Quantum Hallography

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Current Themes in Holography: Exact Results,
Extensions, Applications and Fundamentals
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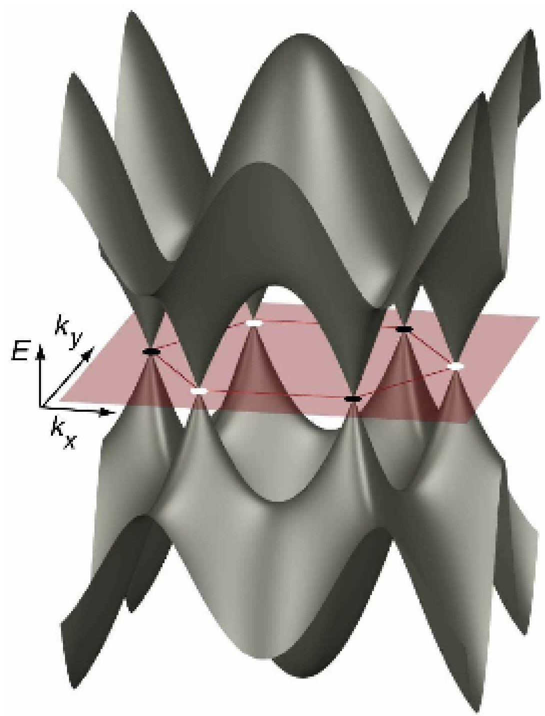
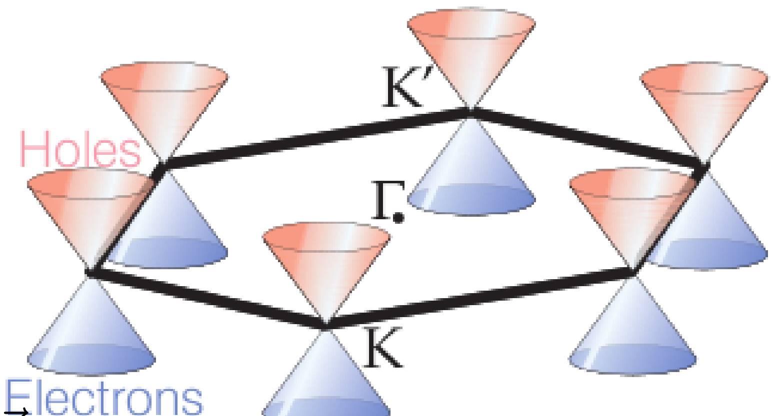
A top-down holographic model of quantum Hall ferromagnetism

1. quantum Hall ferromagnetism (in graphene)
2. defect super-conformal field theory,
   hypermultiplets on 2+1-dim defect,
   $\mathcal{N} = 4$ Yang-Mills in $3 + 1$-dim bulk
3. AdS/CFT with probe D5 brane, oriented to have
   2+1-dimensional Poincare symmetry
   'tHooft limit, probe limit $N_F << N_C = N$, large λ limit
4. super-conformal solution, world-volume is $AdS_4 \otimes S^2$
   deform with $U(1) \subset U(N_F)$ charge density, magnetic field -
   phase diagram
5. mechanism for quantum Hall ferromagnetism, phase diagram

Graphene: relativstic 2+1-D fermions with U(4) symmetry

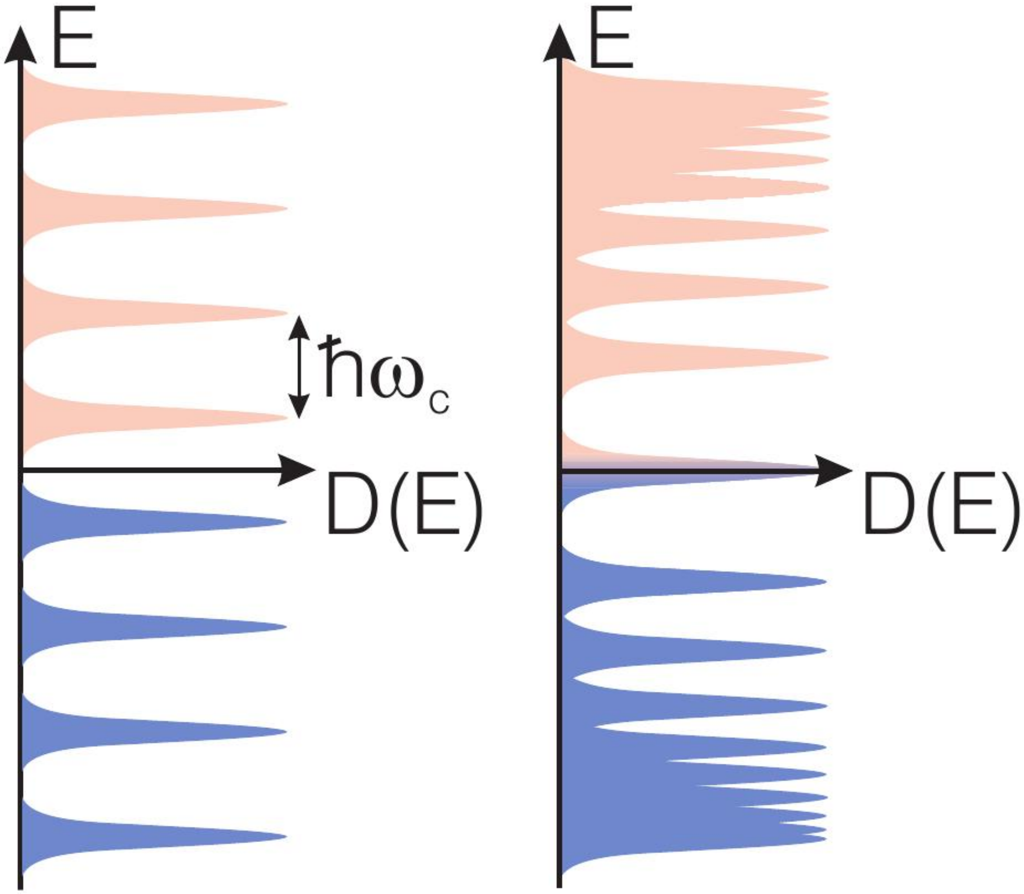




Graphene: $E(k) = \hbar v_F|\vec{k}|$ $v_F \sim c/300$ cutoff $\sim 1ev$, 2 valleys×2 spins

$$S = \int d^3x \sum_{\sigma=1}^{4} \bar{\psi}^\sigma \ i\gamma^\mu \partial_\mu \ \psi^\sigma + \text{interactions}$$

