

Neutrino Theory and Phenomenology:

Lecture II



NBIA PhD School: Neutrinos Underground & in the Heavens (Copenhagen, DK, 2016)

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Outline of lectures:

Lecture I

Pedagogical intro + warm-up case study for oscillations

Lecture II

Standard 3 ν oscillations: evolution and current status

Lecture III

Neutrino absolute masses + open problems in ν physics

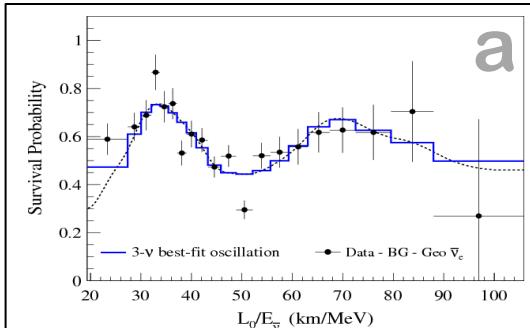
After SBL reactors, let's continue with expt's mainly sensitive to Δm^2

We shall then consider matter effects and oscillations sensitive to δm^2

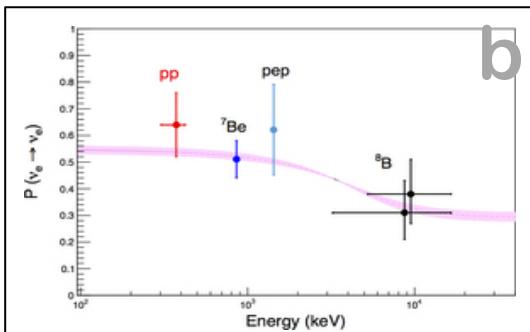
... and combine the whole information on mass-mixing osc. parameters

δm^2 dominated

$e \rightarrow e$ (δm^2 , θ_{12})

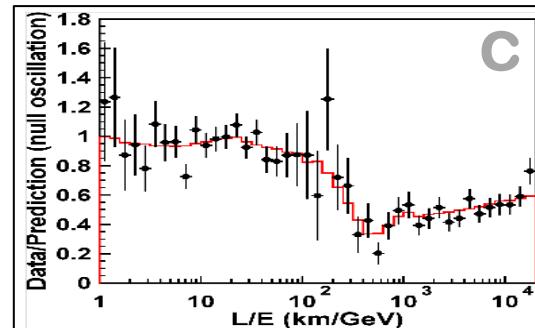


$e \rightarrow e$ (δm^2 , θ_{12})

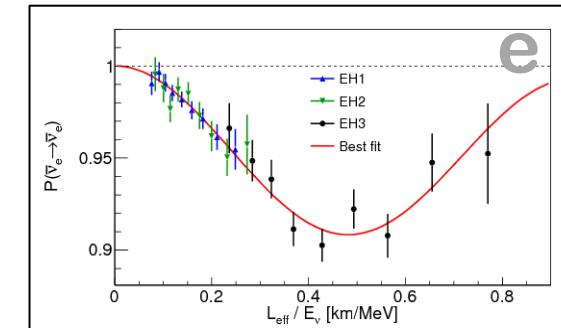


Δm^2 dominated

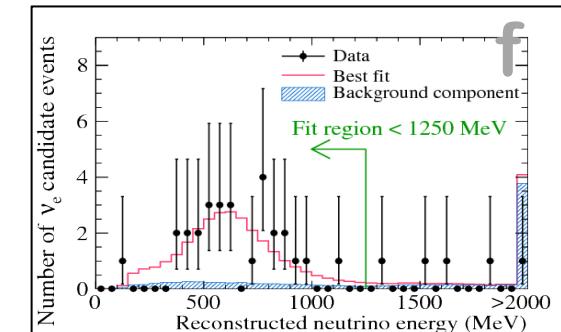
$\mu \rightarrow \mu$ (Δm^2 , θ_{23})



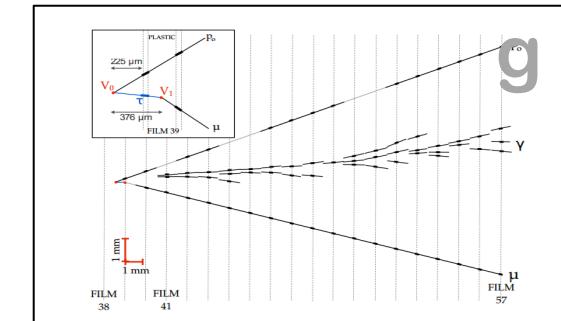
$e \rightarrow e$ (Δm^2 , θ_{13})



$\mu \rightarrow e$ (Δm^2 , θ_{13} , θ_{23})



$\mu \rightarrow \tau$ (Δm^2 , θ_{23})



δm^2 $|\Delta m^2|$ θ_{12} θ_{23} θ_{13}

+ 3 ν unknowns: sign(Δm^2), sign($\theta_{23} - \pi/4$), δ

Propagation

Exercise: One-dominant-mass-scale approximation (vacuum)

Prove that, in experiments mainly sensitive to Δm^2 , i.e.:

$$\frac{\Delta m^2 \chi}{4E} \sim \mathcal{O}(1) \quad \text{and} \quad \frac{\Delta m^2 \chi}{4E} \ll 1$$

the oscillation probabilities depend only on $|\Delta m^2|$ and on the mixing with ν_3 (elements $|U_{\alpha 3}|$, governed by θ_{23} and θ_{13}):

$$P_{\alpha\alpha} = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) \simeq 1 - 4 |U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) \sin^2 \left(\frac{\Delta m^2 \chi}{4E} \right)$$

$$P_{\alpha\beta} = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \simeq 4 |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m^2 \chi}{4E} \right) \quad \alpha \neq \beta$$

$$\text{where } |U_{e 3}|^2 = S_{13}^2, |U_{\mu 3}|^2 = C_{13}^2 S_{23}^2, |U_{\tau 3}|^2 = C_{13}^2 C_{23}^2$$

- no sensitivity to $(\Delta m^2, \theta_{12})$ of course, but also:
- no sensitivity to hierarchy or CP phase δ
- no difference $\nu/\bar{\nu}$

(See tutorial)

In this approximation, one is probing Δm^2 and the mixing matrix elements $|U_{\alpha 3}|^2$ of ν_3 with $\nu_\alpha = (\nu_e, \nu_\mu, \nu_\tau)$

$$\begin{bmatrix} U_{e1} & U_{e2} & \boxed{U_{e3}} \\ U_{\mu 1} & U_{\mu 2} & \boxed{U_{\mu 3}} \\ U_{\tau 1} & U_{\tau 2} & \boxed{U_{\tau 3}} \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & \boxed{s_{13}e^{-i\delta_{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & \boxed{s_{23}c_{13}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & \boxed{c_{23}c_{13}} \end{bmatrix}$$

For the following discussion we need to know that...

$$P(\nu_e \rightarrow \nu_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

$$P(\nu_\mu \rightarrow \nu_e) \simeq s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - 4c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

$$P(\nu_\mu \rightarrow \nu_\tau) \simeq c_{13}^4 \sin^2 2\theta_{23} \left(\frac{\Delta m^2 L}{4E} \right)$$

(Note: approximation not sensitive to δ (CPV) and to mass hierarchy)

... and that, just after the first CHOOZ results, $\theta_{13} \sim 0$ was ~OK

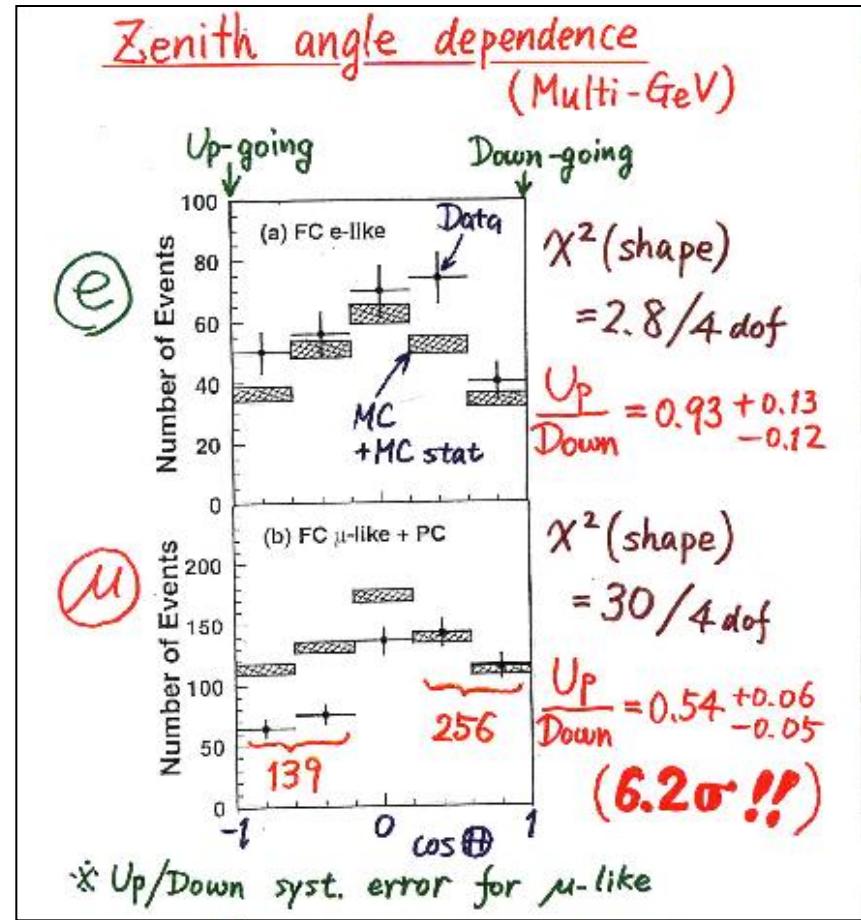
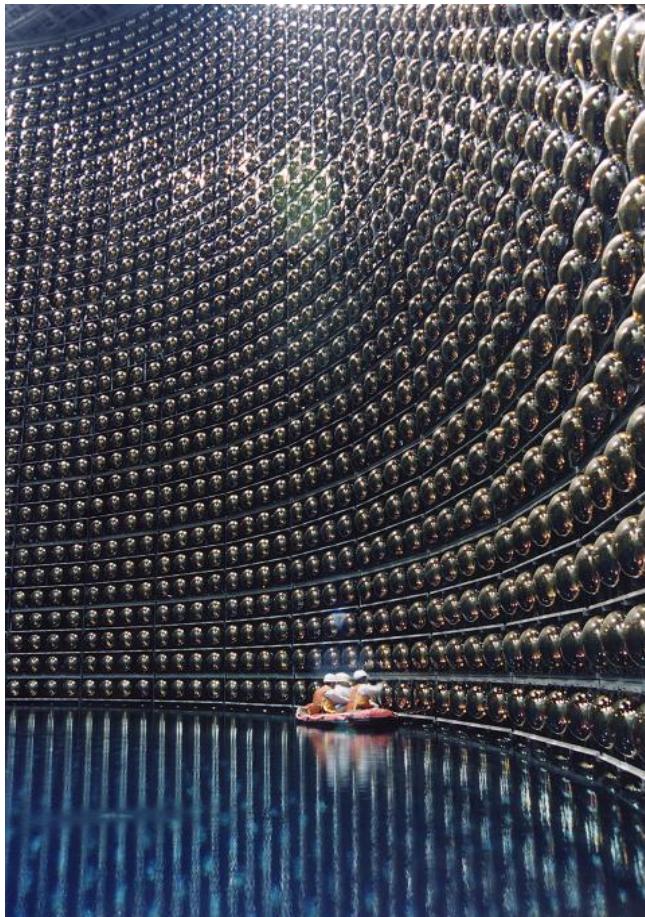
$$P(\nu_e \rightarrow \nu_e) \quad \simeq \quad 1$$

