_2 \approx -0.06(2), \quad (N_c = 3)$$

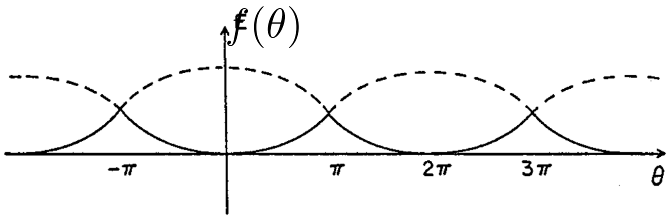
- Deconfinement temperature [D'Elia, Negro, 12, 13]

$$T_c(\theta) = T_c(0)\left[1 + R_\theta\theta^2 + \mathcal{O}(\theta^4)\right], \quad R_\theta \approx -0.0175(7), \quad (N_c = 3)$$

The θ-angle in large N Yang-Mills

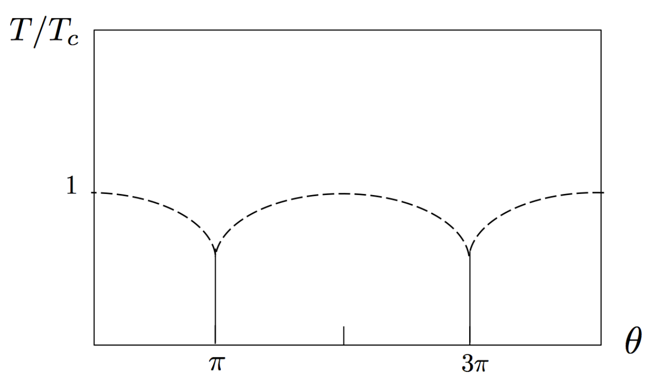
- $\mathcal{L} = \frac{N_c}{2\lambda}\left[\mathrm{Tr}F^2 - i\frac{\lambda}{8\pi^2}\frac{\theta}{N_c}\mathrm{Tr}F\tilde{F}\right]$ 't Hooft limit: must take θ/N fixed.

- Puzzle: energy density should also be periodic in θ with 2π periodicity.
- Solution [Witten, 1980]: f(θ) multi-branched



$$f(\theta) = b\,N^2 \mathrm{min}_k \left(\frac{\theta + 2k\pi}{N}\right)^2 + O\,(\theta^4/N^4)$$

- At θ=(2k+1)π: expect first order transitions. CP spontaneously broken
- Expected phase diagram



Theta term in QCD

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \sum_{f=1}^{N_f} \bar{\psi}_f \left(D + m_f \right) \psi_f$$

a) Massless quarks:

$$\psi \to e^{i\alpha\gamma_5}\, \psi$$

$$[d\psi][d\bar{\psi}] \to \exp\left(\frac{-i\,\alpha\, g^2\, N_f}{32\pi^2} \int d^4x\, \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \right) [d\psi][d\bar{\psi}]$$

$$\theta \to \theta - 2\alpha N_f$$

- Theta rotated away by chiral rotation. No theta dependence

b) Massive quarks: $m_f \to e^{2i\alpha}\, m_f \qquad \theta = \theta_q + \arg\det M$

- Neutron Electric Dipole Moment: $d_n \sim \theta \frac{e m_f}{m_n^2} \sim \theta \frac{e m_\pi^2}{m_n^3} \approx 10^{-16} \theta\, e\,\mathrm{cm}$

- Experiments: $|d_n| < 2.9 \times 10^{-26} e\,\mathrm{cm}$ bound on theta

NEDM in the literature

Year	Approach/model	$c = d_n/(\theta \cdot 10^{-16} e \cdot \mathrm{cm})$
1979	bag model	2.7
1980	ChPT	3.6
1981	ChPT	1
1981	ChPT	5.5
1982	ChPT	20
1984	chiral bag model	3.0
1984	soft pion Skyrme model	1.2
1984	single nucleon contribution	11
1990 [20]	Skyrme model $N_f = 3$	2
1991 [19]	Skyrme model $N_f = 2$	1.4
1991	ChPT	3.3(1.8)
1991	ChPT	4.8
1992	ChPT	−7.2, −3.9
1999	sum rules	2.4(1.0)
2000	heavy baryon ChPT	7.5(3.2)
2004	instanton liquid	10(4)
2007	holographic QCD “hard–wall”	1.08
2015	1502.02295 [hep-lat]	- 3.8 (2)

