Lost in an EFT?

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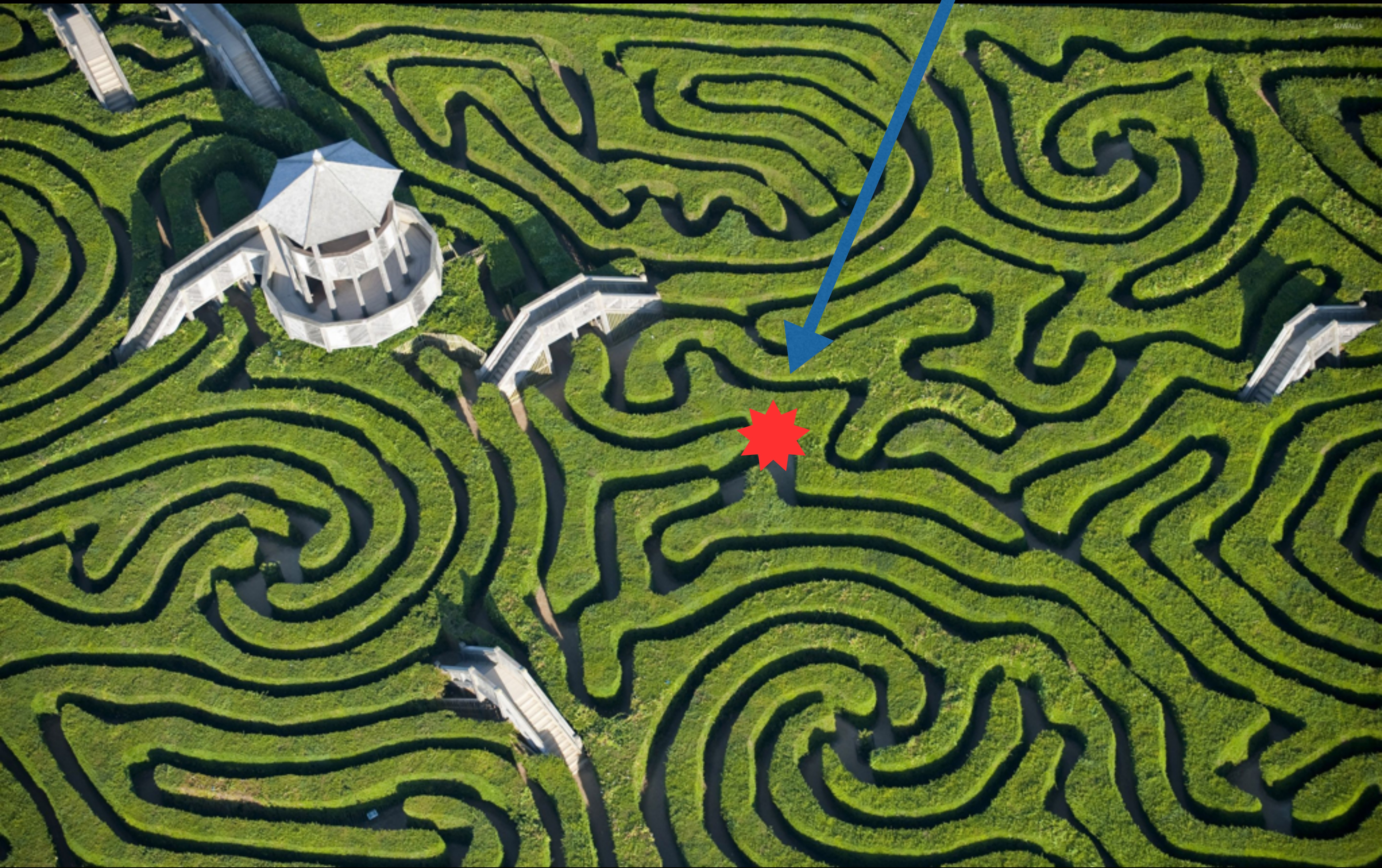
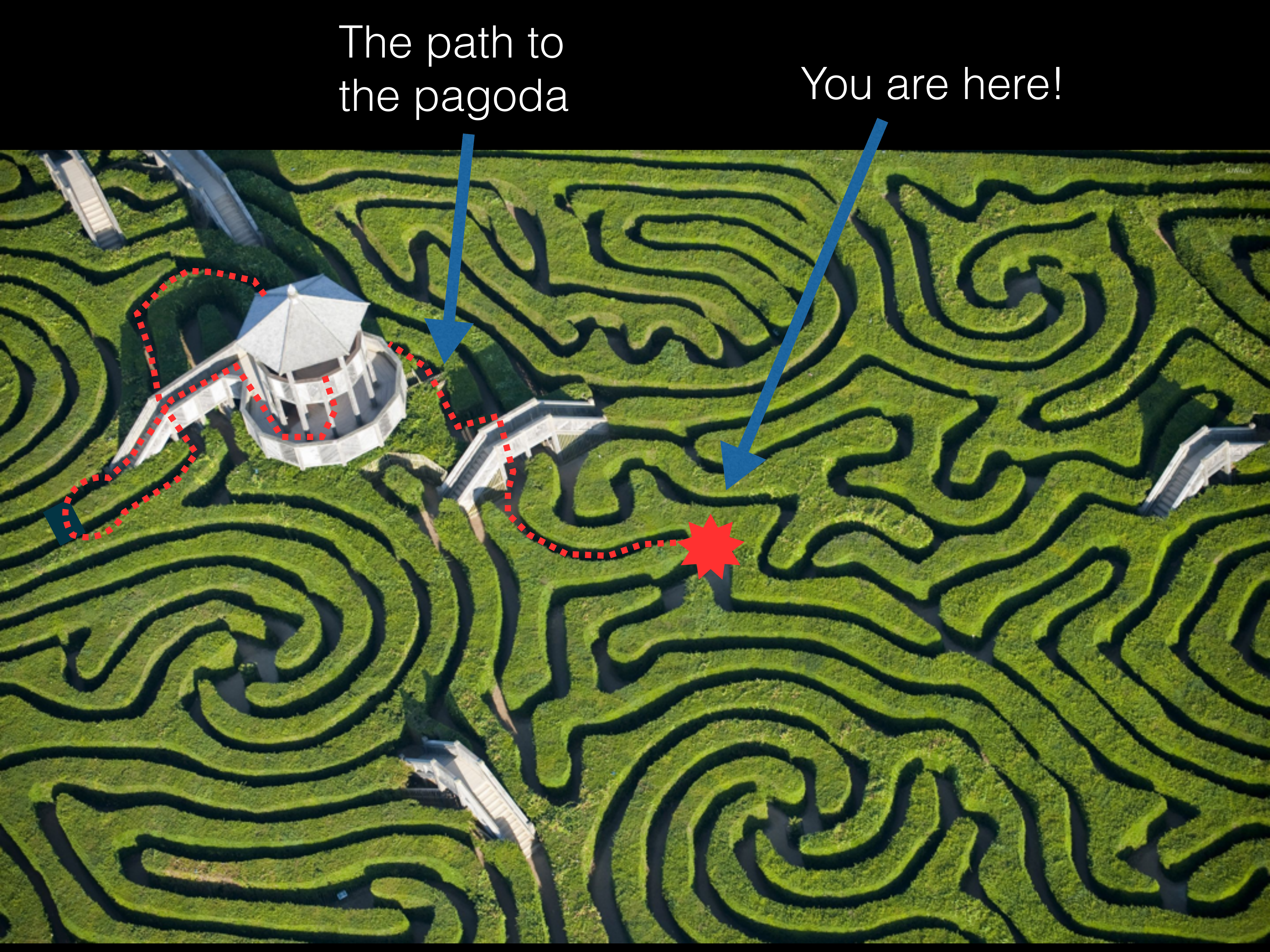
Work with Brian Henning, Xiaochuan Lu, and Hitoshi Murayama

You are here!





The path to
the pagoda
You are here!

Current LHC exploration..

…no obvious signposts so far

What can we understand about the maze in general?

