Copenhagen HEFT, 26th - 28th October 2016

Probing the SMEFT at Future Lepton Colliders

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-*Mixed Heavy-Light Matching in the Universal One-Loop Effective Action*,
Sebastian A.R. Ellis, Jeremie Quevillon, TY, and **Zhengkang Zhang**
PLB accepted [arXiv:1604.02445]

-*The Universal One-Loop Effective Action*,
Aleksandra Drozd, **John Ellis, Jeremie Quevillon** and **TY**,
JHEP 03 (2016) 180 [arXiv:1512.03003]

-*Sensitivities of Prospective Future e+e− Colliders to Decoupled New Physics*,
John Ellis and **TY**
JHEP 03 (2016 089 [arXiv:1510.04561]

-*Comparing EFT and Exact One-Loop Analyses of Non-Degenerate Stops*,
Aleksandra Drozd, **John Ellis, Jeremie Quevillon** and **TY**
JHEP 06 (2015) 028 [arXiv:1504.02409]

-*The Effective Standard Model after LHC Run I*,
John Ellis, **Veronica Sanz** and **TY**
JHEP 29 (2015) 007 [arXiv:1410.7703]

Outline

- Context
- Current LEP and LHC constraints on SMEFT
- Projected sensitivity at ILC, FCC-ee, CLIC
- MSSM stops and one-loop matching
- Conclusion

Context

- All QFTs are really EFTs
- Emphasis on renormalisability a historical accident
- Modern viewpoint: include all terms allowed by given field content and symmetries
- Hence SM is really SMEFT
- Why now? Only use experimentally-established degrees of freedom to construct your theory…

Context

- 1930s-1970s: Beta decay, muon decay etc. -> Fermi theory

$$\mathcal{L}_{FERMI} = -\frac{G_F}{\sqrt{2}}(\bar{\Psi}\mathcal{O}_i\Psi)(\bar{\Psi}\mathcal{O}_i\Psi)$$

$$\mathcal{O}_{scalar} = 1$$
$$\mathcal{O}_{vector} = \gamma_\mu$$
$$\mathcal{O}_{A-V} = \gamma_5\gamma_\mu$$
$$\mathcal{O}_{tensor} = \sigma_{\mu\nu}$$
$$\mathcal{O}_{pseudoscalar} = \gamma_5$$

- Experimental data -> V-A structure

$$\mathcal{L}_{V-A} = -\frac{G_F}{\sqrt{2}}\bar{\Psi}\gamma_\mu(1-\gamma_5)\Psi\,\bar{\Psi}\gamma^\mu(1-\gamma_5)\Psi$$

- Pions: Chiral perturbation theory (non-linear effective Lagrangian)

Context

- 1980s-2012: Discovery of weak bosons -> Non-linear effective Lagrangian for spontaneously-broken global symmetry (breaking mechanism unknown!)
- **Global** symmetry-breaking pattern gives low-energy effective theory regardless of UV mechanism responsible for it

$$\boxed{SU(2) \times SU(2) \to SU(2)_V \qquad (\rho \equiv M_W / M_Z \cos\theta_w \sim 1)}$$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma - m_i \bar{\psi}^i_L \Sigma \psi^i_R + \mathrm{h.c.}$$

$$\Sigma = \exp\left(i \frac{\sigma^a \pi^a}{v} \right)$$

Context

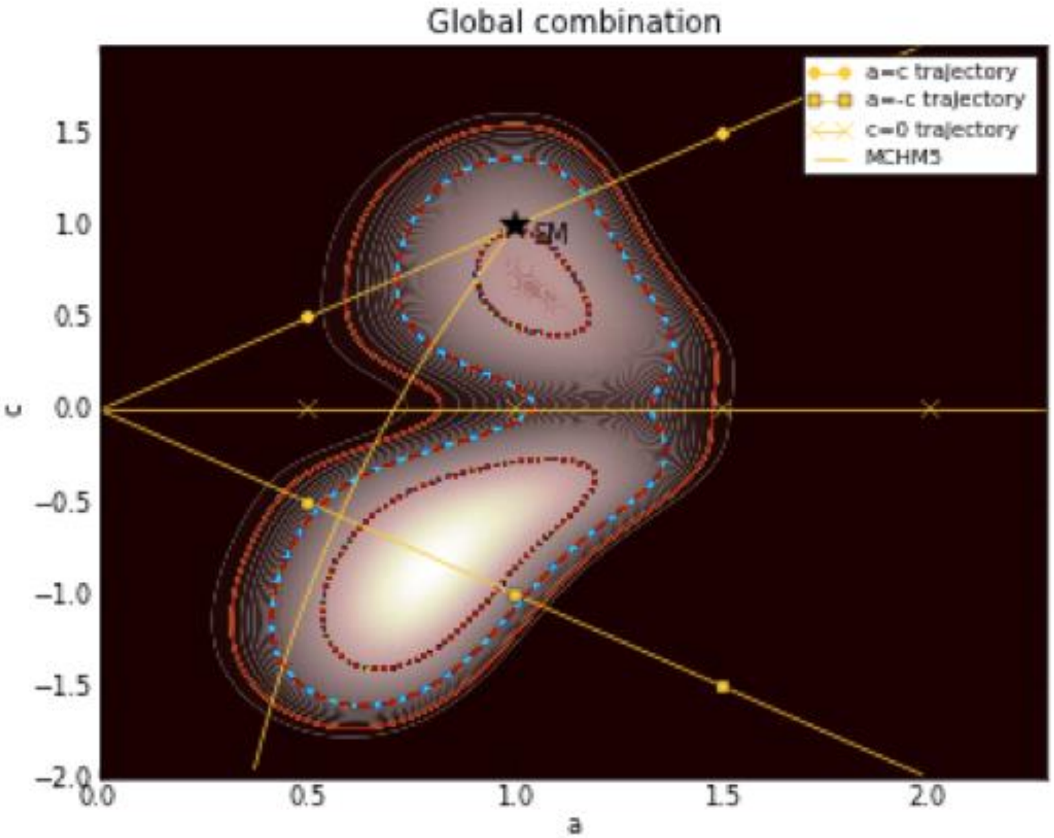
- 2012: Discovery of a scalar -> Non-linear electroweak Lagrangian with general couplings to singlet scalar

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots\right) - m_i \bar{\psi}_L^i \Sigma \left(1 + c\frac{h}{v} + \ldots\right) \psi_R^i + \mathrm{h.c.}$$
$$+ \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}m_h^2 h^2 + d_3 \frac{1}{6}\left(\frac{3m_h^2}{v}\right) h^3 + d_4 \frac{1}{24}\left(\frac{3m_h^2}{v^2}\right) h^4 + \ldots \quad ,$$

$$\Sigma = \exp\left(i\frac{\sigma^a \pi^a}{v}\right)$$

Context

- Could have had very different coupling patterns than SM!

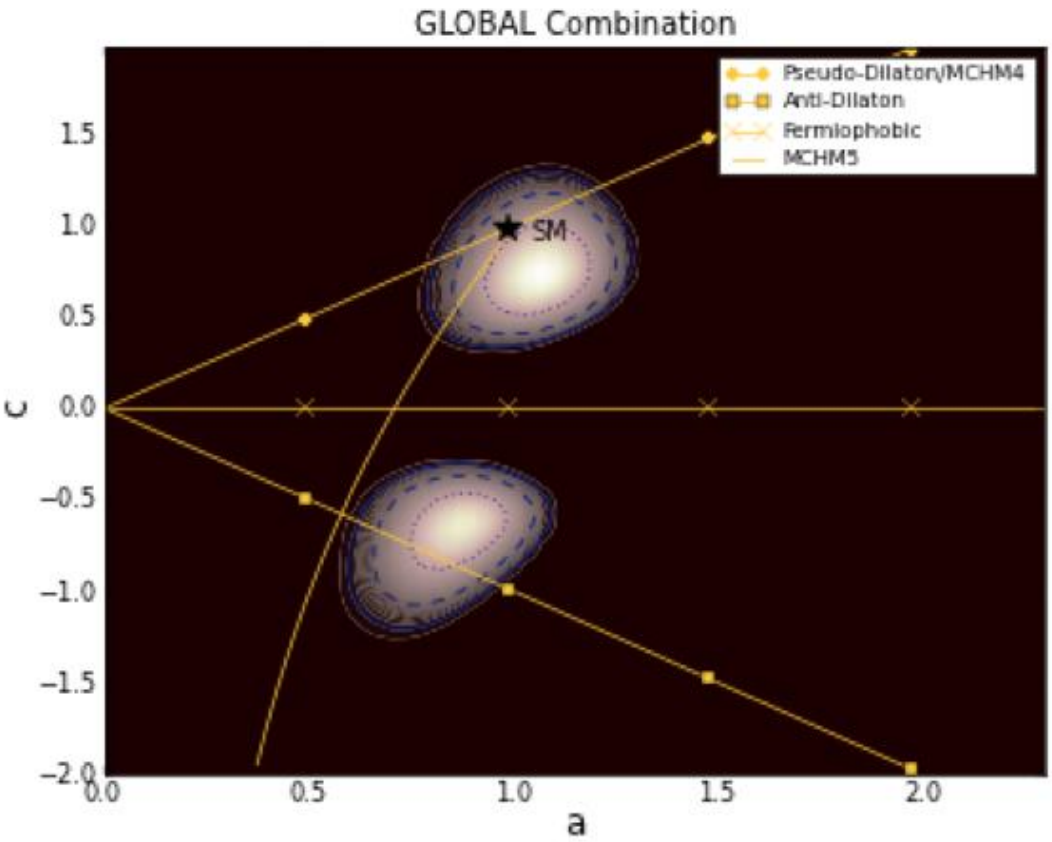


March 2012 pre-discovery

J. Ellis and T.Y. [arXiv:1204.0464]

Context

- Could have had very different coupling patterns than SM!

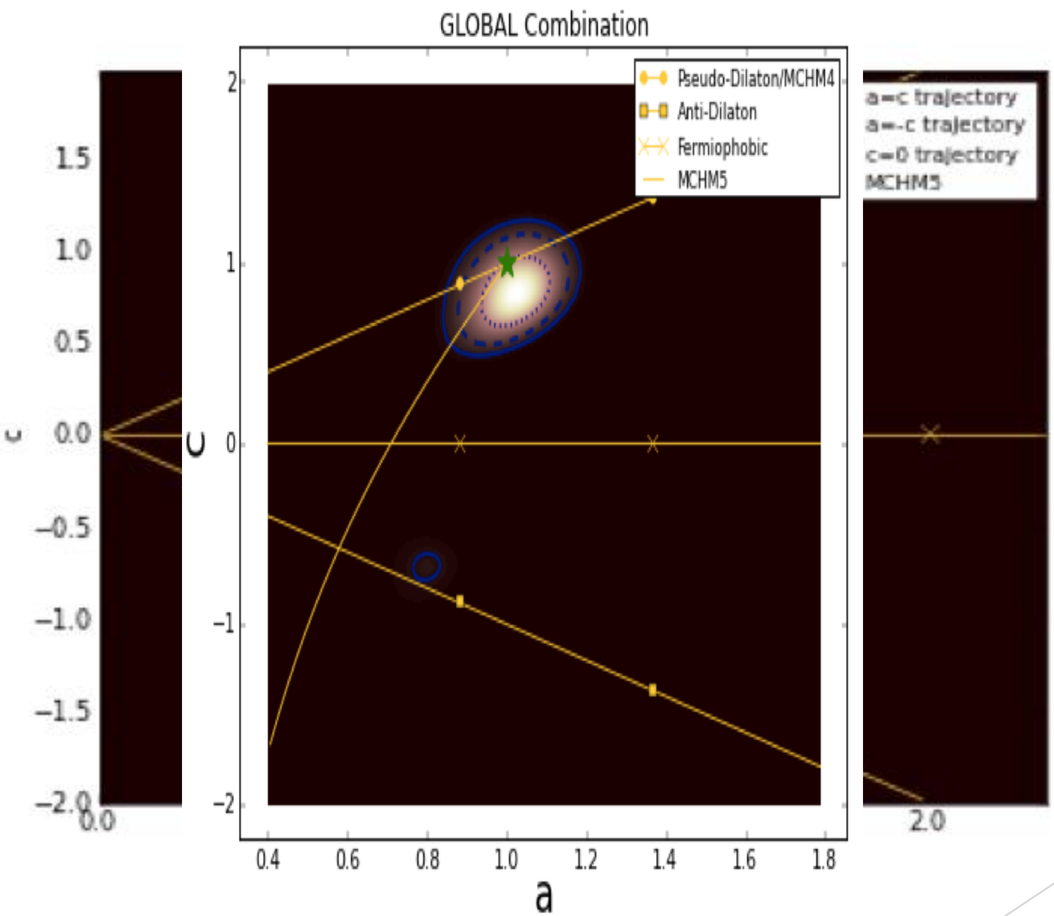


Moriond 2013

J. Ellis and T.Y. [arXiv:1303.1879]

Context

- Could have had very different coupling patterns than SM!
- Many other properties now experimentally measured



July 2012 post-discovery
J. Ellis and T.Y. [arXiv:1207.1693]

Context

Assuming a SM Higgs, the SM EFT is the next phenomenological framework

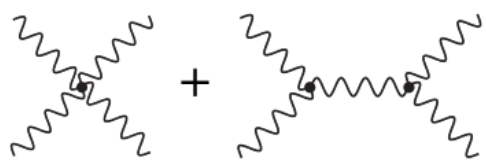
The TeV Scale

What effective theory captures everything we know experimentally about weak interactions?

1933–1982 4-fermion interactions

 $\sim G_F E^2 \quad \Rightarrow \Lambda \sim \text{TeV}$

1982–2011 SM without Higgs

 $\sim \dfrac{g^2 E^2}{m_W^2} \quad \Rightarrow \Lambda \sim \text{TeV}$

2012-now SM + higher-dimension operators?

$\Rightarrow \Lambda \lesssim M_P$?