Non-Relativistic and Relativistic Landau Levels

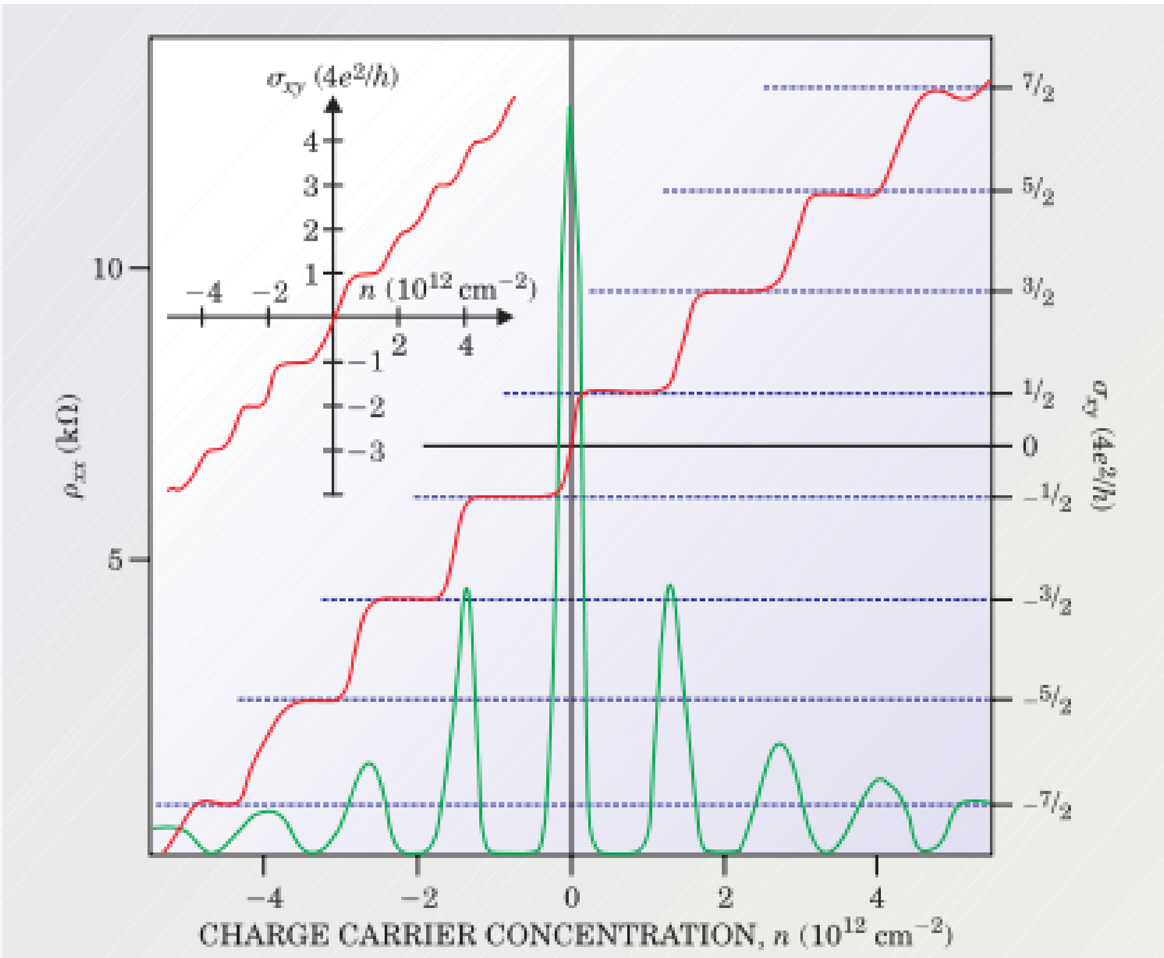


Non-relativistic: $E = \hbar\omega_C \left(n + \frac{1}{2}\right)$, $n = 0, 1, 2, ...$

Relativistic $E = \pm\hbar v_F \sqrt{2|B|n}$ degeneracy $= \frac{e|B|}{2\pi}$

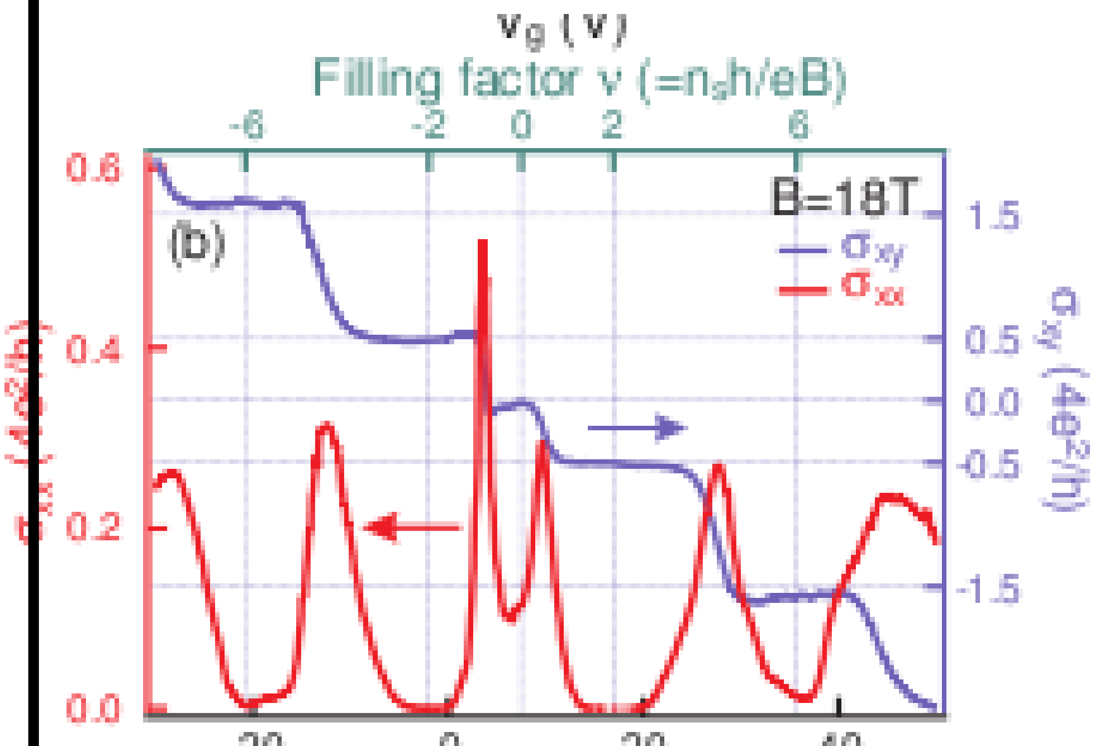
K. Novoselov et. al. *Nature 438, 197 (2005)*

