Operator Bases and Effective Field Theories

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with Xiaochuan Lu, Tom Melia, Hitoshi Murayama

1d EFTs (exactly solvable) → 1507.07240

1512.03433 ← Application to SM EFT

d-dim story → 16xx.xxxxx

HEFT 2016

Ridiculously ubiquitous

EFT consists of local description of d.o.f., possibly constrained by some symmetries

$$\mathcal{L}_{\text{eff}} = \sum_i c_i \mathcal{O}_i(x)$$

Observables in EFT:

S-matrix elements

$$\left(\quad S \quad \right) \longleftarrow \text{physical}$$

We <u>define</u>

Operator basis $\mathcal{K}$: set of operators giving independent S-matrix contributions

$$\mathcal{K} = \{\mathcal{O}_i\}$$

Very little is known about operator bases in general. However, we can infer that

Because $\mathcal{K}$ is defined by dynamical rules, it carries dynamical information

- How to extract dyn info (and its utility) remains an open question
- Present work less ambitious: simply explore structure of $\mathcal{K}$

Conceptually, important step is consideration of entire sets of operators, as opposed to individual specific ops

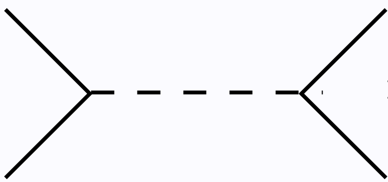
↳ Analogy: consider vector space, not just specific vectors

This type of abstraction (considering classes of objects/functions/maps) generally is fruitful in physics and math

Conceptually, important step is consideration of entire <u>sets</u> of operators

Necessary if we want to:

-Capture analytic info



$$= \phi^2 \frac{1}{-\partial^2 - M^2} \phi^2 = -\frac{1}{M^2} \sum_{n=0}^{\infty} \phi^2 \left(-\frac{\partial^2}{M^2} \right)^n \phi^2$$

-Establish correspondences

<u>HEFT</u>		<u>SM EFT</u>
$hF_{\mu\nu}F^{\mu\nu}$	$\Leftrightarrow$	$\lvert H \rvert^{2n} B_{\mu\nu} B^{\mu\nu}$ $\lvert H \rvert^{2n} W^a_{\mu\nu} W^{a\,\mu\nu}$ $n = 1, 2, \ldots$

A way to study operator bases:

a partition function on $\mathcal{K}$

Hilbert series: a partition function on $\mathcal{K}$

$$H \equiv \mathrm{Tr}_{\mathcal{K}}\, \widehat{w} = \sum_{\mathcal{O} \in \mathcal{K}} \widehat{w}(\mathcal{O})$$

where $\widehat{w}(\mathcal{O})$ is a weighting fxn for an operator

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in CFT: $\widehat{w} = e^{-\beta \widehat{H}}$ $\left(\begin{array}{c}\text{trace over all ops gives} \\ \text{physical partition fxn}\end{array}\right)$

for us: weight by field content and mass dimension

i.e. for $\mathcal{O} \sim \phi_1^{r_1} \ldots \phi_N^{r_N} \partial^k$ with $[\mathcal{O}] = \Delta$

take $\widehat{w}(\mathcal{O}) = p^{\Delta} \phi_1^{r_1} \ldots \phi_N^{r_N}$

Hilbert series: a partition function on $\mathcal{K}$

In other words, the Hilbert series counts operators

$$H(\{\phi_i\}, p) = \sum_{\Delta} \sum_{r_1, \ldots, r_N} c_{\Delta\, \mathbf{r}} p^{\Delta} \phi_1^{r_1} \ldots \phi_N^{r_N}$$

$c_{\Delta\, \mathbf{r}} \in \mathbb{N}$: Number of indep ops in $\mathcal{K}$ of mass dimension Δ and composed of $\mathbf{r} = (r_1, \ldots, r_N)$ powers of $\phi_1, \ldots, \phi_N$

$\{\phi_a\}, p$: "spurions" to label op content—simply complex numbers

The structure of operator bases

What does "independent" mean?

