Wilson loops in ABJ(M): from localization to Bremsstrahlung function through framing

M.S. Bianchi, L. Griguolo, M. Leoni, A. Mauri, S.P., D. Seminara,
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Why BPS Wilson loops?

BPS Wilson Loops in supersymmetric gauge theories

- They are in general non protected operators and their expectation value can be computed exactly by using **localization techniques**.
- They correspond in general to fundamental strings and their expectation value at strong coupling is given by the exponential of a minimal area surface ending on the WL countour. Matching with localization results provides a crucial test of the AdS/CFT correspondence.

The prototype being the 1/2 BPS circular WL in N=4 SYM

$$WL = \mathrm{Tr} P e^{-i \int_\Gamma d\tau (\dot{x}^\mu A_\mu + i |\dot{x}| \theta_I \Phi^I)}$$

It includes scalar couplings.

(Maldacena, PRL80 (1998) 4859)

(Drukker, Gross, Ooguri, PRD60 (1999) 125006)

Why BPS WL in 3D CS-matter theories?

BPS WL in **3D susy CS-matter theories**. They exhibit a richer spectrum of interesting properties compared to the 4D case. Among them:

- Due to dimensional reasons also fermions together with scalars can enter the definition of BPS WL. In general they increase the number of susy charges preserved by WL. Therefore, we have a richer spectrum of BPS WL.

- Framing factors appear as overall complex phases in localization results for $\langle WL \rangle$.

Why framing?

Framing seems to be an important but still not completely understood ingredient of BPS WL in 3D

It originates in topological CS theory as a topological property, but it seems to survive in CS-matter theories though they are not topological.

It is important to understand:

- the origin of framing in non-topological theories.
- How to identify it in localization results.
- How to give it a physical interpretation.

Plan of the talk

- **What is framing? Brief review.**
- **1/6 BPS Wilson loops in ABJ(M) model: framing at three loops. Solution of a puzzle in localization.**
- **New interpretation in terms of the Bremsstrahlung function.**
- **Conclusions and Perspectives**

Pure CS theory

For the $U(N)_k$ pure Chern–Simons theory (topological theory)

$$S_{CS} = -i\frac{k}{4\pi}\int d^3x\,\varepsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho + \tfrac{2}{3}iA_\mu A_\nu A_\rho\right)$$

On a closed path Γ and in fundamental representation

$$\begin{aligned}\langle \mathcal{W}_{\rm CS}\rangle &= \langle \mathrm{Tr}\,P\,e^{-i\int_\Gamma dx^\mu A_\mu(x)}\rangle \\ &= \sum_{n=0}^{+\infty}\mathrm{Tr}P\int dx_1^{\mu_1}\cdots dx_n^{\mu_n}\,\langle A_{\mu_1}(x_1)\cdots A_{\mu_n}(x_n)\rangle\end{aligned}$$

1) either by using semiclassical methods in the large k limit

(Witten, CMP121 (1989) 351)

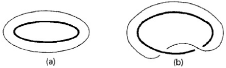
2) or perturbatively (n-pt correlation functions)

(Guadagnini, Martellini, Mintchev, NPB330 (1990) 575)

Regularize singularities in $\langle A_{\mu_1}(x_1)\cdots A_{\mu_n}(x_n)\rangle$ at coincident points.

Using point-splitting regularization

$$\Gamma_f : \quad y^\mu(\tau) \to y^\mu(\tau) + \epsilon\, n^\mu(\tau)$$



$$\lim_{\epsilon\to 0} \oint_\Gamma dx^\mu \oint_{\Gamma_f} dy^\nu \,\langle A_\mu(x) A_\nu(y)\rangle = -i\pi\lambda\,\chi(\Gamma,\Gamma_f) \qquad \lambda = \frac{N}{k}$$

$$\chi(\Gamma,\Gamma_f) = \frac{1}{4\pi}\oint_\Gamma dx^\mu \oint_{\Gamma_f} dy^\nu \; \varepsilon_{\mu\nu\rho}\frac{(x-y)^\rho}{|x-y|^3}$$ Gauss linking number

Higher-order contributions exponentiate the one–loop result

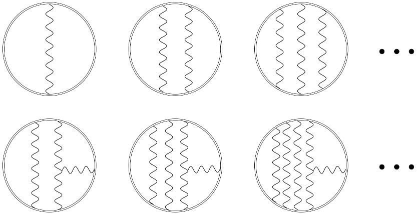
$$\langle \mathcal{W}_{\rm CS}\rangle = \underbrace{e^{-i\pi\lambda\chi(\Gamma,\Gamma_f)}}_{\text{framing factor}}\,\rho(\Gamma)$$

Exponentiation of one–loop framing term relies on the following distinguishing properties (Alvarez, Labastida, NPB395 (1993) 198)

1. The gauge propagator is one–loop exact

$$\langle A_\mu^a(x) A_\nu^b(y) \rangle = \delta^{ab} \frac{i}{2k} \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3}$$

2. Only diagrams with collapsible propagators contribute to framing



3. Factorization theorem

$\mathcal{N} = 2$ susy CS theory

We are primarily interested in supersymmetric theories for which localization can be used.

$$\langle \mathcal{W}_{\rm SCS} \rangle = \langle {\rm Tr}\, P\, e^{-i \int_\Gamma d\tau (\dot{x}^\mu A_\mu(x) - i|\dot{x}|\sigma)} \rangle$$

(Kapustin, Willett, Yaakov, JHEP 1003 (2010) 089)

Localization always provides the result at framing $\chi(\Gamma, \Gamma_f) = -1$. This follows from requiring consistency between point–splitting regularization and supersymmetry used to localize: The only point-splitting compatible with susy is the one where the contour and its frame wrap two different Hopf fibers of S^3

Localization is sensible to framing!

