# Holographic NJL Interactions

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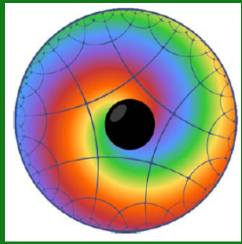


Holograv: Current Themes in Holography

NBI April 2016

Southampton Theory Astrophysics Gravity Research Centre

- Based on work with Keun-Young Kim arXiv:1601.02824
- Witten's multi-trace prescription meets the NJL model

# Introduction

One of the most remarkable aspects of the Standard Model is that the ground state symmetries are less than those of the bare Lagrangian…

- Higgs potential is adhoc and not yet understood
- QCD provides a DYNAMICAL symmetry breaking mechanism

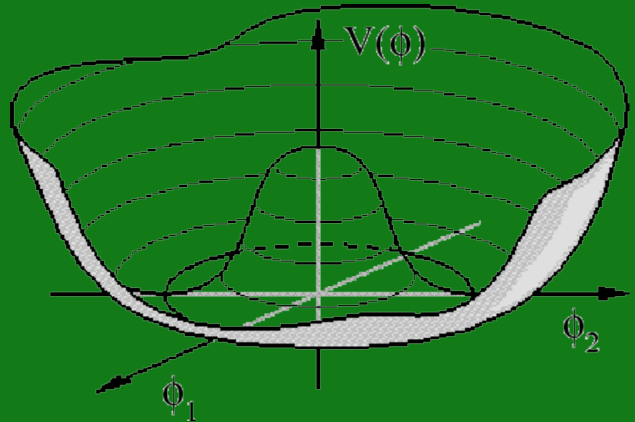
$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

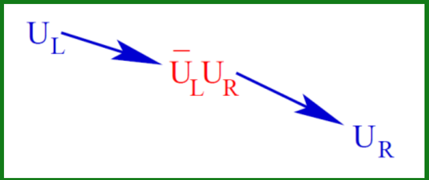
$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + h.c.)$$

$$\bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R$$

Evidence: lack of parity doubling, proton mass, Goldstone pions

$$\langle \bar{u}_L u_R + \bar{d}_L d_R + h.c.\rangle \neq 0$$



# Nambu Jona-Lasinio Model

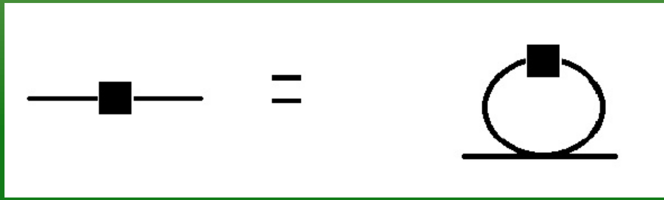
The toy model that encapsulates Nambu's Nobel concepts (before quarks)

$$\mathcal{L} = \bar\psi_L \not\partial \psi_L + \bar\psi_R \not\partial \psi_R + \frac{g^2}{\Lambda_{UV}^2}\bar\psi_L\psi_R\bar\psi_R\psi_L$$

NB L & R symmetries respected

NB Non-renormalizable – explicit UV cut off

The gap equation

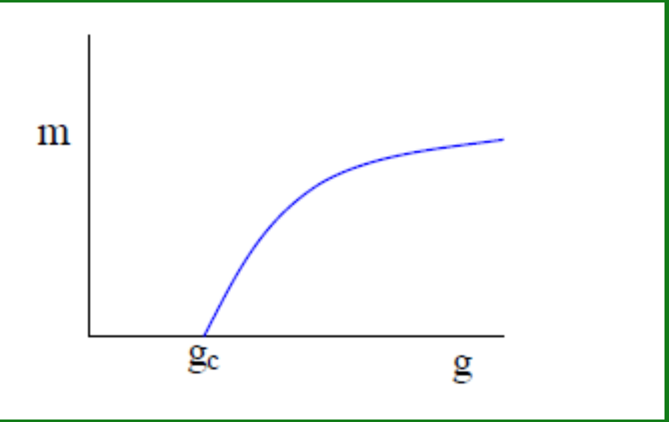


$$-im = -\frac{g^2}{\Lambda_{UV}^2}\int \frac{k^2 dk^2}{16\pi^2}\frac{Tr(\not k + m)}{k^2+m^2}$$

$$1 = \frac{g^2}{4\pi^2}\left(1-\frac{m^2}{\Lambda_{UV}^2}\log\left[(\Lambda_{UV}^2+m^2)/m^2\right]\right)$$