Table partially taken from Panagopoulos, Vicari 2008

- Need to compute

$$\vec{D}_{n,s} = \int \mathrm{d}^3 x \, \vec{x} \, \langle n, s | J^0_{\mathrm{em}} | n, s \rangle$$

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Exact θ-dependence in a top-down holographic Yang-Mills model
Explicit realization of expected YM features.
Qualitative matching with lattice YM at small θ.
Benchmark beyond small θ ?

Witten's holographic Yang-Mills

- N_c D4-branes in IIA string theory
- Low energy physics: 5d $SU(N_c)$ Super-Yang-Mills theory.

- N_c D4-branes on S^1_{x4} of radius $R_4 = 1/M_{KK}$ with antiperiodic fermions.
- Low energy: 4d non-susy $SU(N_c)$ Yang-Mills + adjoint KK modes [Witten 1998]

- Can add θ term to the model, no sign problem.
- To leading order in θ/N_c done in [Witten 1998].

Witten's holographic Yang-Mills

- Gravity action (closed string description), sourced by the N D4-branes

$$S = \frac{1}{2k_0^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(\mathcal{R}+4(\partial\phi)^2\right)-\frac{1}{2}|F_4|^2-\frac{1}{2}|F_2|^2\right]$$

- Gauge theory action (open string description), wrapped D4-branes (IR expansion)

$$S = \frac{1}{8\pi g_s l_s M_{KK}}\int d^4x\ \mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2 l_s}\int_{S_{x^4}} C_1\int d^4x\ \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

$$\boxed{F_2 = d\,C_1} \qquad \lambda_4 = g_{YM}^2 N_c = 2\pi g_s l_s M_{KK} N_c \qquad \boxed{\theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1}$$

- Holography: gravity picture dual to gauge theory at $\lambda_4 \gg 1$, $N_c \gg 1$
- Leading order in θ/N_c : treat F_2 as a probe [Witten 1998].

The θ-backreacted gravity solution

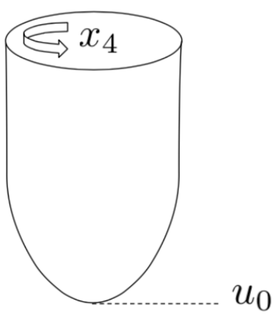
[Barbon, Pasquinucci 99; Dubovsky, Lawrence, Roberts 2011] $(x_4 \sim x_4 + 2\pi/M_{KK})$

$$ds^2_{10} = \left(\frac{u}{R}\right)^{3/2}\left[\sqrt{H_0}\, dx_\mu dx^\mu + \frac{f}{\sqrt{H_0}} dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2} \sqrt{H_0}\left[\frac{du^2}{f} + u^2 d\Omega_4^2\right]$$

$$f = 1 - \frac{u_0^3}{u^3}\,, \qquad H_0 = 1 - \frac{u_0^3}{u^3}\frac{\Theta^2}{1+\Theta^2} \qquad e^\phi = g_s\left(\frac{u}{R}\right)^{3/4} H_0^{3/4}\,, \qquad C_1 = \frac{\Theta}{g_s}\frac{f}{H_0} dx^4\,, \qquad F_4 = 3R^3\omega_4$$

$$\int_{S^4} F_4 = 8\pi^3 l_s^3 g_s N_c\,, \quad R = (\pi g_s N_c)^{1/3} l_s \qquad \theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1$$

$$\boxed{\Theta \equiv \frac{\lambda_4}{4\pi^2}\left(\frac{\theta + 2k\pi}{N_c}\right)} \qquad \boxed{u_0 = \frac{4R^3}{9} M_{KK}^2 \frac{1}{1+\Theta^2}}$$