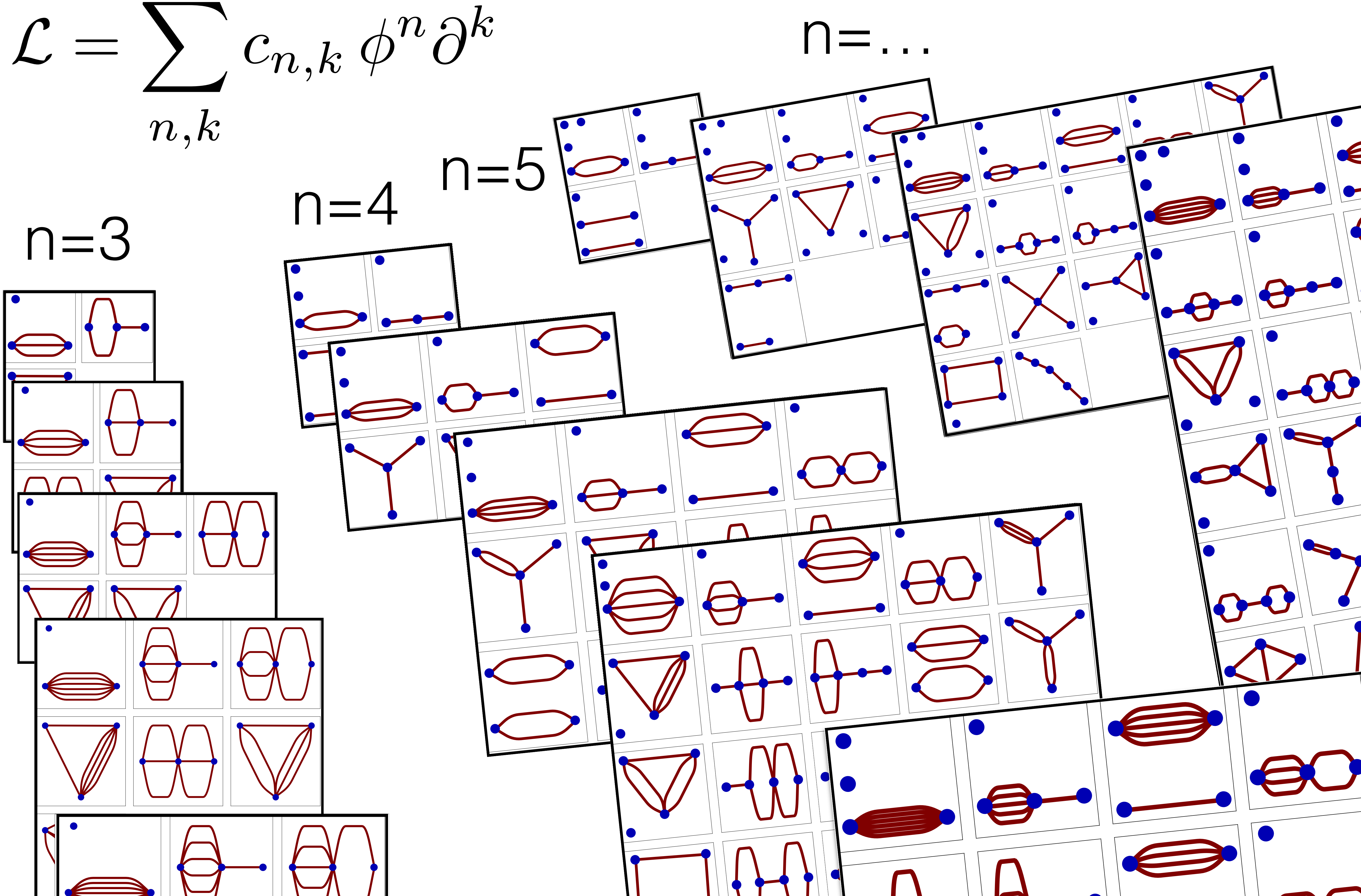
All-order Hilbert series as “S-matrix scintillas”

Orientation: EFT of a real scalar field

Connection to scattering amplitudes

EFT of a real scalar field

$$\mathcal{L} = \sum_{n,k} c_{n,k}\, \phi^n \partial^k$$



Fourier transform

$$\phi(x) = \int d^d p \, \widetilde{\phi}(p) \, e^{i \, p^\mu x_\mu}$$

General operator takes the form

$$\phi^n \partial_{\mu_1} \ldots \partial_{\mu_k} \sim \int d^d p_1 \ldots d^d p_n \, \widetilde{\phi}_1(p_1) \ldots \widetilde{\phi}_n(p_n) F^{(n,k)}(p_1, \ldots, p_n) \exp\left(i \sum_{i=1}^{n} p_i^\mu x_\mu \right)$$

Here, $F^{(n,k)}$ is a degree k polynomial in n momenta

Has to be Lorentz invariant and permutation invariant

$$\phi^n \partial_{\mu_1} \ldots \partial_{\mu_k} \sim \int d^d p_1 \ldots d^d p_n \, \widetilde{\phi}_1(p_1) \ldots \widetilde{\phi}_n(p_n) F^{(n,k)}(p_1, \ldots, p_n) \exp\left(i \sum_{i=1}^{n} p_i^\mu x_\mu \right)$$

Has to be Lorentz invariant and permutation invariant

$$\mathcal{O}_1 \sim \mathcal{O}_2 + \partial^2 \phi \, \mathcal{O}_3 \implies F_1(\{p_i\}) \sim F_2(\{p_i\}) + p_i^2 F_3(\{p_i\}) \,,$$

$$\mathcal{O}_1 \sim \mathcal{O}_2 + \mathrm{d}\mathcal{O}_3 \implies F_1(\{p_i\}) \sim F_2(\{p_i\}) + \left(\sum_i p_i^\mu \right) F_3(\{p_i\})$$

Further EOM and IBP conditions on F

$$\phi^n \partial_{\mu_1} \ldots \partial_{\mu_k} \sim \int d^d p_1 \ldots d^d p_n \, \widetilde{\phi}_1(p_1) \ldots \widetilde{\phi}_n(p_n) F^{(n,k)}(p_1, \ldots, p_n) \exp\left(i \sum_{i=1}^{n} p_i^\mu x_\mu \right)$$

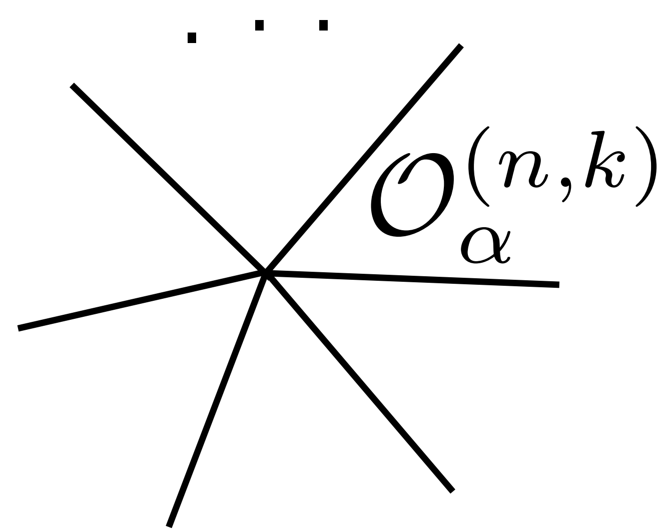
Has to be Lorentz invariant and permutation invariant

$$\begin{aligned} \mathcal{O}_1 \sim \mathcal{O}_2 + \partial^2 \phi \, \mathcal{O}_3 \implies F_1(\{p_i\}) \sim F_2(\{p_i\}) + p_i^2 F_3(\{p_i\}) \,, \\ \mathcal{O}_1 \sim \mathcal{O}_2 + \mathrm{d}\mathcal{O}_3 \implies F_1(\{p_i\}) \sim F_2(\{p_i\}) + \left(\sum_i p_i^\mu \right) F_3(\{p_i\}) \end{aligned}$$

Further EOM and IBP conditions on F

On-shell and momentum conservation

$$A_n = \sum_{k,\alpha}$$

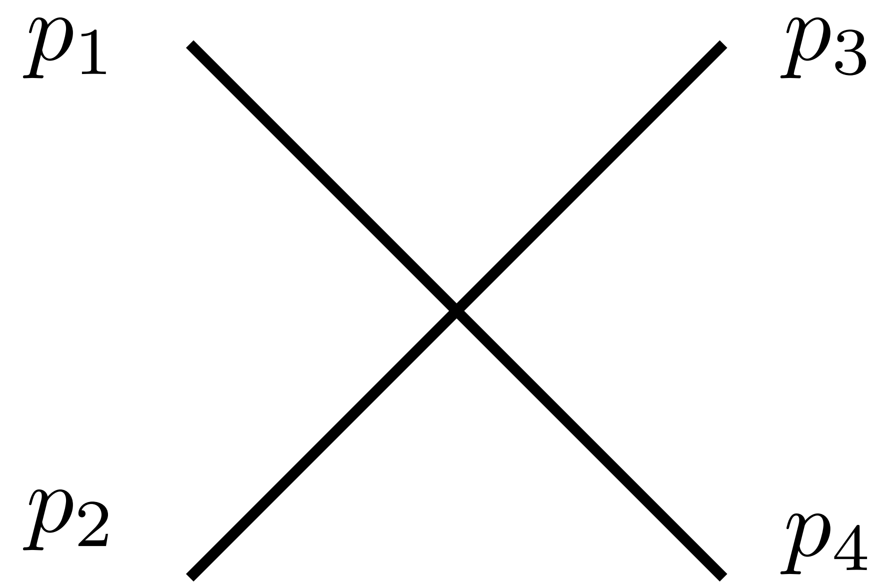


$$+ \dots$$

$$A_n(p_1, \dots, p_n) = \sum_k \sum_\alpha c_\alpha \, F_\alpha^{(n,k)}(p_1, \dots, p_n) + \dots$$

'Finding independent operators' is equiv. to writing down general amplitude (contact terms)