$g^2 < 4\pi^2$ m= 0

$g^2 > 4\pi^2$ m= 0 + another solution



# Holographic Quarks

The simplest holographic model of quarks is D3/ probe D7

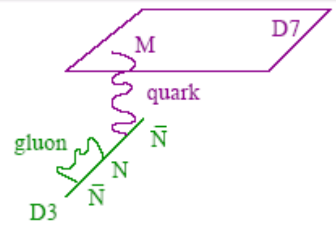
Adds quarks with conformal N=4 gauge interactions

These do not trigger chiral symmetry breaking on their own

Quark mass = T L(ρ)

## Adding Quarks

Bertolini, DiVecchia...; Polchinski, Grana; Karch, Katz...




Quarks can be introduced via D7 branes in AdS

The brane set up is

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D3 | — | — | — | — | • | • | • | • | • | • |
| D7 | — | — | — | — | — | — | — | — | • | • |

fills radial dirn of AdS, $S^3$ embedded in $S^5$ (directions 4–7); $U(1)_A$ (directions 8–9)

We will treat D7 as a probe - quenching in the gauge theory.

Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \qquad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

$$P[G]+F = \begin{pmatrix} -\frac{r^2}{R^2} & & & & & & & \\ & \frac{r^2}{R^2} & B^z & & & & & \\ & -B^z & \frac{r^2}{R^2} & & & & & \\ & & & \frac{r^2}{R^2} & & & & \\ & & & & \frac{R^2}{r^2}\left(1+(\partial_\rho L)^2\right) & & & \\ & & & & & \frac{R^2}{r^2}\rho^2 & & \\ & & & & & & \frac{R^2}{r^2}\rho^2 .. & \\ & & & & & & & \frac{R^2}{r^2} .. \end{pmatrix}$$

$$ds^2 = r^2 dx_{3+1}^2 + \frac{1}{r^2}\left[d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\phi^2\right]$$

$$\mathcal{L} = \rho^3 \sqrt{1 + (\partial_\rho L)^2} \sqrt{1 + \frac{B^z R^4}{r^4}}$$

$$S = \int d\rho \; ^3\sqrt{1+(\partial_\rho L)^2}$$

$$\rho \to \infty, L \to 0 : \mathcal{L} = \rho^3 (\partial_\rho L)^2$$

$$\delta S = 0 = \int d\rho \left( \frac{\partial \mathcal{L}}{\partial L'} \delta L' + \frac{\partial \mathcal{L}}{\partial L} \delta L \right)$$

$$EL: \; \partial_\rho [\rho^3 \partial_\rho L] = 0$$

$$0 = \int d\rho \left( -\partial_\rho \frac{\partial \mathcal{L}}{\partial L'} + \frac{\partial \mathcal{L}}{\partial L} \right) \delta L + \frac{\partial \mathcal{L}}{\partial L'} \delta L|_{UV,IR}$$

$$L = m + \frac{\sigma}{\rho^2}$$

$$\int d^4x \; m \; \bar{q}_L q_R$$

L is a holographic field (as well as a brane embedding). The usual AdS/CFT dictionary says m is the quark mass and $\sigma$ the condensate…

For the N=2 set up

$$\frac{\partial \mathcal{L}}{\partial L'} = -2\sigma = \frac{\rho^3 (\partial_\rho L)}{\sqrt{1+(\partial_\rho L)^2}}$$

$$\partial_\rho L = \frac{-2\sigma}{\sqrt{\rho^6 - 4\sigma^2}}$$

The only regular solution is L = m ($\sigma = 0$)

Check: UV: $\delta L = 0$ by construction IR: $\frac{\partial \mathcal{L}}{\partial L'} = 0$