Y. Zhang et. al. *Nature 438, 201 (2005)*



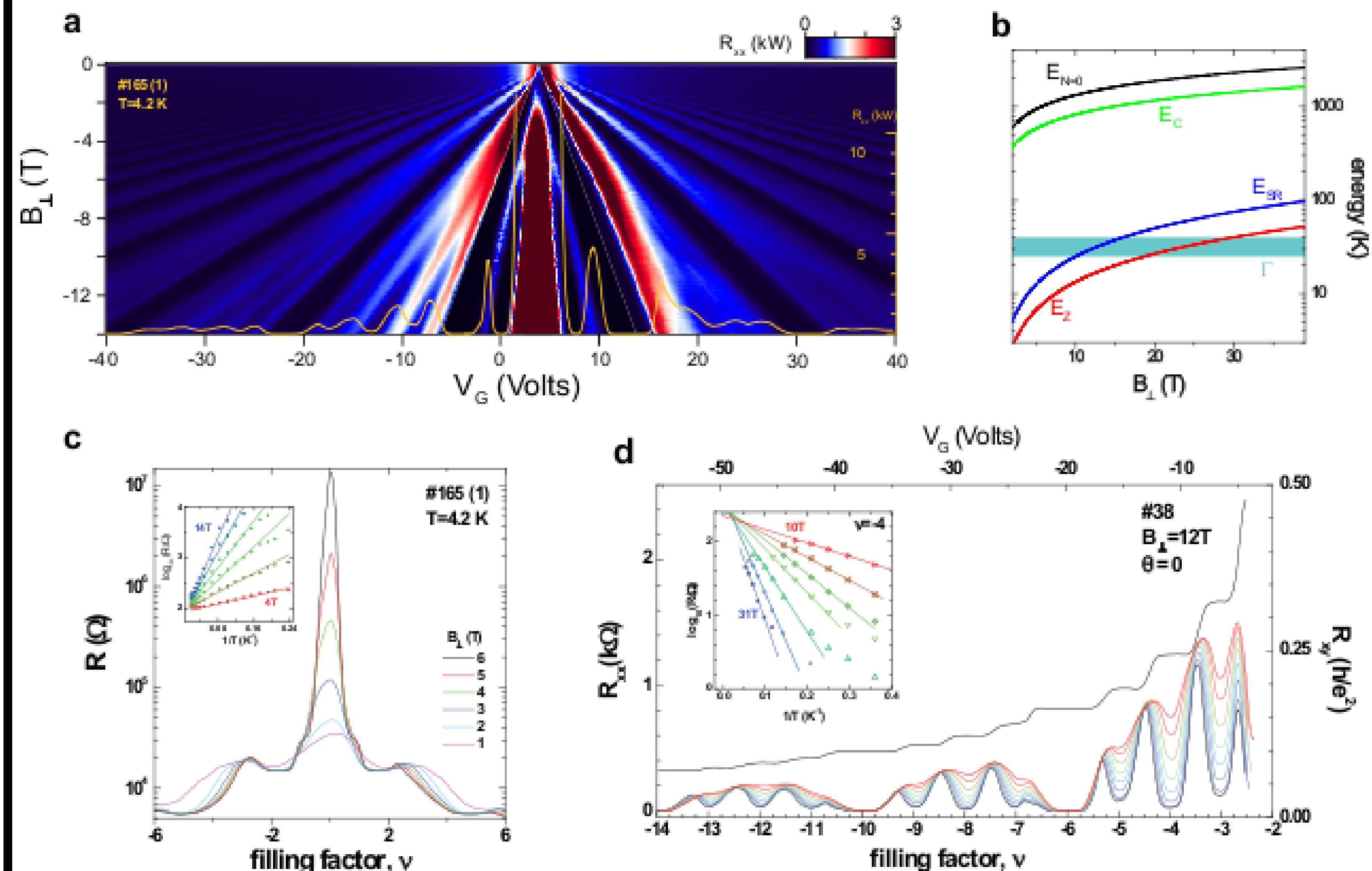
$$\sigma_{xy} = 4\frac{e^2}{h}\left(n + \frac{1}{2}\right)$$

Splitting of $\nu = 0$ Landau level **Zhang et.al. arXiv:1003.2738**



QHE data as a function of the gate voltage V_g, for B = 18 T at T = 0.25 K

Splitting of $\nu = 0$ Landau level A.F.Young et.al., Nat. Phys. 2012

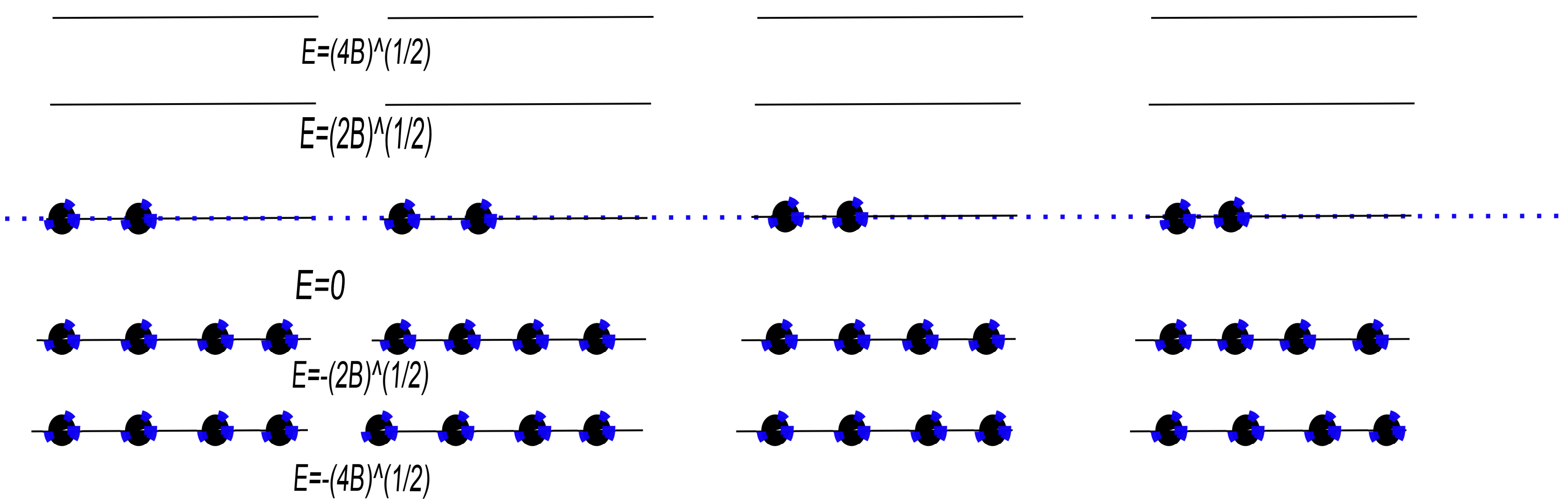


Quantum Hall Ferromagnetism at Weak Coupling

Consider a 4-fold degenerate spectrum of relativistic Landau levels

Ground state has negative energy levels filled

The zero energy states should be half-filled



Quantum Hall Ferromagnetism at Weak Coupling

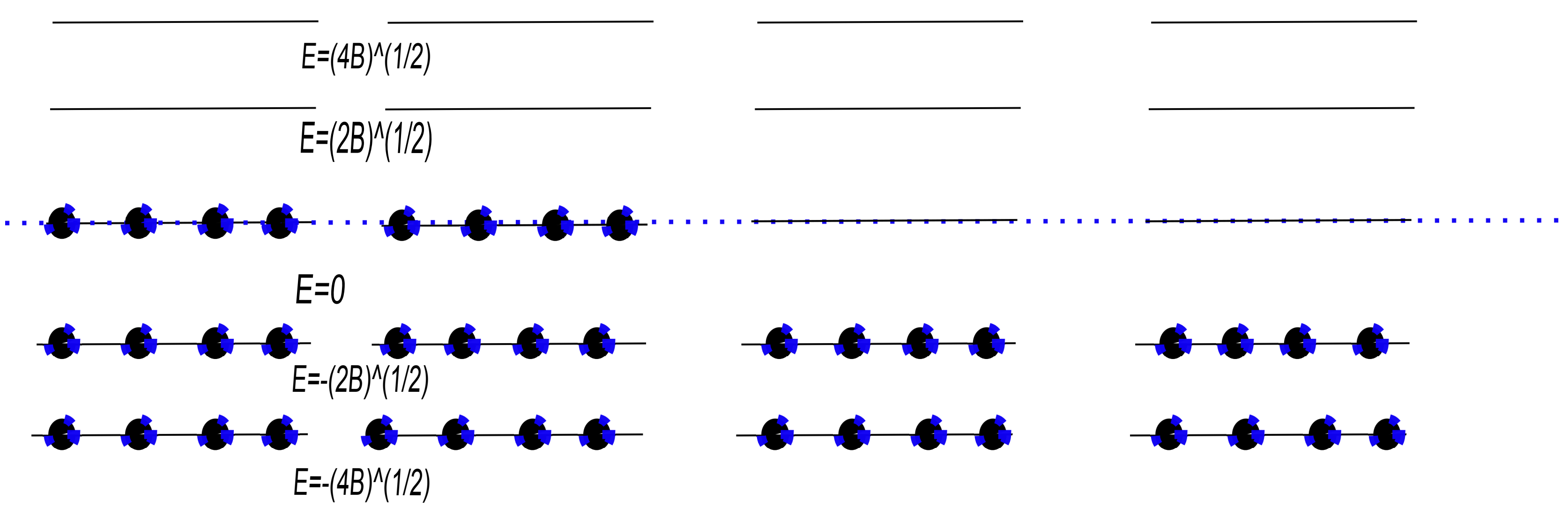
Consider a 4-fold degenerate spectrum of relativistic Landau levels