Four point amplitude

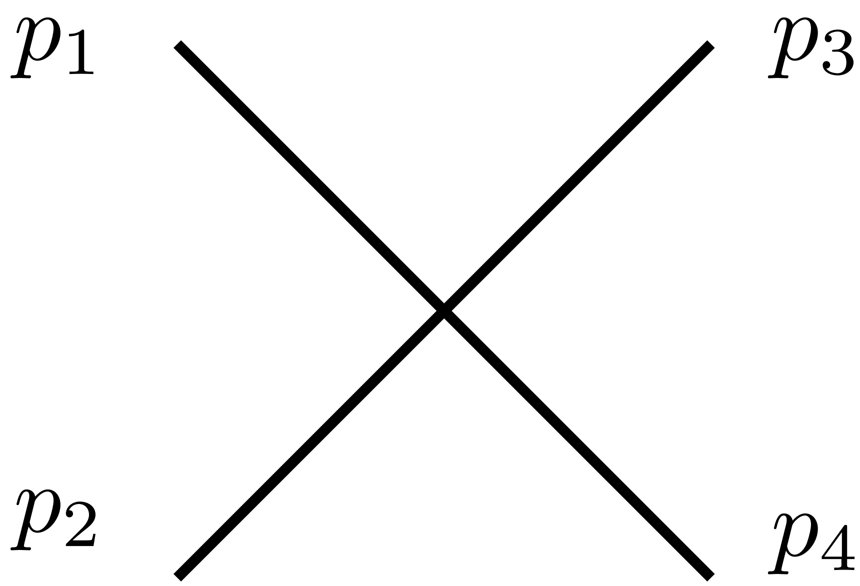


Mandelstam

$$s_{ij} = p_i^{\mu} p_{j\mu}$$

$$s_{ii} = p_i^2 = 0 \quad \text{EOM}$$

Four point amplitude



Mandelstam

$$s_{ij} = p_i^{\mu} p_{j\mu}$$

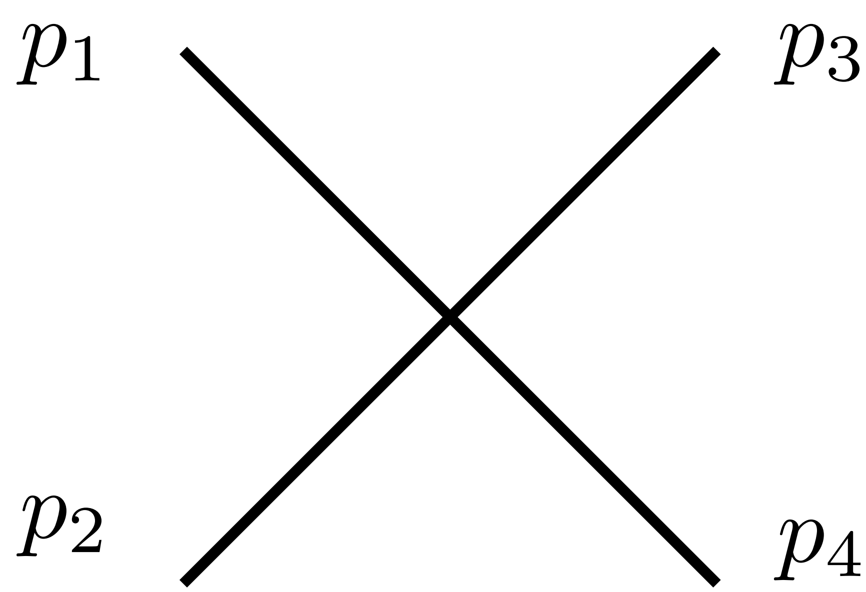
$$s_{ii} = p_i^2 = 0 \qquad \text{EOM}$$

$$\widetilde{F} = \sum_{a_1,\ldots,a_6} c_{a_1..a_6} s_{12}^{a_1} .. s_{34}^{a_6}$$

Eliminate momentum ‘4’ using IBP

$$\{s_{12}, s_{13}, s_{23}\} \equiv \{s, t, u\}$$

Four point amplitude



Mandelstam

$$s_{ij} = p_i^{\mu} p_{j\mu}$$

$$s_{ii} = p_i^2 = 0 \qquad \text{EOM}$$

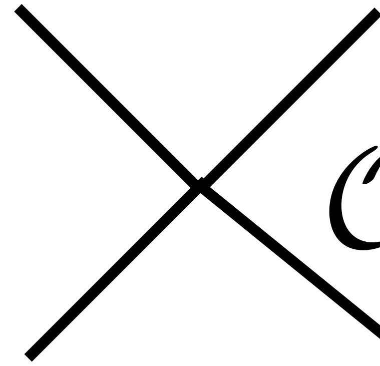
$\mathbb{R}[s,t,u]$ still need to enforce S4 invariance

$$\mathbb{R}[s,t,u]^{S_3} = \mathbb{R}[e_1, e_2, e_3]$$

$$e_1 \equiv s + t + u \qquad e_2 \equiv st + su + tu \qquad e_3 \equiv stu$$

$$e_1 = s + t + u = 0 \qquad \text{(IBP)}$$

$$\mathbb{R}[e_2, e_3]$$

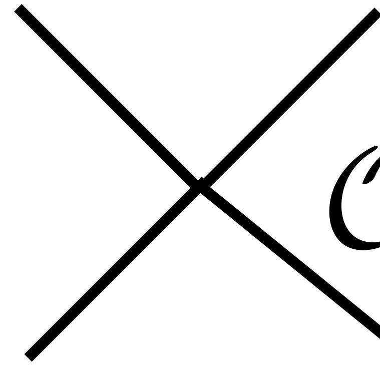
$$\mathcal{A}_4 = \sum_{k,\alpha}$$  $$\mathcal{O}_{\alpha}^{4,k} + \ldots$$

$$A_4(p_1, p_2, p_3, p_4) = \sum_{a,b} c_{a,b}(st + su + tu)^a (stu)^b + \ldots$$

Generating function

$$\frac{1}{(1-(st+su+tu))(1-stu)}$$

$$\mathcal{A}_4 = \sum_{k,\alpha}$$



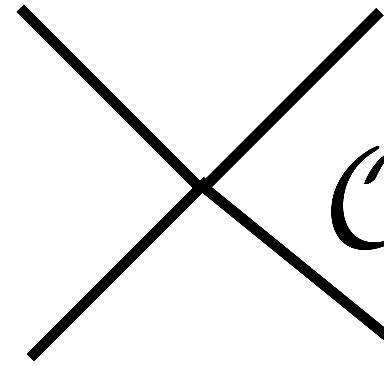
$$\mathcal{O}^{4,k}_{\alpha} + \ldots$$

$$A_4(p_1, p_2, p_3, p_4) = \sum_{a,b} c_{a,b}(st + su + tu)^a (stu)^b + \ldots$$

Generating function

$$\frac{1}{(1 - (st + su + tu))(1 - stu)}$$

$$= (1 + (st + su + tu) + (st + su + tu)^2 + \ldots)(1 + stu + (stu)^2 + \ldots)$$

$$\mathcal{A}_4 = \sum_{k,\alpha}$$  $$\mathcal{O}^{4,k}_{\alpha} + \ldots$$

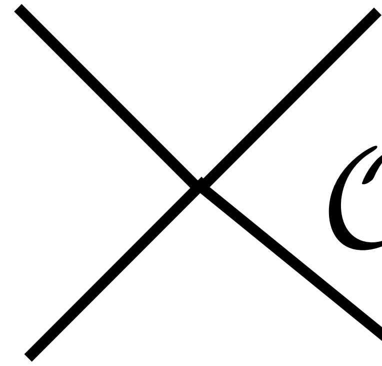
$$A_4(p_1, p_2, p_3, p_4) = \sum_{a,b} c_{a,b}(st + su + tu)^a (stu)^b + \ldots$$

Generating function

$$\frac{1}{(1 - (st + su + tu))(1 - stu)}$$

(Simpler)

$$\frac{1}{(1 - k^4)(1 - k^6)}$$

$$\mathcal{A}_4 = \sum_{k,\alpha}$$  $$\mathcal{O}^{4,k}_{\alpha} + \ldots$$

$$A_4(p_1,p_2,p_3,p_4) = \sum_{a,b} c_{a,b}(st+su+tu)^a(stu)^b + \ldots$$

Generating function

$$\frac{1}{(1-(st+su+tu))(1-stu)}$$

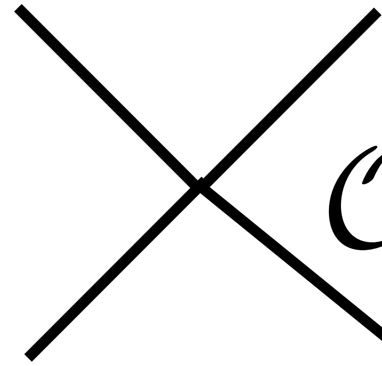
(Simpler)

$$\frac{1}{(1-k^4)(1-k^6)}$$

$$= 1 + k^4 + k^6 + k^8 + k^{10} + 2k^{12} + k^{14} + 2k^{16} + 2k^{18} + 2k^{20} + \ldots$$

e.g. 2 independent operators with 12 derivs..

$$\mathcal{A}_4 = \sum_{k,\alpha}$$



$$\mathcal{O}_\alpha^{4,k} + \dots$$

$$A_4(p_1, p_2, p_3, p_4) = \sum_{a,b} c_{a,b}(st + su + tu)^a (stu)^b + \dots$$

Generating function

$$\frac{1}{(1-(st+su+tu))(1-stu)}$$

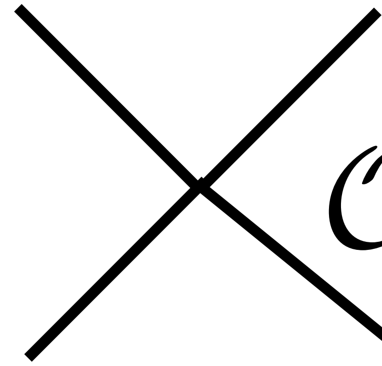
Generates independent terms

(Simpler)

$$\frac{1}{(1-k^4)(1-k^6)}$$

Generates # of independent terms

$$\mathcal{A}_4 = \sum_{k,\alpha}$$



$$\mathcal{O}_{\alpha}^{4,k} + \ldots$$

$$A_4(p_1, p_2, p_3, p_4) = \sum_{a,b} c_{a,b}(st + su + tu)^a (stu)^b + \ldots$$