$$P(\nu_\mu \rightarrow \nu_e) \quad \simeq \quad 0$$

$$P(\nu_\mu \rightarrow \nu_\mu) \quad \simeq \quad 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

$$P(\nu_\mu \rightarrow \nu_\tau) \quad \simeq \quad \sin^2 2\theta_{23} \left(\frac{\Delta m^2 L}{4E} \right)$$

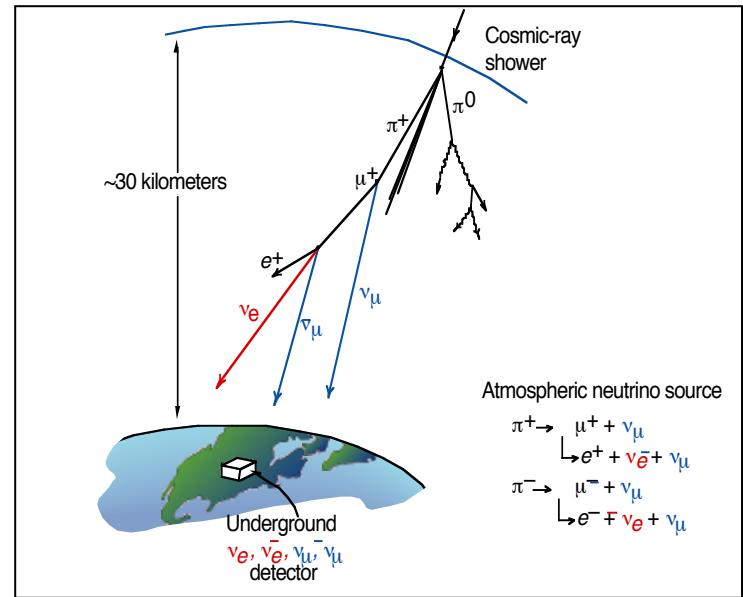
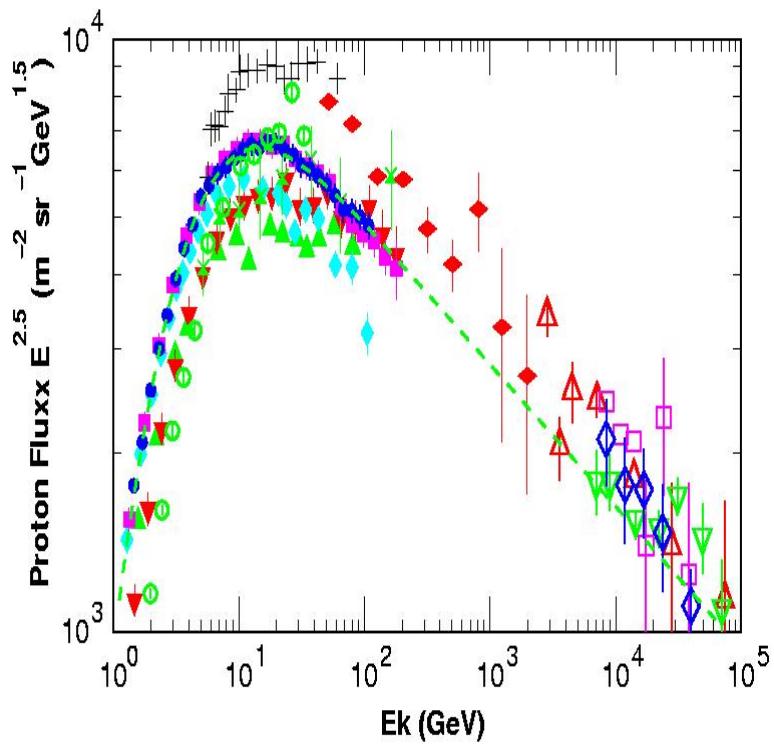
Atmospheric neutrinos: The 1998 Super-Kamiokande breakthrough



(T. Kajita at Neutrino' 98, Takayama)

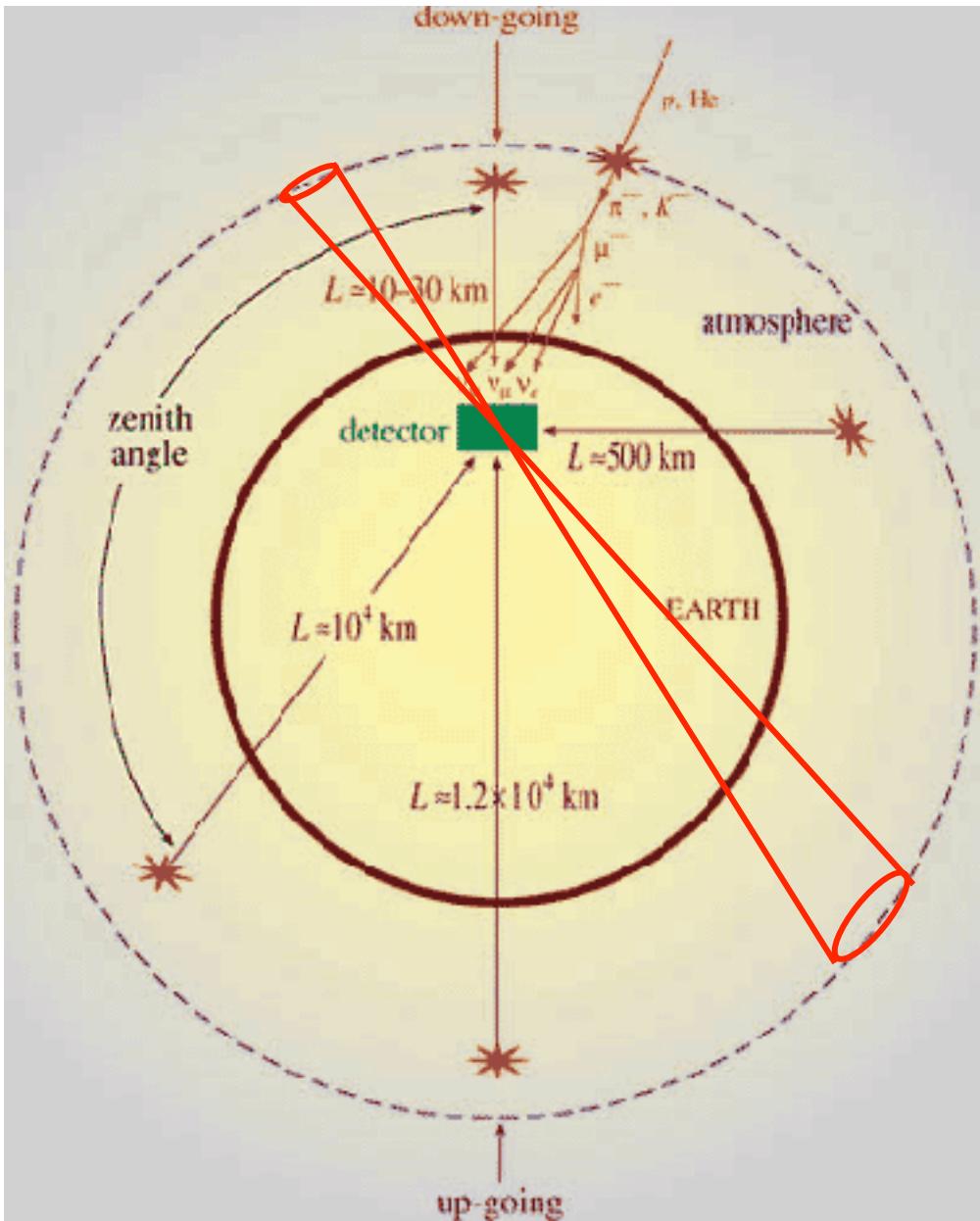
Production

Cosmic rays hitting the atmosphere can generate secondary (anti)neutrinos with electron and muon flavor via meson decays.



Primary flux affected by large normalization uncertainties...