Two operators can lead to the same physical effect due to three basic reasons:

1) Group relations (e.g. Fierz identities)

2) Equations of motion

3) Integration by parts

EOM – on-shell conditions

Can use EOM inside correlation functions w/o changing physical result

$$\partial^2\phi = 0$$

Think of as on-shell condition

Leads to equivalence relation b/w ops

$$\mathcal{O}_1 \sim \mathcal{O}_2 \quad \text{if} \quad \mathcal{O}_1 = \mathcal{O}_2 + \mathcal{O}_3 \frac{\delta S_{\text{kin}}}{\delta \Phi_a}$$

IBP – momentum conservation

Total derivatives don't contribute to scattering

$$\int d^4x \, \partial_\alpha \mathcal{O}^\alpha(x) = 0$$

In Fourier space, simply momentum conservation

$$\sum_i p_{i\mu} = 0$$

Also leads to equivalence relation b/w ops

$$\mathcal{O}_1 \sim \mathcal{O}_2 \quad \text{if} \quad \mathcal{O}_1 = \mathcal{O}_2 + \partial \cdot \mathcal{O}_3$$

Note the kinematic nature of the previous redundancies

↓

No accident! Op basis is defined by S-matrix rules!

(see also talk by T. Melia)

How to handle the derivative

We've found a way to systematically handle EOM and IBP redundancies, putting them on the same footing as group relations

Upshot: Structure of op basis organized by the conformal algebra

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Upshot: Structure of op basis organized by the conformal algebra

Physical intuition: EFTs are perturbative around free theories, and free theories are conformal

Handling the deriv w/ conformal algebra

Have some set of generating operators

$$\left.\begin{array}{r}\phi \\ \partial_\mu \phi \\ \partial_{\mu_1}\partial_{\mu_2}\phi \\ \vdots\end{array}\right\}$$ All local ops formed from products of these

Remove EOM, $\partial^2\phi = 0$, by taking symm, traceless components

L. Lehman and A. Martin
arXiv:1503.07537

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$$\phi$$

descendants are total derivatives

$$\left\{\begin{array}{r}\partial_\mu\phi\\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi\\ \vdots\end{array}\right.$$

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descendants are total derivatives $$\begin{array}{r} \phi \\ \left\{ \begin{array}{r} \partial_\mu \phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{array} \right. \end{array}\Bigg\}$$ forms free-field irrep of conformal group $R_{[(d-2/2);0]}$

BH, Melia, Lu, Murayama
arXiv:1512.03433,16xx.xxxx

Handling the deriv w/ conformal algebra

$$\begin{pmatrix} \phi \\ \partial_\mu \phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$$

Generate local ops by taking tensor products of $R_{[(d-2/2);0]}$

Decomposing back into conformal irreps organizes IBP

$$\underbrace{\text{sym}}^{n} R_{[(d-2)/2;0]} = \sum_{\Delta,l} \underbrace{b^{(n)}_{\Delta,l}} R_{[\Delta;l]}$$

symm tensor prod b/c of Bose statistics

multiplicities

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Only primary ops are not total derivatives!
→ Operator basis consists of scalar (spin-0), conformal primaries

Op bases organized by conformal algebra

May wish to impose invariance under some symmetry group G, in which case:

$$\mathcal{K} = \{\text{scalar, } G\text{-inv, conformal primaries}\}$$

Computing the Hilbert series

$$\mathrm{sym}^n R_{[(d-2)/2;0]} = \sum_{\Delta,l} b^{(n)}_{\Delta,l} R_{[\Delta;l]}$$

To count operators want multiplicities of scalar ($l = 0$) irreps in this decomposition