Generating function

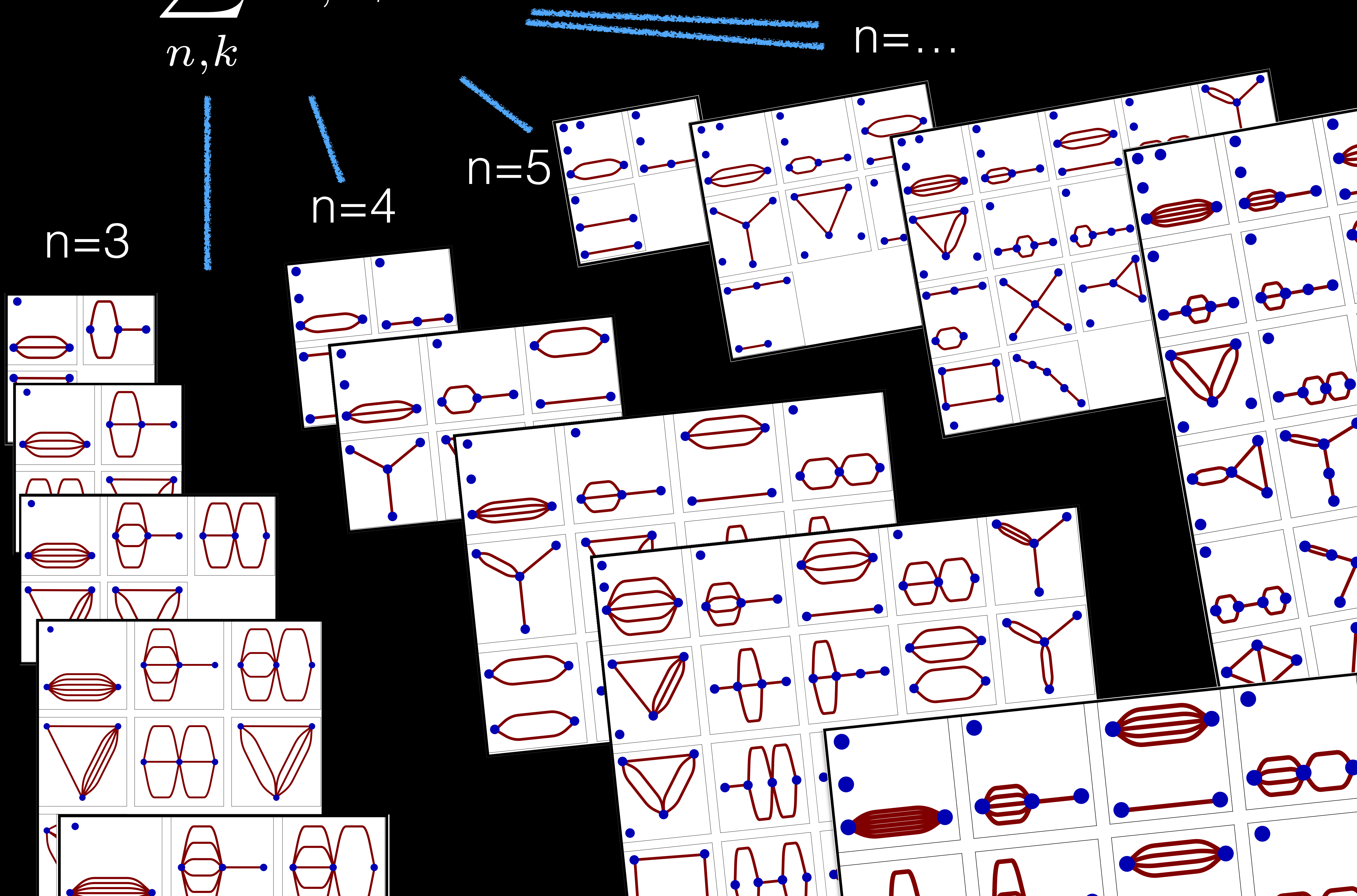
$$\frac{1}{(1 - (st + su + tu))(1 - stu)}$$

(Simpler)