# Witten's Multi-Trace Operator Prescription

hep-th/0112258

In the presence of a quark condensate

$$\frac{g^2}{\Lambda_{UV}^2}\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L \rightarrow \frac{g^2}{\Lambda_{UV}^2}\langle\bar{\psi}_L\psi_R\rangle\bar{\psi}_R\psi_L$$

$$m = \frac{g^2}{\Lambda_{UV}^2}\sigma$$

Witten says we should impose this relation at the UV boundary

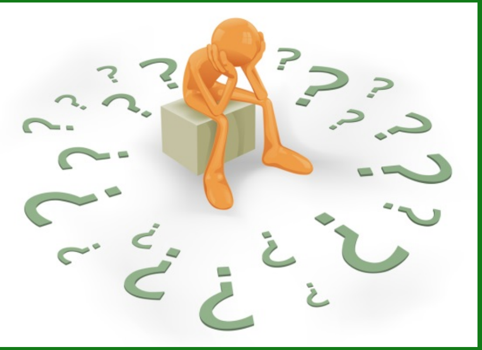
TRY ON N=2 THEORY: Solution that satisfies Euler Lagrange & IR BCs is L=m

The only solution satisfying Witten's UV condition is m=0 for ALL g!!!!

So where's the NJL model dynamics?

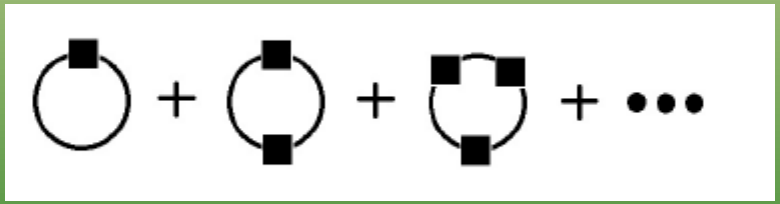
Lost to underlying SUSY (but why?)?

Or is the prescription wrong? ????



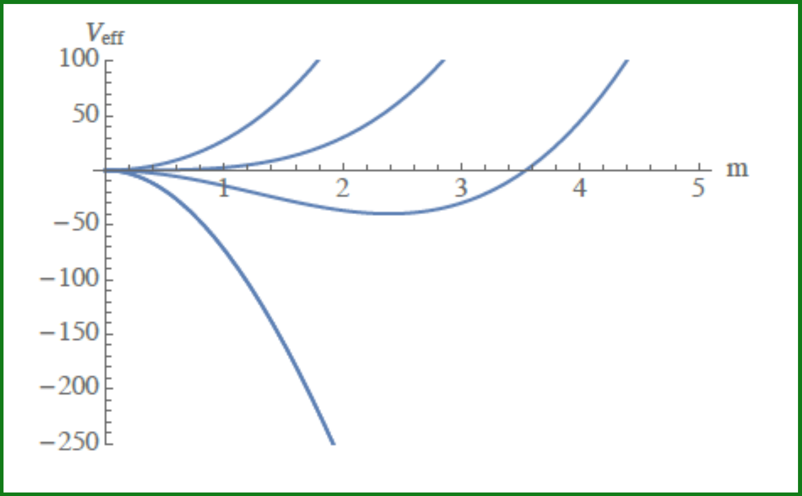
# Nambu Jona-Lasinio Model 2

You can compute the effective potential in the NJL model – the minima are the gap equation solutions for m



Odd terms trace out and the rest resum to the Coleman Weinberg potential

$$\Delta V_{\rm eff} = -\int_0^{\Lambda_{UV}} \frac{d^4 k}{(2\pi)^4} \mathrm{Tr} \log(k^2 + m^2)$$