Can compute using basic group theory techniques involving characters

Obtain expression for Hilbert series as a matrix integral

A matrix formula for H

$$H(p,\phi) = \underbrace{\int d\mu_{SO(d)}(x)}\,\underbrace{\frac{1}{P(p,x)}}\,\underbrace{\mathrm{PE}}\Big[\phi\,\underbrace{\widetilde{\chi}_{[(d-2)/2;0]}}\Big] + \dots$$

Integral over $SO(d)$:
selects out Lorentz scalars

$$\frac{1}{P(p,x)} = \det{}_{\Box}(1 - pg)$$

accounts for IBP

Plethystic exponential:
generating fxn for symmetric products

Conformal character of $R_{[(d\text{-}2)/2;0]}$ - accounts for EOM

A matrix formula for H

$$H(p,\phi) = \int d\mu_{SO(d)}(x) \frac{1}{P(p,x)} \mathrm{PE}\left[\phi \widetilde{\chi}_{[(d-2)/2;0]}\right] \underbrace{+ \ldots}$$

Slight corrections

for $\Delta \leq d$ ops

Application: SM EFT

Want independent gauge invariant, Lorentz invariant ops built from SM fields

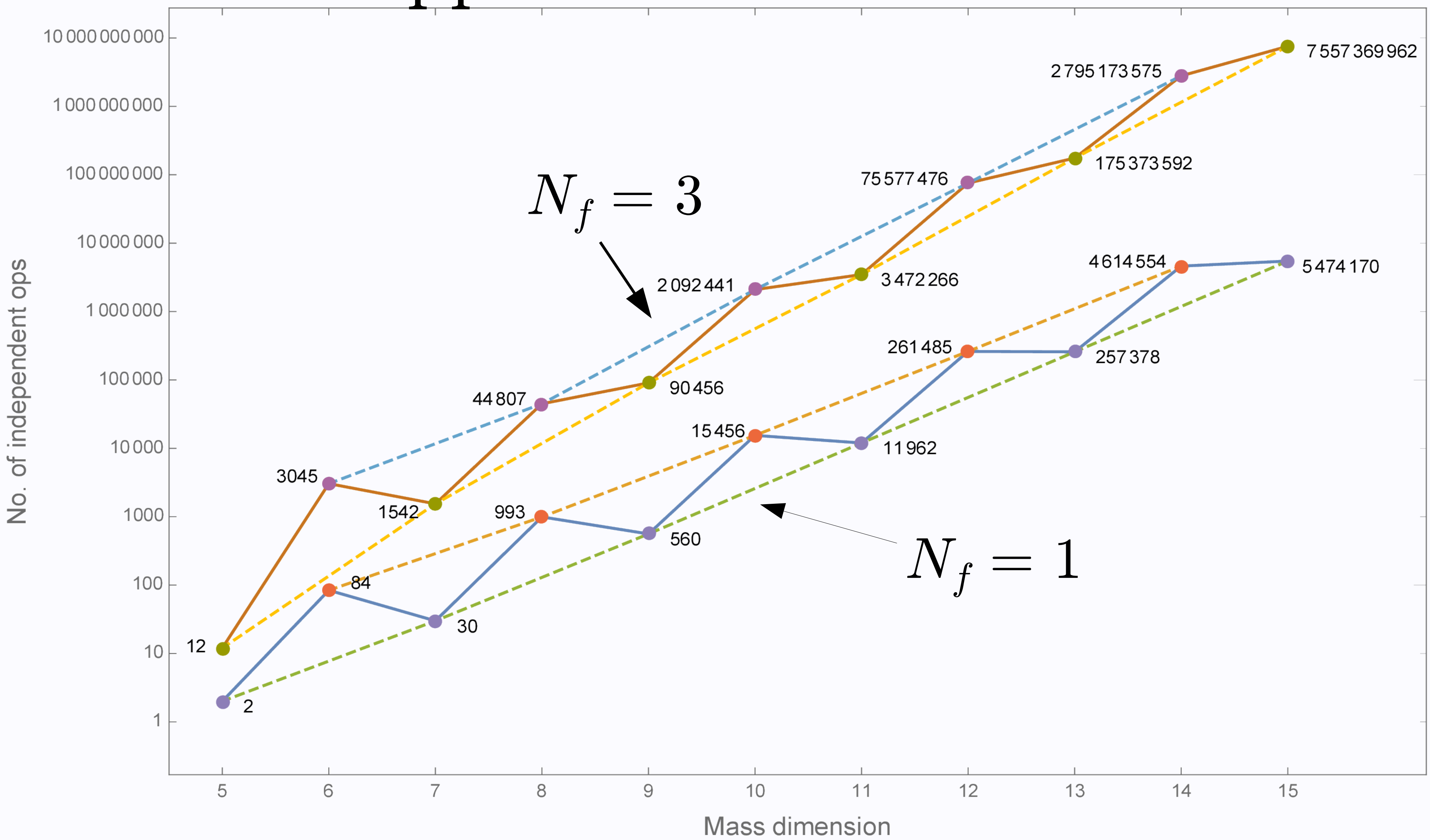
	$SU(2)_L$	$SU(2)_R$	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$
H	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$
Q	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$
u_c	$\mathbf{2}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$
d_c	$\mathbf{2}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$
L	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
e_c	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	1
G_L	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$	0
W_L	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0
B_L	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0