Markus Luty PASCOS 2015 slide

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Dimension-6 Operators

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	**3**	**2**	$\frac{1}{6}$
q_R^u	**3**	**1**	$\frac{2}{3}$
q_R^d	**3**	**1**	$-\frac{1}{3}$
L_L	**1**	**2**	$-\frac{1}{2}$
l_R	**1**	**1**	-1
ϕ	**1**	**2**	$\frac{1}{2}$

$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y \quad + \quad \boxed{\mathcal{L}_{\text{SM}}^{\text{dim-6}} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i}$$

$$\begin{aligned}
\mathcal{L}_m &= \bar{Q}_L i\gamma^\mu D_\mu^L Q_L + \bar{q}_R i\gamma^\mu D_\mu^R q_R + \bar{L}_L i\gamma^\mu D_\mu^L L_L + \bar{l}_R i\gamma^\mu D_\mu^R l_R \\
\mathcal{L}_G &= -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} \\
\mathcal{L}_H &= (D_\mu^L\phi)^\dagger(D^{L\mu}\phi) - V(\phi) \\
\mathcal{L}_Y &= y_d\bar{Q}_L\phi q_R^d + y_u\bar{Q}_L\phi^c q_R^u + y_L\bar{L}_L\phi l_R + \text{h.c.} \qquad ,
\end{aligned}$$

- First classified systematically by Buchmuller and Wyler (Nucl. Phys. B 268 (1986) 621)
- 59 dim-6 CP-even operators in a **non-redundant** basis, assuming no flavor structure (Gradkowski et al [arXiv:1008.4884])(2499 in general, Alonso et al [arXiv:1312.2014])

$$\begin{aligned}
\mathcal{O}_H &= \tfrac{1}{2}(\partial^\mu|H|^2)^2 \\
\mathcal{O}_T &= \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 \\
\mathcal{O}_6 &= \lambda|H|^6 \\
\mathcal{O}_W &= \tfrac{ig}{2}\left(H^\dagger\sigma^a \overleftrightarrow{D^\mu} H\right) D^\nu W_{\mu\nu}^a \\
\mathcal{O}_B &= \tfrac{ig'}{2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right) \partial^\nu B_{\mu\nu}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{BB} &= g'^2|H|^2 B_{\mu\nu}B^{\mu\nu} \\
\mathcal{O}_{GG} &= g_s^2|H|^2 G_{\mu\nu}^A G^{A\mu\nu} \\
\mathcal{O}_{HW} &= ig(D^\mu H)^\dagger\sigma^a(D^\nu H)W_{\mu\nu}^a \\
\mathcal{O}_{HB} &= ig'(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu} \\
\mathcal{O}_{3W} &= \tfrac{1}{3!}g\epsilon_{abc}W_\mu^{a\,\nu}W_{\nu\rho}^b W^{c\,\rho\mu}
\end{aligned}$$

Basis adopted from Pomarol and Riva 1308.1426

(SILH basis Giudice et al. hep-ph/0703164)

$\mathcal{O}_{y_u} = y_u\|H\|^2\bar{Q}_L\widetilde{H}u_R + \text{h.c.}$	$\mathcal{O}_{y_d} = y_d\|H\|^2\bar{Q}_L H d_R + \text{h.c.}$	$\mathcal{O}_{y_e} = y_e\|H\|^2\bar{L}_L H e_R + \text{h.c.}$
$\mathcal{O}_R^u = (iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu u_R)$ $\mathcal{O}_L^q = (iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)$ $\mathcal{O}_L^{(3)\,q} = (iH^\dagger\sigma^a\overleftrightarrow{D}_\mu H)(\bar{Q}_L\sigma^a\gamma^\mu Q_L)$	$\mathcal{O}_R^d = (iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}_R^e = (iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu e_R)$
$\mathcal{O}_{LL}^{(3)\,ql} = (\bar{Q}_L\sigma^a\gamma_\mu Q_L)(\bar{L}_L\sigma^a\gamma^\mu L_L)$		$\mathcal{O}_{LL}^{(3)\,l} = (\bar{L}_L\sigma^a\gamma^\mu L_L)(\bar{L}_L\sigma^a\gamma_\mu L_L)$

Modifications of EWPO from dim-6 Operators

- (Pseudo-)Observables

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_\ell + 3\Gamma_\nu \qquad R_\ell = \frac{\Gamma_{had}}{\Gamma_\ell} \qquad \sigma_{had} = 12\pi \frac{\Gamma_e \Gamma_{had}}{\hat{m}_Z^2 \Gamma_Z^2} \qquad A^f_{FB} = \frac{3}{4} A_e A_f \qquad M_W = c_W M_Z$$

$$R_q = \frac{\Gamma_q}{\Gamma_{had}}$$

- Depends on

$$\Gamma_f = \frac{\sqrt{2} G_F M_Z^2 \hat{M}_Z}{6\pi} \left[(g_L^f)^2 + (g_R^f)^2 \right] \qquad A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}$$

$$g^f = T_f^3 - Q_f s_W^2 \qquad s_W^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} G_F m_Z^2}}$$

- Dim-6 operators can modify observables directly through Zff couplings contributions or indirectly through redefinitions of input observables

$$m_Z^2 = (m_Z^2)^0 (1 + \Pi_{ZZ}) \qquad G_F = G_F^0 (1 - \Pi_{WW}^0) \qquad \alpha(m_Z) = \alpha^0(m_Z)(1 + \Pi'_{\gamma\gamma})$$

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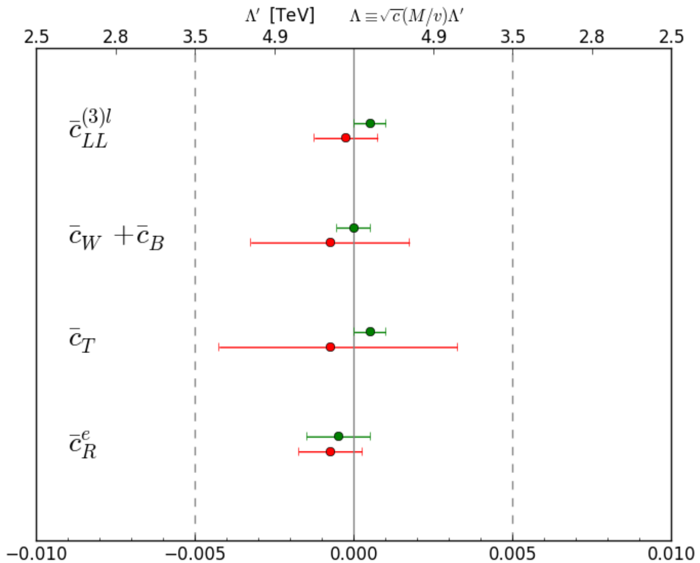
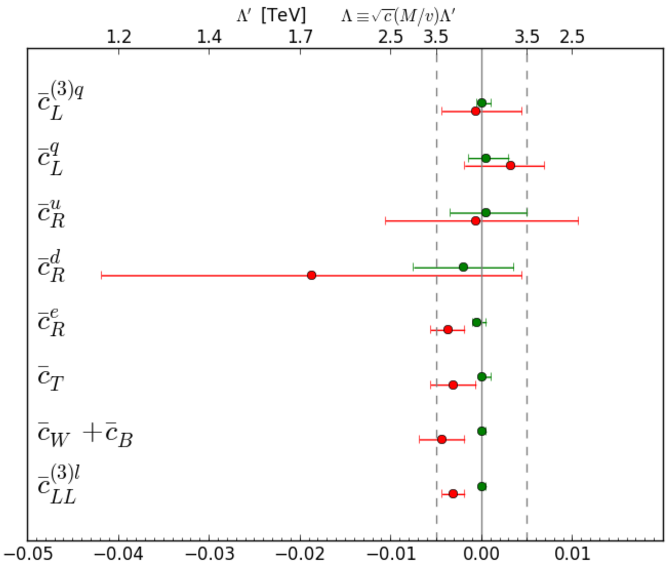
SM EFT Present Constraints

- Marginalized constraints on a complete non-redundant basis of dim-6 operators affecting EWPTs

Ellis, Sanz and T.Y. 1410.7703

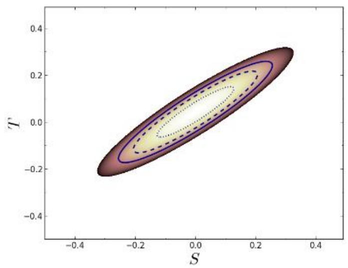




- S,T parameter corresponds to $(c_W + c_B), c_T$ subset

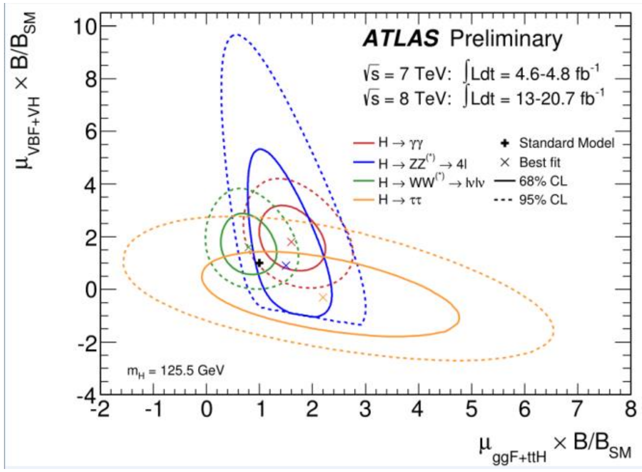
$$S = \frac{4\sin^2\theta_W}{\alpha(m_Z)}(\bar{c}_W + \bar{c}_B) \approx 119(\bar{c}_W + \bar{c}_B)$$

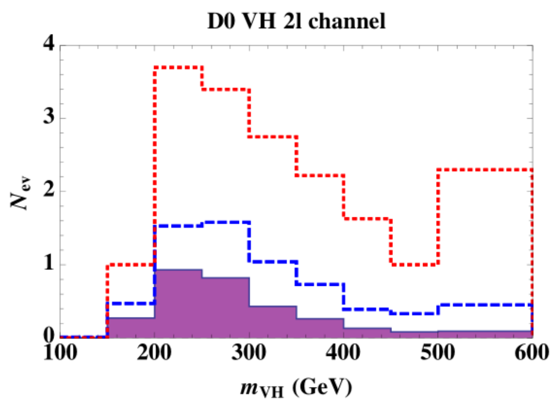
$$T = \frac{1}{\alpha(m_Z)}\bar{c}_T \approx 129\bar{c}_T\,.$$

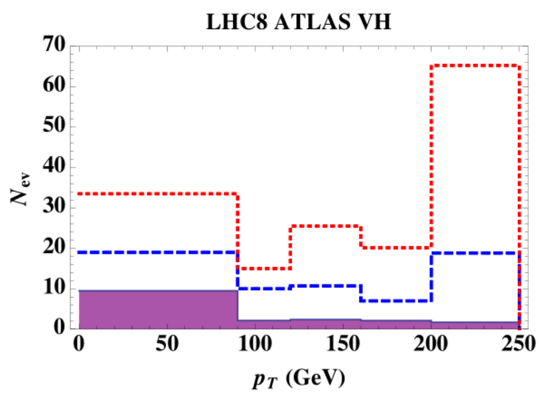


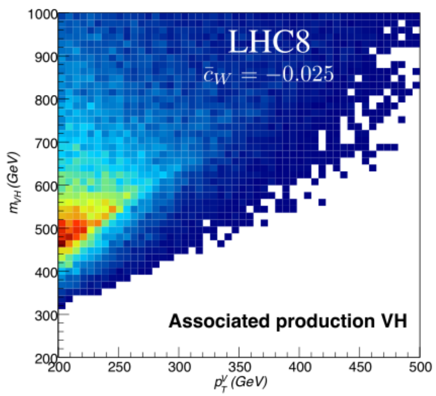
Higgs constraints on dim-6 operators

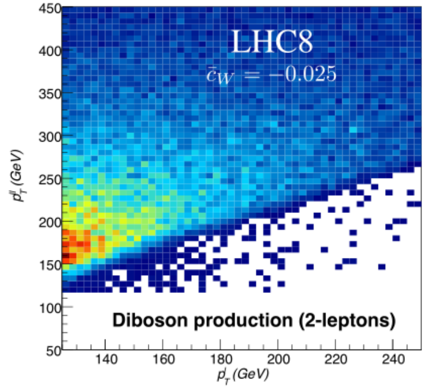
- Operators affect Higgs signal strength measurements, differential distributions





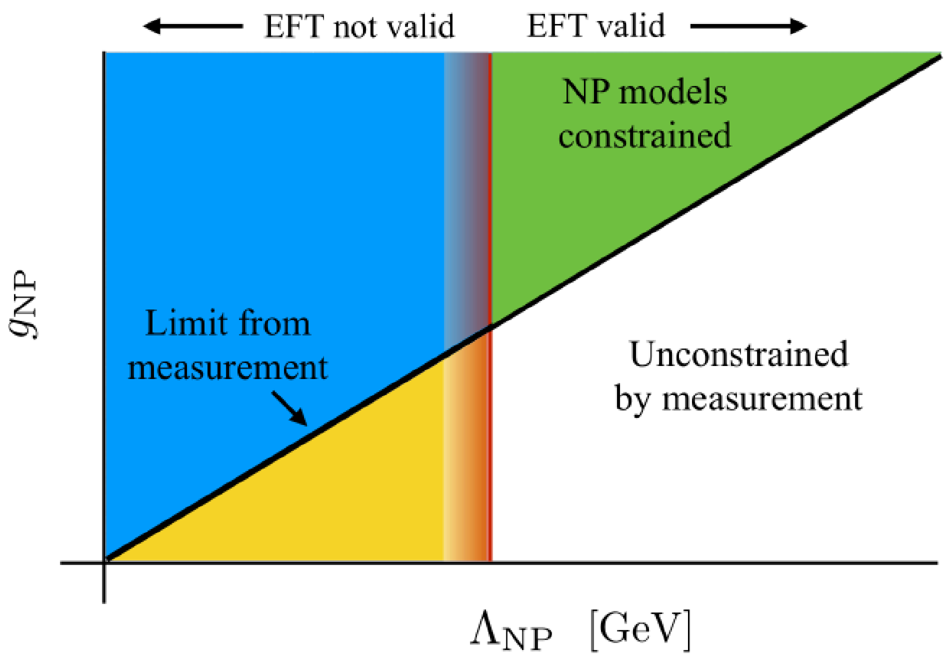






Ellis, Sanz and T.Y. 1410.7703

Higgs constraints on dim-6 operators



Englert and Spannowsky [arXiv:1408.5147]