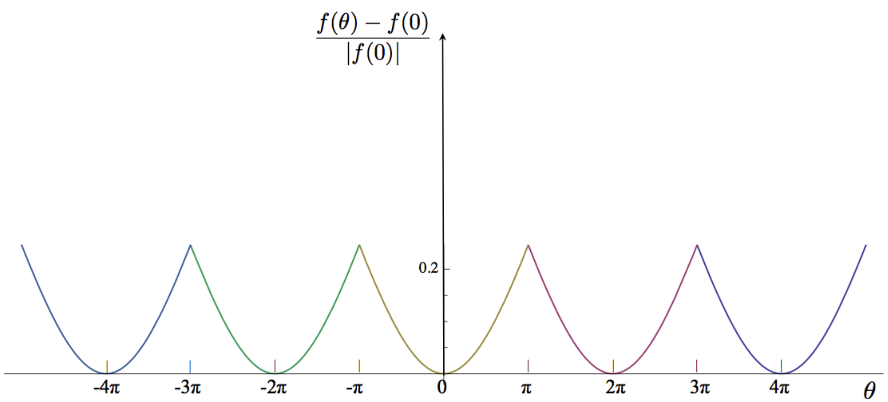


- (u,x_4) subspace is a cigar
- $g_{00}(u_0) \neq 0$ (regular) : confinement
- KK modes NOT decoupled
- Small curvature if $\Theta << \lambda_4^{1/4}$

The ground-state energy density

From holographic relation $Z = e^{-V_4 f(\theta)} \approx e^{-S^{\rm ren}_{E\,\rm on-shell}}$

$$\boxed{f(\Theta) = -\frac{2N_c^2\lambda_4}{3^7\pi^2}\frac{M_{KK}^4}{(1+\Theta^2)^3}} \qquad \Theta \equiv \frac{\lambda_4}{4\pi^2}\left(\frac{\theta+2k\pi}{N_c}\right) \qquad \boxed{f(\theta) = \min_k f(\Theta)}$$



Expected structure explicitly realized

$$f(\theta) - f(0) = \frac{1}{2}\chi_g\theta^2\left[1 + \bar{b}_2\frac{\theta^2}{N_c^2} + \bar{b}_4\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right] \qquad \chi_g = \frac{\lambda_4^3 M_{KK}^4}{4(3\pi)^6} \qquad \bar{b}_2 = -\frac{\lambda_4^2}{8\pi^4}\,, \quad \bar{b}_4 = \frac{5\lambda_4^4}{384\pi^8}$$

- Qualitative agreement with Lattice
- Prediction: $b_4 > 0$ (it would be nice to check it on the lattice)

Other observables

- String tension (from rectangular Wilson loop) $\boxed{T_s = \frac{2\lambda_4}{27\pi} M_{KK}^2 \frac{1}{(1+\Theta^2)^2}}$

$$T_s = \frac{2\lambda_4}{27\pi} M_{KK}^2 \left(1 - \frac{\lambda_4^2}{8\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{256\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right)$$

- Light scalar glueball mass $\boxed{M(\Theta) = \frac{M(\Theta=0)}{\sqrt{1+\Theta^2}}}$

$$M(\theta) = M(\theta=0)\left(1 - \frac{\lambda_4^2}{32\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{2048\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right)$$

- ‘t Hooft loop and oblique confinement (from minimal area D2 on S_1)

$$T_m = \frac{1}{27\pi^2} M_{KK}^2 \lambda_4 \frac{|\theta + 2k\pi|}{(1+\Theta^2)^2} \equiv T_s \frac{|\theta+2k\pi|}{2\pi}$$ area law at finite theta