... but (anti)neutrino flavor ratio ($\mu/e \sim 2$) robust within few %



Moreover: same v flux
from opposite solid angles
(up-down symmetry)

[Flux dilution ($\sim 1/r^2$) is
compensated by larger
production surface ($\sim r^2$)]

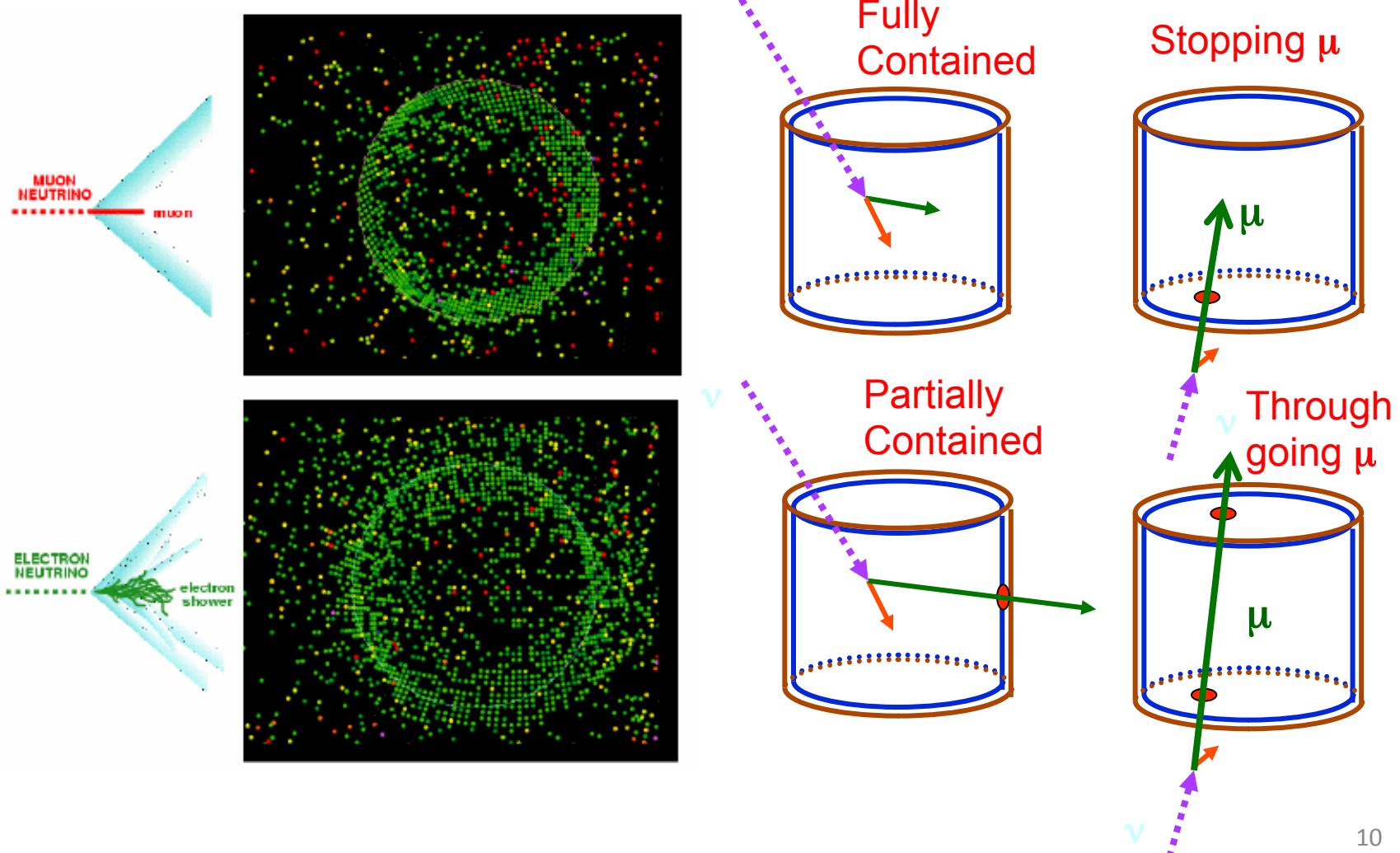
Should be reflected in
symmetry of event
zenith spectra, if
energy & angle can be
reconstructed well enough

Detection in SK

Parent neutrinos detected via CC interactions in the target (water).

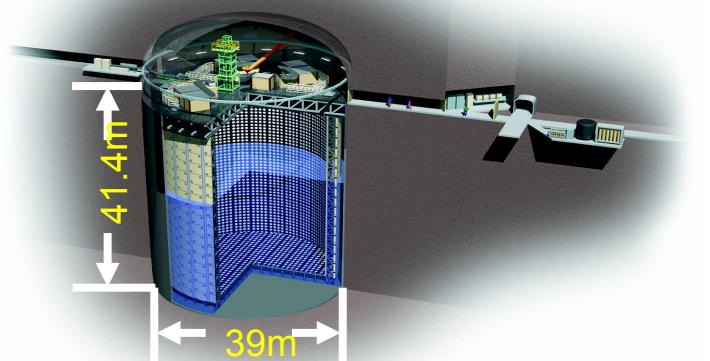
Final-state μ and e distinguished by \neq Cherenkov ring sharpness.

(But: no charge discrimination, no τ event reconstruction). Topologies:

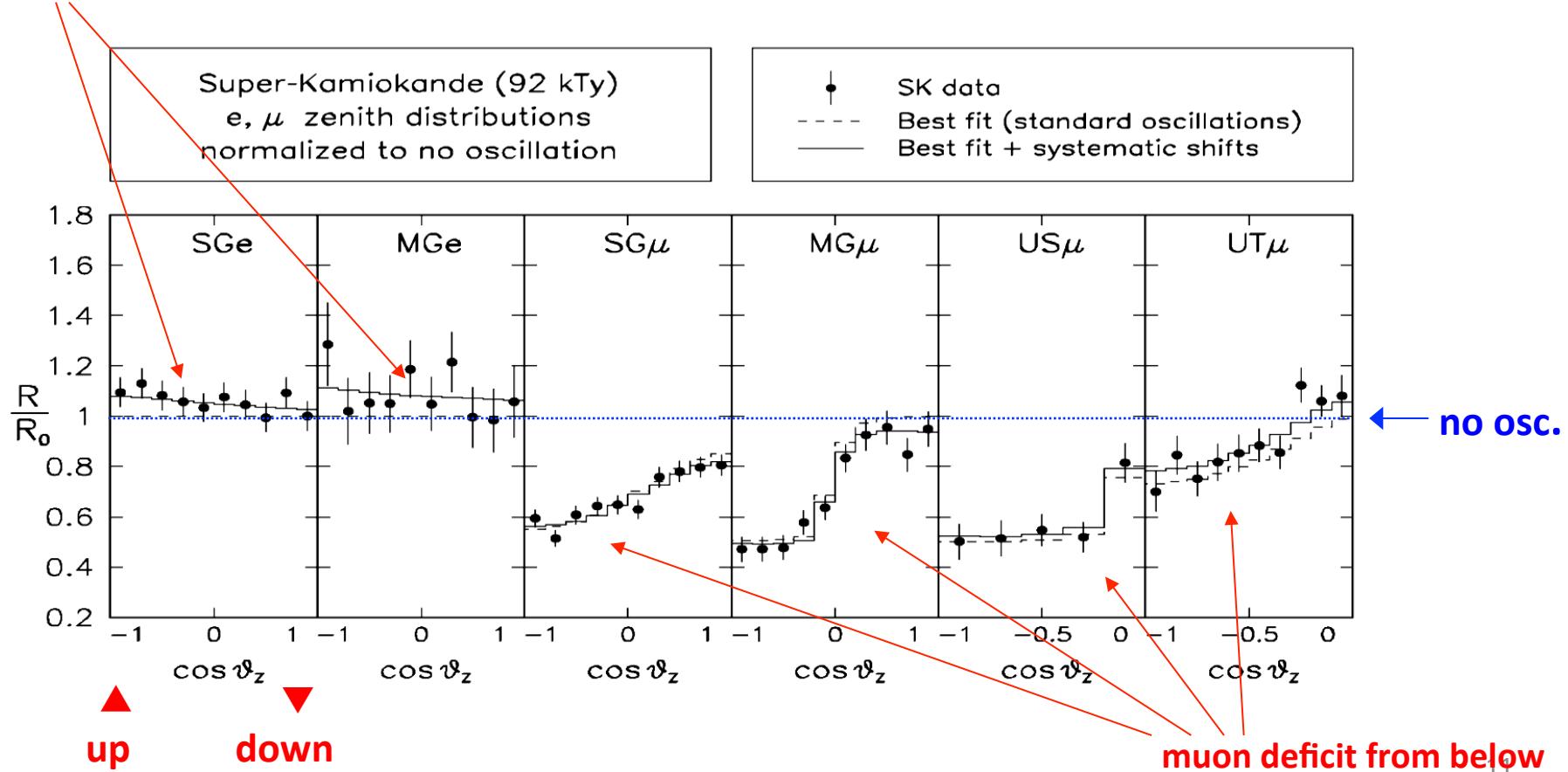


Results - SK zenith distributions

SGe	Sub-GeV electrons
MGe	Multi-GeV electrons
SG μ	Sub-GeV muons
MG μ	Multi-GeV muons
US μ	Upward Stopping muons
UT μ	Upward Through-going muons



electrons ~OK



Observations over several decades in L/E:

ν_e induced events: \sim as expected

ν_μ induced events: disappearance from below

Interpretation in terms of oscillations:

Channel $\nu_\mu \rightarrow \nu_e$? No (or subdominant) \leftarrow CHOOZ OK!