Ground state has negative energy levels filled

The zero energy states should be half-filled

Highly degenerate ground state

Interaction resolves degeneracy



But, graphene is strongly coupled

Graphene with Coulomb interaction $\quad V(r) = \frac{e^2}{4\pi r}$

$$S = \int d^3x \sum_{k=1}^{4} \bar{\psi}_k \left[\gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A}) \right] \psi_k$$

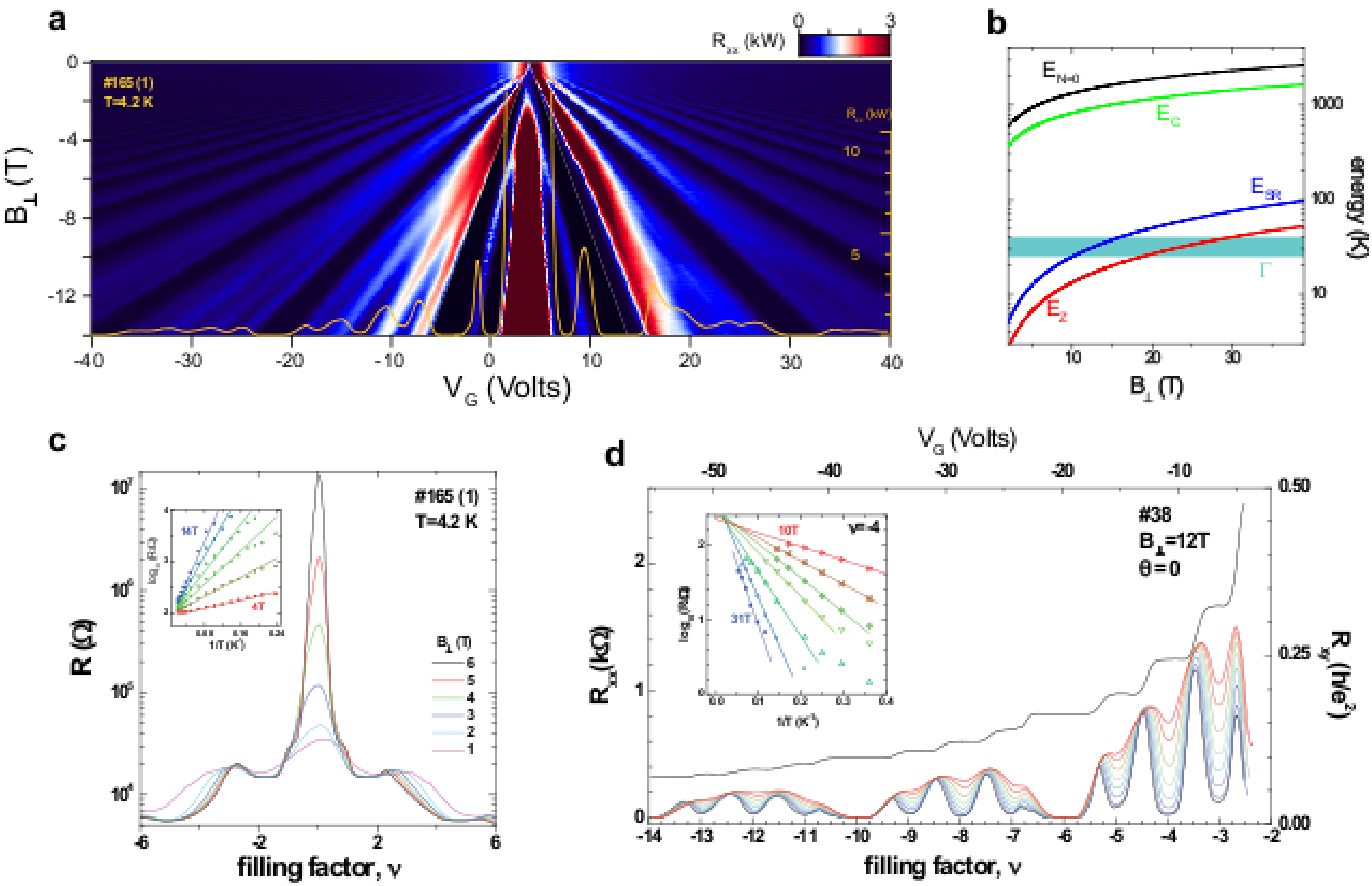
$$+ \frac{1}{4e^2} \int dt d^2x \left[F_{0i} \frac{1}{2\sqrt{\partial_t^2 - c^2\nabla^2}} F_{0i} - F_{ij} \frac{c^2}{2\sqrt{\partial_t^2 - c^2\nabla^2}} F_{ij} \right]$$

- The interaction is not relativistic since speeds of light are different
- This theory is strongly coupled: the graphene fine structure constant is larger than one,

$$\alpha_{\rm graphene} = \frac{\frac{e^2}{4\pi\lambda}}{\hbar v_F/\lambda} = \frac{e^2}{4\pi\hbar v_F} = \frac{e^2}{4\pi\hbar c}\frac{c}{v_F} \approx \frac{300}{137}$$

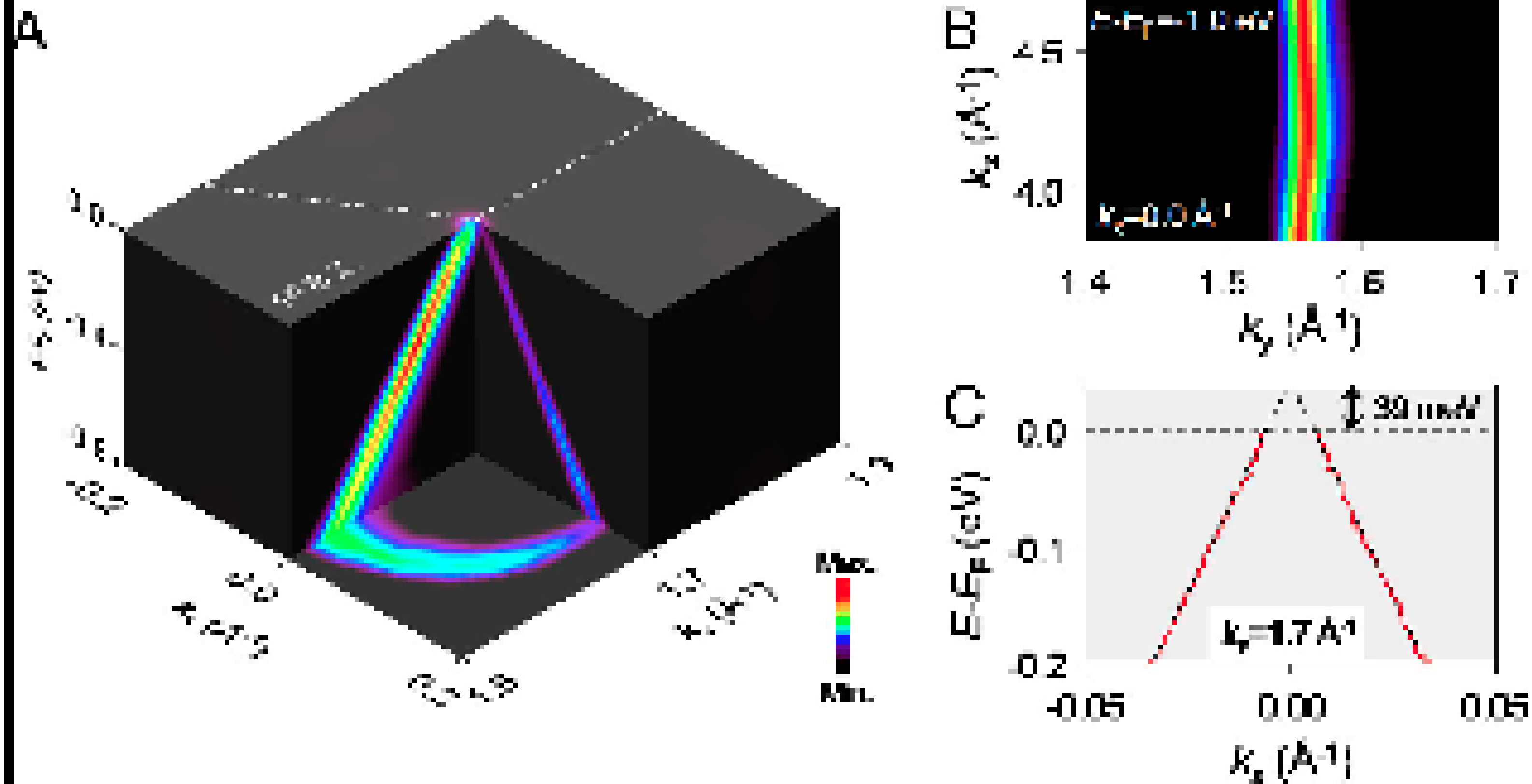
$$\beta_{v_F} = \frac{1}{4}\alpha_{\rm gr} v_F - \left(\frac{1}{3} - (0.03) \right) \alpha_{\rm gr}^2 v_F + \ldots$$

