Topology of the Electroweak Vacua

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BMG & Oscar Randal-Williams, 1610.05623

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...but which $U(1) < SU(2) \times U(1)$?!

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... for others, it does!

e.g. Every choice not homeomorphic to the SM one has stable EW string solutions!

Main results

- $\pi_1(SU(2) \times U(1)/U(1)) = \mathbb{Z}/p$ for some p
- Measure p using astro/cosmo/collider expts
- $p \neq 1$ is a smoking gun for BSM
- $\exists$ a consistent NLSM $\forall\, p$
- $\implies p$ is an unconstrained parameter of BSM EFTs
- $\exists$ plausible UV-complete(r) models with $p \neq 1$

Outline

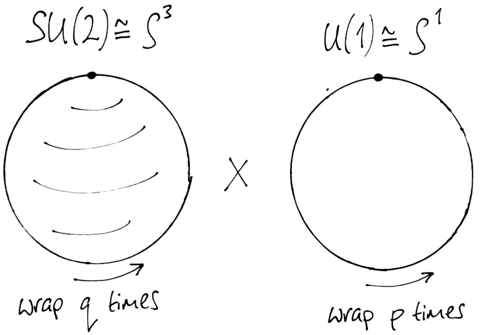
- Topology of $SU(2) \times U(1)/U(1)$
- Electroweak strings
- Non-linear Sigma Models
- Linear Sigma Models
- Composite Higgs Models

Topology of $SU(2) \times U(1)/U(1)$

Recall that

- $\pi_2 \neq 0 \implies$ stable monopole solutions
- $\pi_1 \neq 0 \implies$ stable string solutions

cf. e.g., Weinberg, QFT



$$U(1) \cong H_{p,q} = \{(\mathrm{diag}(z^q, z^{-q}), z^p)\} < \{(U, z)\} = SU(2) \times U(1)$$

p, q coprime w.l.o.g.

Algebraic topology for dummies I

Homogeneous space $G/H_{p,q}$ is a principal fibre bundle

- $H_{p,q} \overset{i}{\hookrightarrow} G \overset{\pi}{\to} G/H_{p,q}$

with an associated long exact sequence $(\ker = \operatorname{im})$ of homotopy groups

- $\ldots \pi_1(H_{p,q}) \to \pi_1(G) \to \pi_1(G/H_{p,q}) \to \pi_0(H_{p,q}) \ldots$

Su doku

- $\mathbb{Z} \overset{i}{\to} \mathbb{Z} \overset{\pi}{\to} \pi_1(G/H_{p,q}) \to 0$

So

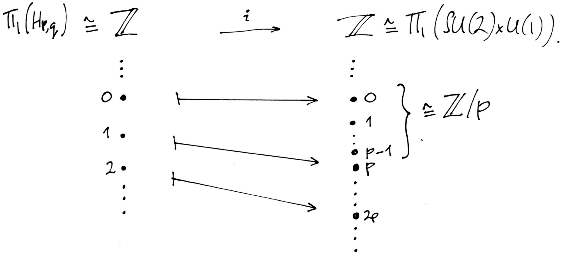
- $\pi_1(G/H_{p,q}) \cong \mathbb{Z}/\ker \pi \cong \operatorname{im} i$

Algebraic topology for dummies II

- $\pi_1(G/H_{p,q}) \cong \mathbb{Z}/\ker\pi \cong \mathrm{im}\, i$

A loop wound once around $H_{p,q}$ is wound p times around $G \cong SU(2) \times U(1)$. So $\mathrm{im}\, i \cong p\mathbb{Z}$ and

- $\pi_1(G/H_{p,q}) \cong \mathbb{Z}/p\mathbb{Z} = \mathbb{Z}/p$.



So $\exists$ strings when $p \neq 1$:-)

Algebraic topology for dummies III

More gymnastics yield

- $\pi_2(G/H_{p,q}) \cong \pi_2(S^3) \cong 0$.

So $\nexists$ monopoles :-(

The SM as a special case

In the SM, $\exists$ a Higgs field s.t. $\phi \to U z^q \phi$. The VEV $\langle \phi \rangle = (0 v)^T$ is stabilized by $H_{1,q} = \{(\mathrm{diag}(z^q, z^{-q}), z)\}$. So

- $\pi_1 \cong \mathbb{Z}/1 \cong 0 \implies$ no strings :-()
- $\pi_2 \cong 0 \implies$ no monopoles :-(

Electroweak strings

EW cosmic strings

Any BSM model that breaks $SU(2) \times U(1)$ to $H_{p \neq 1, q}$ features stable, EW-scale strings, with charge integer mod p.