Channel $\nu_\mu \rightarrow \nu_\tau$? Yes (dominant)

One-mass-scale approximation (for $\theta_{13} \sim 0$):

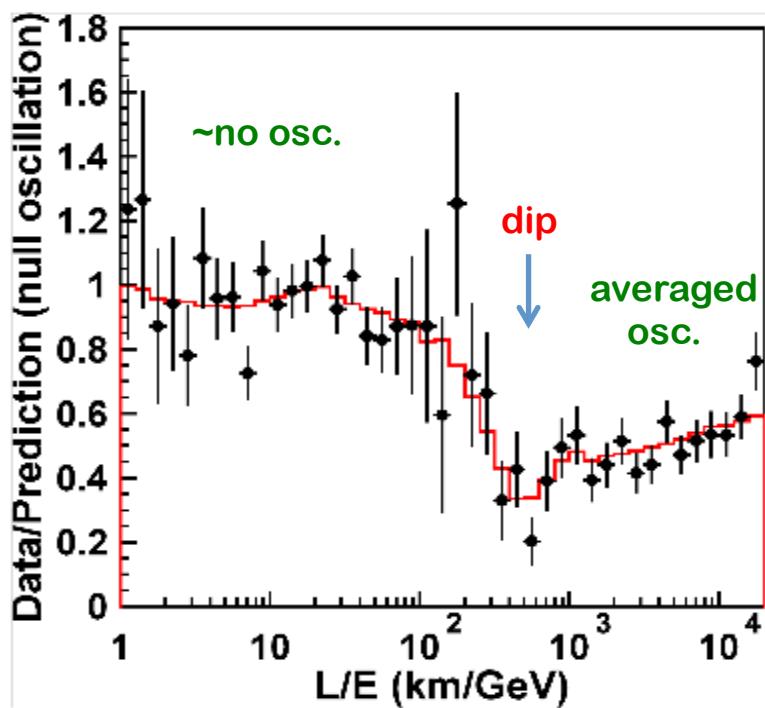
$$P_{\mu\tau} \approx \sin^2(2\theta_{23}) \sin^2(\Delta m^2 L / 4E)$$

[In this channel, flavor evolution is \sim vacuum-like, despite propagation in Earth matter for upgoing events – see Lect. II]

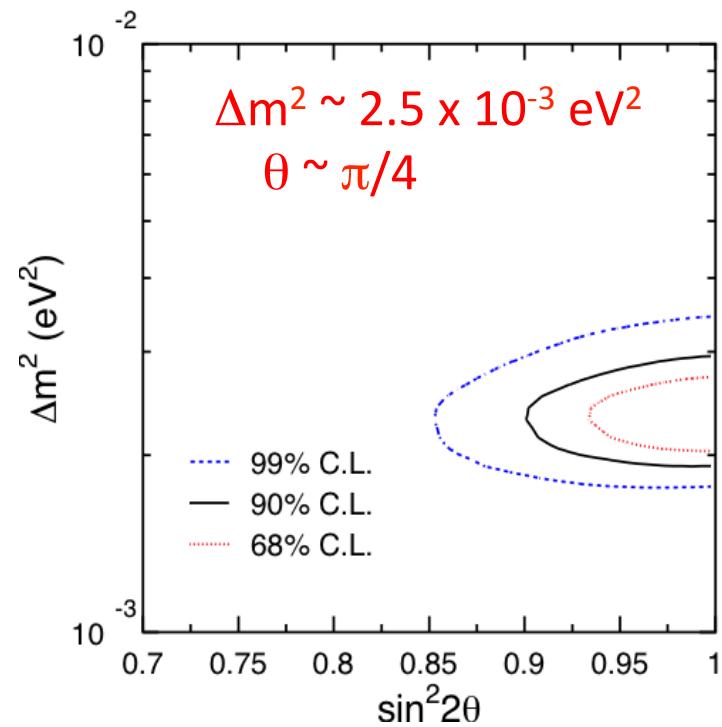
Results were consistent with other atmos. expts. using different techniques (MACRO, Soudan2) but characterized by lower statistics

Dedicated L/E analysis in SK “sees” half-period of oscillations

1st oscillation dip still visible
despite large L & E smearing



Strong constraints on the
parameters ($\Delta m^2, \theta$)

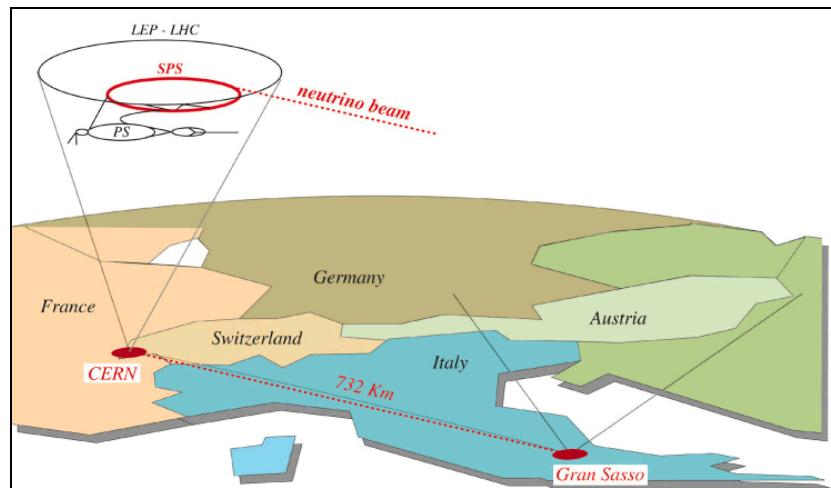
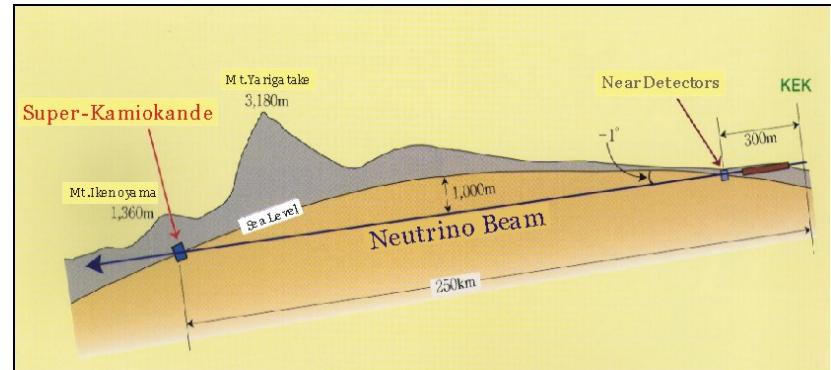
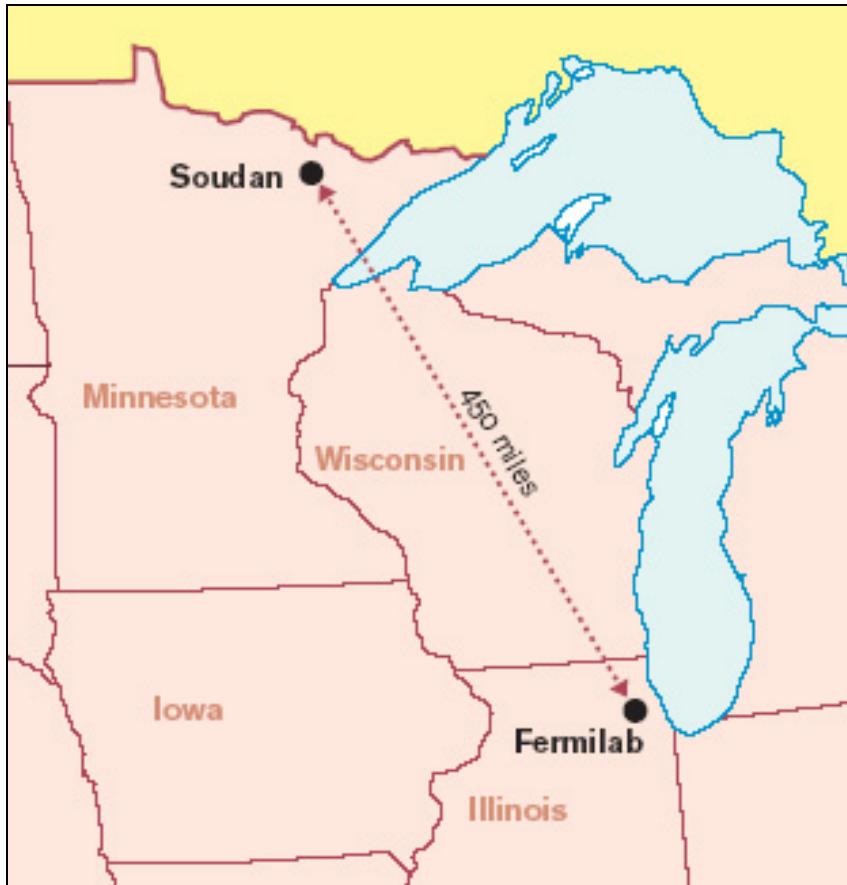


[Latest SK data analyses more refined: include many bins and syst.
in order to “squeeze” subleading effects beyond dominant L/E]

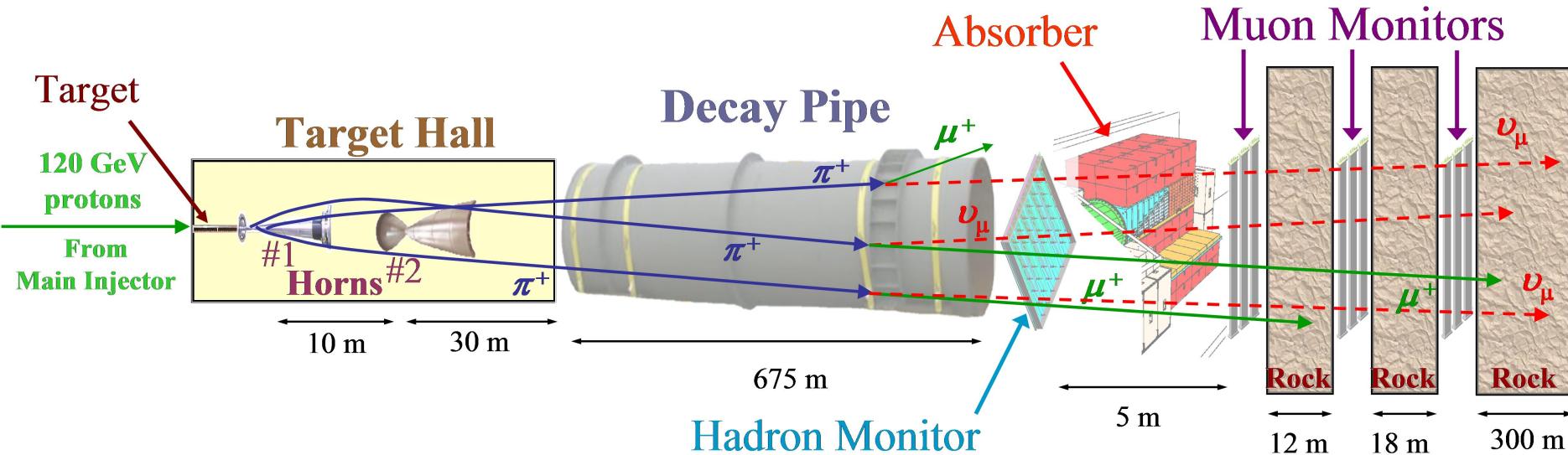
Long-baseline neutrino experiments

K2K, T2K (JP) , MINOS, NOvA (US), OPERA (CERN)