Splitting of $\nu = 0$ Landau level **A.F.Young et.al., Nat. Phys. 2012**



Electron dispersion relation with ARPES

D.A. Siegel et. al. PNAS,1100242108



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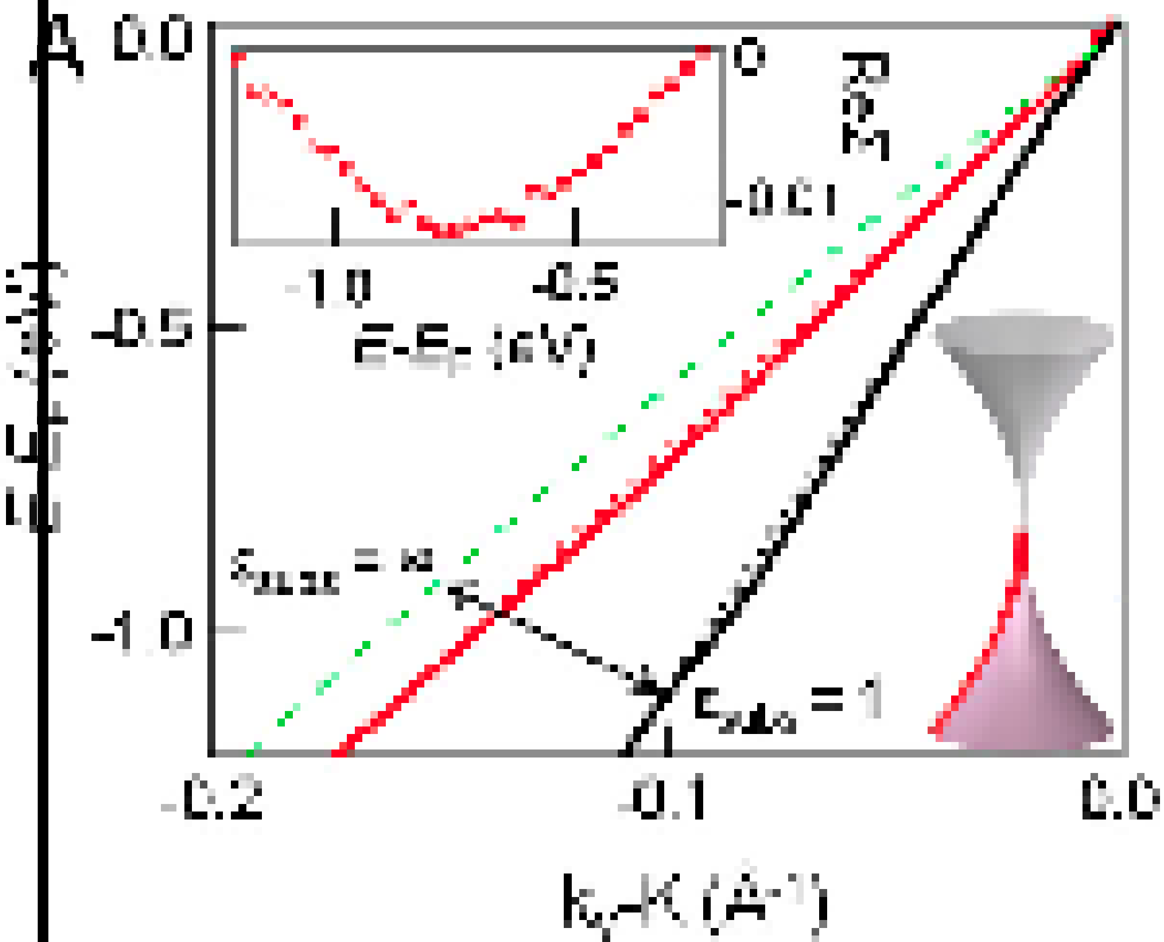


D3 - Probe D5 brane System

- N coincident D3 branes and N_5 coincident D5 branes oriented as

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D5$	X	X	X	O	X	X	X	O	O	O

  brane extends in directions X, sits at point in directions O

- $\#ND = 4$ system – preserves 1/2 of supersymmetries
- 't Hooft limit: $N \to \infty$, $\lambda = 4\pi g_s N$ fixed: D3's $\to AdS_5 \times S^5$
- probe limit $N_5 << N$ embed D5's in $AdS_5 \times S^5$
- flat space $\sim$ strong coupling $R^2 = \sqrt{\lambda}\alpha' >> 1$
- "2DEG" = D3-D5 strings hypermultiplet - fund. reps. of $SU(N)$, $U(N_5)$

Probe D5 brane

- Probe brane geometry from solving Dirac-Born-Infeld action plus Wess-Zumino terms

$$S_5 = N_5 T_5 \int d^6\sigma \left[-\sqrt{-\det(g + 2\pi\alpha' F)} + 2\pi\alpha' F \wedge \omega^{(4)} \right]$$

- $\exists$ a supersymmetric solution with $SO(3) \times SO(3)$ R-symmetry where worldsheet is $AdS_4 \times S^2$,

$$F = 0\ ,\ ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2} + d\Omega_2^2 \right]$$

- $AdS_5 \times S^5$ coordinates and 4-form

$$\frac{dS^2}{\sqrt{\lambda}\alpha'} = r^2(-dt^2+dx^2+dy^2+dz^2)+\frac{dr^2}{r^2}+d\psi^2+\sin^2\psi d^2\Omega_2+\cos^2\psi d^2\tilde{\Omega}_2$$

$$\omega^{(4)} = \lambda\alpha'^2 r^4 dt \wedge dx \wedge dy \wedge dz + \lambda\alpha'^2 \frac{c(\psi)}{2} d\Omega_2 \wedge d\tilde{\Omega}_2$$

Dual to superconformal defect field theory

- Field theory dual is bulk $\mathcal{N} = 4$ Yang-Mills plus a hypermultiplet defect theory with SO(3)×SO(3) R-symmetry
  O.DeWolfe D.Z.Freedman H.Ooguri hep-th/0111135
  J.Erdmenger Z.Guralnik I.Kirsch hep-th/0203020

$$S = \int d^4x \left\{ -\frac{1}{2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} + \ldots \right\}$$
$$+ \int d^3x \sum_{\sigma=1}^{N_5} \sum_{\alpha=1}^{N} \left[\bar{\psi}^\sigma_\alpha i\gamma^\mu \partial_\mu \psi^\sigma_\alpha + \partial_\mu \bar{\varphi}^\sigma_\alpha \partial^\mu \varphi^\sigma_\alpha \right] + \text{interactions}$$

- Fermion ψ, scalar φ are SO(3) spinors (with different SO(3)'s), fundamental rep. of global $U(N_5)$ and fundamental rep. of SU(N) gauge group.