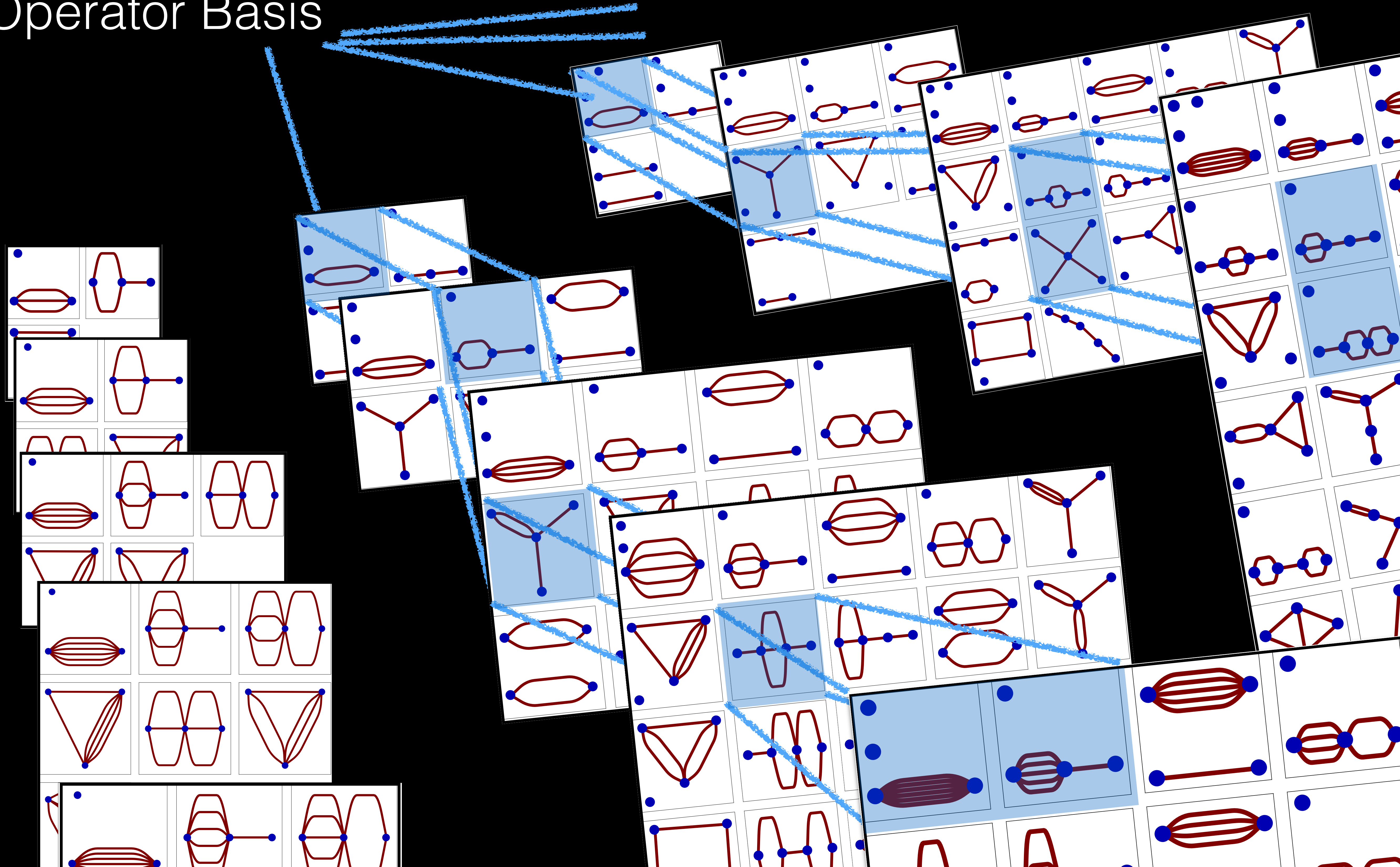
$$\frac{1}{(1 - k^4)(1 - k^6)}$$

Hilbert series $H(\phi, k)\Big|_{\mathcal{O}(\phi^4)}$

$$\mathcal{L} = \sum_{n,k} c_{n,k} \phi^n \partial^k$$



Operator Basis



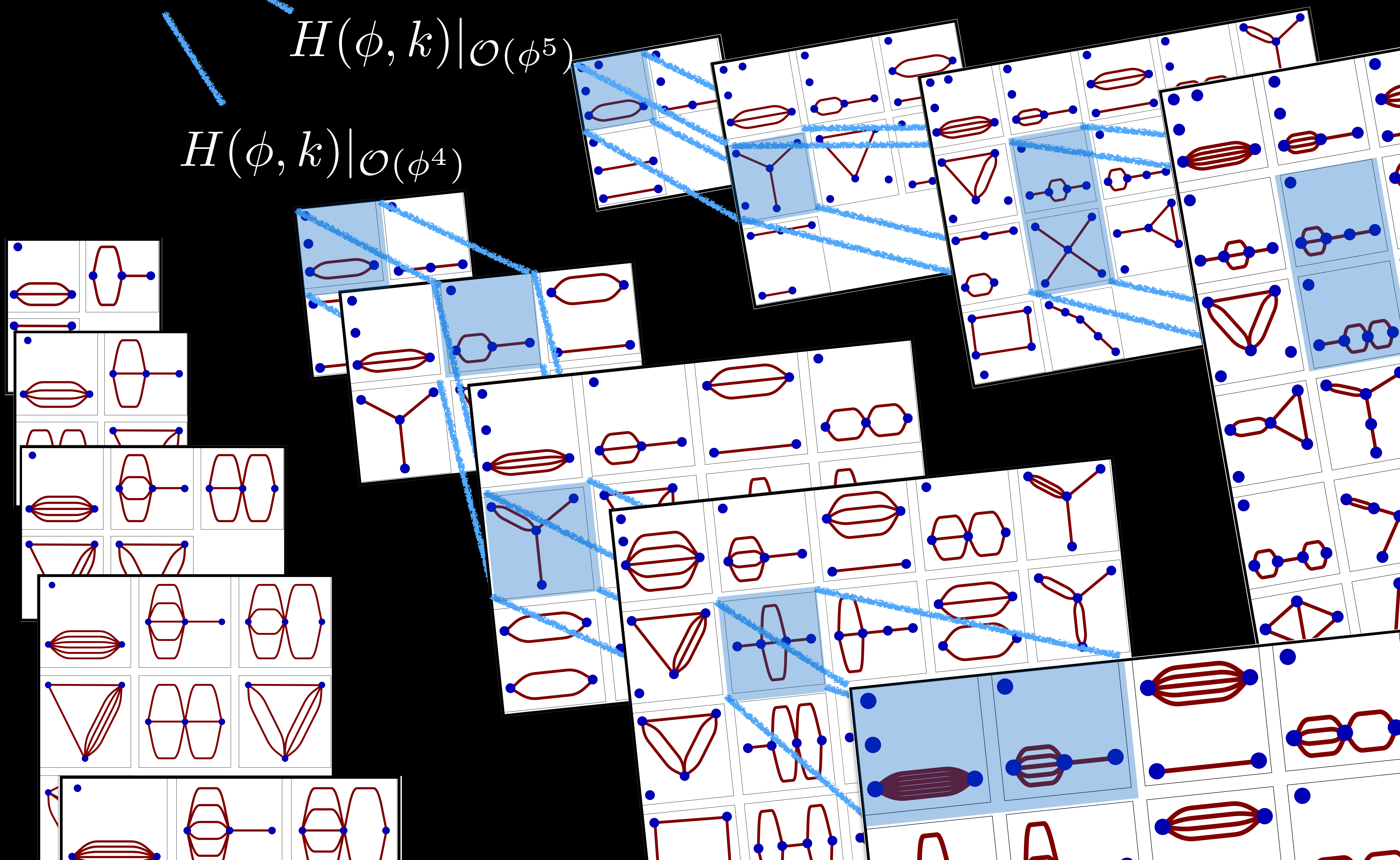
Hilbert series over operator basis

$$H(\phi, k)$$

$H(\phi, k)|_{\mathcal{O}(\phi^n)}$

$H(\phi, k)|_{\mathcal{O}(\phi^5)}$

$H(\phi, k)|_{\mathcal{O}(\phi^4)}$



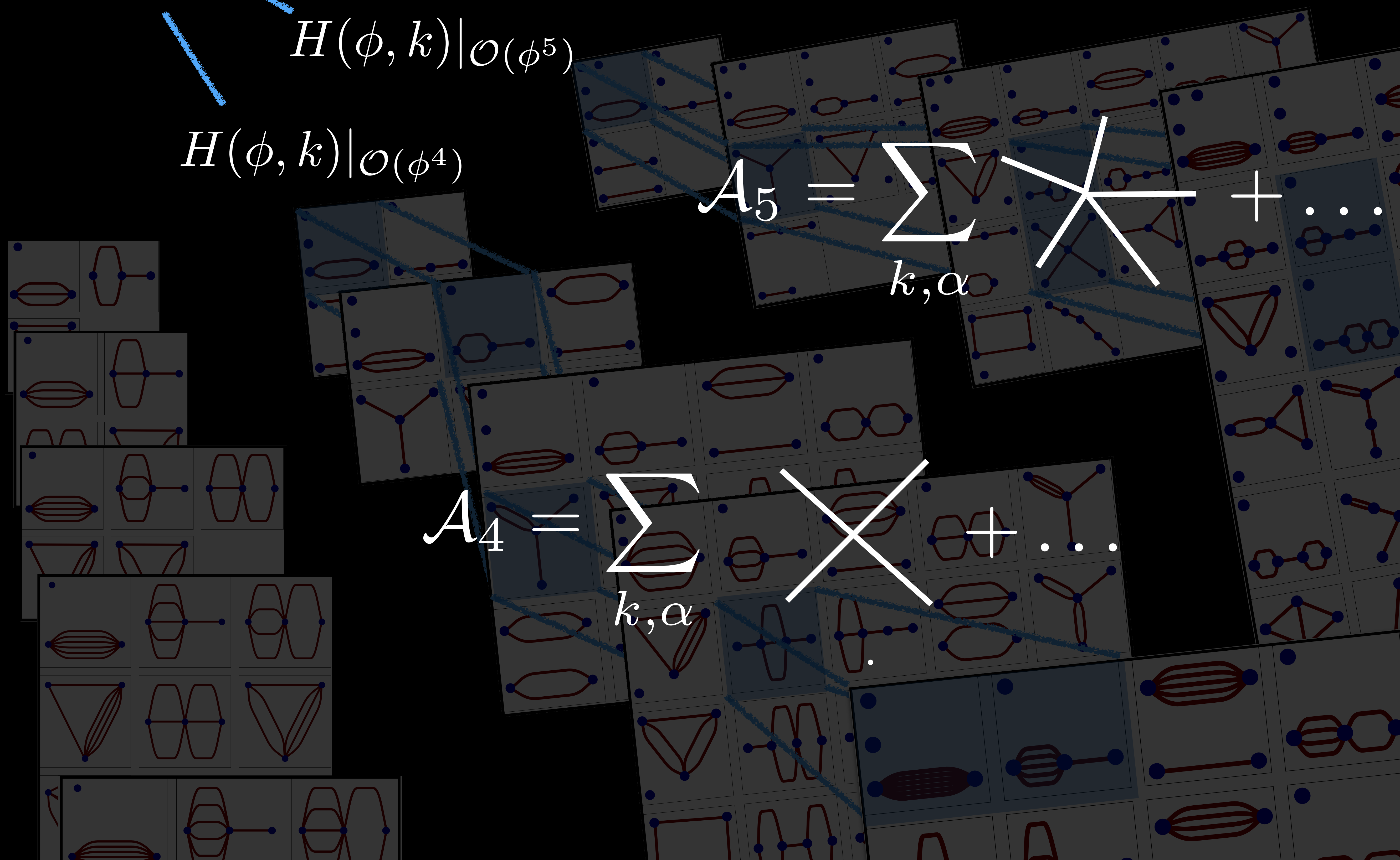
Hilbert series over operator basis

$$H(\phi, k)$$

$$H(\phi, k)|_{\mathcal{O}(\phi^5)}$$

$$H(\phi, k)|_{\mathcal{O}(\phi^4)}$$

$$H(\phi, k)|_{\mathcal{O}(\phi^n)} \qquad \mathcal{A}_n = \sum_{k,\alpha}$$



$$\mathcal{A}_5 = \sum_{k,\alpha} + \ldots$$

$$\mathcal{A}_4 = \sum_{k,\alpha} + \ldots$$

The maze of the real scalar

The maze of the real scalar

$$H(\phi, k)?$$

The maze of the real scalar

$$H(\phi, \boldsymbol{t})?$$

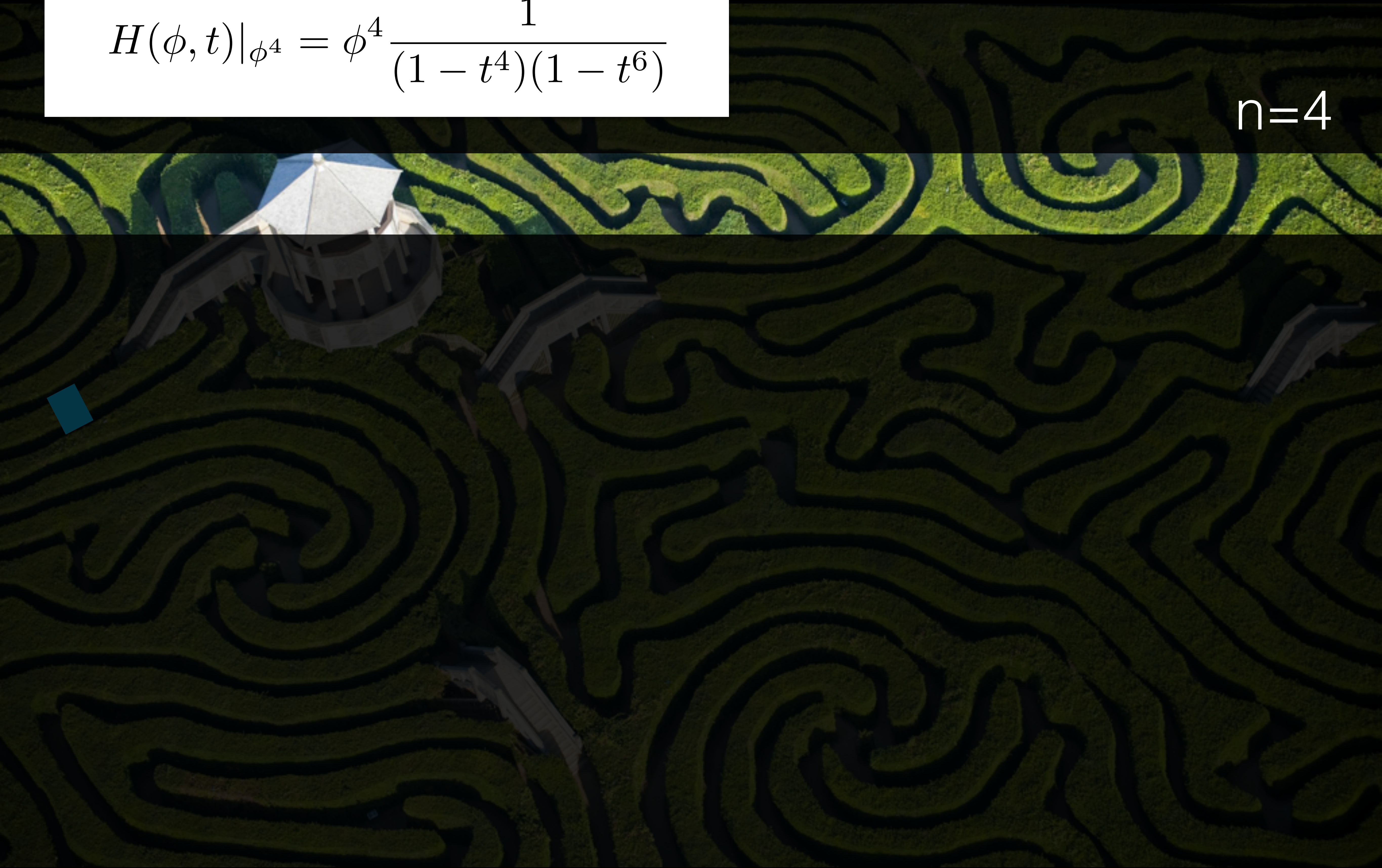
The maze of the real scalar

$$H(\phi, t)|_{\phi^4} = \phi^4 \frac{1}{(1-t^4)(1-t^6)}$$

The maze of the real scalar

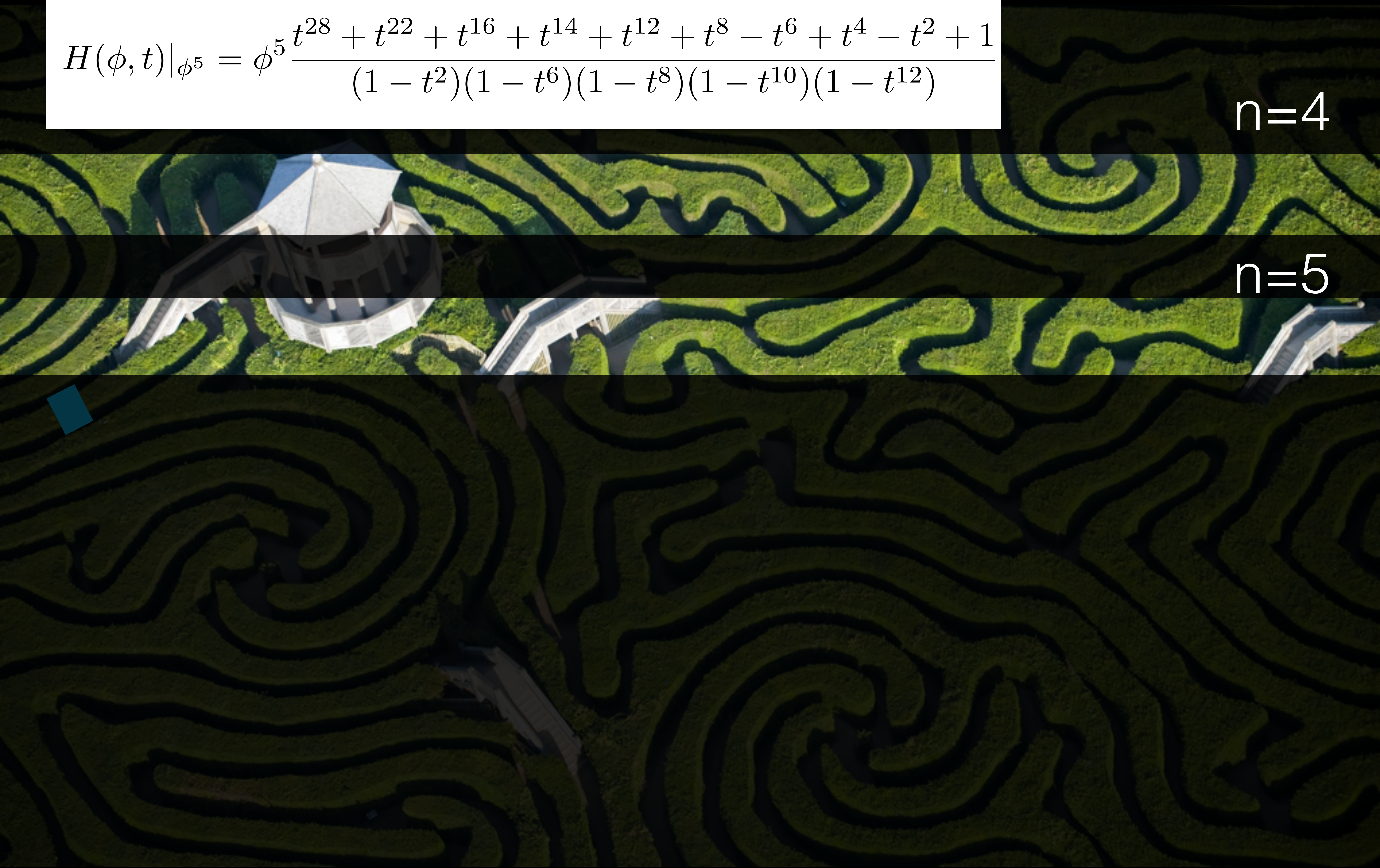
$$H(\phi,t)|_{\phi^4} = \phi^4 \frac{1}{(1-t^4)(1-t^6)}$$

n=4



The maze of the real scalar

$$H(\phi,t)|_{\phi^5} = \phi^5 \frac{t^{28}+t^{22}+t^{16}+t^{14}+t^{12}+t^8-t^6+t^4-t^2+1}{(1-t^2)(1-t^6)(1-t^8)(1-t^{10})(1-t^{12})}$$



The maze of the real scalar

$r=6$

$$
\begin{aligned}
D &= \left(1-t^2\right)\left(1-t^4\right)\left(1-t^6\right)^2\left(1-t^8\right)^2\left(1-t^{10}\right)\left(1-t^{12}\right)\\