“Reproducing atmospheric ν_μ physics” in controlled conditions

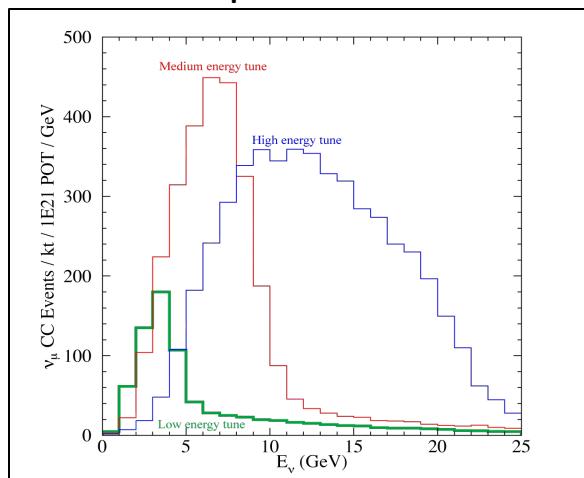


Production (e.g., MINOS)



π decay: ν energy is only function of $\nu\pi$ angle and π energy

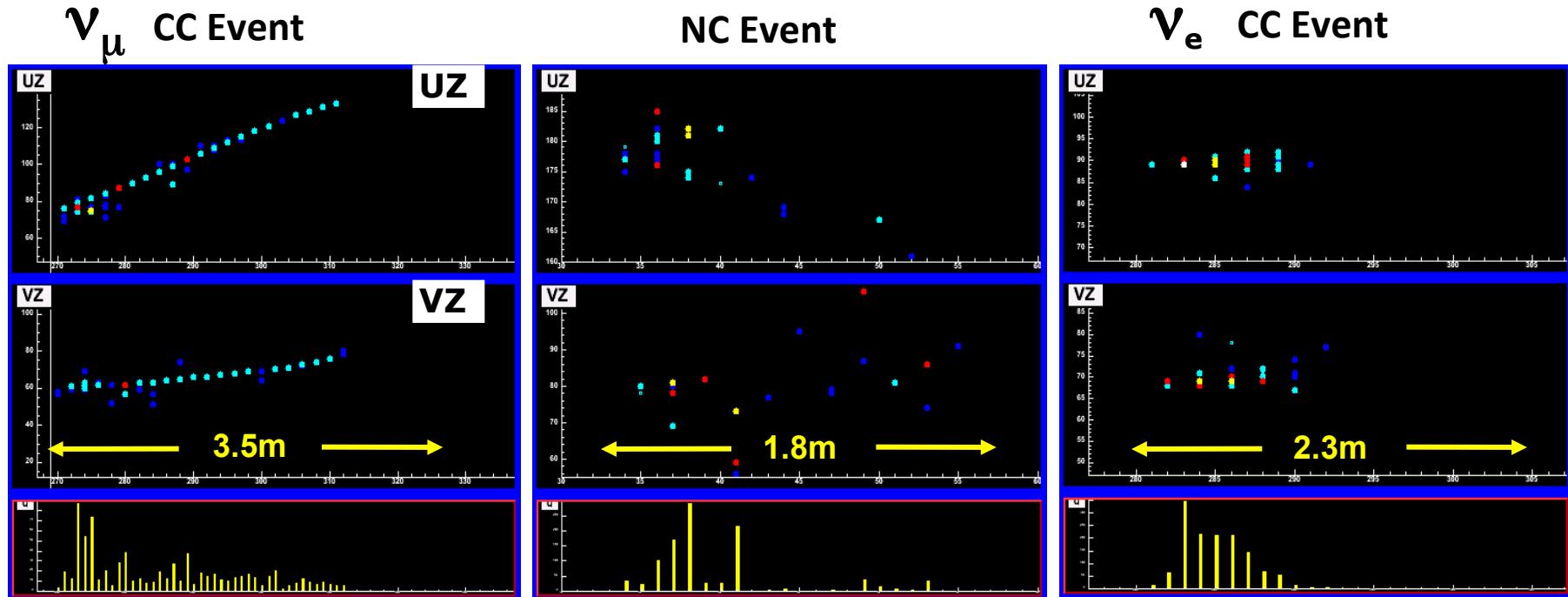
Spectra:



(Far) Detection

K2K, T2K: Cherenkov technique in SK

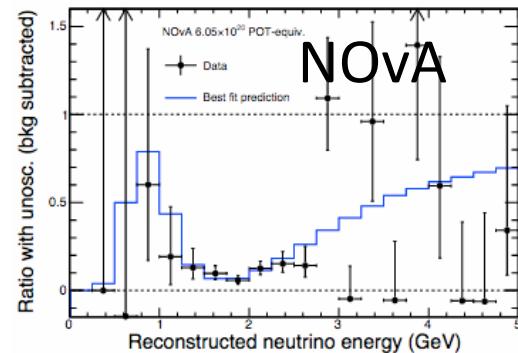
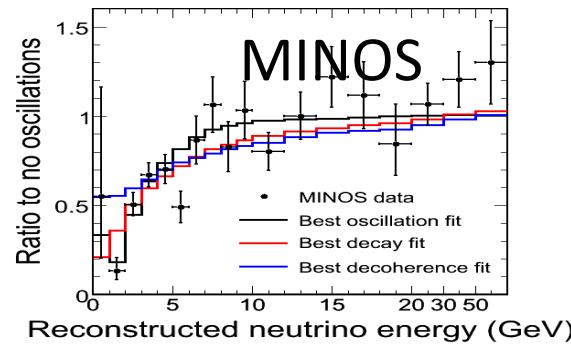
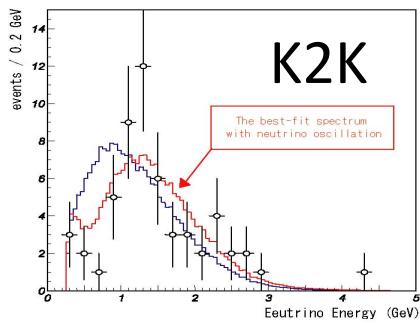
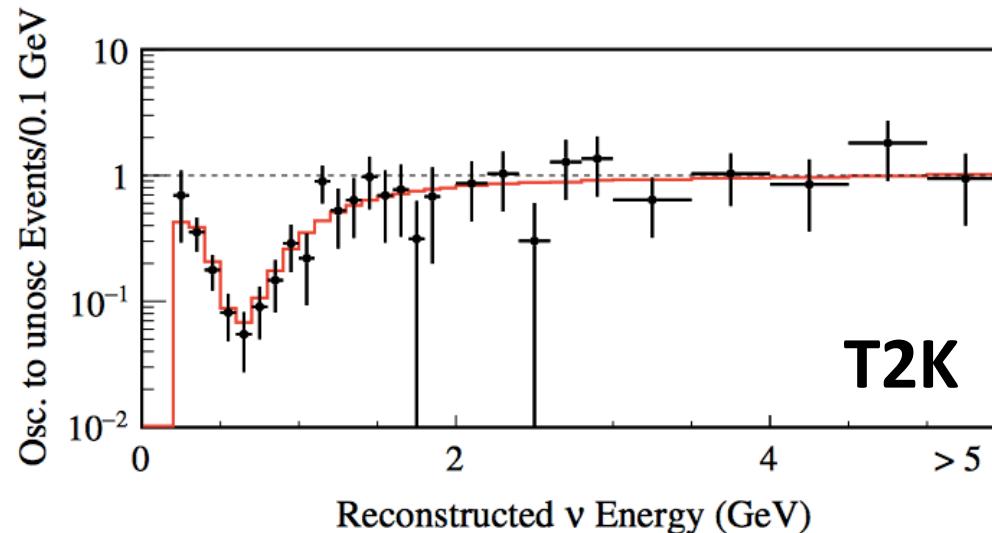
MINOS, NOvA: Scintillator detectors



- Long muon track + hadronic activity at vertex
- Short showering event, often diffuse
- Short event with typical EM shower profile