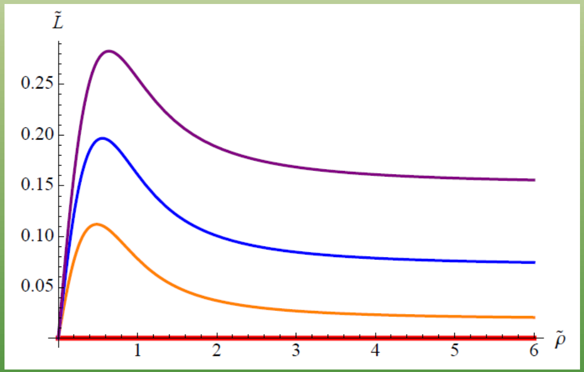

The potential is unbounded… but normally m is not a parameter…

We add the NJL operator

$$\frac{g^2}{\Lambda_{UV}^2} \bar\psi_L \psi_R \bar\psi_R \psi_L \rightarrow \frac{g^2}{\Lambda_{UV}^2} \langle \bar\psi_L \psi_R \rangle \bar\psi_R \psi_L \rightarrow \frac{m^2 \Lambda_{UV}^2}{g^2}$$

The potential is bounded… and for large enough g there is a non-zero m at the minimum.

# Witten's Multi-Trace Operator Prescription 2




We were supposed to impose on whatever embedding we get

$$m = \frac{g^2}{\Lambda_{UV}^2}\sigma$$

Let's make the action do it for us via a UV boundary term

$$S = \int d\rho\, \rho^3 \sqrt{1 + (\partial_\rho L)^2} + \frac{L^2 \Lambda_{UV}^2}{g^2}|_{UV}$$

$$L = m + \frac{\sigma}{\rho^2}$$

$$0 = EL\ eqn\ + \frac{\partial \mathcal{L}}{\partial L'}\delta L|_{UV,IR} + \frac{2L\Lambda_{UV}^2}{g^2}\delta L|_{UV}$$

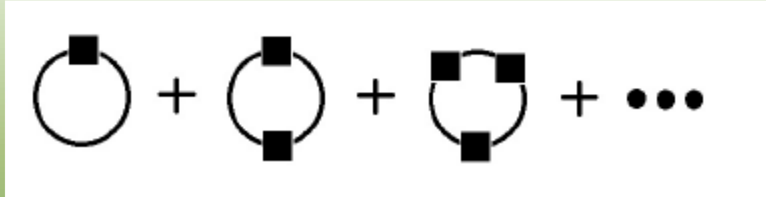
$$\frac{\partial \mathcal{L}}{\partial L'} = -2\sigma = \frac{\rho^3(\partial_\rho L)}{\sqrt{1 + (\partial_\rho L)^2}}$$

which correctly gives

$$\sigma \simeq \frac{m\Lambda_{UV}^2}{g^2}$$

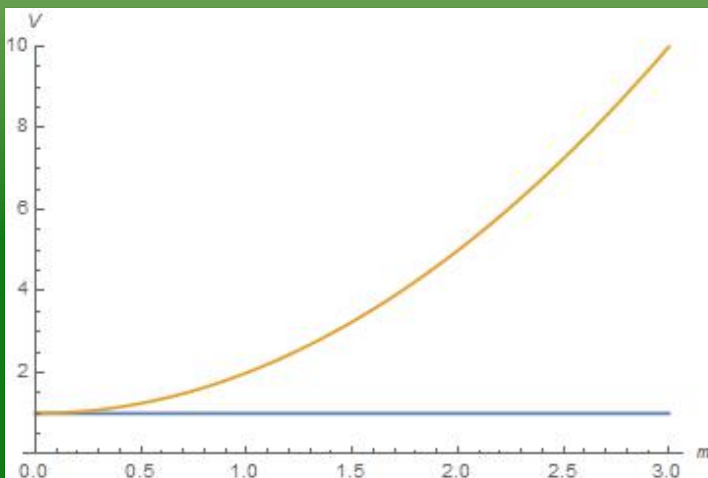
and we've effectively added the same term to the potential as in the NJL model

$$\frac{m^2\Lambda_{UV}^2}{g^2}$$



$$S = \int d\rho\, \rho^3 \sqrt{1 + (\partial_\rho L)^2}$$

evaluated on the solutions L=m (all our embeddings share a common UV divergence in the energy which we subtract)