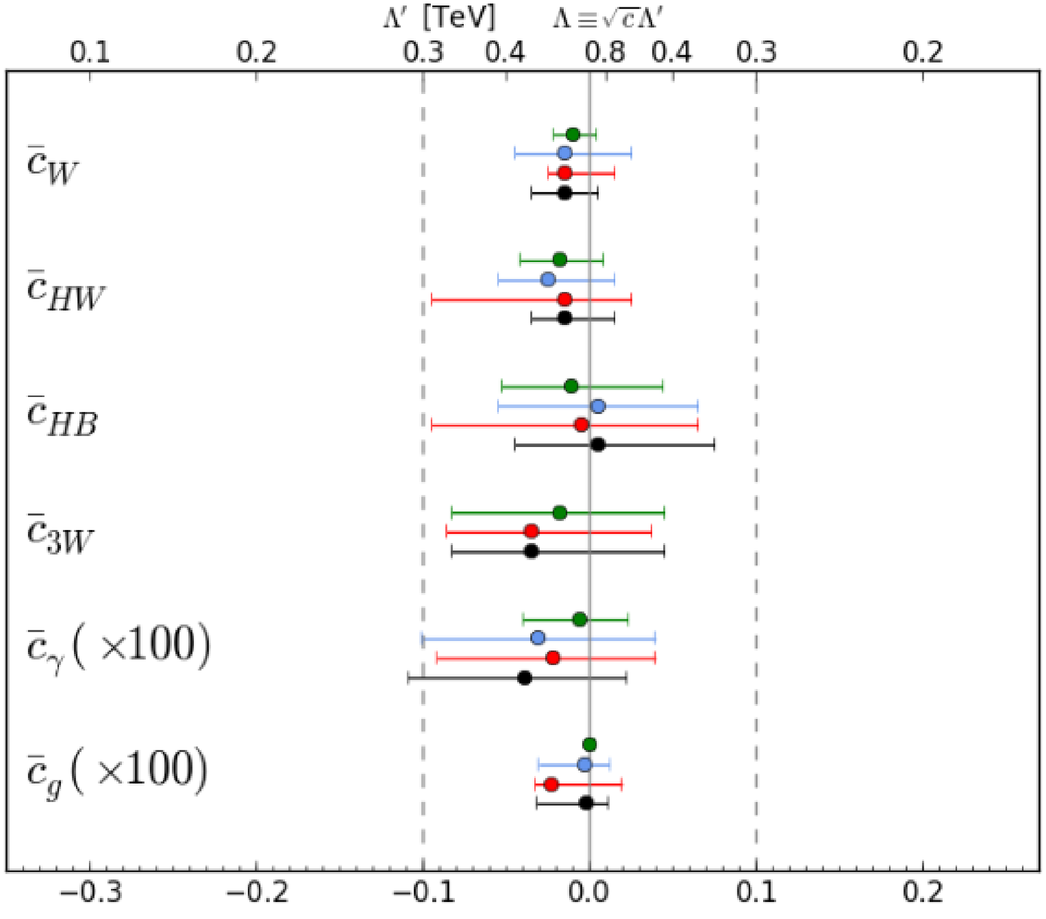
- Validity of EFT depends on interpretation (and extraction)

See e.g.
-Contino, Falkowski, Goertz, Grojean, Riva 1604.06444
-Da Liu, Pomarol, Rattazzi, Riva 1603.03064
-Falkowski, Gonzales-Alonso, Greljo, Marzocca, Son 1609.06312

SM EFT Present Constraints

- Constraints from LHC triple-gauge coupling measurements and Higgs physics

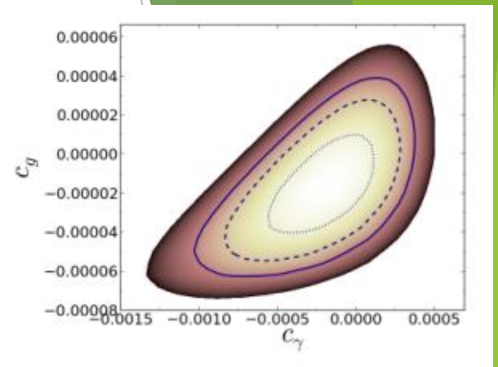
x 10 TeV

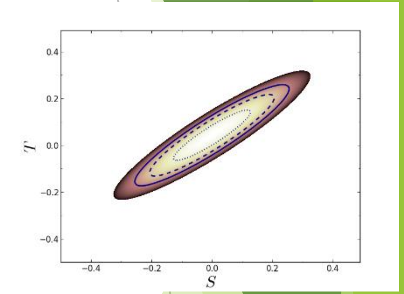


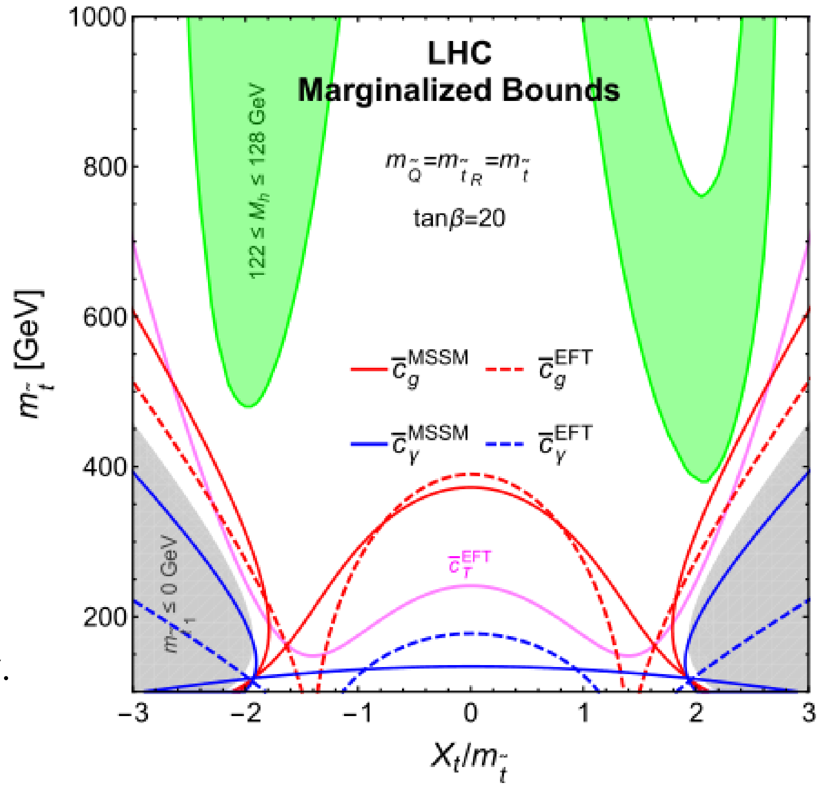
Ellis, Sanz and T.Y. 1410.7703

Translating EFT Constraints to MSSM Stops

Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$, $X_t = 0$
$\bar{c}_g$	LHC	marginalized	$[-4.5, 2.2] \times 10^{-5}$	~ 410 GeV
		individual	$[-3.0, 2.5] \times 10^{-5}$	~ 390 GeV
$\bar{c}_\gamma$	LHC	marginalized	$[-6.5, 2.7] \times 10^{-4}$	~ 215 GeV
		individual	$[-4.0, 2.3] \times 10^{-4}$	~ 230 GeV
$\bar{c}_T$	LEP	marginalized	$[-10, 10] \times 10^{-4}$	~ 290 GeV
		individual	$[-5, 5] \times 10^{-4}$	~ 380 GeV
$\bar{c}_W + \bar{c}_B$	LEP	marginalized	$[-7, 7] \times 10^{-4}$	~ 185 GeV
		individual	$[-5, 5] \times 10^{-4}$	~ 195 GeV



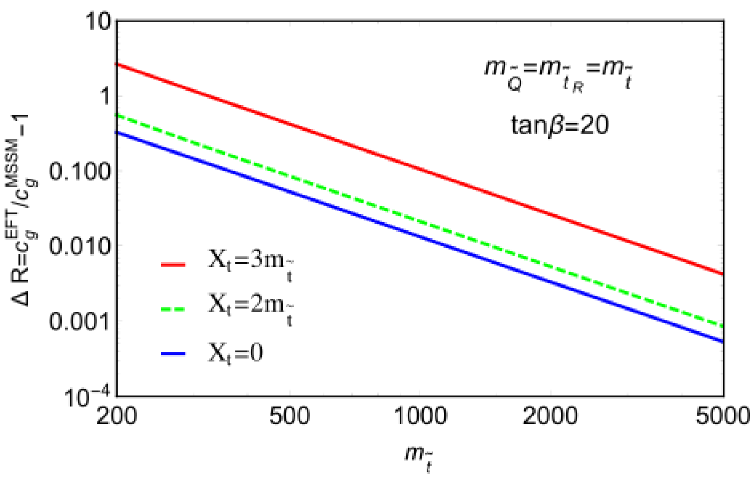


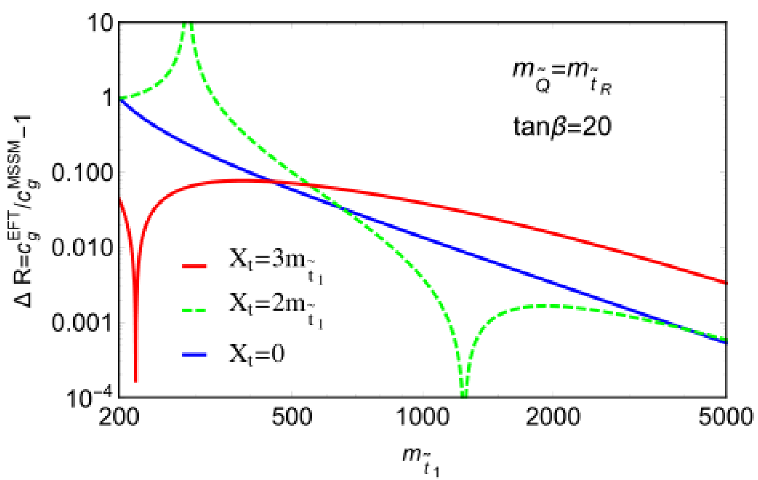


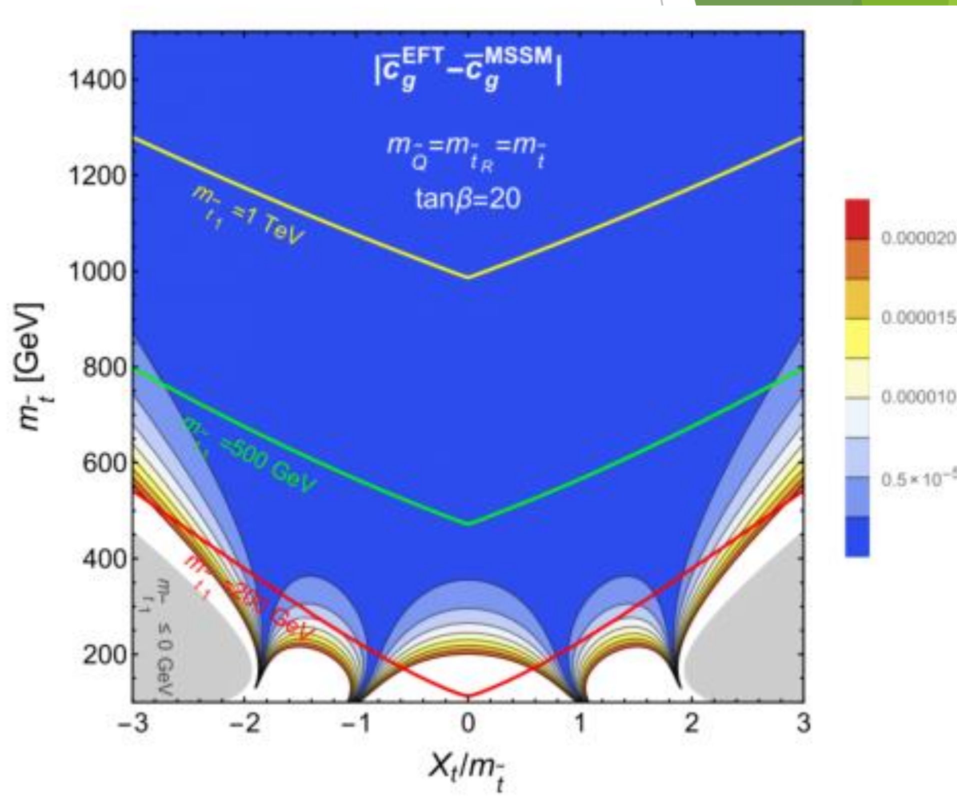
Drozd, Ellis, Quevillon and T.Y.
1504.02409

EFT Validity for Stops

- Operators > dim-6 become important when EFT cut-off/stop mass is too low
- Compare EFT dim-6 vs full MSSM amplitude