K2K, MINOS, T2K, NOvA supplemented by near detectors to measure $P_{\mu\mu}$

Results in muon neutrino disappearance mode, $P_{\mu\mu}$

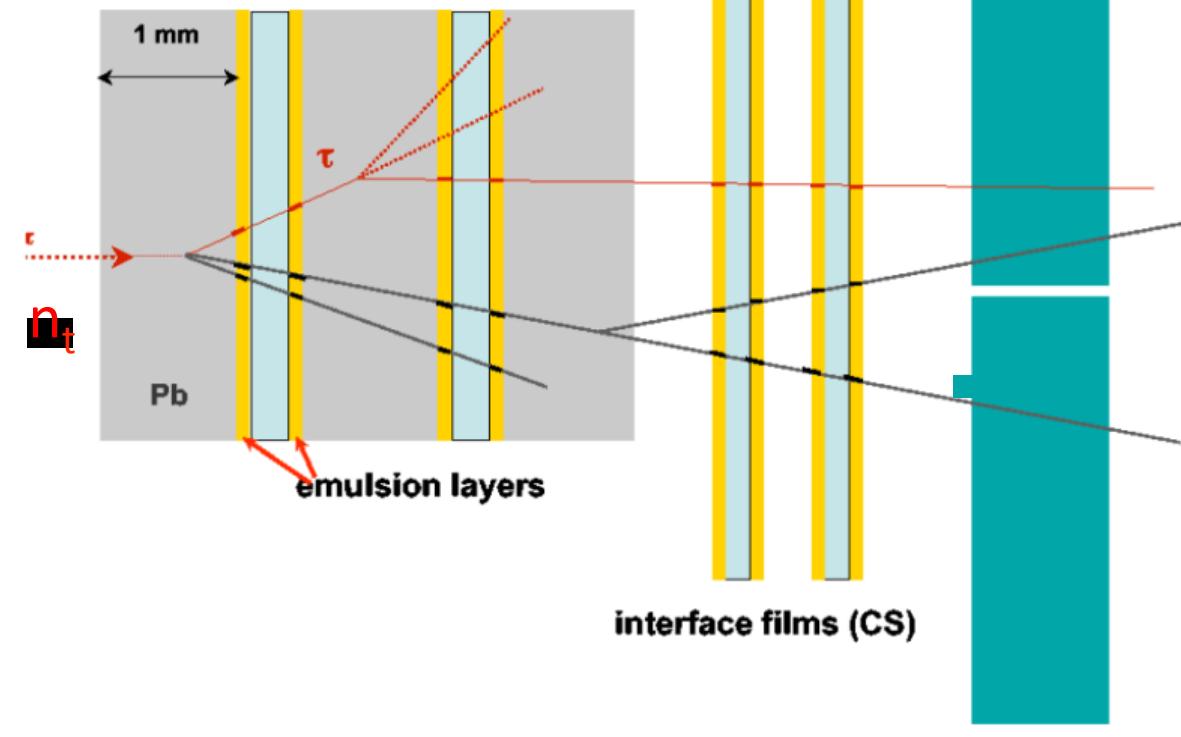


**1st oscillation dip observed in energy spectrum
(equivalent to L/E spectrum since L is fixed).**

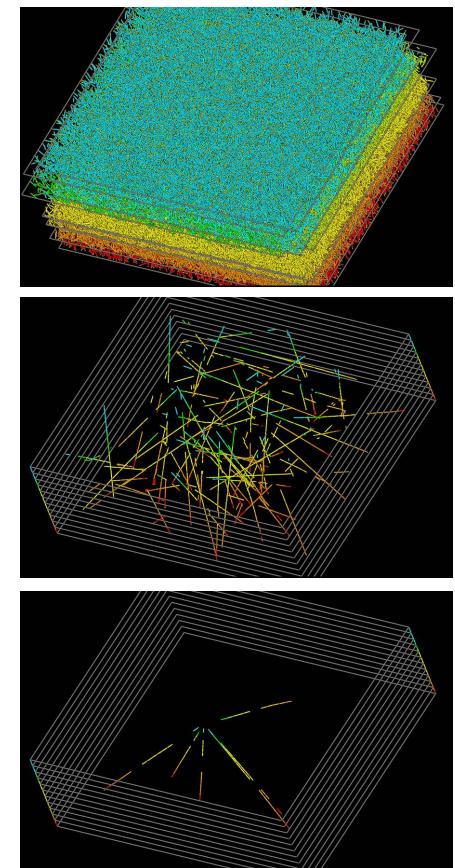
[Exotic explanations without dip (decay, decoherence) excluded]

Testing dominant oscillations via direct τ appearance: OPERA

- the OPERA hybrid detector



Finding needles
in a haystack...



Five “ τ needles” found! (consistent with expected signal)

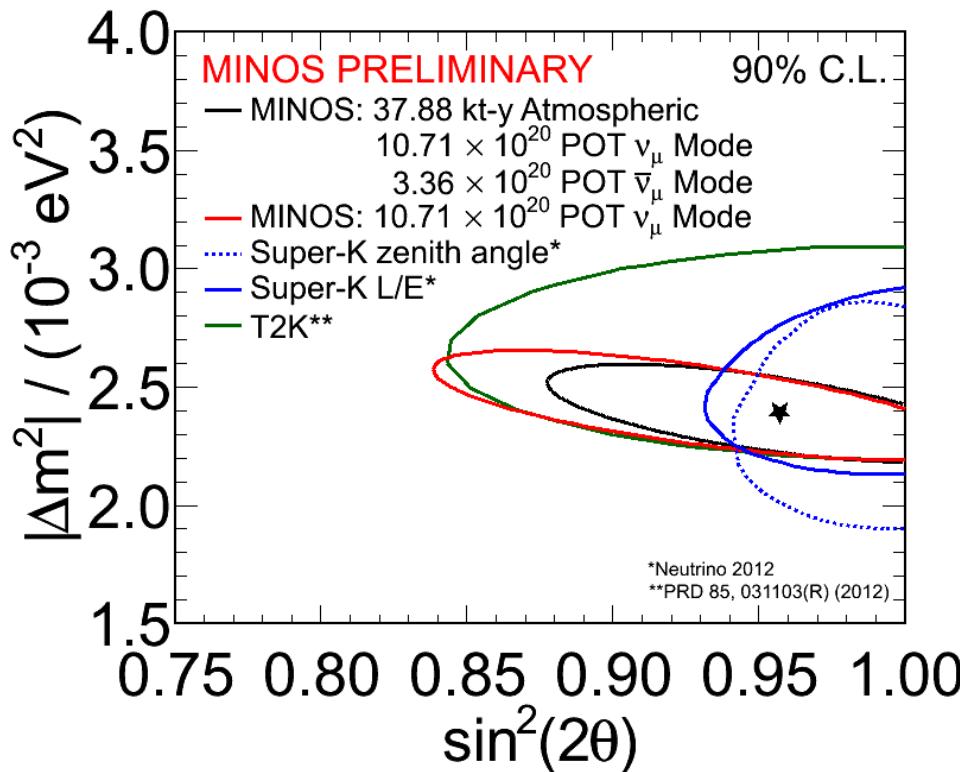
Interpretation of LBL accel. data

Once more... dominant $P_{\mu\tau} = \sin^2(2\theta_{23}) \sin^2(\Delta m^2 L / 4E_\nu)$

Dip position and depth determine Δm^2 and θ_{23}

Osc. parameters consistent among atm and LBL experiments

Old-fashioned way to present such mass-mixing constraints:

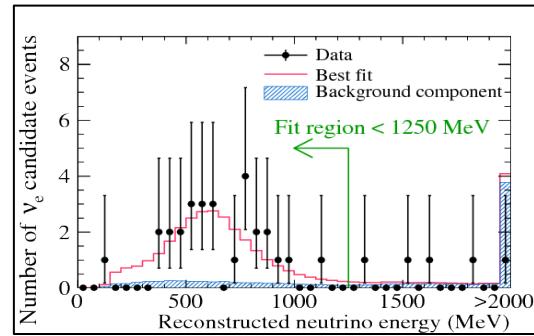


The format of such “2ν” plots is, however, obsolete...

In particular, we know that $\theta_{13} > 0$ from SBL reactors:
one expects also $\mu \rightarrow e$ flavor appearance in LBL experiments.

→ Found in T2K & NOvA; e-like event rate consistent with reactors' θ_{13}

e.g., T2K e-like appearance data:
(but note relatively low statistics wrt disappear. data)



For $\theta_{13} > 0$, relevant vacuum probabilities are θ_{23} -octant asymmetric,

$$P(\nu_\mu \rightarrow \nu_e) \simeq s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \quad \leftarrow \text{strongly asym. (appearance)}$$

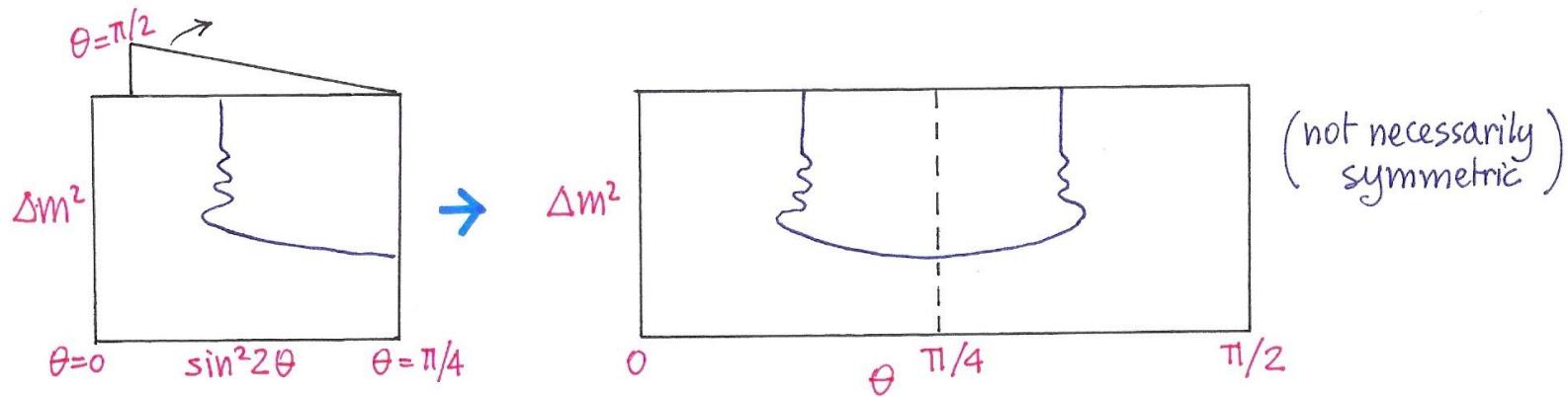
$$P(\nu_\mu \rightarrow \bar{\nu}_\mu) \simeq 1 - 4c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \quad \leftarrow \text{weakly asym. (disappearance)}$$

Combination of LBL dispapp. + appear. results is (weakly) octant asymmetric;
and similarly for atmospheric neutrino results →

$(\Delta m^2, \theta_{23})$ parameters...

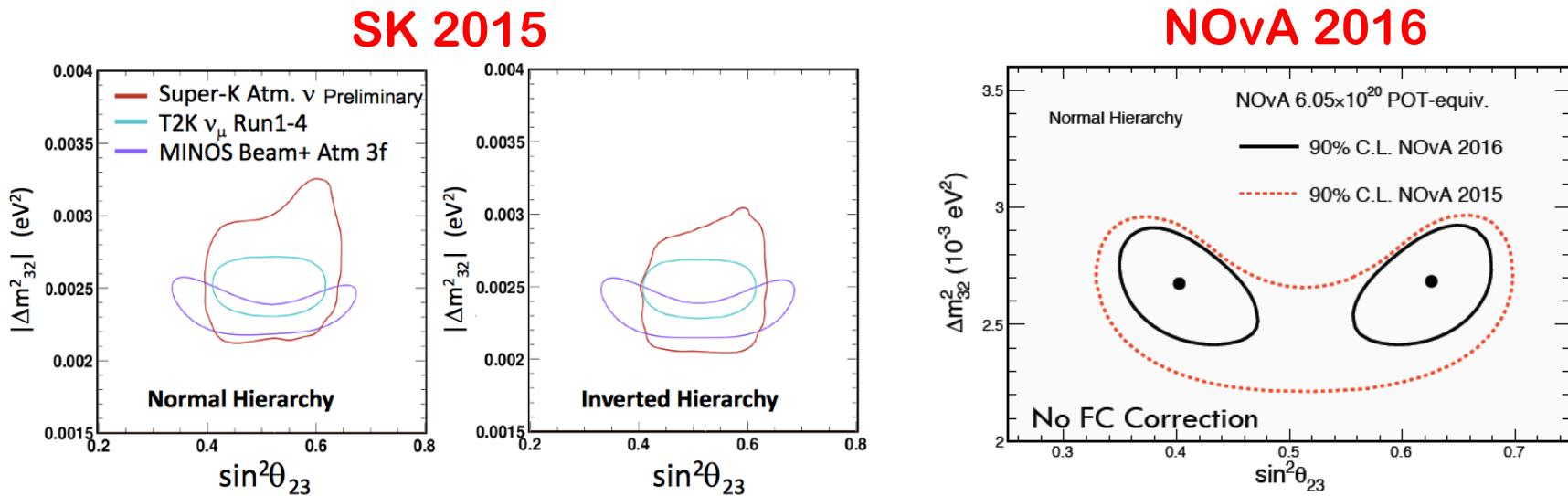
... are mainly determined by ATM+LBL expts. via $P(\nu_\mu \rightarrow \nu_\mu)$.