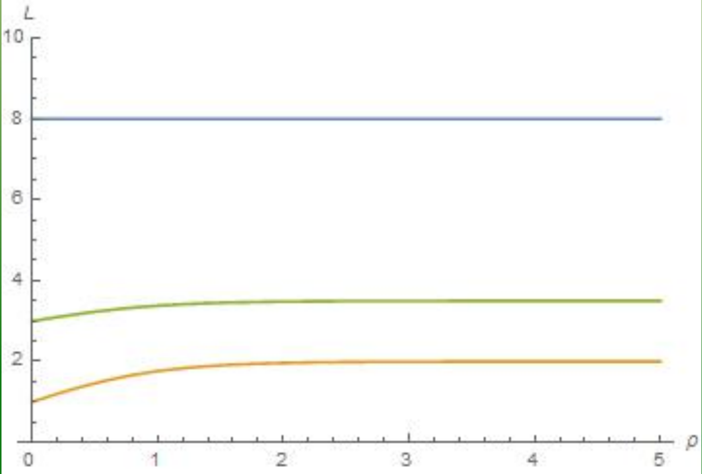

But this underlying piece of the action is therefore m independent in contrast with the basic NJL model unbounded potential – presumably due to the N=2 SUSY – adding the NJL potential term clearly gives m=0…

Conclusion: the Witten prescription looks fine…

the NJL dynamics is as you would expect…

# Breaking SUSY

A little thought suggests any IR susy breaking will reintroduce traditional NJL dynamics

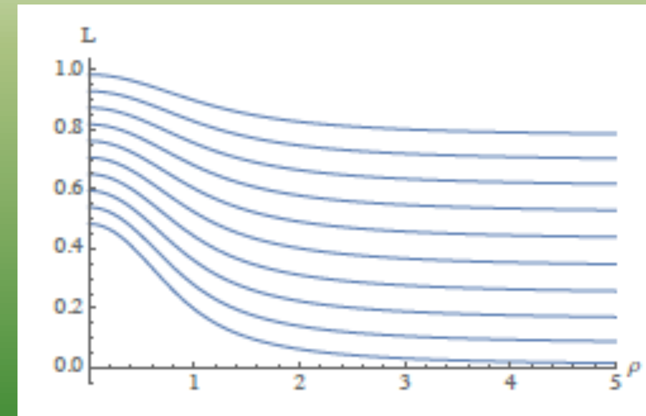



$$S = \int d\rho\, \rho^3 \sqrt{1 + (\partial_\rho L)^2}$$

At large m the embedding doesn't see the IR breaking and the solutions asymptote to the minimal susy action value…. Any dynamics at small m will introduce non-zero derivatives and raise S…

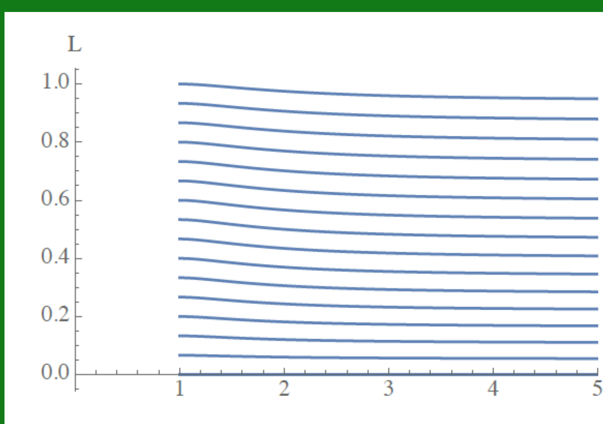
# Controlled Example – B field

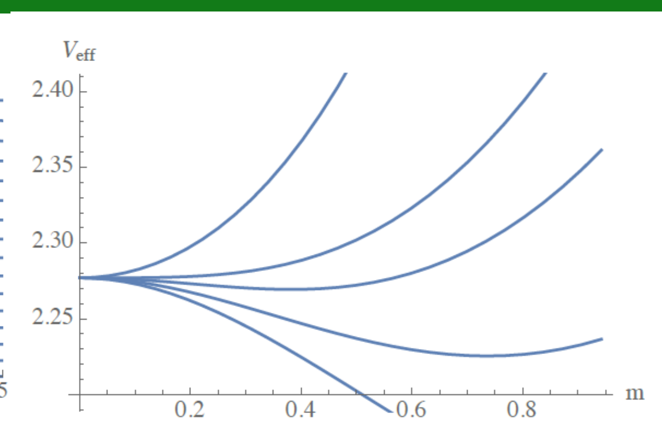
$$\mathcal{L} = \rho^3\sqrt{1+(\partial_\rho w_6)^2}\sqrt{1+\frac{B^z R^4}{r^4}}$$