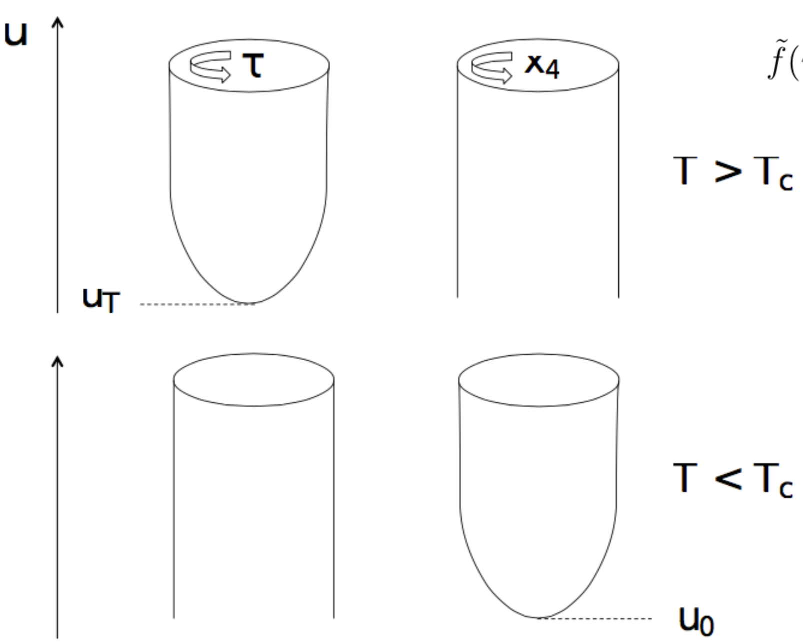
$$T_{dy} = -pT_s + qT_m = \left(-p + \frac{\theta}{2\pi}q\right) T_s$$ dyons are screened if $\theta = 2\pi(p/q)$

Finite temperature

Take Euclidean time periodic with period 1/T. Two possible gravity solutions

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[\tilde{f}(u)dx_0^2 + dx_a dx^a + dx_4^2\right] + \left(\frac{u}{R}\right)^{-3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right],$$

$$\tilde{f}(u) = 1 - \frac{u_T^3}{u^3}.$$



- black hole solution
- $g_{00}(u_T) = 0$: deconfinement
- no theta dependence

$$\boxed{C_1 \sim \theta dx_4, \;\; F_2 = 0 \quad [(u, x_4) : \text{cylinder}]}$$

- Just Euclidean extension of one discussed before
- Theta-dependence
- confinement

Deconfinement temperature

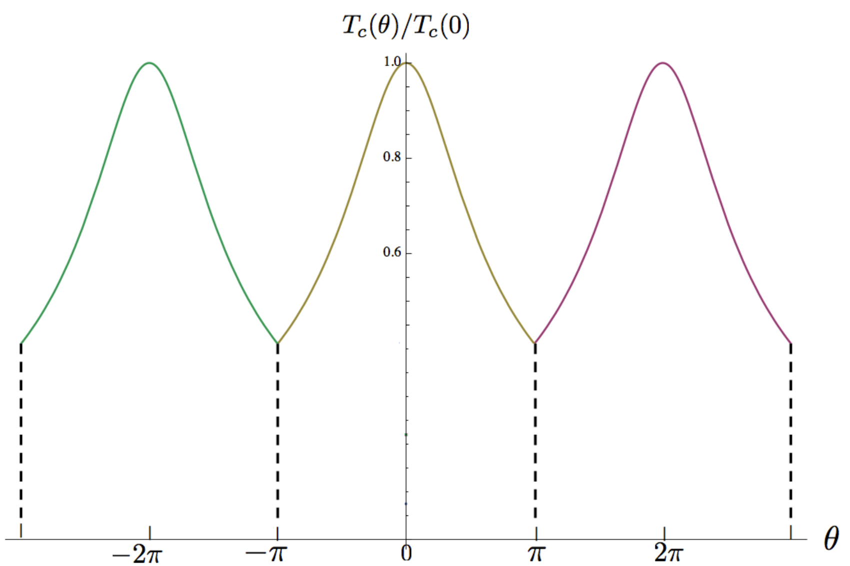
Compare the free energy densities of confined and deconfined phase

$$f = -p = -\frac{2N_c^2\lambda_4}{3^7\pi^2}\frac{M_{KK}^4}{(1+\Theta^2)^3} \equiv \frac{f(0)}{(1+\Theta^2)^3}$$

$$f_{dec} = -p_{dec} = -\frac{1}{6}\frac{256 N_c^2\pi^4\lambda_4}{729 M_{KK}^2}T^6$$

$$\boxed{T_c(\Theta) = \frac{M_{KK}}{2\pi}\frac{1}{\sqrt{1+\Theta^2}}}$$

$$T_c(\theta) = \frac{M_{KK}}{2\pi}\left[1 - \frac{\lambda_4^2}{32\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{2048\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right]$$



$$T_c(\theta)_{lat} = T_c(0)_{lat}\left[1 - R_\theta\theta^2 + \mathcal{O}(\theta^4)\right]\ , \quad R_\theta = 0.0175(7)$$