- Holographic description introduces temperature T, $U(1) \subset U(N_5)$ charge density ρ, magnetic field B

Weak Coupling

$$S = \int d^3x \sum_{\sigma=1}^{N_5} \sum_{\alpha=1}^{N} \left[\bar{\psi}_\alpha^\sigma i\gamma^\mu D_\mu \psi_\alpha^\sigma + D_\mu \bar{\varphi}_\alpha^\sigma D^\mu \varphi_\alpha^\sigma \right] + \text{interactions}$$

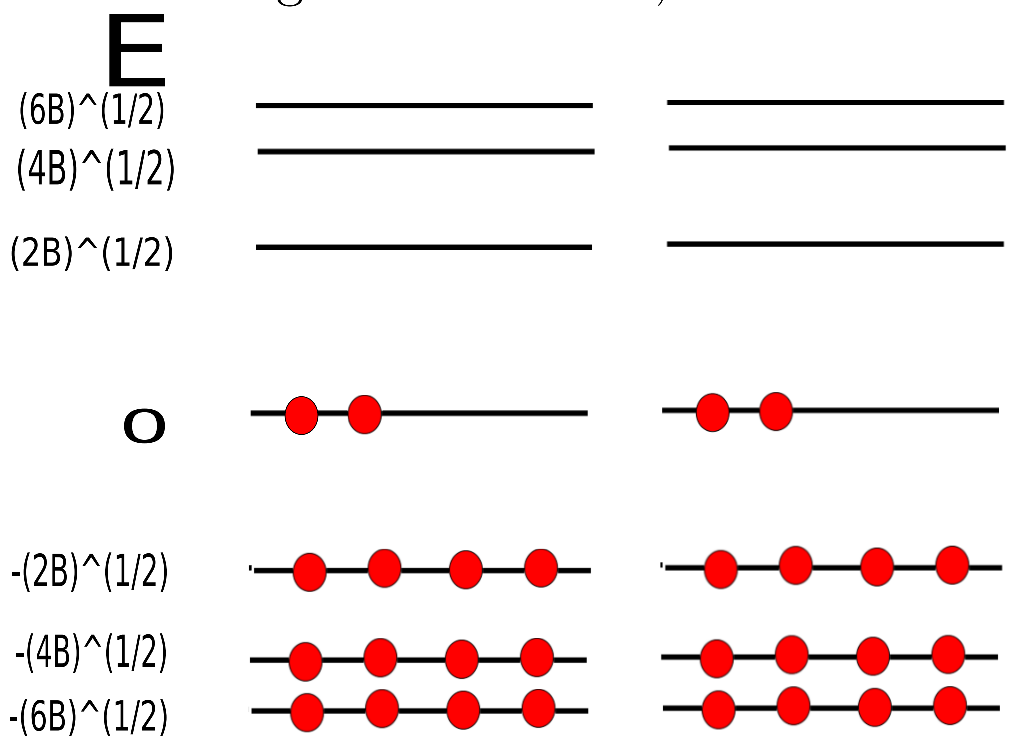
External Magnetic field

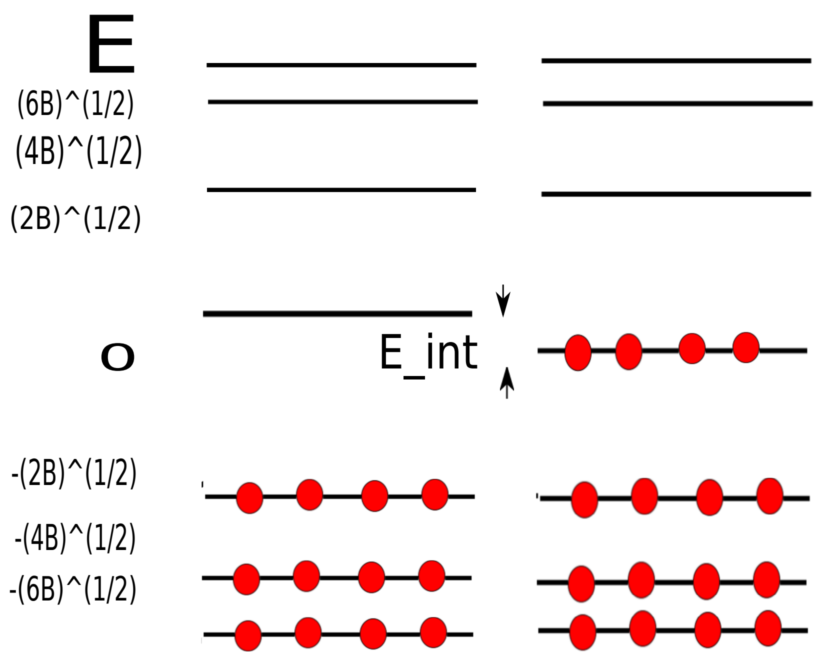
- $D_\mu = \partial_\mu + iA_\mu$ with a background magnetic field $\vec{\nabla} \times \vec{A} = B$
- Landau levels
  - Fermions $E_n = \sqrt{2Bn}$
  - Boson $\omega_n = \sqrt{(2n+1)B}$
  - $n = 0, 1, 2, ...$; Landau level density is $\frac{B}{2\pi} \cdot 2 \cdot N \cdot N_5$
- There exist $\frac{B}{2\pi} 2NN_5$ fermion zero modes.
- The lowest energy non-zero modes are scalars.

Landau levels

$N_5 = 1$, fermion is an $SO(3)$ doublet

In the charge neutral state, half of the zero modes are filled




E
(6B)^(1/2)
(4B)^(1/2)
(2B)^(1/2)
o
E_int
-(2B)^(1/2)
-(4B)^(1/2)
-(6B)^(1/2)

Hall States

The gapped states have charge densities and Hall conductivities

$$\rho = \frac{B}{2\pi} N\cdot(0, \pm 1, \pm 2, \ldots, \pm N_5) \ , \ \ \sigma_{xy} = N\cdot(0, \pm 1, \pm 2, \pm 3, \ldots, \pm N_5)$$

or filling fractions

$$\nu \equiv \frac{2\pi}{N}\frac{\rho}{B} \ = \ 0, \pm 1, \pm 2, \ldots, \pm N_5$$

All other quantum Hall states are beyond the threshold for creating scalars.

Do the quantum Hall states survive when we turn on the coupling? Do they survive at strong coupling?