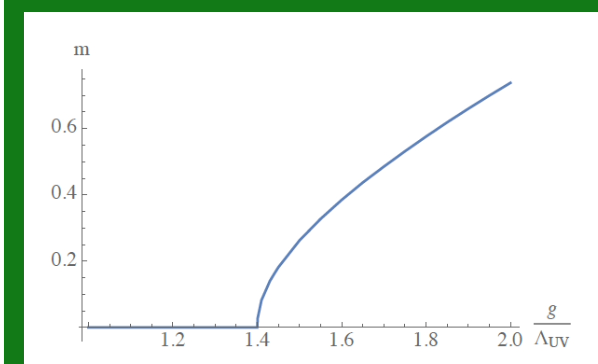



Johnson, Kundu, Filev: the action grows as L approaches r=0 and chiral symmetry breaking is triggered

Initially lets switch off the deep IR so that B doesn't cause $\chi$SBing – impose an IR cut off

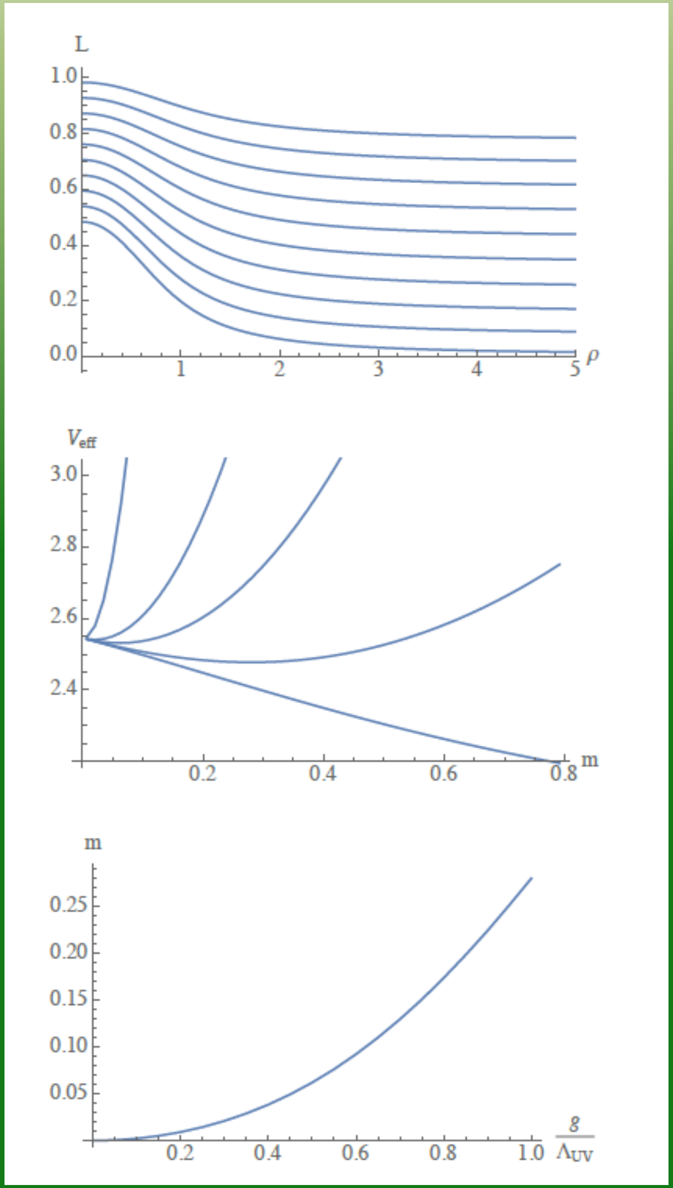



Dynamics looks just like NJL (critical coupling is different)

# B field, no IR Cut Off




Here B triggers a condensate without the NJL term – the NJL term converts it to a mass at any g…