Cusps: tri-critical points
Colored: deconf. first order transition
Dashed: CP-breaking first order transition

Phase diagram as expected

Overview

- Holographic YM results exact in θ, large N
- Observables are those at $\theta=0$ multiplied by powers of $(1+\Theta^2)$

- A rule to compute theta-corrected CP even observables
  - a factor of $(1+\Theta^2)^{-1/2}$ for each power of M_{KK}
  - a factor of $(1+\Theta^2)^{-1}$ for each power of λ_4

- Mass scales reduced by θ (checked also baryon vertex mass)
- Structure of (T,θ) phase diagram explicitly
- Agreement with lattice trends at small θ
- Benchmarks for subleading coefficients in θ expansion

- Other observables computed: e.g. entanglement entropy.

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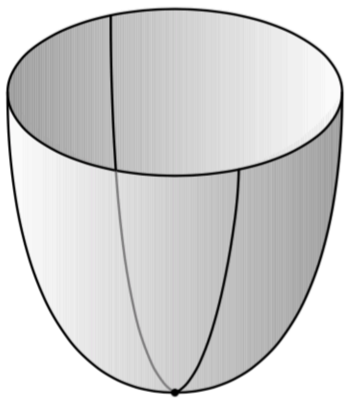
Work in progress
θ-dependence in Witten-Sakai-Sugimoto model
with massive flavors
Towards a holographic computation of Neutron Electric Dipole Moment

Theta term in Witten-Sakai-Sugimoto



$U_{\mathrm{KK}} \equiv U_0$

Quenched flavors: probe D8-anti-D8
χSB = joining of the two branches
Flavors are massless

Gauge field on D8 gives meson tower:

- A_z : Goldstone bosons
- Aμ : (axial) vector mesons

F_2 mixes with $U(1)_A$ gauge field on D8

$$S^{(3)} = -\frac{1}{4\pi(2\pi l_s)^6}\int \widetilde{F}_{(2)} \wedge {}^{\star}\widetilde{F}_{(2)} - \frac{1}{2\pi}\int C_{(7)} \wedge \left(\mathrm{d}\widetilde{F}_{(2)} - \mathrm{Tr}\,\mathcal{F} \wedge \omega_y\right)$$

$$\widetilde{F}_{(2)} = \mathrm{d}C_{(1)} + \mathrm{Tr}\,(\mathcal{A}) \wedge \delta(y)\mathrm{d}y \qquad \delta_\Lambda C_{(1)} = \mathrm{Tr}\,(\Lambda)\delta(y)\mathrm{d}y\ , \qquad \delta_\Lambda \mathcal{A} = i[\Lambda, \mathcal{A}] - \mathrm{d}\Lambda$$

$$\mathcal{A} = \widehat{A}\frac{\mathbb{1}}{\sqrt{2N_f}} + A^a T^a$$

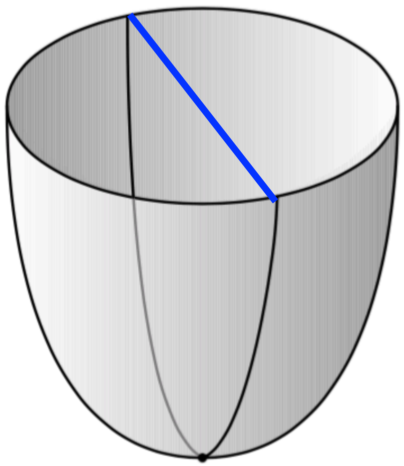
Vacuum solution: A pure gauge

$$\int \tilde{F}_2 = \int dC_1 - \int Tr\mathcal{A} \wedge \omega_y = 0 \ \rightarrow \ S^3 = 0$$

$$\psi \rightarrow e^{i\alpha\gamma_5}\,\psi \qquad \theta \rightarrow \theta - 2\alpha N_f$$

No θ dependence.