N_{SO(4)} &= t^{40}+t^{38}+2t^{36}+2t^{34}+9t^{32}+10t^{30}+17t^{28}+12t^{26}+19t^{24}+10t^{22}+12t^{20}\\
&\quad +7t^{18}+7t^{16}+5t^{14}+3t^{12}+2t^{10}+t^8-t^2+1\\
N_{O(4)} &= t^{38}+2t^{34}+4t^{32}+7t^{30}+9t^{28}+6t^{26}+10t^{24}+3t^{22}+5t^{20}+3t^{18}+3t^{16}+2t^{14}\\
&\quad +3t^{12}+t^{10}+t^8-t^2+1
\end{aligned}
$$

$r=7$

$$
\begin{aligned}
D &= \left(1-t^2\right)\left(1-t^4\right)\left(1-t^6\right)^2\left(1-t^8\right)^2\left(1-t^{10}\right)\left(1-t^{12}\right)\left(1-t^{14}\right)\left(1-t^{20}\right)\left(1-t^{24}\right)\\
N_{SO(4)} &= 3t^{92}+10t^{90}+31t^{88}+65t^{86}+123t^{84}+223t^{82}+387t^{80}+584t^{78}+902t^{76}+1257t^{74}\\
&\quad +1736t^{72}+2255t^{70}+2892t^{68}+3482t^{66}+4226t^{64}+4785t^{62}+5440t^{60}+5885t^{58}\\
&\quad +6344t^{56}+6482t^{54}+6678t^{52}+6471t^{50}+6343t^{48}+5864t^{46}+5433t^{44}+4747t^{42}\\
&\quad +4214t^{40}+3447t^{38}+2889t^{36}+2228t^{34}+1745t^{32}+1246t^{30}+915t^{28}+579t^{26}\\
&\quad +396t^{24}+225t^{22}+132t^{20}+67t^{18}+40t^{16}+15t^{14}+9t^{12}+4t^{10}+t^8-t^2+1\\
N_{O(4)} &= 2t^{92}+5t^{90}+16t^{88}+32t^{86}+61t^{84}+111t^{82}+192t^{80}+290t^{78}+450t^{76}+623t^{74}+869t^{72}\\
&\quad +1126t^{70}+1446t^{68}+1743t^{66}+2114t^{64}+2393t^{62}+2725t^{60}+2946t^{58}+3176t^{56}+3248t^{54}\\
&\quad +3341t^{52}+3240t^{50}+3171t^{48}+2935t^{46}+2712t^{44}+2375t^{42}+2103t^{40}+1719t^{38}+1440t^{36}\\
&\quad +1110t^{34}+867t^{32}+622t^{30}+456t^{28}+286t^{26}+199t^{24}+111t^{22}+68t^{20}+35t^{18}+23t^{16}\\
&\quad +8t^{14}+7t^{12}+3t^{10}+t^8-t^2+1
\end{aligned}
$$