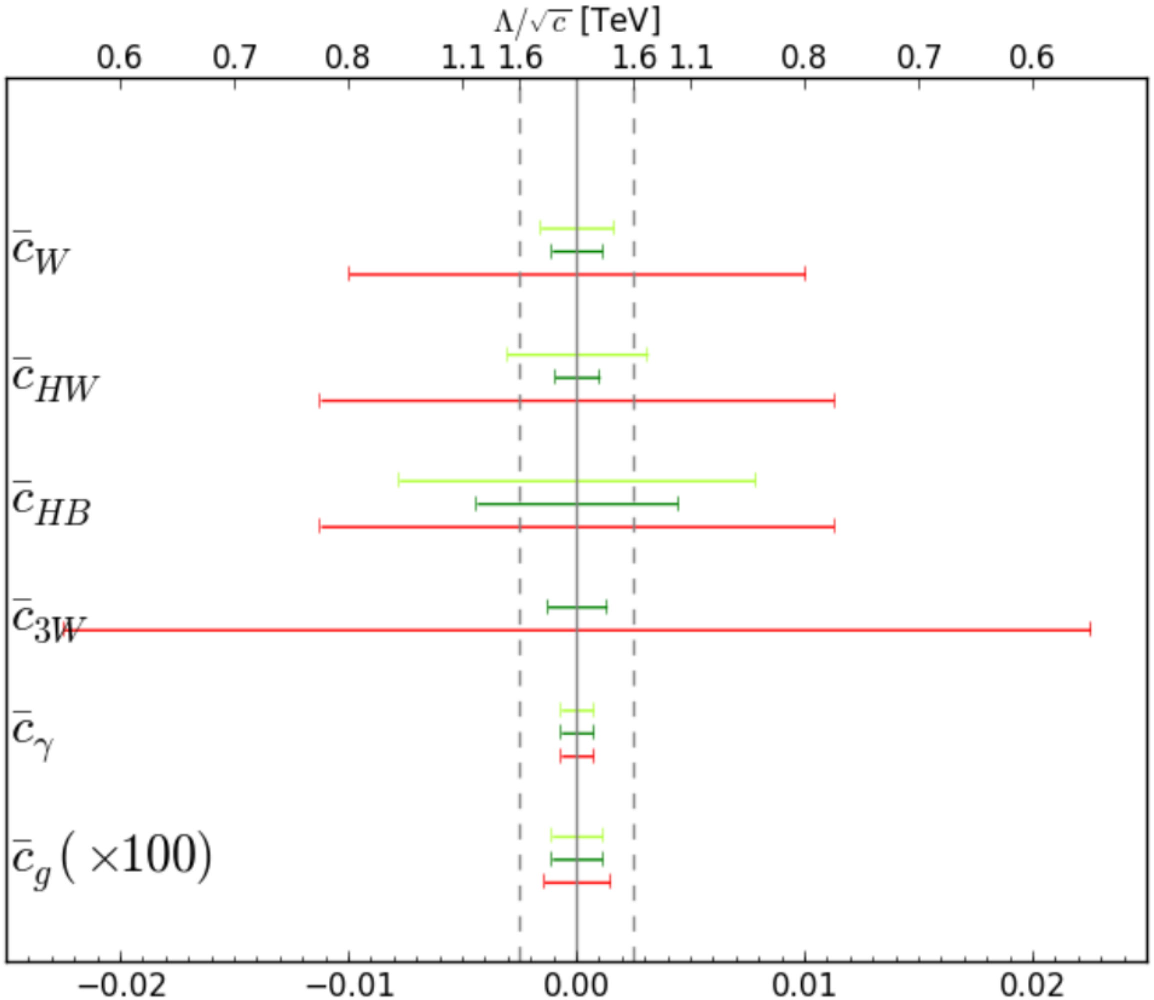






CLIC

- Marginalised and individual constraints from Higgs channels for CLIC @ 350 GeV

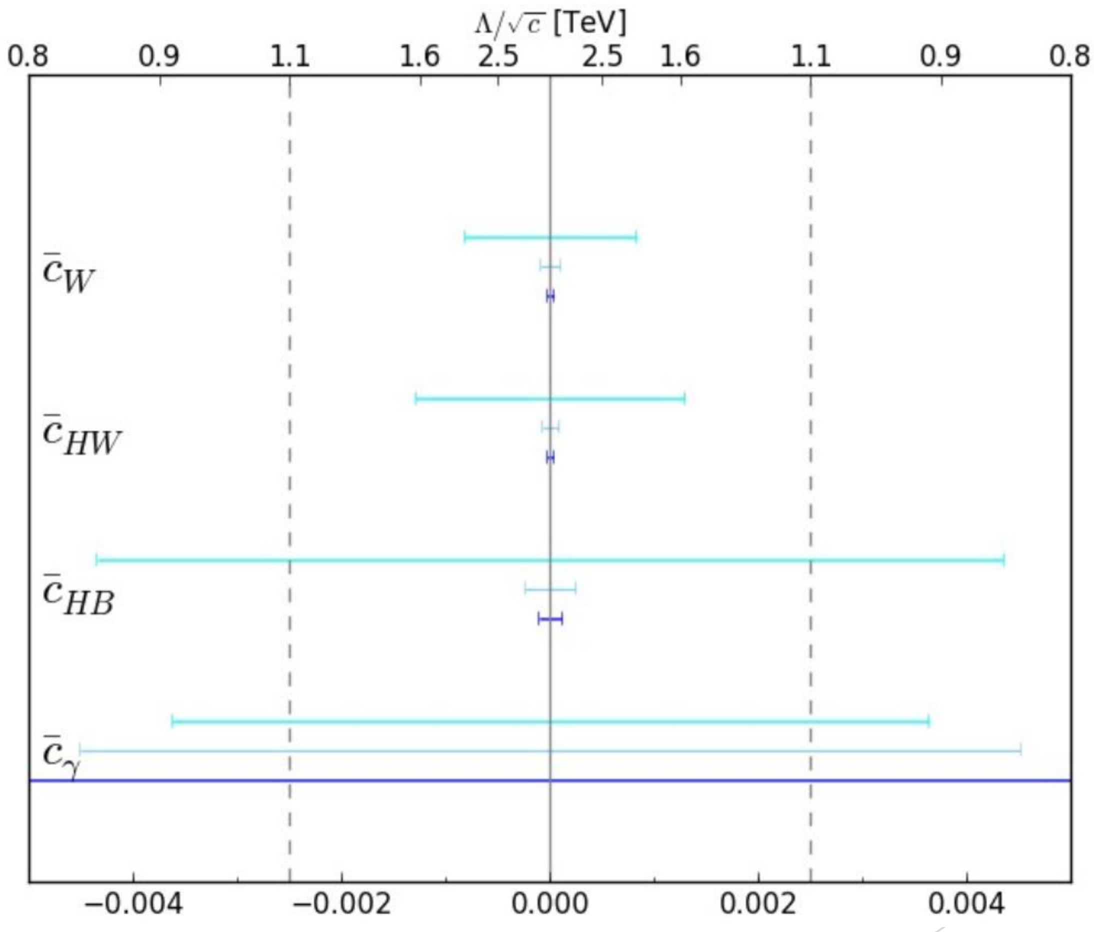


See Z. Zhang 1610.05771 for interplay between EWPT and TGC measurements

Ellis, Roloff, Sanz, You (Preliminary)

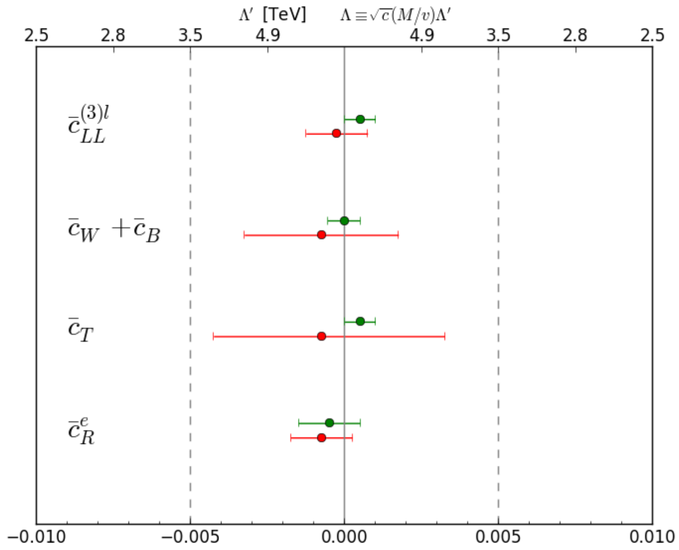
CLIC

- Individual constraints from ZH production xsection for CLIC @ 350 GeV, 1.4 TeV, 3 TeV



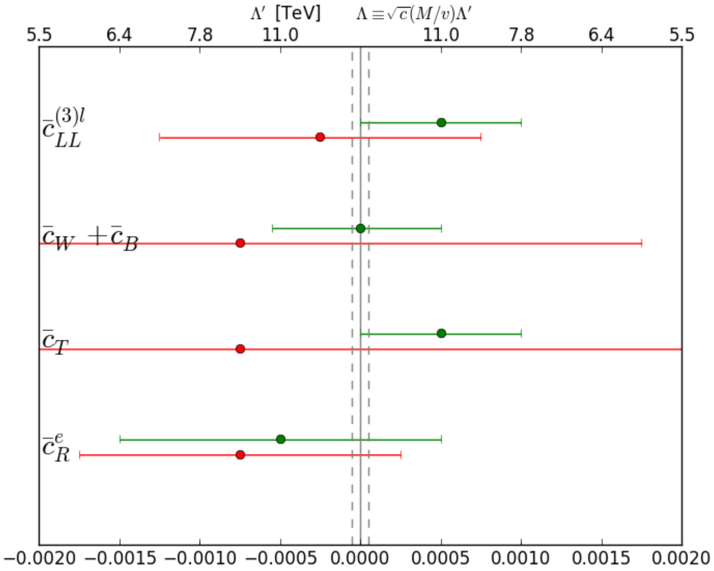
Ellis, Roloff, Sanz, You (Preliminary)

FCC-ee EWPT Constraints



LEP

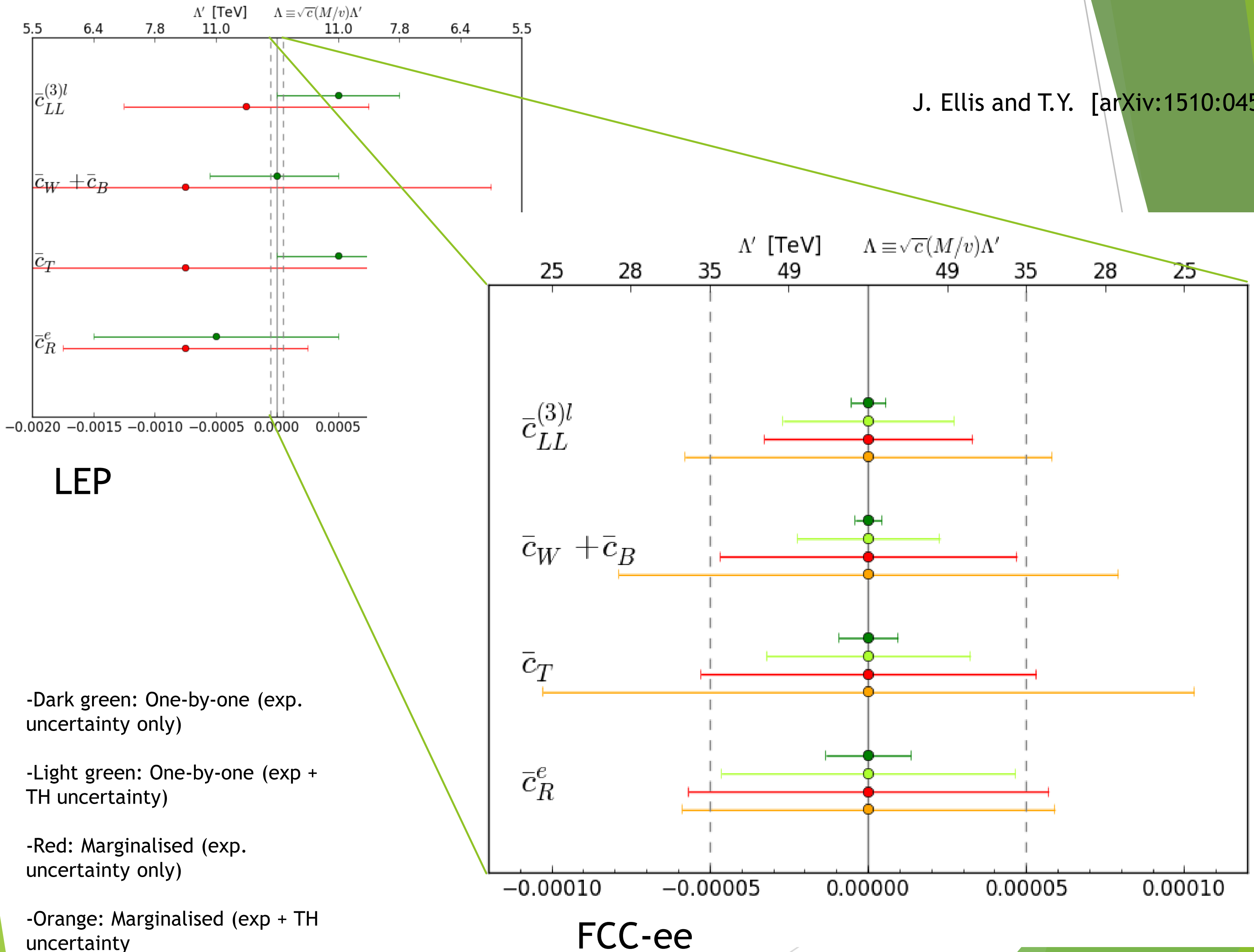
FCC-ee EWPT Constraints



LEP

FCC-ee EWPT Constraints

J. Ellis and T.Y. [arXiv:1510:04561]



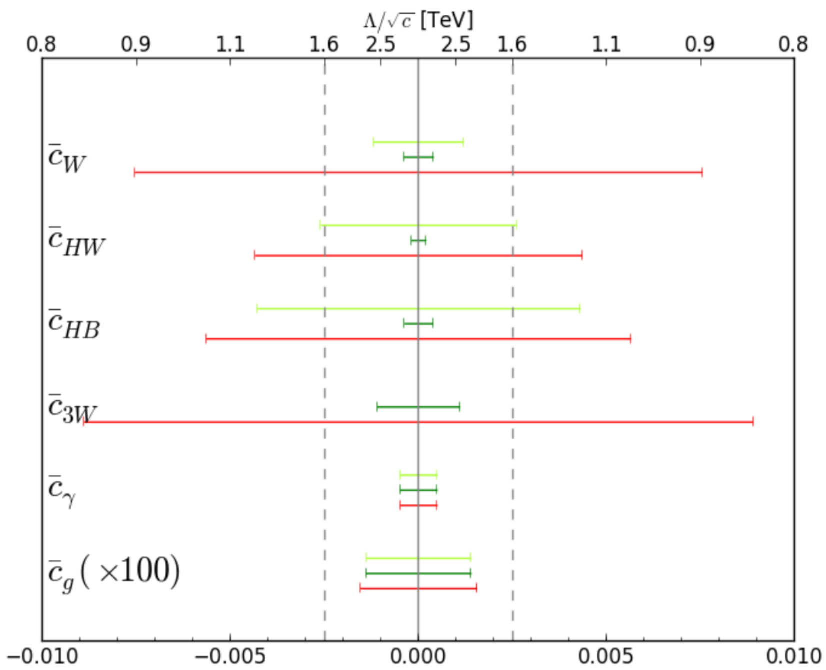
-Dark green: One-by-one (exp. uncertainty only)