Mass term in Witten-Sakai-Sugimoto



Worldsheet instanton
[Aharony-Kutasov; Hashimoto –Hirayama-Lin_hee 2008]

$$S_{\rm mass} = c \int d^4x \, {\rm Tr}\, \mathcal{P} \left[M \exp\left(-i \int_{-\infty}^{\infty} \mathcal{A}_z dz \right) + {\rm c.c.} \right]$$

$$c = \frac{1}{3^{9/2}\pi^3} g_{\rm YM}^3 N_c^{3/2} M_{\rm KK}^3 \mathcal{N}^{-1} \qquad \mathcal{A} = \widehat{A} \frac{\mathbb{1}}{\sqrt{2N_f}} + A^a T^a$$

- On previous solution: $\int \hat{A}_z dz \sim \theta, \text{ for } M = m_q \mathbf{1}$
- Ground-state energy density (from on-shell action)

$$F(\theta) - F(0) = N_f m_q \Sigma \left[1 - \cos(\theta/N_f) \right] \ , \quad \Sigma = \frac{\langle \bar{\psi}\psi \rangle}{N_f}$$

- Same result as from chiral Lagrangian since

$$\mathcal{L}_{mass} \sim \Sigma \, {\rm Tr}[e^{i\theta/N_f} M U + c.c.] \, , \ U \sim \exp\left(-i \int \mathcal{A}_z dz \right)$$

Baryons in Witten-Sakai-Sugimoto

$$\frac{1}{8\pi^2}\int_{D8} C_{(5)} \wedge \mathrm{Tr}\,\mathcal{F}^2 \qquad\qquad \frac{1}{8\pi^2}\int_B \mathrm{Tr}\,\mathcal{F}^2 = n_B$$

Baryon = D4-brane wrapped on S^4; instanton of 5d SS action
[Hata,Sakai,Sugimoto,Yamato. 2007]

$$\begin{aligned} S_{\mathrm{bulk+D8}} = -\kappa \int \mathrm{d}^4x\mathrm{d}z\, \left(\frac{1}{2}h(z)\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + k(z)\mathrm{Tr}\,F_{\mu z}F^{\mu}{}_{z}\right) + \\ -\frac{\kappa}{2}\int \mathrm{d}^4x\mathrm{d}z\, \left(\frac{1}{2}h(z)\,\widehat{F}_{\mu\nu}\widehat{F}^{\mu\nu} + k(z)\widehat{F}_{\mu z}\widehat{F}^{\mu}{}_{z}\right) + \\ +\frac{N_c}{24\pi^2}\int\left[\omega_5^{SU(N_f)}(A) + \frac{3}{\sqrt{2N_f}}\widehat{A}\,\mathrm{Tr}\,F^2 + \frac{1}{2\sqrt{2N_f}}\widehat{A}\,\widehat{F}^2\right] \end{aligned}$$

BPST-like instanton (Nf=2)

$$A_M^{\mathrm{cl}} = -if(\xi)g\partial_M g^{-1}\,, \quad \widehat{A}_0^{\mathrm{cl}} = \frac{1}{8\pi^2 a}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right]\,, \quad A_0^{\mathrm{cl}} = \widehat{A}_M = 0$$

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2}\,, \quad g(x) = \frac{(z-Z)\mathbb{1} - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi}$$

Small size: $\rho^2 \sim 1/\lambda$
Spectrum (baryon states) from quantization of collective coordinates

$$A_M \longmapsto \quad A'_M = W(t)A_M W(t)^{-1}$$

Baryons in Witten-Sakai-Sugimoto

Adding mass and theta term

$$S_{\text{mass}} = c \int \mathrm{d}^4x \,\mathrm{Tr}\,\mathcal{P}\left[M \exp\left(-i\int_{-\infty}^{\infty} \mathcal{A}_z \mathrm{d}z\right) + \text{c.c.}\right]$$

$$S_{\text{kin}} = -\frac{\chi_g}{2}\int \mathrm{d}^4x \left(\theta + \sqrt{\frac{N_f}{2}}\int_{-\infty}^{\infty} \mathrm{d}z\,\widehat{A}_z\right)^2$$