As g grows the NJL dynamics adds to the symmetry breaking and m grows…

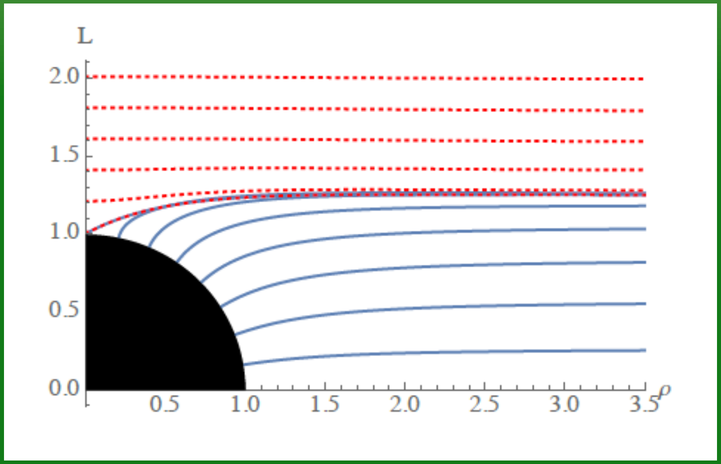
# Controlled Example - Temperature

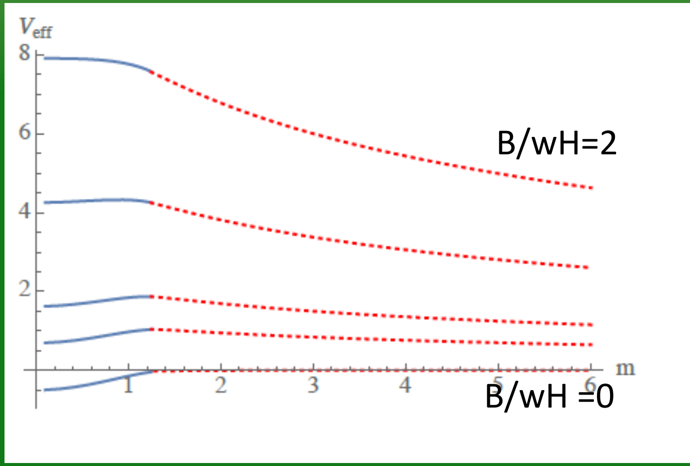
AdS Schwarzschild geometry in r and L coordinates

$$ds^2 = w^2\left(-g_t dt^2 + g_x dx_3^2\right) + \frac{1}{w^2}\left(d\rho^2 + \rho^2\delta\Omega_3^2 + dL^2 + L^2 d\varphi^2\right)$$

$$g_t = \frac{(w^4 - w_H^4)^2}{w^4(w^4 + w_H^4)}, \qquad g_x = \frac{w^4 + w_H^4}{w^4}$$

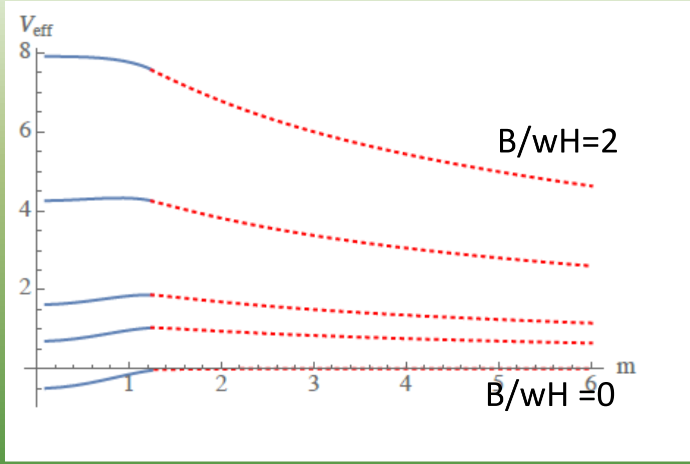
$$\mathcal{L} = \sqrt{\left(1 - \frac{w_H^8}{(\rho^2+L^2)^4}\right)^2 + \frac{B^2}{(\rho^2+L^2)^2}\left(1 - \frac{w_H^4}{(\rho^2+L^2)^2}\right)^2}\,\rho^3\sqrt{1+L'^2}$$