SM field content

$$X_{L/R} = \frac{1}{2}(X \pm \widetilde{X})$$

Each field has generating multiplet corresponding to a free field conformal irrep

Application: SM EFT



Number of indep ops in SM EFT up to dim 15

Non-linear realizations

Non-linear realizations

$$\xi(x) = e^{i\pi^i(x)X^i/f}$$

Pions parameterize coset G/H

The building blocks (CCWZ)

Maurer-Cartan forms

$$\xi^{-1}\partial_\mu \xi = u_\mu^i X^i + \underbrace{v_\mu^a T^a}$$

$$D_\mu = \partial_\mu + v_\mu$$

Transforms like gauge field

Lowest order Lagrangian

$$\mathcal{L} = f_\pi^2 \mathrm{Tr}(u_\mu^2)$$

Non-linear realizations

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<u>The building blocks (CCWZ)</u>

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$$D_\mu = \partial_\mu + v_\mu$$

Transforms like gauge field

$$\left.\begin{matrix} u_\mu \\ D_\mu \\ F_{\mu\nu} \end{matrix}\right\}$$

"Naive" building blocks for higher order terms... there are constraints!

Multiplet of generators

$$F_{\mu\nu} = -[u_\mu, u_\nu]_{\mathfrak{h}}$$
$$D_\mu u_\nu - D_\nu u_\mu = -[u_\mu, u_\nu]_{\mathfrak{g}/\mathfrak{h}}$$

Constraints: imply we can eliminate $F_{\mu\nu}$ and curl of u_μ (as well as $D^2 u_\mu$)

$$D^\mu u_\mu = 0$$ ← EOM

Above implies generators of local ops to be

$$u_\mu$$
$$D_{\{\mu_1} u_{\mu_2\}}$$
$$D_{\{\mu_1} D_{\mu_2} u_{\mu_3\}}$$
$$\vdots$$

Hilbert series for non-linear realization

$$\left.\begin{array}{c} u_\mu \\ D_{\{\mu_1} u_{\mu_2\}} \\ D_{\{\mu_1} D_{\mu_2} u_{\mu_3\}} \\ \vdots \end{array}\right\}$$

Associate character χ_V

Get Hilbert series

$$H(p,V) = \int d\mu_H(y) \int d\mu_{SO(d)}(x) \frac{1}{P(p,x)} \mathrm{PE}\Big[V \chi_V(p,x,y)\Big] + \ldots$$

(V is a spurion for u_μ)

Déjà vu

Determining # of HDOs in SM EFT has long (and tricky) history.