- To first order in mass, just O(θ^2) correction to Mass (Hamiltonian)
- Baryon states do not receive O(θ) corrections
- New instanton solution to leading order in mass and theta

$$\boxed{\widehat{A}_z^{\text{mass}} = \frac{1}{1+z^2}u(r)}$$

$$u(r) = \frac{c m_q \theta}{\kappa}\int_0^{\infty} \mathrm{d}r'\, u_G(r,r')\left(1+\cos\frac{\pi}{\sqrt{1+\rho^2/r'^2}}\right)$$

$$\boxed{A^0_{\text{mass}} = W(r,z)(\vec{x}-\vec{X})\cdot\vec{\tau}}$$

$$h(z)\left(\partial_r^2 W(r,z) + \frac{4}{r}\partial_r W(r,z) + \frac{8\rho^2}{(\xi^2+\rho^2)^2}W(r,z)\right) + \partial_z(k(z)\partial_z W(r,z)) =$$
$$= \frac{27\pi}{\lambda}\frac{\rho^2}{(\xi^2+\rho^2)^2}\frac{1}{r}\frac{u'(r)}{1+z^2} \equiv \mathscr{F}(r,z)$$

NEDM in Witten-Sakai-Sugimoto

$J^0{}_{em} \sim F^{0z}|_{boundary}$

$$\vec{D}_{n,s} = \int \mathrm{d}^3x\, \vec{x}\, \langle n, s|J^0_{\mathrm{em}}|n, s\rangle$$

$$\vec{\mathcal{D}}_{n,s} = \frac{2}{3}\int_0^\infty \mathrm{d}r\, r^4\, \kappa[k(z)\,\partial_z W(r,z)]_{z\to-\infty}^{z\to\infty}\, \langle s|\vec{\sigma}|s\rangle = -\vec{\mathcal{D}}_{p,s}$$

$$d_n = \frac{2}{3}\int_0^\infty \mathrm{d}r\, r^4\, \kappa[k(z)\,\partial_z W(r,z)]_{z\to-\infty}^{z\to\infty}$$

The NEDM obtained performing the numerical analysis for $N_c = 3$, $\lambda = 16.632$ and $M_{\mathrm{KK}} = 949\,\mathrm{MeV}$ yields

$$d_n = 3.25 \cdot 10^{-17}\, \theta\, e \cdot \mathrm{cm}$$

- Holographic result takes into account contribution from whole meson tower
- NEDM proportional to $N_c\, m_\pi^2$ as in Skyrme model

Coupling with pions

From old-fashioned pion-nucleon effective Lagrangian

$$\mathcal{L}_{\pi NN} = \boldsymbol{\pi} \cdot \bar{N} \boldsymbol{\tau} (i\gamma_5 g_{\pi NN} + \bar{g}_{\pi NN}) N$$

$$\langle \pi^a(x) \rangle = -\frac{g_{\pi NN}}{8\pi M_N} \frac{x_i}{r^3} \langle \sigma_i \tau^a \rangle$$ + CP breaking term

$\langle \pi^a \rangle = A^a{}_z$ Large r limit of our instanton solution

$$g_{\pi NN} \sim N_c^{\frac{3}{2}} \qquad \bar{g}_{\pi NN} \sim \theta N_c^{??}$$

No linear in theta term found in Az : $\bar{g}_{\pi NN} = 0$ at leading order in 1/N

Thank you

The scalar glueball mass

- 0^{++} glueball spectrum $\longrightarrow$ $\langle \mathrm{Tr}F^2(x)\mathrm{Tr}F^2(y)\rangle = \sum_n c_n\, e^{-M_n|x-y|}$
- Solve e.o.m of dual gravity scalar field: a metric fluctuation in 11d. Reducing on S^4:

$$h_{ab} = H_{ab}(u)e^{-ik\cdot x} \qquad H_{ab}(u) = \frac{u}{R}H(u)\,\mathrm{diag}\left(0,1,1,1,0,-\frac{3}{1+\Theta^2},0\right)$$

$$H''(u) + \frac{4u^3-u_0^3}{u(u^3-u_0^3)}H'(u) - \frac{M^2R^3}{u^3-u_0^3}H(u) = 0$$

- Regularity at $u=u_0$ and UV normalizability only if $M^2>0$ and discrete

$$\boxed{M(\Theta) = \frac{M(\Theta=0)}{\sqrt{1+\Theta^2}}} \qquad M(\theta) = M(\theta=0)\left(1-\frac{\lambda_4^2}{32\pi^4}\frac{\theta^2}{N_c^2}+\frac{3\lambda_4^4}{2048\pi^8}\frac{\theta^4}{N_c^4}+\mathcal{O}(\theta^6)\right)$$

$$M_{lat}(\theta) = M_{lat}(0)\left(1+g_2\theta^2+\mathcal{O}(\theta^4)\right)\ , \quad g_2 = -0.06(2)$$