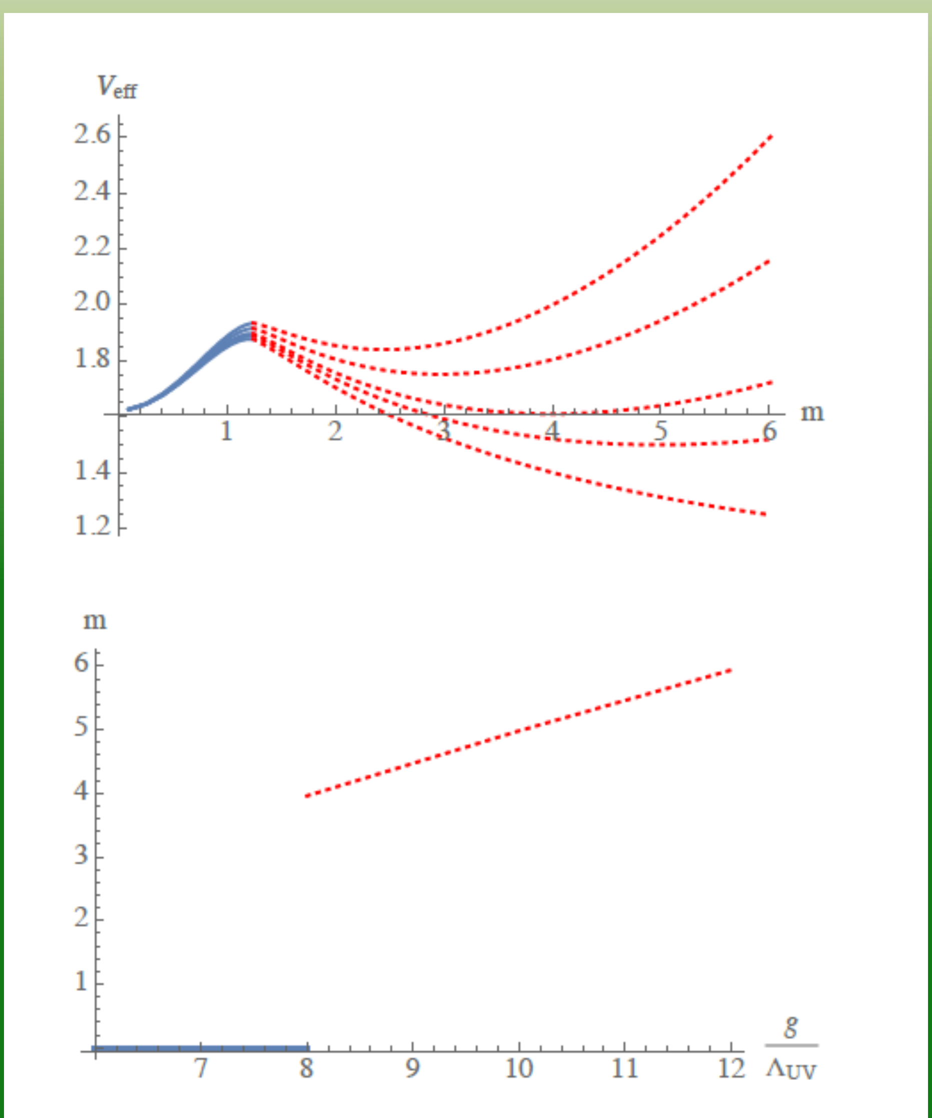




B=0: the black hole “eats” the space and reduces the action of the m=0 embedding.

m=0 is the minimum – adding NJL doesn’t change that – temperature disfavours condensation…




At B/wH=2: there is a first order transition in the NJL coupling from non-zero m to zero m – it is also the meson melting transition.



# Conclusions

- Witten's prescription for the NJL operator is to add a surface term to the potential that matches the expected $\frac{m^2 \Lambda_{UV}^2}{g^2}$
- The N=2 D3/D7 system has no m dependence in its effective potential before the NJL term is introduced so standard NJL behaviour is missing
- With an IR deformation standard NJL behaviour is reproduced
- Temperature blocks gap formation
- Onwards to flavour physics etc