Probe D5 brane with a magnetic field

- Introduce a magnetic field B $(m = 0, \rho = 0, T = 0)$
- **V.Filev C.Johnson J.Shock arXiv:0903.5345**
  For any B, D5-brane is no longer $AdS_4 \times S^2$

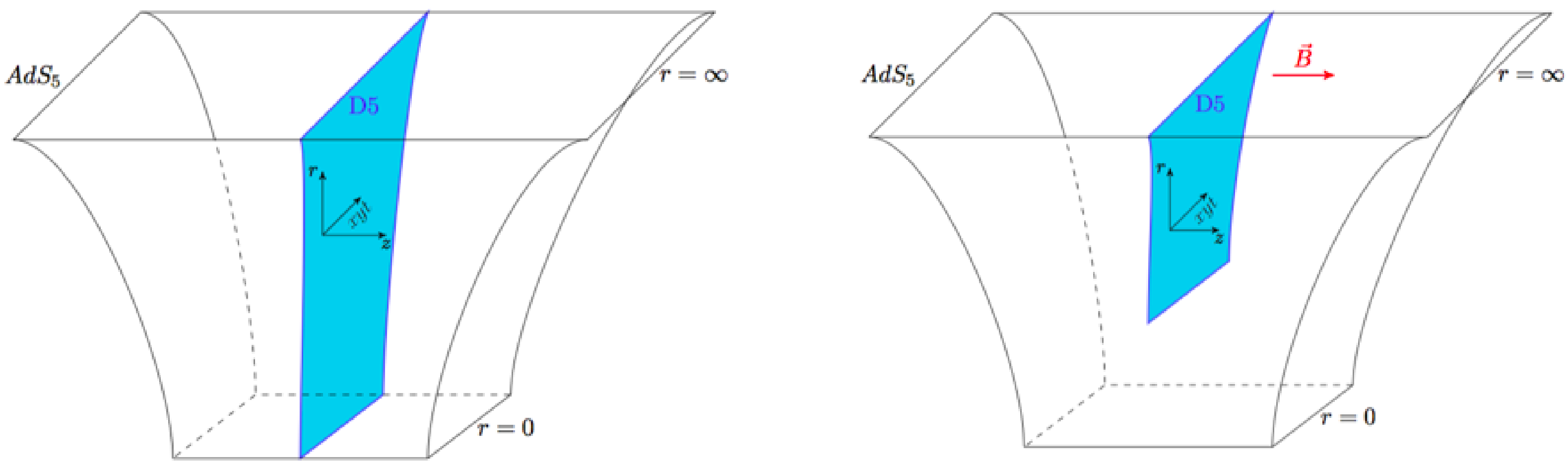
$$F = B dx \wedge dy$$

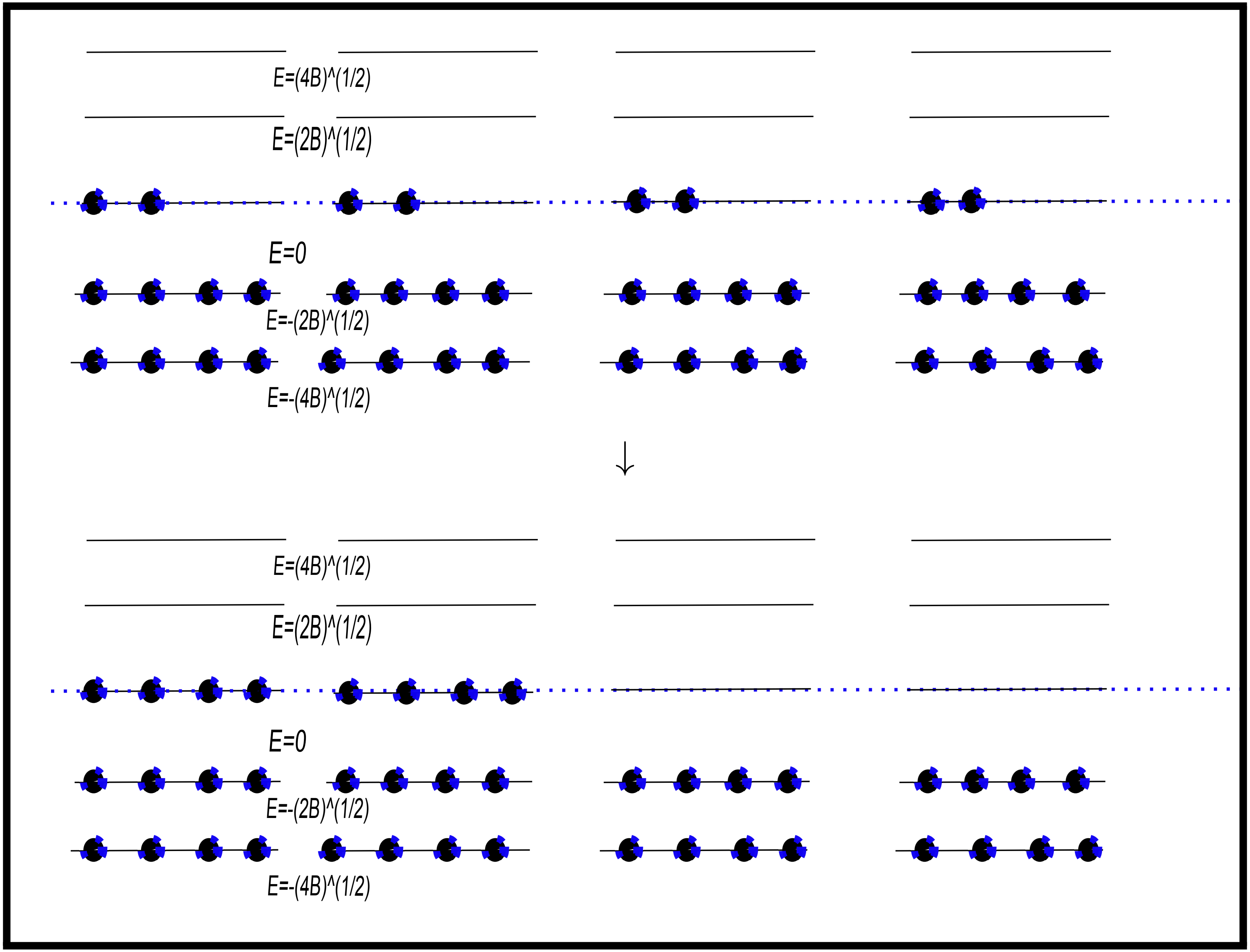
$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}(1 + r^2\dot{\psi}(r)^2) + \sin^2\psi(r) d\Omega_2^2 \right]$$

$$\psi(r \to \infty) = \frac{\pi}{2} + \frac{m=0}{r} + \frac{<\bar{\psi}\vec{\tau}\psi>}{r^2} + \ldots \quad , \quad \psi(r = r_0) = 0$$

- Mass gap for D3-D5 strings
- Spontaneously broken SO(3) chiral symmetry for any nonzero magnetic field (at zero temperature and density).
- Quantum Hall Ferromagnetism/Magnetic catalysis at strong coupling, $\rho = 0$.