‘t Hooft loop and oblique confinement

- Monopole-antimonopole potential from minimal action of D2 wrapping S_{x4}

$$S_{D2} = -T_2 \int d^3\xi e^{-\hat{\phi}} \sqrt{-\det(g+\mathcal{F})} + T_2 \int \hat{C}_1 \wedge \mathcal{F}$$

- Equivalent to M2-brane wrapping the $y = \Theta\, x_4$ cycle in

$$ds_{11}^2 = e^{-\frac{2}{3}\hat{\phi}} ds_{10}^2 + e^{\frac{4}{3}\hat{\phi}} (dy - \hat{C}_{x_4} dx_4)^2$$

- ‘t Hooft loop has an area law at finite theta (magnetic screening only at theta=0)

$$T_m = \frac{1}{27\pi^2} M_{KK}^2 \lambda_4 \frac{|\theta + 2k\pi|}{(1+\Theta^2)^2} \equiv T_s \frac{|\theta + 2k\pi|}{2\pi}$$

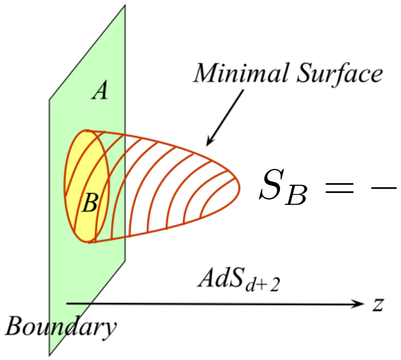
- Dyons are screened under certain conditions (oblique confinement)

$$T_{dy} = -pT_s + qT_m = \left(-p + \frac{\theta}{2\pi} q\right) T_s \qquad \theta = 2\pi(p/q)$$

See also [Gross, Ooguri, 98]

Entanglement entropy

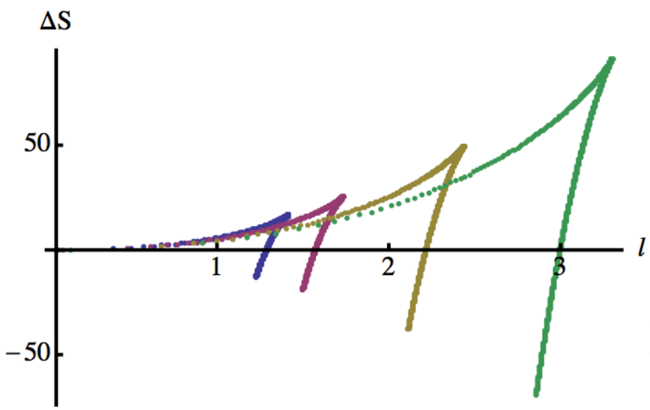
[Ryu, Takayanagi 2006]



$$S_B = -Tr(\rho_B \log \rho_B) \approx \min_{\gamma_B} \frac{A(\gamma_B)}{4G_N}$$

$$S = \frac{1}{4G_N} \int d^8\sigma e^{-2\phi} \sqrt{-\det(G_{\rm ind})}$$

Slab geometry: B={x in [-l/2, l/2]} [Klebanov, Kutasov, Murugan 07 at θ=0]



$$S_{dis} = V_2 \frac{2^2}{\pi 3^5} \frac{\lambda_4 N_c^2}{(1+\Theta^2)^2} M_{KK}^2 \left(\frac{1}{\epsilon} - 1\right)$$

$\partial S_{dis}/\partial l \sim \mathcal{O}(N_c^0)$

$$S_{conn,fin} = -V_2 \frac{2^6}{3} \frac{\pi^{3/2}\Gamma[\frac{3}{5}]^5}{\Gamma[\frac{1}{10}]^5} \frac{\lambda_4 N_c^2}{(l\ M_{KK})^4} M_{KK}^2$$

First order phase transition $\boxed{l_c = l_{c,0}\sqrt{1+\Theta^2}}$