- $P_{\nu\mu}$ is octant symmetric (i.e., invariant for $\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}$)
only in the limit $\delta m^2 \rightarrow 0$ and $\theta_{13} \rightarrow 0$: $P_{\nu\mu} \approx 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2 x}{4E} \right)$
- For $\theta_{13} \neq 0$ it is no longer octant-symmetric :
 $P_{\nu\mu} \approx 1 - 4 c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \sin^2 \left(\frac{\Delta m^2 x}{4E} \right)$
- Further effects ($\delta m^2 \neq 0$, matter) also contribute to asymmetry
→ need to unfold 2nd octant in general :



- Typical abscissa: either $\sin^2 \theta$ (linear scale) or $\tan^2 \theta$ (log scale)

Examples of recent (slightly asym.) plots in terms of $\sin^2\theta_{23}$



Not yet established if $\theta_{23} \sim$ maximal or not. If nonmaximal:
 first or second octant? → “octant ambiguity”

Next frontier in LBL/Atm oscillation searches: probe subleading effects related to octant, matter, hierarchy, δ_{CP} , δm^2 , θ_{12} , ...

So far we have mainly discussed Δm^2 -driven oscillations...

**Let us now discuss oscillation
searches mainly sensitive to δm^2**

Typically they involve relatively large L and small E.

For $E \sim O(\text{MeV})$, below μ and τ production via CC,
one probes mainly ν_e disappearance probabilities →

Exercise: Expts. sensitive to δm^2 in the limit $\Delta m^2 \rightarrow \infty$

Previously we have considered experiments with sensitivity to Δm^2 in the limit $\Delta m^2 \rightarrow 0$. At the other end of the spectrum, there are expts. with leading sensitivity to δm^2 , for which one can take $\Delta m^2 \rightarrow \infty$:

$$\frac{\delta m^2 x}{4E} \sim \mathcal{O}(1) \quad \text{and} \quad \frac{\Delta m^2 x}{4E} \gg 1$$

This is the case, for instance, of long-baseline reactor experiments (KamLAND) with large x and relatively low E . At low $E \sim \text{few MeV}$, the main observable is the disappearance probability P_{ee} . Prove that:

$$P_{ee} \simeq \cos^4 \theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\delta m^2 x}{4E} \right) \right] + \sin^4 \theta_{13}$$

(which does not depend on hierarchy, $\nu/\bar{\nu}$, CP)

(See Tutorial)

Important note: The Δm^2 -averaged form for P_{ee} ,

$$P_{ee}^{3\nu} = C_{13}^4 P_{ee}^{2\nu}(\delta m^2, \theta_{12}) + S_{13}^4$$

holds not only for KamLAND, but also for solar neutrinos (proof omitted) where, however, $P_{ee}^{2\nu}$ takes a very different form due to matter effects in the Sun.

In this approximation, one is probing δm^2 and the mixing matrix elements $|U_{e i}|^2$ of ν_e with $\nu_i = (\nu_1, \nu_2, \nu_3)$

$$\begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{bmatrix}$$

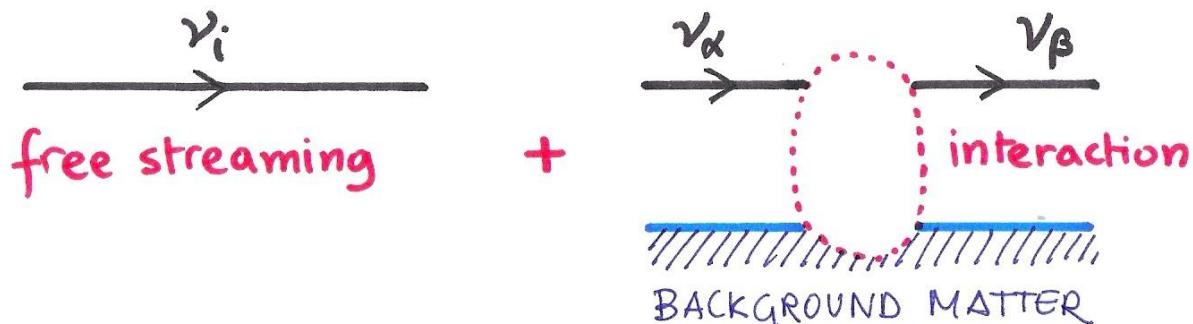
Evolution of electron flavor affected by background matter... →

Hamiltonian for ν oscillations in matter (msw)

It was first realized by Wolfenstein, and later elaborated by Mykheev and Smirnov, that neutrinos traveling in matter receive a contribution to coherent forward scattering, in the form of a tiny interaction energy $V_{\alpha\beta}$:

$$H_{\text{flavor}} = \frac{1}{2E} \left[\begin{pmatrix} m_1^2 & m_2^2 & m_3^2 \\ m_2^2 & m_3^2 & m_1^2 \\ m_3^2 & m_1^2 & m_2^2 \end{pmatrix} + \begin{pmatrix} V_{ee} & V_{e\mu} & V_{e\tau} \\ V_{\mu e} & V_{\mu\mu} & V_{\mu\tau} \\ V_{\tau e} & V_{\tau\mu} & V_{\tau\tau} \end{pmatrix} \right]$$

VACUUM (KINEMATICS) MATTER (DYNAMICS)



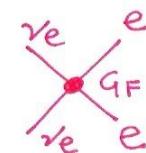
Within the Standard Model, and in ordinary matter:

$$V_{\alpha\beta} = \left(\begin{array}{ccc|cc} \nu_e & \nu_e & & 0 & 0 \\ Z & & & & \\ \hline p, n, e & & \nu_\mu & \nu_\mu & \\ & & Z & & \\ & & p, n, e & & \\ & & & \nu_\tau & \nu_\tau \\ & & & Z & \\ & & & p, n, e & \end{array} \right)_{NC} + \left(\begin{array}{ccc|cc} \nu_e & & & 0 & 0 \\ W & & & & \\ \hline e & & e & & \\ & & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \end{array} \right)_{CC}$$

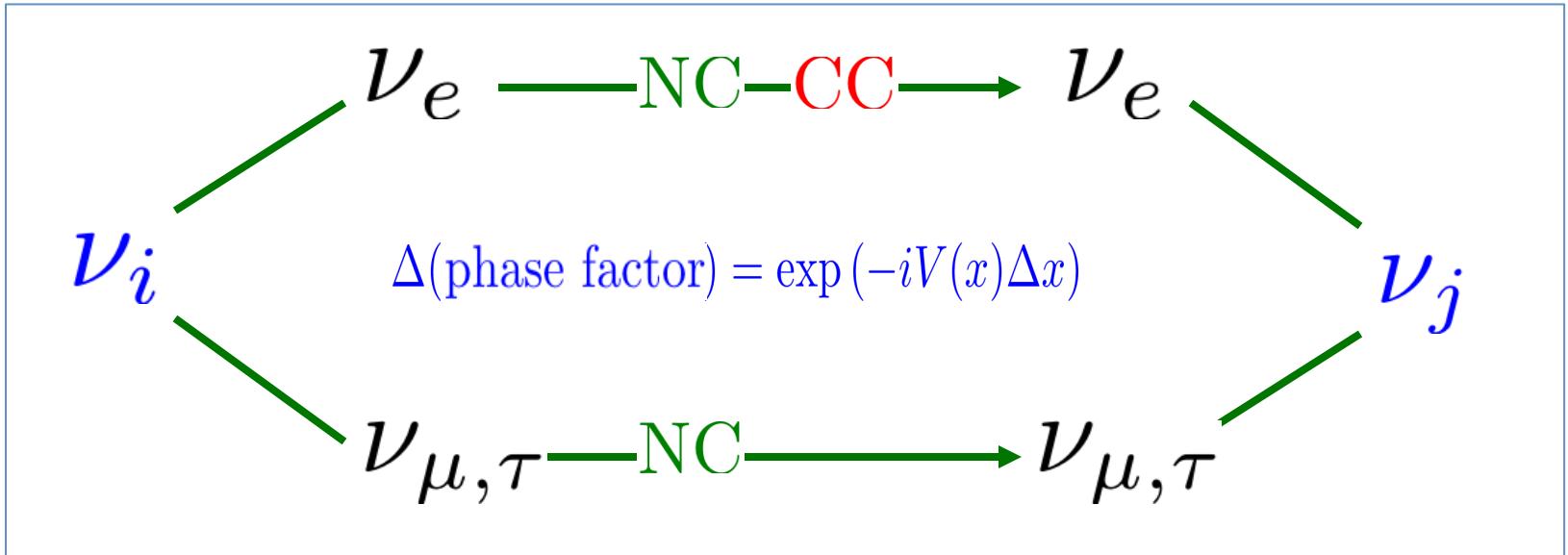
↑
proportional to unity
and unobservable

↑
observable in ν_e
oscillations

Relevant term is the $\nu_e - \nu_{\mu,\tau}$ energy difference: $V_{CC} \approx$
(No analogous for μ, τ , which are absent in ordinary matter)



**Analogy of matter effects with two-slit experiment:
one “arm” (flavor) feels a different “refraction index”**



governed by the local ν “interaction energy” or “potential” V
($V \rightarrow -V$ for antineutrinos)

- It turns out that the V_{cc}^{ee} interaction energy is

$$V = \sqrt{2} G_F N_e$$

where N_e = electron number density, and $V \rightarrow -V$ for $\nu \rightarrow \bar{\nu}$