$r=8$

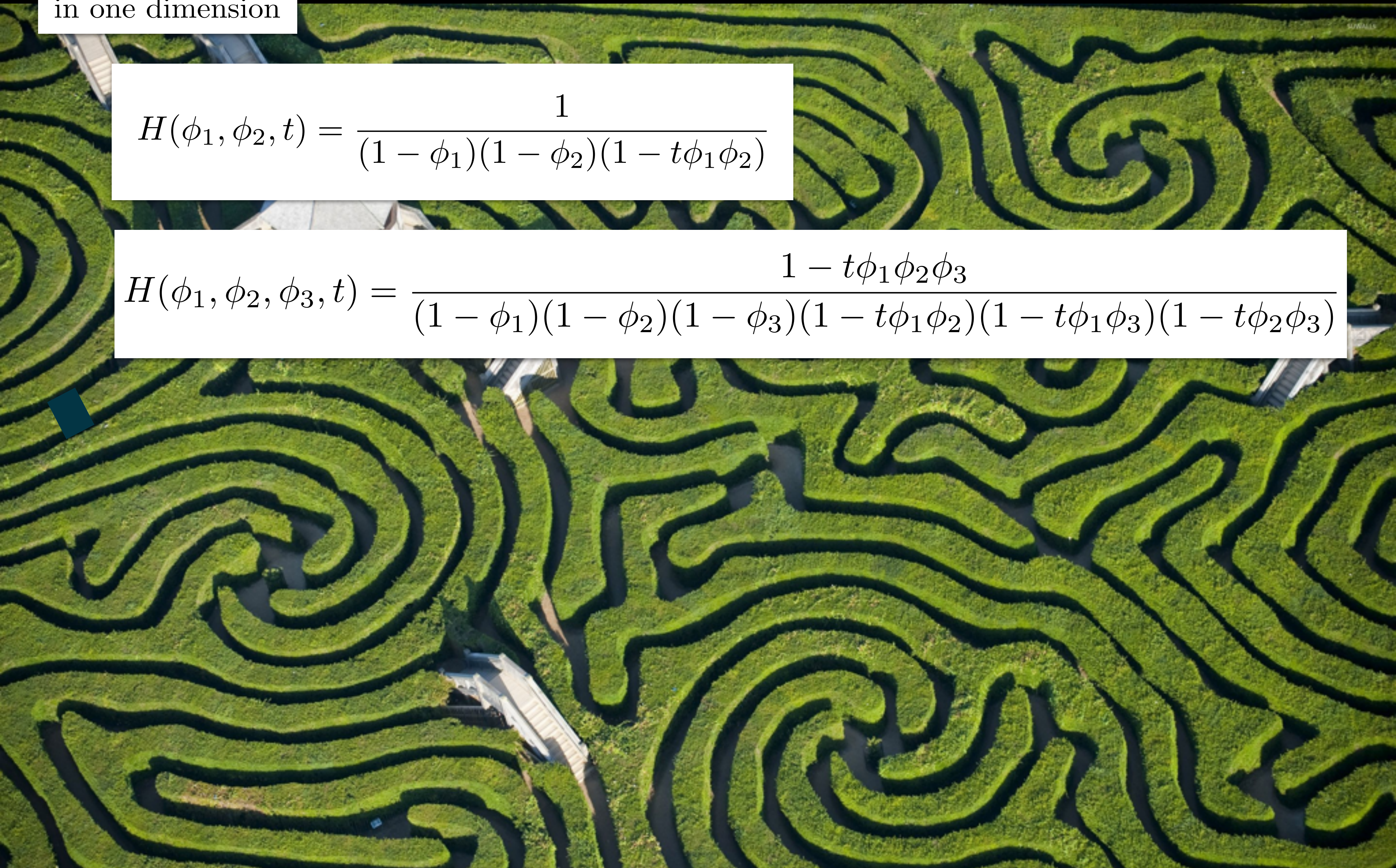
$$
\begin{aligned}
D &= \left(1-t^2\right)\left(1-t^4\right)^2\left(1-t^6\right)^2\left(1-t^8\right)^2\left(1-t^{10}\right)\left(1-t^{12}\right)\left(1-t^{14}\right)\left(1-t^{16}\right)\left(1-t^{20}\right)\left(1-t^{24}\right)\left(1-t^{30}\right)\\
N_{SO(4)} &= 2t^{140}+10t^{138}+36t^{136}+111t^{134}+271t^{132}+669t^{130}+1407t^{128}+2795t^{126}+5230t^{124}\\
&\quad +9232t^{122}+15449t^{120}+24891t^{118}+38348t^{116}+57041t^{114}+82132t^{112}+114575t^{110}\\
&\quad +155450t^{108}+205584t^{106}+265155t^{104}+334161t^{102}+412525t^{100}+498510t^{98}+591155t^{96}\\
&\quad +688016t^{94}+786655t^{92}+883876t^{90}+977349t^{88}+1062559t^{86}+1137707t^{84}+1199160t^{82}\\
&\quad +1244896t^{80}+1272653t^{78}+1282305t^{76}+1272005t^{74}+1243654t^{72}+1197349t^{70}\\
&\quad +1135523t^{68}+1060045t^{66}+974725t^{64}+881151t^{62}+784165t^{60}+685657t^{58}+589292t^{56}\\
&\quad +496902t^{54}+411517t^{52}+333452t^{50}+265063t^{48}+205662t^{46}+156048t^{44}+115222t^{42}\\
&\quad +83089t^{40}+57902t^{38}+39348t^{36}+25693t^{34}+16282t^{32}+9828t^{30}+5766t^{28}+3145t^{26}\\
&\quad +1694t^{24}+828t^{22}+394t^{20}+167t^{18}+75t^{16}+24t^{14}+11t^{12}+5t^{10}+2t^8+t^6-t^4-t^2+1\\
N_{SO(4)} &= 2t^{140}+5t^{138}+17t^{136}+55t^{134}+136t^{132}+333t^{130}+704t^{128}+1396t^{126}+2613t^{124}+4624t^{122}\\
&\quad +7727t^{120}+12449t^{118}+19185t^{116}+28527t^{114}+41075t^{112}+57305t^{110}+77725t^{108}+102791t^{106}\\
&\quad +132582t^{104}+167073t^{102}+206252t^{100}+249244t^{98}+295553t^{96}+343975t^{94}+393313t^{92}\\
&\quad +441911t^{90}+488647t^{88}+531262t^{86}+568832t^{84}+599572t^{82}+622455t^{80}+636334t^{78}\\
&\quad +641159t^{76}+636026t^{74}+621863t^{72}+598709t^{70}+567793t^{68}+530057t^{66}+487388t^{64}\\
&\quad +440609t^{62}+392120t^{60}+342830t^{58}+294647t^{56}+248455t^{54}+205749t^{52}+166713t^{50}\\
&\quad +132516t^{48}+102798t^{46}+77994t^{44}+57599t^{42}+41517t^{40}+28929t^{38}+19665t^{36}+12832t^{34}\\
&\quad +8135t^{32}+4921t^{30}+2881t^{28}+1578t^{26}+858t^{24}+419t^{22}+205t^{20}+91t^{18}+41t^{16}+14t^{14}\\
&\quad +9t^{12}+4t^{10}+2t^8+t^6-t^4-t^2+1
\end{aligned}
$$

The maze of multiple flavors of real scalars

in one dimension

$$H(\phi_1, \phi_2, t) = \frac{1}{(1-\phi_1)(1-\phi_2)(1-t\phi_1\phi_2)}$$

The maze of multiple flavors of real scalars



in one dimension

$$H(\phi_1, \phi_2, t) = \frac{1}{(1-\phi_1)(1-\phi_2)(1-t\phi_1\phi_2)}$$

$$H(\phi_1, \phi_2, \phi_3, t) = \frac{1-t\phi_1\phi_2\phi_3}{(1-\phi_1)(1-\phi_2)(1-\phi_3)(1-t\phi_1\phi_2)(1-t\phi_1\phi_3)(1-t\phi_2\phi_3)}$$

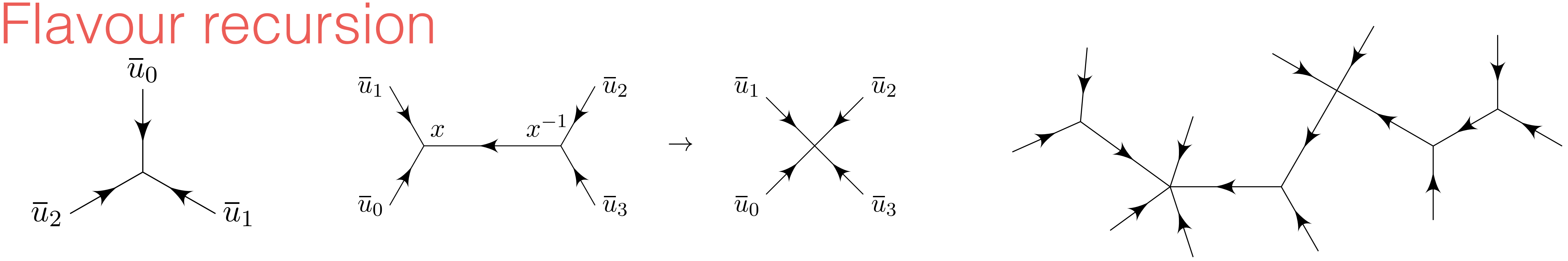
The maze of multiple flavors of real scalars

in one dimension

$$H(\phi_1, \phi_2, t) = \frac{1}{(1-\phi_1)(1-\phi_2)(1-t\phi_1\phi_2)}$$

$$H(\phi_1, \phi_2, \phi_3, t) = \frac{1-t\phi_1\phi_2\phi_3}{(1-\phi_1)(1-\phi_2)(1-\phi_3)(1-t\phi_1\phi_2)(1-t\phi_1\phi_3)(1-t\phi_2\phi_3)}$$