Adding matter

What happens when we couple $\mathcal{N}=2$ CS multiplet to chiral matter?

$\mathcal{N}=6$ **susy ABJ(M) model** for $U(N_1)_k \times U(N_2)_{-k}$ CS-gauge vectors minimally coupled to

$SU(4)$ complex scalars C_I, $\bar{C}^I$ and fermions ψ_I, $\bar{\psi}^I$

in the (anti)bifundamental representation of the gauge group with non-trivial potential.

$$\langle \mathcal{W} \rangle = \langle \mathrm{Tr} P \exp \left[-i \int_\Gamma d\tau (A_\mu \dot{x}^\mu - \tfrac{2\pi i}{k} |\dot{x}| M_J{}^I C_I \bar{C}^J) \right] \rangle$$

For $M_I{}^J = \mathrm{diag}(+1,+1,-1,-1) \quad \longrightarrow \quad$ **1/6-BPS Wilson loop**

(Drukker, Plefka, Young, JHEP 0811 (2008) 019
Chen, Wu, NPB 825 (2010) 38, Rey, Suyama, Yamaguchi, JHEP 0903 (2009))

Localization result for ABJ(M)

- Using localization, $\langle \mathcal{W} \rangle$ reduces to a non-gaussian Matrix Model that can be computed exactly (Drukker, Marino, Putrov, CMP 306 (2011))

Weak coupling expansion and **planar limit** $(\lambda_1 = N_1/k, \lambda_2 = N_2/k \ll 1)$

$$\langle \mathcal{W} \rangle = \underbrace{e^{i\pi\lambda_1}} \left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) \underbrace{-i\frac{\pi^3}{2}\lambda_1\lambda_2^2} + \mathcal{O}(\lambda^4)\right)$$

$\Downarrow$ pure CS framing (-1) factor

$\Downarrow$ extra imaginary term **???**

- Using ordinary perturbation theory (framing $= 0$) $\rightarrow$ no contributions at odd orders (Rey, Suyama, Yamaguchi, JHEP 0903 (2009))

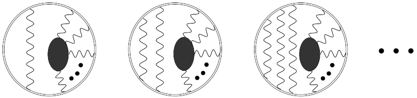
Conjecture: Matter contributes to framing

PROOF $\rightarrow$ **perturbative 3-loop calculation at framing (-1)**

Matter contributes in two different ways

1) Matter gives non-trivial corrections to the the gauge propagator (FINITE at two loops)

$$\langle A_\mu(x) A_\nu(y) \rangle \to \frac{i}{2k}\left[1 - \frac{\pi^2}{2}\left(\underbrace{\lambda_2^2} + \underbrace{\lambda_1\lambda_2\left(\frac{1}{4}+\frac{2}{\pi^2}\right)}\right)\right]\varepsilon_{\mu\nu\rho}\frac{(x-y)^\rho}{|x-y|^3}$$

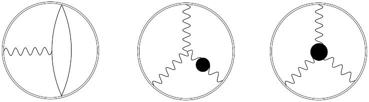


Exponentiation still works, so we can write

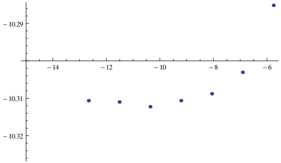
$$\langle \mathcal{W} \rangle = \underbrace{e^{i\pi\left(\lambda_1 - \frac{\pi^2}{2}\lambda_1\lambda_2^2 + \mathcal{O}(\lambda^5)\right)}}\left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) + \mathcal{O}(\lambda^4)\right)$$

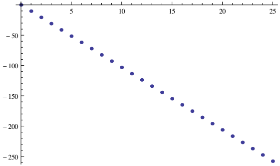
framing function

2) Lower-transcendentality term needs to be cancelled by some other comtribution. Therefore, not only collapsible propagators contribute, also matter vertex-like diagrams



Numerical investigation





Bremsstrahlung function

In 4D N=4 SYM BPS WL depending on some deformation parameters can be used to compute the Bremsstrahlung function

$$\Delta E = 2\pi B \int dt(\dot{v})^2 \qquad \Gamma_{cusp}(\phi,\theta) \sim B(\lambda)(\theta^2 - \phi^2)$$

Given a "θ-latitude" Wilson loop $B(\lambda) = -\frac{1}{4\pi^2}\partial_\theta^2 \log \langle W(\lambda,\theta)\rangle|_{\theta=0}$

(Correa, Henn, Maldacena, Sever, JHEP 1206 (2012) 048)

(Fiol, Garolera, Lewkowycz, JHEP 1205 (2012) 093)

Conjecture for ABJM (Bianchi, Griguolo, Leoni, SP, Seminara, JHEP 1406 (2014) 123)

$$B_{1/2}(\lambda) = \frac{1}{8\pi}\,\mathrm{tg}\,\Phi_{1/6}(\lambda) \qquad \langle \mathcal{W}\rangle = e^{i\pi\Phi_{1/6}(\lambda)}\rho(\lambda)$$

Framing function !!

New physical interpretation of the framing phase for non–topological 3D models.

Conclusions and Perspectives

We have identified higher-loop framing terms in the Matrix Model result for the 1/6-BPS Wilson loop in ABJ(M) and related it to the Bremsstrahlung function. As a consequence, $B(\lambda)$ is given in terms of an odd power series.

What's next:

- Better understand contributions from vertex-like diagrams.
- What happens at higher orders? Divergences?
- Better understand the relation between the fermionic cusp and the bosonic framing.
- Framing in fermionic 1/2-BPS Wilson loops.
- Framing in less supersymmetric CS-matter theories. $\mathcal{N} = 4$ case is under investigation.
- Framing at strong coupling?