-Light green: One-by-one (exp + TH uncertainty)

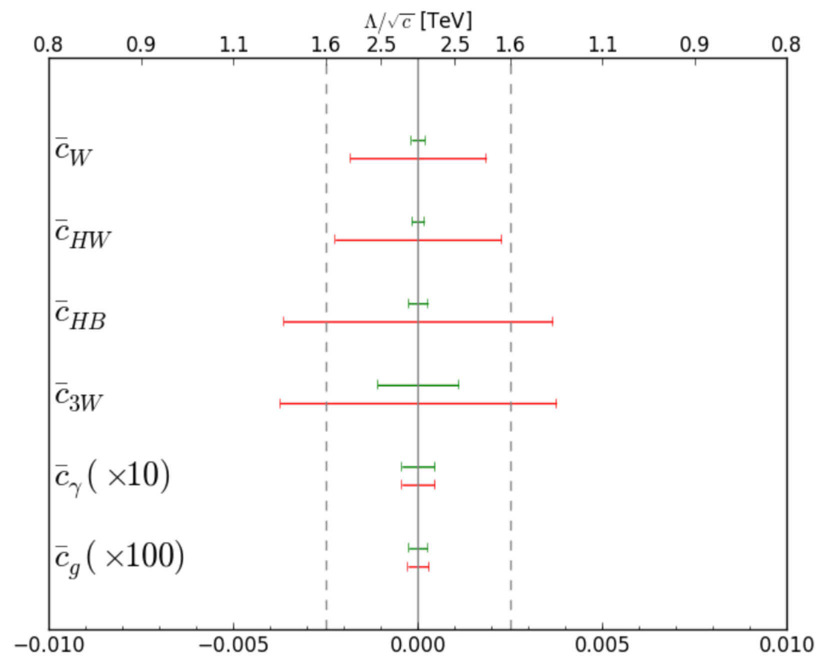
-Red: Marginalised (exp. uncertainty only)

-Orange: Marginalised (exp + TH uncertainty

Future Higgs Constraints



ILC250



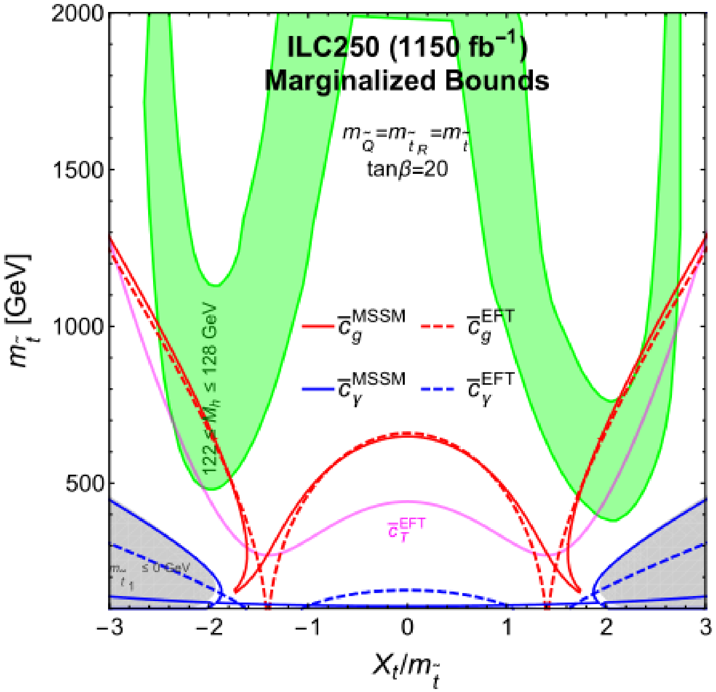
FCC-ee

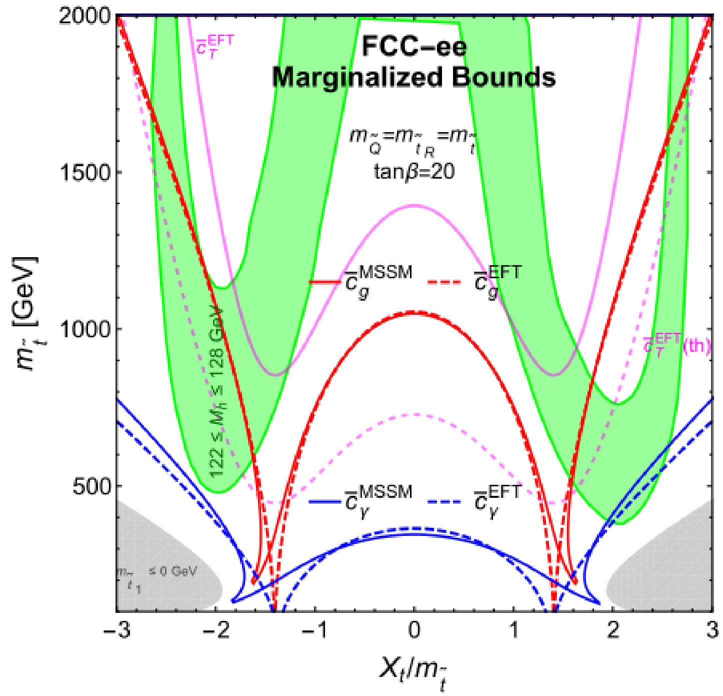
Future Constraints to MSSM Stops

Tevong You

Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$ $X_t = 0$	deg. $m_{\tilde{t}_1}$ $X_t = m_{\tilde{t}}/2$
$\bar{c}_g$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-7.7, 7.7] \times 10^{-6}$	~ 675 GeV	~ 520 GeV
		individual	$[-7.5, 7.5] \times 10^{-6}$	~ 680 GeV	~ 545 GeV
	FCC-ee	marginalized	$[-3.0, 3.0] \times 10^{-6}$	~ 1065 GeV	~ 920 GeV
		individual	$[-3.0, 3.0] \times 10^{-6}$	~ 1065 GeV	~ 915 GeV
$\bar{c}_\gamma$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-3.4, 3.4] \times 10^{-4}$	~ 200 GeV	~ 40 GeV
		individual	$[-3.3, 3.3] \times 10^{-4}$	~ 200 GeV	~ 35 GeV
	FCC-ee	marginalized	$[-6.4, 6.4] \times 10^{-5}$	~ 385 GeV	~ 250 GeV
		individual	$[-6.3, 6.3] \times 10^{-5}$	~ 390 GeV	~ 260 GeV
$\bar{c}_T$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-3, 3] \times 10^{-4}$	~ 480 GeV	~ 285 GeV
		individual	$[-7, 7] \times 10^{-5}$	~ 930 GeV	~ 780 GeV
	FCC-ee	marginalized	$[-3, 3] \times 10^{-5}$	~ 1410 GeV	~ 1285 GeV
		individual	$[-0.9, 0.9] \times 10^{-5}$	~ 2555 GeV	~ 2460 GeV
$\bar{c}_W + \bar{c}_B$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-2, 2] \times 10^{-4}$	~ 230 GeV	~ 170 GeV
		individual	$[-6, 6] \times 10^{-5}$	~ 340 GeV	~ 470 GeV
	FCC-ee	marginalized	$[-2, 2] \times 10^{-5}$	~ 545 GeV	~ 960 GeV
		individual	$[-0.8, 0.8] \times 10^{-5}$	~ 830 GeV	~ 1590 GeV

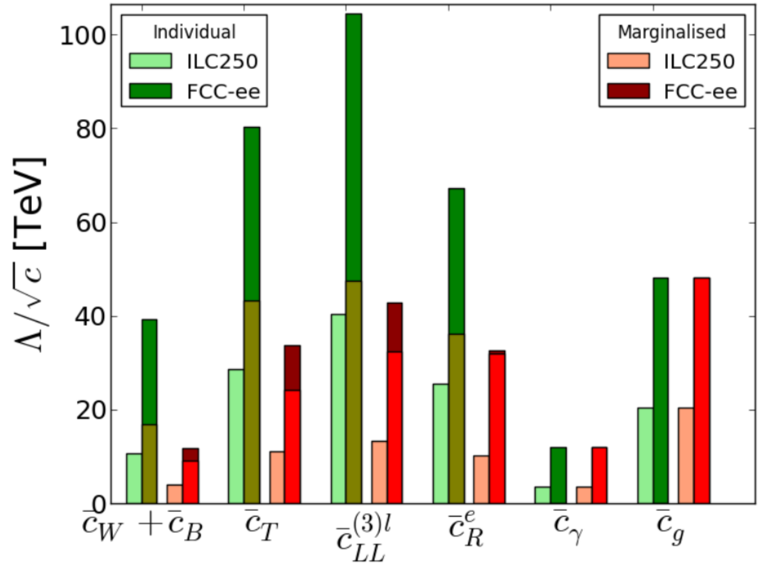
Drozd, Ellis, Quevillon and T.Y.
1504.02409

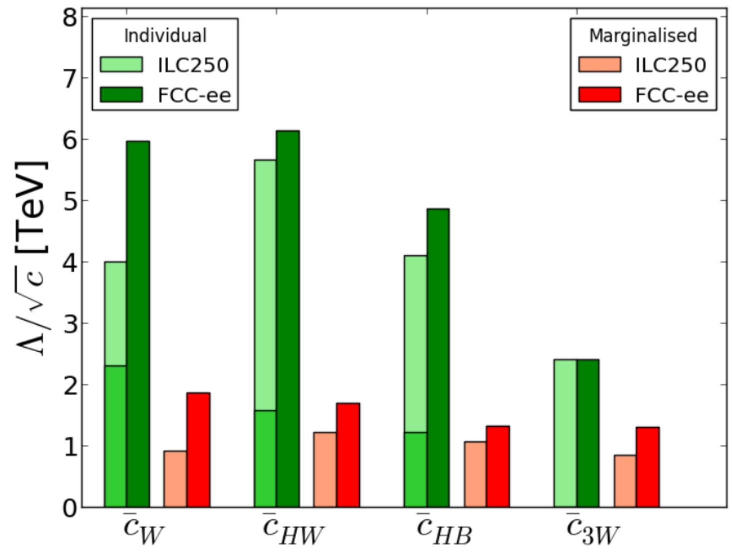




Future e+e- Constraints

J. Ellis and T.Y. [arXiv:1510:04561]





- Future precision sensitive to TeV scale, even for loop-induced operators

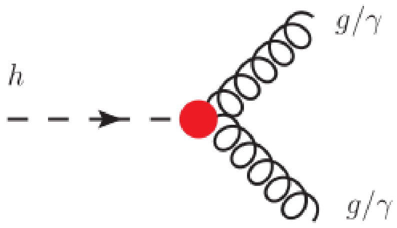
- Need calculations of higher-order corrections to SM EFT at NLO, see e.g.

-Hartmann, Trott 1507.03568
-Gauld, Pecjak, Scott 1512.02508, 1607.06354
-Maltoni, Vryonidou, Zhang 1607.0533

- Matching at one-loop

One-Loop Effective Action

- Feynman diagrams vs path integral
  - Calculate observable in EFT



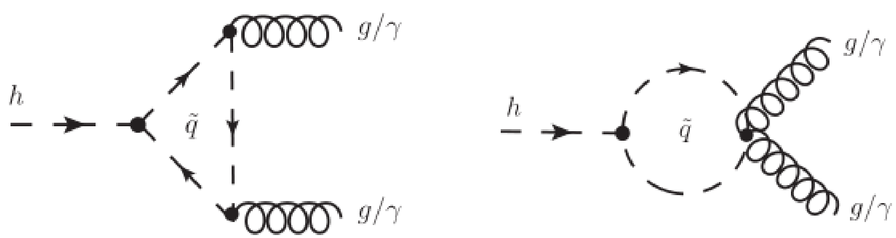
$$iV^{\mu\nu}_{hgg}(p_2,p_3) = -4ig_3^2\sqrt{2}v\frac{\bar{c}_g}{m_W^2}\left(p_2p_3g^{\mu\nu} - p_2^\nu p_3^\mu\right)$$

$$iV^{\mu\nu}_{h\gamma\gamma}(p_2,p_3) = -4ie^2\sqrt{2}v\frac{\bar{c}_\gamma}{m_W^2}\left(p_2p_3g^{\mu\nu} - p_2^\nu p_3^\mu\right)$$

$$\mathcal{A}^{hgg}_{EFT} = -16g_s^2\sqrt{2}v\frac{\bar{c}_g}{m_W^2}\left(\xi_2^*.\xi_3^*M_h^2 - 2(\xi_2^*.p_1)(\xi_3^*.p_1)\right)\,,$$

$$\mathcal{A}^{h\gamma\gamma}_{EFT} = -2g_1^2\cos^2\theta_W\sqrt{2}v\frac{\bar{c}_\gamma}{m_W^2}\left(\xi_2^*.\xi_3^*M_h^2 - 2(\xi_2^*.p_1)(\xi_3^*.p_1)\right)$$