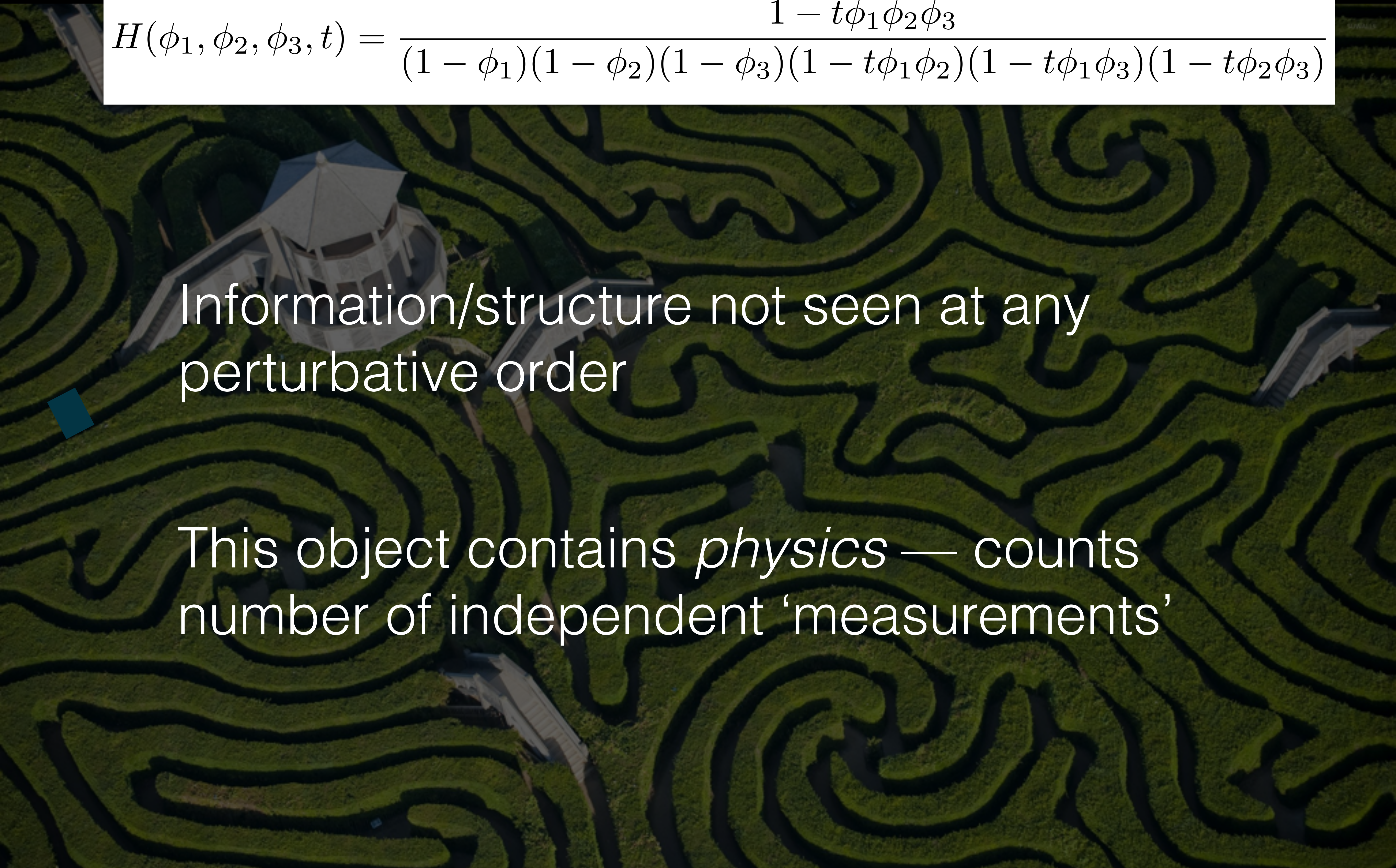
Flavour recursion



$$H_{N+1}\left(\bar{u}_0, \bar{u}_1, \cdots, \bar{u}_{N+1}\right) = \oint_{|x|=1} \frac{dx}{2\pi i}\frac{1}{x} H_N\left(\bar{u}_0, \cdots, \bar{u}_{N-1}, x\right) H_2\left(x^{-1}, \bar{u}_N, \bar{u}_{N+1}\right)$$

The maze of multiple flavors of real scalars

$$H(\phi_1, \phi_2, \phi_3, t) = \frac{1 - t\phi_1\phi_2\phi_3}{(1-\phi_1)(1-\phi_2)(1-\phi_3)(1-t\phi_1\phi_2)(1-t\phi_1\phi_3)(1-t\phi_2\phi_3)}$$

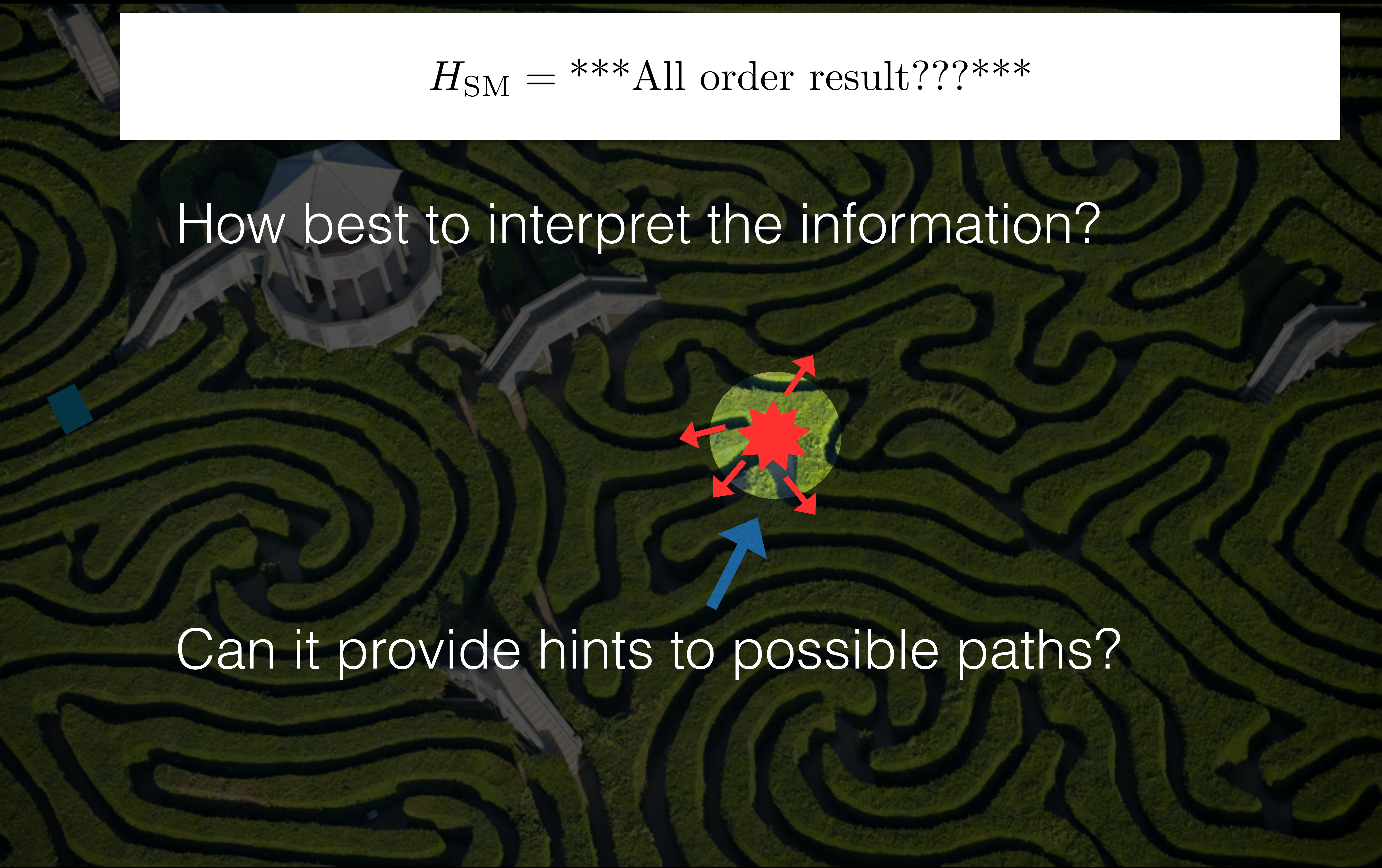


Information/structure not seen at any perturbative order

This object contains *physics* — counts number of independent ‘measurements’

Some dreaming…

$$H_{\mathrm{SM}} = \text{***All order result???***}$$



How best to interpret the information?

Can it provide hints to possible paths?

are there any deep underlying patterns?

are there any deep underlying patterns?

Connection with scattering amplitudes

Structure already in 1D

…makes us excited!