D5 brane & magnetic field breaks chiral symmetry




E=(4B)^(1/2)
E=(2B)^(1/2)
E=0
E=-(2B)^(1/2)
E=-(4B)^(1/2)
E=(4B)^(1/2)
E=(2B)^(1/2)
E=0
E=-(2B)^(1/2)
E=-(4B)^(1/2)

Probe D5 brane with a magnetic field and density

- Introduce a magnetic field B and density ρ $(m = 0, T = 0)$

$$F = A_t'(r)dr \wedge dt + Bdx \wedge dy$$

$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}(1 + r^2\dot{\psi}(r)^2) + \sin^2\psi(r)d\Omega_2^2 \right]$$

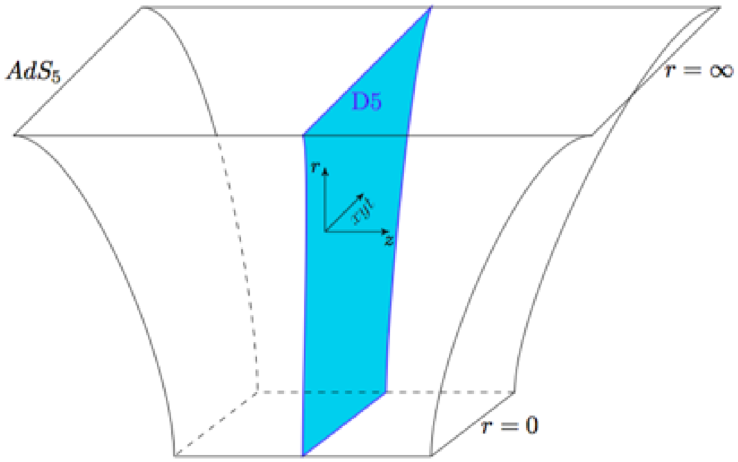
$$\psi(r \to \infty) = \frac{\pi}{2} + \frac{m = 0}{r} + \frac{< \bar{\psi}\vec{\tau}\psi >}{r^2} + \ldots$$

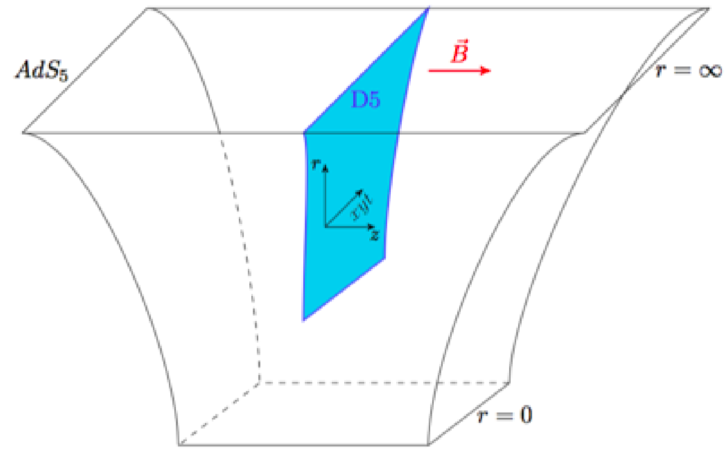
- Probe D5 must reach Poincare horizon at $r = 0 \to$ **all finite density states are ungapped (compressible)**.

- Chiral symmetry restored at critical density
K.Jensen A.Karch D.T.Son E.G.Thompson arXiv:1002.3159

$$\nu \equiv \frac{2\pi\rho}{NB} \quad , \quad \nu_{\text{crit.}} = 1.68 N_5/\sqrt{\lambda}$$

D5 brane & magnetic field breaks chiral symmetry





Hall states of the D5 brane

- As N_5 D5 branes enter the bulk of AdS_5, they blow up to D7 brane with magnetic flux

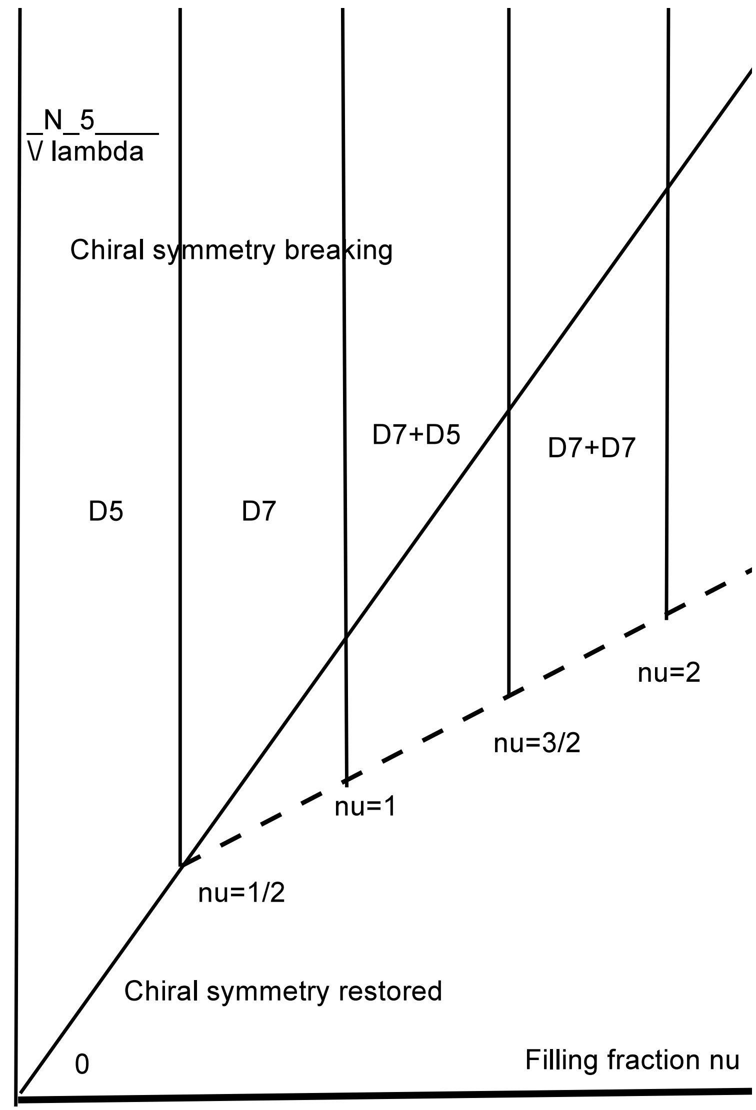
$$S_7 = T_7 \int d^8\sigma \left[-\sqrt{-\det(g + 2\pi\alpha' F)} + \frac{(2\pi\alpha')^2}{2} F \wedge F \wedge c^{(4)} \right]$$

$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}\left(1 + r^2\psi'(r)^2\right) + \right.$$
$$\left. + \sin^2\psi d^2\Omega_2 + \cos^2\psi d^2\tilde{\Omega}_2 \right]$$

$$F = \frac{d}{dr} A_t^7(r) dr \wedge dt + B dx \wedge dy + \frac{N_5}{2} d\tilde{\Omega}_2 \qquad (1)$$

- $F \wedge F \wedge c^{(4)}(r)$ term in D7 brane action dissolves electric charge – completely only when $\nu = 1$
- For $\nu = 1$, D7 brane has Minkowski embedding and incompressible charge gapped state.

- For $\nu = 2, 3, ..., N_5$, N_5 D5 branes blow up into $2, 3, ..., N_5$ D7 branes, each with $\nu = 1$ which subsequently have Minkowski embeddings and incompressible charge gapped state.
- How many of the states $\nu = 0, \pm 1, \pm 2, ..., \pm N_5$ are stable still open question.


_N_5____
\/ lambda
Chiral symmetry breaking
D7+D5
D7+D7
D5
D7
nu=2
nu=3/2
nu=1
nu=1/2
Chiral symmetry restored
0
Filling fraction nu

Conclusions

- $\exists$ integer Hall states of the D5 brane
- qualitative comparison with weak coupling is surprisingly good
- Symmetry breaking pattern,
  D5: $SU(2) \times SU(2) \rightarrow U(1) \times SU(2)$ breaks valley symmetry $\prod_n \psi^\dagger_{n,1\uparrow}\psi^\dagger_{n,1\downarrow}|0>$
  D7: $SU(2) \times SU(2) \rightarrow SU(2)$ preserves locked valley and spin symmetry $\prod_n [\psi^\dagger_{n,1\uparrow} - \psi^\dagger_{n,2\downarrow}]|0>$
- When ν divides N_5, the Hall state has ν identical D7's $\rightarrow$ SU(ν) symmetry