  - Calculate observable in MSSM



$$(\bar{c}_g^{\mathrm{MSSM}})^{\tilde{t}} = \frac{m_W^2}{6(4\pi)^2}\frac{N_g^{\tilde{t}}}{D_g^{\tilde{t}}}\,,$$

$$N_g^{\tilde{t}} = \frac{c_{2\beta}g_1^2}{s_W^2}\left[v^2c_{2\beta}g_1^2\left(2c_{2W}+1\right) + 3\left(3v^2h_t^2 + 2\left(m_{\tilde{t}_R}^2 - m_{\tilde{Q}}^2\right)c_{2W} + 2m_{\tilde{Q}}^2 + m_{\tilde{t}_R}^2\right)\right] + 36h_t^2\left(v^2h_t^2 + m_{\tilde{Q}}^2 + m_{\tilde{t}_R}^2 - X_t^2\right)\,,$$

$$D_g^{\tilde{t}} = \frac{v^2c_{2\beta}g_1^2}{s_W^2}\left[v^2c_{2\beta}g_1^2\left(2c_{2W}+1\right) + 3\left(3v^2h_t^2 + 4\left(m_{\tilde{t}_R}^2 - m_{\tilde{Q}}^2\right)c_{2W} + 4m_{\tilde{Q}}^2 + 2m_{\tilde{t}_R}^2\right)\right] + 36\left(v^2h_t^2 + 2m_{\tilde{Q}}^2\right)\left(v^2h_t^2 + 2m_{\tilde{t}_R}^2\right) - 72v^2h_t^2X_t^2\,,$$

$$(\bar{c}_g^{\mathrm{MSSM}})^{\tilde{b}} = \frac{m_W^2}{6(4\pi)^2}\frac{c_{2\beta}g_1^2\left\{6\left[\left(m_{\tilde{b}_R}^2 - m_{\tilde{Q}}^2\right)c_{2W} + m_{\tilde{Q}}^2 + 2m_{\tilde{b}_R}^2\right] - v^2c_{2\beta}g_1^2\left(c_{2W}+2\right)\right\}}{\left(12m_{\tilde{b}_R}^2 - v^2c_{2\beta}g_1^2\right)\left[v^2c_{2\beta}g_1^2\left(c_{2W}+2\right) - 24m_{\tilde{Q}}^2s_W^2\right]}$$

  - Match the two to obtain Wilson coefficient

One-Loop Effective Action

- Feynman diagrams vs path integral
  - Start with path integral of UV theory

$$P_\mu \equiv iD_\mu$$

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger(P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

$$e^{iS_{\text{eff}}[\phi](\mu)} = \int \mathcal{D}\Phi\, e^{iS[\phi,\Phi](\mu)}.$$

  - Expand action around minimum

$$S_{\text{eff}} \approx S[\Phi_c] + \frac{i}{2}\text{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi_c}\right)$$

$$\begin{aligned} e^{iS_{\text{eff}}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\ &= \int [D\eta] e^{i\left(S[\phi,\Phi_c]+\frac{1}{2}\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\eta^2+\mathcal{O}(\eta^3)\right)} \\ &\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\ &\approx e^{iS[\phi,\Phi_c]-\frac{1}{2}\text{Tr}\,\ln\left(-\left.\frac{\delta^2 S}{\delta\Phi}\right|_{\Phi=\Phi_c}\right)}, \end{aligned}$$

  - Use Gaillard-Cheyette method to write one-loop part in the form

$$\mathcal{L}^{\text{eff}}_{\text{1-loop}} = i\int \frac{d^4q}{(2\pi)^4}\text{Tr}\ln[-(\tilde{G}_{\nu\mu}\partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}]\,,$$

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0} \frac{n+1}{(n+2)!}[P_{\alpha_1},[\ldots[P_{\alpha_n},G'_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}\,,$$

$$\tilde{U} = \sum_{n=0}\frac{1}{n!}[P_{\alpha_1},[\ldots[P_{\alpha_n},U]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

See
-Gaillard Nucl. Phys. B 268 (1986) 669
-Cheyette Nucl. Phys. B 297 (1988) 183
-Henning, Lu & Murayama [arXiv:1412.1837]

One-Loop Effective Action

- Universality in the one-loop effective action assuming degenerate mass matrix M:

-Henning, Lu & Murayama [arXiv:1412.1837]

$$
\begin{aligned}
\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2}\,\mathrm{tr}\Bigg\{ \\
&+ m^4\left[-\frac{1}{2}\left(\log\frac{m^2}{\mu^2}-\frac{3}{2}\right)\right] \\
&+ m^2\left[-\left(\log\frac{m^2}{\mu^2}-1\right)U\right] \\
&+ m^0\left[-\frac{1}{12}\left(\log\frac{m^2}{\mu^2}-1\right)G'^2_{\mu\nu}-\frac{1}{2}\log\frac{m^2}{\mu^2}U^2\right] \\
&+ \frac{1}{m^2}\left[-\frac{1}{60}\left(P_\mu G'_{\mu\nu}\right)^2-\frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu}-\frac{1}{12}\left(P_\mu U\right)^2-\frac{1}{6}U^3-\frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right] \\
&+ \frac{1}{m^4}\bigg[\frac{1}{24}U^4+\frac{1}{12}U\left(P_\mu U\right)^2+\frac{1}{120}\left(P^2U\right)^2+\frac{1}{24}\left(U^2G'_{\mu\nu}G'_{\mu\nu}\right) \\
&\qquad -\frac{1}{120}\left[\left(P_\mu U\right),\left(P_\nu U\right)\right]G'_{\mu\nu}-\frac{1}{120}\left[U\left[U,G'_{\mu\nu}\right]\right]G'_{\mu\nu}\bigg] \\
&+ \frac{1}{m^6}\left[-\frac{1}{60}U^5-\frac{1}{20}U^2\left(P_\mu U\right)^2-\frac{1}{30}\left(UP_\mu U\right)^2\right] \\
&+ \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned}
\tag{2.55}
$$

Tevong You

One-Loop Effective Action

- e.g. MSSM stop

$$G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

(R-parity)

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix} \qquad \Phi = (\tilde{Q}\,, \tilde{t}^*_R).$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

$$\begin{aligned}
\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2}\,\mathrm{tr}\Big\{ & \\
&+ m^4\left[-\frac{1}{2}\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right)\right] \\
&+ m^2\left[-\left(\log\frac{m^2}{\mu^2} - 1\right)U\right] \\
&+ m^0\left[-\frac{1}{12}\left(\log\frac{m^2}{\mu^2} - 1\right)G'^2_{\mu\nu} - \frac{1}{2}\log\frac{m^2}{\mu^2}U^2\right] \\
&+ \frac{1}{m^2}\left[-\frac{1}{60}(P_\mu G'_{\mu\nu})^2 - \frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu} - \frac{1}{12}(P_\mu U)^2 - \frac{1}{6}U^3 - \frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right] \\
&+ \frac{1}{m^4}\left[\frac{1}{24}U^4 + \frac{1}{12}U(P_\mu U)^2 + \frac{1}{120}(P^2U)^2 + \frac{1}{24}\left(U^2 G'_{\mu\nu}G'_{\mu\nu}\right)\right. \\
&\qquad\left. - \frac{1}{120}[(P_\mu U),(P_\nu U)]G'_{\mu\nu} - \frac{1}{120}[U[U,G'_{\mu\nu}]]G'_{\mu\nu}\right] \\
&+ \frac{1}{m^6}\left[-\frac{1}{60}U^5 - \frac{1}{20}U^2(P_\mu U)^2 - \frac{1}{30}(UP_\mu U)^2\right] \\
&+ \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Big\}.
\end{aligned} \tag{2.55}$$

$$c_{GG} = \frac{h_t^2}{(4\pi)^2}\frac{1}{12}\left[\left(1 + \frac{1}{12}\frac{g'^2 c_{2\beta}}{h_t^2}\right) - \frac{1}{2}\frac{X_t^2}{m_{\tilde{t}}^2}\right] \qquad c_{WB} = -\frac{h_t^2}{(4\pi)^2}\frac{1}{24}\left[\left(1 + \frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right) - \frac{4}{5}\frac{X_t^2}{m_{\tilde{t}}^2}\right]$$

$$c_{WW} = \frac{h_t^2}{(4\pi)^2}\frac{1}{16}\left[\left(1 - \frac{1}{6}\frac{g'^2 c_{2\beta}}{h_t^2}\right) - \frac{2}{5}\frac{X_t^2}{m_{\tilde{t}}^2}\right] \qquad c_W = \frac{h_t^2}{(4\pi)^2}\frac{1}{40}\frac{X_t^2}{m_{\tilde{t}}^2}$$

$$c_{BB} = \frac{h_t^2}{(4\pi)^2}\frac{17}{144}\left[\left(1 + \frac{31}{102}\frac{g'^2 c_{2\beta}}{h_t^2}\right) - \frac{38}{85}\frac{X_t^2}{m_{\tilde{t}}^2}\right] \qquad c_B = \frac{h_t^2}{(4\pi)^2}\frac{1}{40}\frac{X_t^2}{m_{\tilde{t}}^2}$$

$$c_{3G} = \frac{g_s^2}{(4\pi)^2}\frac{1}{20} \qquad c_{3W} = \frac{g^2}{(4\pi)^2}\frac{1}{20} \qquad c_{2G} = \frac{g_s^2}{(4\pi)^2}\frac{1}{20} \qquad c_{2W} = \frac{g^2}{(4\pi)^2}\frac{1}{20} \qquad c_{2B} = \frac{g'^2}{(4\pi)^2}\frac{1}{20}$$

$$c_H = \frac{h_t^4}{(4\pi)^2}\frac{3}{4}\left[\left(1 + \frac{1}{3}\frac{g'^2 c_{2\beta}}{h_t^2} + \frac{1}{12}\frac{g'^4 c_{2\beta}^2}{h_t^4}\right) - \frac{7}{6}\frac{X_t^2}{m_{\tilde{t}}^2}\left(1 + \frac{1}{14}\frac{(g^2+2g'^2)c_{2\beta}}{h_t^2}\right) + \frac{7}{30}\frac{X_t^4}{m_{\tilde{t}}^2}\right]$$

$$c_T = \frac{h_t^4}{(4\pi)^2}\frac{1}{4}\left[\left(1 + \frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right)^2 - \frac{1}{2}\frac{X_t^2}{m_{\tilde{t}}^2}\left(1 + \frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right) + \frac{1}{10}\frac{X_t^4}{m_{\tilde{t}}^4}\right]$$

$$c_R = \frac{h_t^4}{(4\pi)^2}\frac{1}{2}\left[\left(1 + \frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right)^2 - \frac{3}{2}\frac{X_t^2}{m_{\tilde{t}}^2}\left(1 + \frac{1}{12}\frac{(3g^2+g'^2)c_{2\beta}}{h_t^2}\right) + \frac{3}{10}\frac{X_t^4}{m_{\tilde{t}}^4}\right]$$

$$c_D = \frac{h_t^2}{(4\pi)^2}\frac{1}{20}\frac{X_t^2}{m_{\tilde{t}}^2}$$

$$c_6 = -\frac{h_t^6}{(4\pi)^2}\frac{1}{2}\left\{\begin{aligned} &\left[1 + \frac{1}{12}\frac{(3g^2-g'^2)c_{2\beta}}{h_t^2}\right]^3 + \left[-\frac{1}{12}\frac{(3g^2+g'^2)c_{2\beta}}{h_t^2}\right]^3 + \left(1 + \frac{1}{3}\frac{g'^2 c_{2\beta}}{h_t^2}\right)^3 \\ &-\frac{X_t^2}{m_{\tilde{t}}^2}\left[2\left(1 + \frac{1}{12}\frac{(3g^2-g'^2)c_{2\beta}}{h_t^2}\right)\left(1 + \frac{1}{8}\frac{(g^2+g'^2)c_{2\beta}}{h_t^2}\right) + \left(1 + \frac{1}{3}\frac{g'^2 c_{2\beta}}{h_t^2}\right)^2\right] \\ &+\frac{X_t^4}{m_{\tilde{t}}^4}\left[1 + \frac{1}{8}\frac{(g^2+g'^2)c_{2\beta}}{h_t^2}\right] - \frac{X_t^6}{m_{\tilde{t}}^6}\frac{1}{10} \end{aligned}\right\}$$

Tevong You

One-Loop Effective Action

- Non-degenerate case: Start over path integral calculation from beginning

$$\mathcal{L}^{\text{eff}}_{\text{1-loop}} = i\int \frac{d^4q}{(2\pi)^4} \operatorname{Tr}\ln[-(\tilde{G}_{\nu\mu}\partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}]\,,$$

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0} \frac{n+1}{(n+2)!}[P_{\alpha_1},[\ldots[P_{\alpha_n},G'_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}\,,$$

$$\tilde{U} = \sum_{n=0} \frac{1}{n!}[P_{\alpha_1},[\ldots[P_{\alpha_n},U]]]\,.$$

$$\Rightarrow \quad \mathcal{L}^{\text{eff}}_{\text{1-loop}} = -ic_s \operatorname{tr}\left\{M^2\sum_{n=0}^{\infty}\mathcal{I}_n\right\} \qquad \mathcal{I}_n = \int\frac{d^4q}{(2\pi)^4}\int d\xi \left[-\Delta_\xi(\{q_\mu,\tilde{G}_{\nu\mu}\}\frac{\partial}{\partial q_\nu} + \tilde{G}_{\sigma\mu}\tilde{G}^{\sigma}{}_{\nu}\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q_\nu} - \tilde{U})\right]^n \Delta_\xi$$

- e.g.

$$I_3 = \operatorname{tr}\int d^4q\int d\xi\, \Delta X \Delta X \Delta X \Delta (-M^2)$$

$X = U + A$ where $A = -\{q_\mu, \tilde{G}_{\nu\mu}\}\frac{\partial}{\partial q_\nu} - \tilde{G}_{\nu\mu}\tilde{G}_{\alpha\mu}\frac{\partial}{\partial q_\nu}\frac{\partial}{\partial q_\alpha} + \delta\tilde{U}(x)$

$$\Rightarrow A = A_1^{\nu}\frac{\partial}{\partial q_\nu} + A_2^{\alpha_1\nu}\frac{\partial}{\partial q_{\alpha_1}}\frac{\partial}{\partial q_\nu} + A_3^{\alpha_1\alpha_2\nu}\frac{\partial}{\partial q_{\alpha_1}}\frac{\partial}{\partial q_{\alpha_2}}\frac{\partial}{\partial q_\nu} + A_4^{\alpha_1\alpha_2\alpha_3\nu}\frac{\partial^4}{\partial q_{\alpha_1}\partial q_{\alpha_2}\partial q_{\alpha_3}\partial q_\nu} + \ldots$$

$$A_1^{\nu} = -q_\mu G'_{\nu\mu} - \frac{1}{3}[P_\mu, G'_{\nu\mu}] + [P_\nu, U]$$

$$A_2^{\alpha_1\nu} = -\frac{1}{3}\cdot 2q_\mu[P_{\alpha_1}, G'_{\nu\mu}] - \frac{1}{8}\left([P_\mu,[P_{\alpha_1},G'_{\nu\mu}]] + [P_{\alpha_1},[P_\mu,G'_{\nu\mu}]]\right) - \frac{1}{2}\cdot\frac{1}{2}G'_{\alpha_1\mu}G'_{\nu\mu} + \frac{1}{2}[P_{\alpha_1},[P_\nu,U]]$$

$$A_3^{\alpha_1\alpha_2\nu} = -\frac{1}{8}\cdot 2q_\mu[P_{\alpha_1},[P_{\alpha_2},G'_{\nu\mu}]] - \frac{1}{30}\left([P_\mu,[P_{\alpha_1},[P_{\alpha_2},G'_{\nu\mu}]]] + [P_{\alpha_1},[P_\mu,[P_{\alpha_2},G'_{\nu\mu}]]] + [P_{\alpha_1},[P_{\alpha_2},[P_\mu,G'_{\nu\mu}]]]\right)$$
$$-\frac{1}{2}\cdot\frac{1}{3}\left(G'_{\alpha_1\mu}[P_{\alpha_2},G'_{\nu\mu}] + [P_{\alpha_1},G'_{\alpha_2\mu}]G'_{\nu\mu}\right) + \frac{1}{3!}[P_{\alpha_1},[P_{\alpha_2},[P_\nu,U]]]$$

- etc.