- A cosmic network of these will form during the EWPhT
- Can we see them?!
- Purely gravitational effects go like $v^2/m_P^2 \sim 10^{-34}$:-(

Superconducting EW cosmic strings

Any BSM model that breaks $SU(2) \times U(1)$ to $H_{p \neq 1,q}$ features stable, EW-scale strings, with charge integer mod p.

- But quarks and leptons have massless modes on string

Witten, 85

- Superconducting currents in astrophysical EM backgrounds
- And violation of B and L!
- Astro signatures: CMB, radio bursts, cosmic rays, galactic and stellar dynamics, ...

cf. Hindmarsh & Kibble, 94

EW strings at colliders

Can presumably make loops of string at multi-TeV colliders, e.g. LHC

- Not much studied

cf. Nambu, 77

- Mass/size 'easy' to compute
- But what about production and decay?

cf. Affleck & Manton, 82

But can a BSM model with $p \neq 1$ be consistent with all other data?!

Non-linear Sigma Models

Consider a NLSM based on $G/H_{p,q}$.

Only p/q can matter locally, but even this is irrelevant when we gauge $\implies$ couplings ok

A different story globally. e.g. What is the largest group containing G that can act (almost) effectively on $G/H_{p,q}$?

- $G/H_{p=1,q} \cong S^3 \cong SU(2)$: $(SU(2) \times SU(2)) \rtimes \mathbb{Z}/2$
- $G/H_{p=2,q} \cong \mathbb{R}P^3 \cong SO(3)$: $(SO(3) \times SO(3)) \rtimes \mathbb{Z}/2$
- $G/H_{p>2,q}$: a group with $d < 6$.

Only $p = 1, 2$ yield custodial symmetries for both m_W/m_Z and $Z \to b\bar{b}$.

Sikivie & al., 80

Agashe & al., 0605341

Where is the Higgs?

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Add a singlet scalar matter field.

Contino & al., 1002.1011

$G/H_{p,q}$ NLSM plus singlet scalar Higgs

- Looks contrived
- But very much in the spirit of HEFT!
- There is 1 additional parameter: $p \in \mathbb{Z}$
- It behoves us to try to measure or bound p.

Let’s try for a UV complete(r) model . . .

Linear Sigma Models

Linear Sigma Models I

Consider SM $-H+\Phi$ a $(2j+1,q)$ of $SU(2)\times U(1)$

- With VEV $(0\ldots0v)^T$, get $p=2j/\mathrm{gcd}(2j,q)$:-)
- Couplings to gauge bosons and fermions come out right :-)
- No custodial symmetry $\implies \frac{m_W^2}{m_Z^2}=\frac{g_2^2}{2j(g_2^2+g_1^2)}$:-(

Linear Sigma Models II

Consider SM $-H+\Phi$, a $(3,3)$ of $O(3)\times O(3)$

- Potential
  $V(\Phi) = a\mathrm{tr}(\Phi^T\Phi - c^2)^2 + b(\mathrm{tr}\Phi^T\Phi - 3c^2)^2 \implies \Phi^T\Phi = c^2 1$
  in vacuo
- The vacuum manifold is $O(3)$ with connected component $SO(3) \cong \mathbb{R}P^3$:-)
- But no fermion masses :-(
- Reinstating H returns the vacuum manifold to S^3 :-(

Georgi & Mahacek, 85

A more tantalising model …

Composite Higgs Models

Consider the almost minimal model based on $SO(5)/O(4)$

Agashe & al., 0605341

- The Higgs is now a co-ordinate in $SO(5)/O(4) \cong \mathbb{R}P^4$
- The potential is at least $SU(2) \times U(1)$ invariant, but let's take the $O(4)$ invariant $V = \frac{x_1^2}{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2}$
- $0 \leq V \leq 1$, with minimum at $x_1 = 0$: vacuum manifold is $\mathbb{R}P^3$: $p = 2$
- This is '$v = f$' in composite Higgs language :-(

Summary

- BSM models of EWSB can feature $\mathbb{Z}/p$-charged strings
- Such strings are a smoking gun for physics BSM
- Are we sensitive to them, either above ground or below?
- From the EFT viewpoint, p is just an extra parameter
- The success of the SM elsewhere means that $p \neq 1$ is unlikely
- A noteworthy theoretical curiosity nevertheless