- Buchmuller and Wyler (1986)
- Grzadkowski et. al. (2010)
- UCSD group (2013)
- Lehman and Martin (2014/2015)

Buchmuller and Wyler (1986), Grzadkowski et. al. (2010): Dim 6, $N_f = 1$

UCSD group (2013) ← Dim 6, general N_f

Lehman and Martin (2014/2015): Dim 7, general N_f; Dim 8, $N_f = 1$

Déjà vu

Determining # of HDOs in Chiral Lagrangian has long (and tricky) history.

- Gasser and Leutwyler (1984/85) ← $\mathcal{O}(p^4)$
- Fearing and Scherer (1994) ← $\mathcal{O}(p^6)$
- Bijnens et. al. (1999/2001) { $\mathcal{O}(p^6)$ Parity even/odd sectors

Déjà vu

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We have found further redundancies

in $\mathcal{O}(p^6)$ terms!

What can we hope to learn?

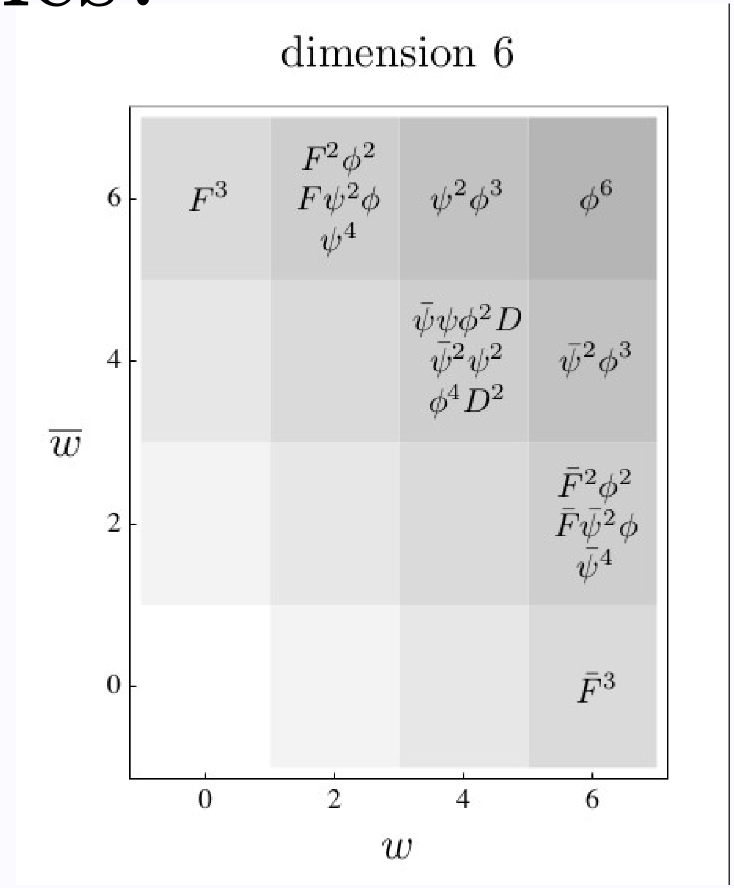
Selection rules?

Non-renormalization of dim-6 ops

Alonso, Jenkins, Manohar 1409.0868

Cheung and Shen 1505.01844

Operator mixing can go "right or up" but not "down or left"



Some "right ingredients" in our framework

- Character multiplication $\leftrightarrow$ OPE
- weight $w = n - h$ similar to twist $\tau = \Delta - l$

Correspondence b/w non-linear & linear

On general grounds

$$S_{\text{HEFT}}[h, \dots] \qquad \& \qquad S_{\text{SMEFT}}[H, \dots]$$

can parameterize the same physics

$$\downarrow$$

Can we be more precise in this statement?

- number of distinct physical effects, power counting schemes, etc.

Thanks

Operator bases...

...still not as cool as the ringle