One-Loop Effective Action

- The universal one-loop effective action without assuming mass degeneracy

$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2 + f_8^{ijk} (U_{ij} U_{jk} U_{ki}) \\
& + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij} \, [P_\mu, [P_\nu, U_{ij}]] \, [P_\mu, [P_\nu, U_{ji}]] + f_{12,b}^{ij} \, [P_\mu, [P_\nu, U_{ij}]] \, [P_\nu, [P_\mu, U_{ji}]] \\
& + f_{12,c}^{ij} \, [P_\mu, [P_\mu, U_{ij}]] \, [P_\nu, [P_\nu, U_{ji}]] \\
& + f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} \, [P_\mu, U_{ij}] \, [P_\nu, U_{jk}] \, G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] \\
& + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} .
\end{aligned}
$$

-A. Drozd, J. Ellis, J. Quevillon and TY [arXiv:1512.03003]

One-Loop Effective Action

$$\mathrm{I}[\mathrm{q}^{2\alpha}]^{nm\cdots p}_{i\,j\,\cdots l} = \int \frac{d^4\mathrm{q}}{(2\pi)^4} \int d\xi\, \mathrm{q}^{2\alpha} \left(\Delta_{\xi,i}\right)^n \left(\Delta_{\xi,j}\right)^m \cdots \left(\Delta_{\xi,l}\right)^p$$

$$\Delta_{\xi,i} = 1/(q^2 - \xi m_i^2)$$

- Universal coefficients

$$\begin{aligned}
f_9^{ij} &= \frac{1}{2}\Big(\big(\mathrm{I}_{i\,j}^{31} - \mathrm{I}[\mathrm{q}^2]_{i\,j}^{41}\big)\, m_i^2 + \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j}^{14} - \mathrm{I}[\mathrm{q}^2]_{i\,j}^{23} - \mathrm{I}[\mathrm{q}^2]_{i\,j}^{32} + \mathrm{I}_{i\,j}^{13} + \mathrm{I}_{i\,j}^{22}\big)\, m_j^2 \Big)\,, \\
f_{10}^{ijkl} &= \mathrm{I}_{i\,j\,k\,l}^{2111} m_i^2\,, \\
f_{11}^{ijk} &= \big(\mathrm{I}_{i\,j\,k}^{212} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{213}\big)\, m_i^2 + \big(\mathrm{I}_{i\,j\,k}^{122} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{123}\big)\, m_j^2 + \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{132} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{222} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{312} + \mathrm{I}_{i\,j\,k}^{122} + \mathrm{I}_{i\,j\,k}^{212}\big)\, m_k^2\,, \\
f_{12,a}^{ij} &= \frac{1}{3}\big(2\mathrm{I}[\mathrm{q}^4]_{i\,j}^{25} - 3\mathrm{I}[\mathrm{q}^2]_{i\,j}^{24} + \mathrm{I}[\mathrm{q}^2]_{i\,j}^{33} + \mathrm{I}[\mathrm{q}^2]_{i\,j}^{42} + \mathrm{I}_{i\,j}^{23} - \mathrm{I}_{i\,j}^{32}\big)\, m_i^2\,, \\
f_{12,b}^{ij} &= f_{12,a}^{ij}\,, \\
f_{12,c}^{ij} &= \frac{1}{3}\big(-3\mathrm{I}[\mathrm{q}^2]_{i\,j}^{24} + \mathrm{I}_{i\,j}^{23} + 2\mathrm{I}[\mathrm{q}^4]_{i\,j}^{25} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j}^{33} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j}^{42} + 2\mathrm{I}_{i\,j}^{32}\big)\, m_i^2\,, \\
f_{13}^{ijk} &= \frac{1}{2}\Big(\big(\mathrm{I}_{i\,j\,k}^{311} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{411}\big)\, m_i^2 + \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{141} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{231} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{321} + \mathrm{I}_{i\,j\,k}^{131} + \mathrm{I}_{i\,j\,k}^{221}\big)\, m_j^2 \\
&\qquad + \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{114} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{123} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{132} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{213} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{222} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{312} + \mathrm{I}_{i\,j\,k}^{113} + \mathrm{I}_{i\,j\,k}^{122} + \mathrm{I}_{i\,j\,k}^{212}\big)\, m_k^2 \Big)\,, \\
f_{14}^{ijk} &= \big(\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{132} + 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{141} + \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{231} - 2\mathrm{I}_{i\,j\,k}^{131}\big)\, m_j^2\,, \\
f_{15a}^{ijk} &= \frac{1}{3}\big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{312} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{411} + 2\mathrm{I}_{i\,j\,k}^{311}\big)\, m_i^2 \\
&\quad + \frac{1}{3}\big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{132} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{141} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{222} + 2\mathrm{I}_{i\,j\,k}^{131} + 2\mathrm{I}_{i\,j\,k}^{221} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{231} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{321}\big)\, m_j^2 \\
&\quad + \frac{1}{3}\big(2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{114} + \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{123} + \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{213} - 2\mathrm{I}_{i\,j\,k}^{113}\big)\, m_k^2\,, \\
f_{15b}^{ijk} &= \frac{1}{3}\big(\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{312} + \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{321} + 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{411} - 2\mathrm{I}_{i\,j\,k}^{311}\big)\, m_i^2 \\
&\quad + \frac{1}{3}\big(-2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{141} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{231} + 2\mathrm{I}_{i\,j\,k}^{131}\big)\, m_j^2 \\
&\quad + \frac{1}{3}\big(-2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{114} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{123} - 2\mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{132} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{213} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k}^{222} + 2\mathrm{I}_{i\,j\,k}^{113} + 2\mathrm{I}_{i\,j\,k}^{122}\big)\, m_k^2\,, \\
f_{16}^{ijklm} &= \mathrm{I}_{i\,j\,k\,l\,m}^{21111} m_i^2\,, \\
f_{17}^{ijkl} &= \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{2113} + \mathrm{I}_{i\,j\,k\,l}^{2112}\big)\, m_i^2 + \big(\mathrm{I}_{i\,j\,k\,l}^{1212} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{1213}\big)\, m_j^2 + \big(\mathrm{I}_{i\,j\,k\,l}^{1122} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{1123}\big)\, m_k^2 \\
&\quad + \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{1132} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{1222} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{1312} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{2122} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{2212} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{3112} \\
&\quad + \mathrm{I}_{i\,j\,k\,l}^{1122} + \mathrm{I}_{i\,j\,k\,l}^{1212} + \mathrm{I}_{i\,j\,k\,l}^{2112}\big) m_l^2\,, \\
f_{18}^{ijkl} &= \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{2113} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{2122} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{2131} + \mathrm{I}_{i\,j\,k\,l}^{2112} + \mathrm{I}_{i\,j\,k\,l}^{2121}\big)\, m_i^2 \\
&\quad + \big(-\mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{1312} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{2212} - \mathrm{I}[\mathrm{q}^2]_{i\,j\,k\,l}^{3112} + \mathrm{I}_{i\,j\,k\,l}^{1212} + \mathrm{I}_{i\,j\,k\,l}^{2112}\big)\, m_l^2\,, \\
f_{19}^{ijklmn} &= \mathrm{I}_{i\,j\,k\,l\,m\,n}^{211111} m_i^2\,.
\end{aligned} \tag{2.5}$$

etc.

One-Loop Effective Action

$$\mathrm{I}[\mathrm{q}^{2\alpha}]^{nm\cdots p}_{i\,j\,\cdots l} = \int \frac{d^4\mathrm{q}}{(2\pi)^4} \int d\xi\, \mathrm{q}^{2\alpha} \left(\Delta_{\xi,i}\right)^n \left(\Delta_{\xi,j}\right)^m \cdots \left(\Delta_{\xi,l}\right)^p$$

$$\Delta_{\xi,i} = 1/(q^2 - \xi m_i^2)$$

- Universal coefficients

$$\tilde{f}_5^{ij} = \frac{6m_i^4 m_j^2 - 3m_i^2 m_j^4 - 6m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - 2m_j^6}{54\Delta m_{ij}^8} + \frac{1}{270}\left(\frac{7}{m_i^2} - \frac{9}{m_j^2}\right),$$

$$\tilde{f}_6^{ij} = \frac{7}{180m_i^2} - \frac{1}{20m_j^2},$$

$$\tilde{f}_7^{ij} = \frac{-2m_i^4 m_j^2 + \frac{5}{2}m_i^2 m_j^4 + 2m_i^4 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - \frac{m_i^6}{2}}{\Delta m_{ij}^8},$$

$$\tilde{f}_8^{ijk} = \frac{m_i^2 m_j^2 m_k^2 + m_i^2 m_j^2 m_k^2 \ln\left(m_i^2\right) - m_i^4 m_j^2 - m_i^4 m_k^2 + m_i^6 - m_i^6 \ln\left(m_i^2\right)}{\Delta m_{ij}^4 \Delta m_{ik}^4} + \frac{m_i^2 m_j^2 \ln\left(m_j^2\right)}{\Delta m_{ij}^4 \Delta m_{jk}^2} - \frac{m_i^2 m_k^2 \ln\left(m_k^2\right)}{\Delta m_{ik}^4 \Delta m_{jk}^2},$$

$$\tilde{f}_9^{ij} = \frac{m_i^4 m_j^2 + m_i^2 m_j^4 + 2m_i^4 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 2m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - m_j^6}{4\Delta m_{ij}^8},$$

$$\tilde{f}_{10}^{ijkl} = \left\{\frac{-m_i^2 m_j^2 + m_i^2 m_j^2 \ln\left(m_j^2\right) + m_i^4 - m_i^4 \ln\left(m_i^2\right)}{2\Delta m_{ij}^4 \Delta m_{jk}^2 \Delta m_{jl}^2} + (j \leftrightarrow k) - (j \leftrightarrow l)\right\} + \frac{m_i^2 \ln\left(m_i^2\right)}{2\Delta m_{ij}^2 \Delta m_{ik}^2 \Delta m_{il}^2},$$

$$\begin{aligned}\tilde{f}_{11}^{ijk} = \frac{1}{2\Delta m_{ij}^2 \Delta m_{ik}^6 \Delta m_{jk}^6}\Bigg(&3m_i^2 m_j^4 m_k^4 - 3m_i^4 m_j^2 m_k^4 + 6m_i^2 m_j^2 m_k^6 \ln\left(\frac{m_i^2}{m_j^2}\right) - 6m_i^2 m_j^4 m_k^4 \ln\left(\frac{m_i^2}{m_k^2}\right)\\ &+6m_i^4 m_j^2 m_k^4 \ln\left(\frac{m_j^2}{m_k^2}\right) + 2m_i^2 m_j^6 m_k^2 \ln\left(\frac{m_i^2}{m_k^2}\right) - 2m_i^6 m_j^2 m_k^2 \ln\left(\frac{m_j^2}{m_k^2}\right) - m_i^4 m_j^6 + m_i^6 m_j^4\\ &-3m_i^2 m_k^8 + 4m_i^4 m_k^6 - m_i^6 m_k^4 - 2m_i^2 m_k^8 \ln\left(\frac{m_i^2}{m_k^2}\right) + 3m_j^2 m_k^8 - 4m_j^4 m_k^6 + m_j^6 m_k^4 + 2m_j^2 m_k^8 \ln\left(\frac{m_j^2}{m_k^2}\right)\Bigg),\end{aligned}$$

$$\tilde{f}_{12,a}^{ij} = \frac{28m_i^6 m_j^2 - 28m_i^2 m_j^6 - 12m_i^6 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 36m_i^4 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - 12m_i^2 m_j^6 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^8 - m_j^8}{36\Delta m_{ij}^{12}},$$

$$\tilde{f}_{12,b}^{ij} = \tilde{f}_{12,a}^{ij},$$

etc.

One-Loop Effective Action

$$\mathrm{I}[\mathrm{q}^{2\alpha}]^{nm\cdots p}_{i\,j\,\cdots l} = \int \frac{d^4\mathrm{q}}{(2\pi)^4} \int d\xi\, \mathrm{q}^{2\alpha} \left(\Delta_{\xi,i}\right)^n \left(\Delta_{\xi,j}\right)^m \cdots \left(\Delta_{\xi,l}\right)^p$$

$$\Delta_{\xi,i} = 1/(q^2 - \xi m_i^2)$$

- Universal coefficients

$$\tilde{f}_5^{ij} = \frac{6m_i^4 m_j^2 - 3m_i^2 m_j^4 - 6m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - 2m_j^6}{54\Delta m_{ij}^8} + \frac{1}{270}\left(\frac{7}{m_i^2} - \frac{9}{m_j^2}\right),$$

$$\tilde{f}_6^{ij} = \frac{7}{180m_i^2} - \frac{1}{20m_j^2},$$

$$\tilde{f}_7^{ij} = \frac{-2m_i^4 m_j^2 + \frac{5}{2} m_i^2 m_j^4 + 2m_i^4 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - \frac{m_i^6}{2}}{\Delta m_{ij}^8},$$

$$\tilde{f}_8^{ijk} = \frac{m_i^2 m_j^2 m_k^2 + m_i^2 m_j^2 m_k^2 \ln\left(m_i^2\right) - m_i^4 m_j^2 - m_i^4 m_k^2 + m_i^6 - m_i^6 \ln\left(m_i^2\right)}{\Delta m_{ij}^4 \Delta m_{ik}^4} + \frac{m_i^2 m_j^2 \ln\left(m_j^2\right)}{\Delta m_{ij}^4 \Delta m_{jk}^2} - \frac{m_i^2 m_k^2 \ln\left(m_k^2\right)}{\Delta m_{ik}^4 \Delta m_{jk}^2},$$

$$\tilde{f}_9^{ij} = \frac{m_i^4 m_j^2 + m_i^2 m_j^4 + 2m_i^4 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 2m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - m_j^6}{4\Delta m_{ij}^8},$$

$$\tilde{f}_{10}^{ijkl} = \left\{\frac{-m_i^2 m_j^2 + m_i^2 m_j^2 \ln\left(m_j^2\right) + m_i^4 - m_i^4 \ln\left(m_i^2\right)}{2\Delta m_{ij}^4 \Delta m_{jk}^2 \Delta m_{jl}^2} + (j \leftrightarrow k) - (j \leftrightarrow l)\right\} + \frac{m_i^2 \ln\left(m_i^2\right)}{2\Delta m_{ij}^2 \Delta m_{ik}^2 \Delta m_{il}^2},$$

$$\begin{aligned}\tilde{f}_{11}^{ijk} = \frac{1}{2\Delta m_{ij}^2 \Delta m_{ik}^6 \Delta m_{jk}^6}\Bigg(& 3m_i^2 m_j^4 m_k^4 - 3m_i^4 m_j^2 m_k^4 + 6m_i^2 m_j^2 m_k^6 \ln\left(\frac{m_i^2}{m_j^2}\right) - 6m_i^2 m_j^4 m_k^4 \ln\left(\frac{m_i^2}{m_k^2}\right) \\ & + 6m_i^4 m_j^2 m_k^4 \ln\left(\frac{m_j^2}{m_k^2}\right) + 2m_i^2 m_j^6 m_k^2 \ln\left(\frac{m_i^2}{m_k^2}\right) - 2m_i^6 m_j^2 m_k^2 \ln\left(\frac{m_j^2}{m_k^2}\right) - m_i^4 m_j^6 + m_i^6 m_j^4 \\ & - 3m_i^2 m_k^8 + 4m_i^4 m_k^6 - m_i^6 m_k^4 - 2m_i^2 m_k^8 \ln\left(\frac{m_i^2}{m_k^2}\right) + 3m_j^2 m_k^8 - 4m_j^4 m_k^6 + m_j^6 m_k^4 + 2m_j^2 m_k^8 \ln\left(\frac{m_j^2}{m_k^2}\right)\Bigg),\end{aligned}$$

$$\tilde{f}_{12,a}^{ij} = \frac{28m_i^6 m_j^2 - 28m_i^2 m_j^6 - 12m_i^6 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 36m_i^4 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - 12m_i^2 m_j^6 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^8 - m_j^8}{36\Delta m_{ij}^{12}},$$

$$\tilde{f}_{12,b}^{ij} = \tilde{f}_{12,a}^{ij},$$

etc.

Degenerate mass limit:

$$f_5 = -\frac{i}{(4\pi)^2 60m^2}, \quad f_{11} = \frac{i}{(4\pi)^2 12m^4}, \quad f_{15a} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_6 = -\frac{i}{(4\pi)^2 90m^2}, \quad f_{12,a} = 0, \quad f_{15b} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_7 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{12,b} = 0, \quad f_{16} = -\frac{i}{(4\pi)^2 60m^6},$$

$$f_8 = -\frac{i}{(4\pi)^2 6m^2}, \quad f_{12,c} = \frac{i}{(4\pi)^2 120m^4}, \quad f_{17} = -\frac{i}{(4\pi)^2 20m^6},$$

$$f_9 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{13} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{18} = -\frac{i}{(4\pi)^2 30m^6},$$

$$f_{10} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{14} = \frac{-i}{(4\pi)^2 60m^4}, \quad f_{19} = \frac{i}{(4\pi)^2 120m^8}.$$

One-Loop Effective Action

- e.g. MSSM stop

$$G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

(R-parity)

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix} \qquad \Phi = (\tilde{Q}\,, \tilde{t}^*_R).$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

$$\begin{aligned}
{}^{\rm eff}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U^2_{ij} \\
& + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij}]^2 + f_8^{ijk}(U_{ij}U_{jk}U_{ki}) \\
& + f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f^{ij}_{12,a}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\mu,[P_\nu,U_{ji}]] + f^{ij}_{12,b}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\nu,[P_\mu,U_{ji}]] \\
& + f^{ij}_{12,c}\,[P_\mu,[P_\mu,U_{ij}]]\,[P_\nu,[P_\nu,U_{ji}]] \\
& + f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}\,[P_\mu,U_{ij}]\,[P_\nu,U_{jk}]\,G'_{\nu\mu,ki} \\
& + \left(f^{ijk}_{15a}U_{i,j}[P_\mu,U_{j,k}] - f^{ijk}_{15b}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] + f_{18}^{ijkl}U_{ij}[P_\mu,U_{jk}]U_{kl}[P_\mu,U_{l} \\
& + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}) \Big\}.
\end{aligned}$$

One-Loop Effective Action

- e.g. MSSM stop

$$G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

(R-parity)

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix} \qquad \Phi = (\tilde{Q}\,, \tilde{t}^*_R).$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2} g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2} g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2} g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2} g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

$$\begin{aligned}
{}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U^2_{ij} \\
& + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij} \\
& + f_9^{ij}(U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk} U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij}[P_\mu,[P_\nu,U_{ij}]]\,[P_\mu,[P_\nu,U_{ji}]] + f_{12,b}^{ij}[P_\mu,[P_\nu,U_{ij}]]\,[P_\nu, \\
& + f_{12,c}^{ij}[P_\mu,[P_\mu,U_{ij}]]\,[P_\nu,[P_\nu,U_{ji}]] \\
& + f_{13}^{ijk} U_{ij}U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk}\,[P_\mu, U_{ij}]\,[P_\nu, U_{jk}]\,G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j}[P_\mu, U_{j,k}] - f_{15b}^{ijk}[P_\mu, U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl} U_{ij}U_{jk}[P_\mu, U_{kl}][P_\mu, U_{li}] + \\
& + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}) \Big\}.
\end{aligned}$$

$$c_{WB} = -\frac{g_2^2 c_{2\beta} + 2h_t^2}{48 m^2_{\tilde{Q}}} + \bar{X}_t^2 \left(\frac{33 m^4_{\tilde{Q}} m^2_{\tilde{t}_R} - 3 m^2_{\tilde{Q}} m^4_{\tilde{t}_R} + 5 m^6_{\tilde{Q}} + m^6_{\tilde{t}_R}}{24 m^2_{\tilde{Q}} \left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{m^2_{\tilde{Q}} m^2_{\tilde{t}_R}\left(2m^2_{\tilde{Q}} + m^2_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5} \right),$$

$$c_W = \bar{X}_t^2 \left(\frac{-8 m^2_{\tilde{Q}} m^2_{\tilde{t}_R} + m^4_{\tilde{Q}} - 17 m^4_{\tilde{t}_R}}{12\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} + \frac{\left(3 m^2_{\tilde{Q}} m^4_{\tilde{t}_R} + m^6_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5} \right),$$

$$c_B = \bar{X}_t^2 \left(\frac{-8 m^2_{\tilde{Q}} m^2_{\tilde{t}_R} - 23 m^4_{\tilde{Q}} + 7 m^4_{\tilde{t}_R}}{12\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{\left(-12 m^4_{\tilde{Q}} m^2_{\tilde{t}_R} + 3 m^2_{\tilde{Q}} m^4_{\tilde{t}_R} - 4 m^6_{\tilde{Q}} + m^6_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{6\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5} \right),$$

$$c_D = \bar{X}_t^2 \left(\frac{10 m^2_{\tilde{Q}} m^2_{\tilde{t}_R} + m^4_{\tilde{Q}} + m^4_{\tilde{t}_R}}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{3 m^2_{\tilde{Q}} m^2_{\tilde{t}_R}\left(m^2_{\tilde{Q}} + m^2_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5} \right),$$

etc.

One-Loop Effective Action

- e.g. MSSM stop

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \Phi^\dagger(P^2 - M$$

(R-parity)

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix}$$

$$U = \Big(\; (h_t^2 + \tfrac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger \quad s_\beta^2 H H^\dagger - \tfrac{1}{2}(g_1 \quad h_t X_t \tilde{H}^\dagger$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	$f_{17},\ f_{18}$	-
c_T	f_7	f_{11}	$f_{17},\ f_{18}$	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	$f_{13},\ f_{14}$	-	-
c_{BB}	f_9	$f_{13},\ f_{14}$	-	-
c_{WB}	f_9	$f_{13},\ f_{14}$	-	-
c_W	-	$f_{15a},\ f_{15b}$	-	-
c_B	-	$f_{15a},\ f_{15b}$	-	-
c_D	-	f_{12c}	-	-

$$\mathcal{L}^{\rm eff}_{\text{1-loop}}[\phi] \supset -ic_s\Big\{ f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2$$
$$+ f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij}$$
$$+ f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki})$$
$$+ f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}]$$
$$+ f_{12,a}^{ij}[P_\mu,[P_\nu,U_{ij}]][P_\mu,[P_\nu,U_{ji}]] + f_{12,b}^{ij}[P_\mu,[P_\nu,U_{ij}]][P_\nu,$$
$$+ f_{12,c}^{ij}[P_\mu,[P_\mu,U_{ij}]][P_\nu,[P_\nu,U_{ji}]]$$
$$+ f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}[P_\mu,U_{ij}][P_\nu,U_{jk}]G'_{\nu\mu,ki}$$
$$+ \left(f_{15a}^{ijk}U_{i,j}[P_\mu,U_{j,k}] - f_{15b}^{ijk}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}]$$
$$+ f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] +$$
$$+ f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni})\Big\}.$$

$$c_{WB} = -\frac{g_2^2 c_{2\beta} + 2h_t^2}{48 m_{\tilde{Q}}^2} + \bar{X}_t^2\left(\frac{33m_{\tilde{Q}}^4 m_{\tilde{t}_R}^2 - 3m_{\tilde{Q}}^2 m_{\tilde{t}_R}^4 + 5m_{\tilde{Q}}^6 + m_{\tilde{t}_R}^6}{24m_{\tilde{Q}}^2\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^4} - \frac{m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2\left(2m_{\tilde{Q}}^2 + m_{\tilde{t}_R}^2\right)\ln\left(\frac{m_{\tilde{Q}}^2}{m_{\tilde{t}_R}^2}\right)}{2\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^5}\right),$$

$$c_W = \bar{X}_t^2\left(\frac{-8m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2 + m_{\tilde{Q}}^4 - 17m_{\tilde{t}_R}^4}{12\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^4} + \frac{\left(3m_{\tilde{Q}}^2 m_{\tilde{t}_R}^4 + m_{\tilde{t}_R}^6\right)\ln\left(\frac{m_{\tilde{Q}}^2}{m_{\tilde{t}_R}^2}\right)}{2\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^5}\right),$$

$$c_B = \bar{X}_t^2\left(\frac{-8m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2 - 23m_{\tilde{Q}}^4 + 7m_{\tilde{t}_R}^4}{12\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^4} - \frac{\left(-12m_{\tilde{Q}}^4 m_{\tilde{t}_R}^2 + 3m_{\tilde{Q}}^2 m_{\tilde{t}_R}^4 - 4m_{\tilde{Q}}^6 + m_{\tilde{t}_R}^6\right)\ln\left(\frac{m_{\tilde{Q}}^2}{m_{\tilde{t}_R}^2}\right)}{6\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^5}\right),$$

$$c_D = \bar{X}_t^2\left(\frac{10m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2 + m_{\tilde{Q}}^4 + m_{\tilde{t}_R}^4}{2\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^4} - \frac{3m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2\left(m_{\tilde{Q}}^2 + m_{\tilde{t}_R}^2\right)\ln\left(\frac{m_{\tilde{Q}}^2}{m_{\tilde{t}_R}^2}\right)}{\left(m_{\tilde{Q}}^2 - m_{\tilde{t}_R}^2\right)^5}\right),$$

etc.

One-Loop Effective Action

- UOLEA is a general result, not just SMEFT, for evaluating path integral at one-loop

- Functional methods also applicable to mixed heavy-light one-loop matching

  -Henning, Lu, Murayama 1604.01019
  -S.A.R. Ellis, J. Quevillon, TY, Z. Zhang 1604.02445

- Functional method can be simplified

  -Fuentes-Martin, Portoles, Ruiz-Femenia 1607.02142
  -Z. Zhang 1610.00710

- Universality of one-loop effective action allows once-and-for-all computations

- To be completed…

  See also:
  -Anastasiou, Lazopoulos, Santiago - MatchMaker (in preparation)

Conclusion

- What does lack of new physics at LHC mean?
  - Model-building: partially-tuned naturalness (business as usual)
  - Model-building: hidden naturalness (e.g. neutral naturalness)
  - Model-building: decoupled naturalness (e.g. relaxation)
  - Model-building: accidental naturalness (e.g. landscape)
  - **Phenomenology: SMEFT**
  - (Formal theory: reformulate QFT?)

- What is SMEFT useful for?
  - Classification of experimental effects of new physics
  - Classification of new physics models
  - Systematic matching computations
  - Systematic observable computations
  - Benchmark/target precision for measurements
  - Encapsulates experimental constraints