- Then, the Hamiltonian of ν propagation in matter reads:

$$H_{\text{flavor}} = \frac{1}{2E} \left[\begin{pmatrix} m_1^2 & m_2^2 & m_3^2 \\ m_2^2 & m_3^2 & m_1^2 \\ m_3^2 & m_1^2 & m_2^2 \end{pmatrix} \right] \left[\begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

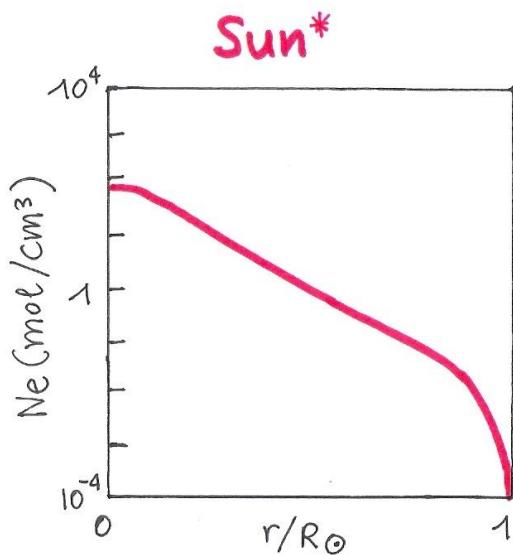
where

$$A = 2\sqrt{2} G_F N_e E$$

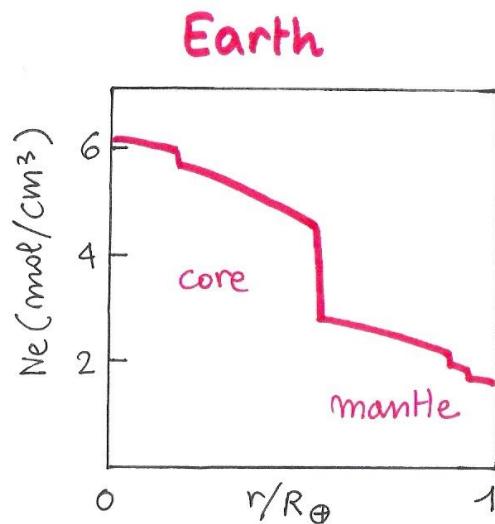
- The relative size of matter/vacuum terms is given by $A/\Delta m_{ij}^2$. Roughly speaking, one may expect sizable effects for $A/\Delta m_{ij}^2 \sim \mathcal{O}(1)$.
- The dependence $A = A(x)$ makes the evolution nontrivial in many cases.

(see also tutorial)

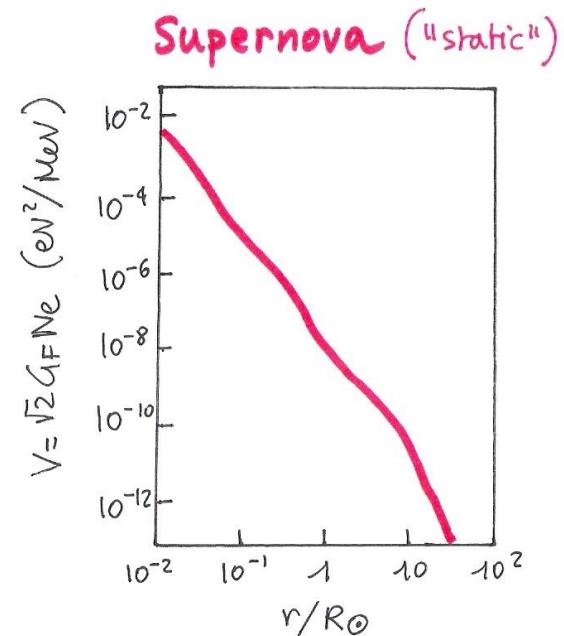
Examples of matter density profiles:



Sun*



Earth



Supernova ("static")

\sim exponential

\sim step-like

\sim power law

There is a huge literature about (semi)analytical solutions of flavor evolution equations for (approximations of) these and other profiles, in 2ν , 3ν or $N\nu$ cases. Analytical understanding is useful, because numerical solutions are prone to artifacts.

* For solar ν_e at $r \approx 0$: $\frac{A}{\delta m^2} \gtrsim 1$ for $E \gtrsim$ few MeV \rightarrow expect large matter effects!

Exercise : 2ν oscillations in matter at constant density

Prove that, in the 2ν limit ($\theta_{13}=0$), the ν_e survival probability reads :

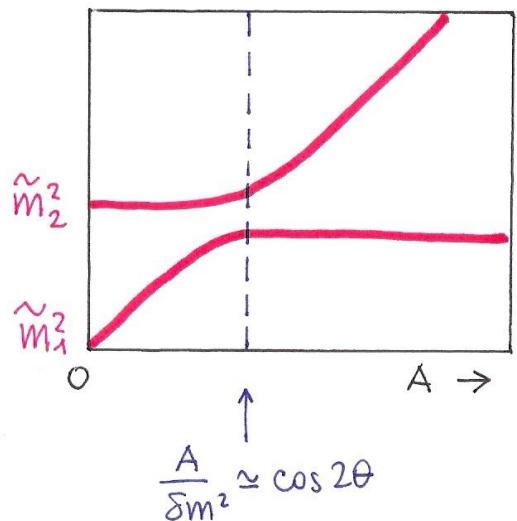
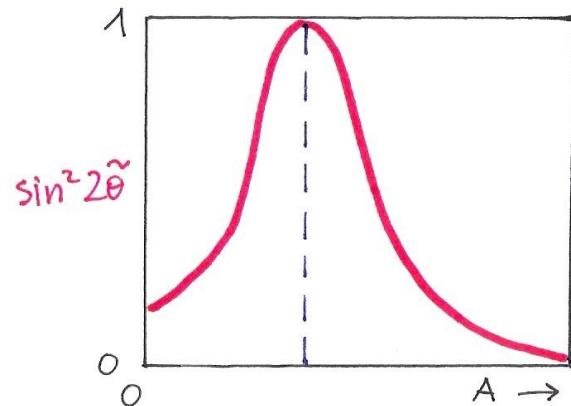
$$P_{\text{ee}}^{2\nu}(\text{mat}) = 1 - \sin^2 2\tilde{\theta}_{12} \sin^2 \left(\frac{\delta\tilde{m}^2 x}{4E} \right) \quad \text{for } N_e = \text{const}$$

i.e., it has the same vacuum-like structure, but with the replacements :

$$\sin 2\tilde{\theta}_{12} = \frac{\sin 2\theta_{12}}{\sqrt{\left(\cos 2\theta_{12} - \frac{A}{\delta m^2}\right)^2 + \sin^2 2\theta_{12}}} , \quad \delta\tilde{m}^2 = \delta m^2 \frac{\sin 2\theta_{12}}{\sin 2\tilde{\theta}_{12}} \quad \begin{pmatrix} A = \pm\sqrt{2} G_F N_e \\ + : \nu \\ - : \bar{\nu} \end{pmatrix}$$

(see tutorial)

Comments : ($\theta_{12} \equiv \theta$ for simplicity)



$$(A = 2\sqrt{2} G_F N_e E)$$

Mykheen-Smirnov-Wolfenstein (MSW) resonance:

For $A/\delta m^2 > 0$, the effective parameters have a resonant behavior around :

$$\frac{A}{\delta m^2} \approx \cos 2\theta$$

(only for r : no resonance for \bar{r} , since $A < 0$ for \bar{r})

Limiting cases:

$A/\delta m^2 \ll 1$: $(\delta \tilde{m}^2, \tilde{\theta}) \approx (\delta m^2, \theta)$ ← vacuum-like

$A/\delta m^2 \approx \cos 2\theta$: $(\delta \tilde{m}^2, \tilde{\theta}) \approx (\delta m^2 \sin 2\theta, \pi/4)$ ← reson.

$A/\delta m^2 \gg 1$: $(\delta \tilde{m}^2, \tilde{\theta}) \approx (A, \pi/2)$ ← matter dominance

Confirms expectations of large matter effects for $A/\delta m^2 \sim \mathcal{O}(1)$.

Exercise: 2ν osc. in matter with slowly varying density

If $N_e(x)$ changes slowly from $x=x_i$ (with $\tilde{\theta}=\tilde{\theta}_i$) to $x=x_f$ (with $\tilde{\theta}=\tilde{\theta}_f$) while oscillations are fast, then the averaged Pee probability takes the form:

$$Pee^{2\nu} \approx \cos^2 \tilde{\theta}_i \cos^2 \tilde{\theta}_f + \sin^2 \tilde{\theta}_i \sin^2 \tilde{\theta}_f$$

← "adiabatic" approximation

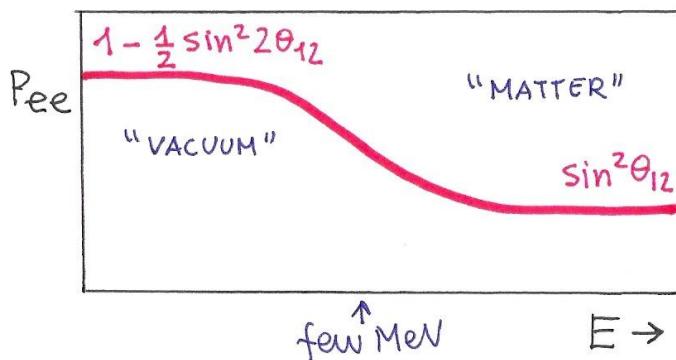
(see tutorial)

Application to solar ν .

It turns out that, for the $(\delta m^2, \theta_{12})$ values chosen by nature, the adiabatic approximation can be applied to solar ν_e . In this case, $\tilde{\theta}_{12}(x_f) = \theta_{12}$ (vacuum value at the exit from the Sun), while $\tilde{\theta}_{12}(x_i)$ must be evaluated at the production point x_i . Limiting cases:

- $E \lesssim \text{few MeV}$ (vacuum dominance): $A/\delta m^2 \lesssim 1$ and $\tilde{\theta}_{12}(x_i) \approx \theta_{12}$
- $P_{ee} \approx C_{12}^4 + S_{12}^4 = 1 - \frac{1}{2} \sin^2 2\theta_{12}$
- This is the averagest vacuum probability, octant symmetric.

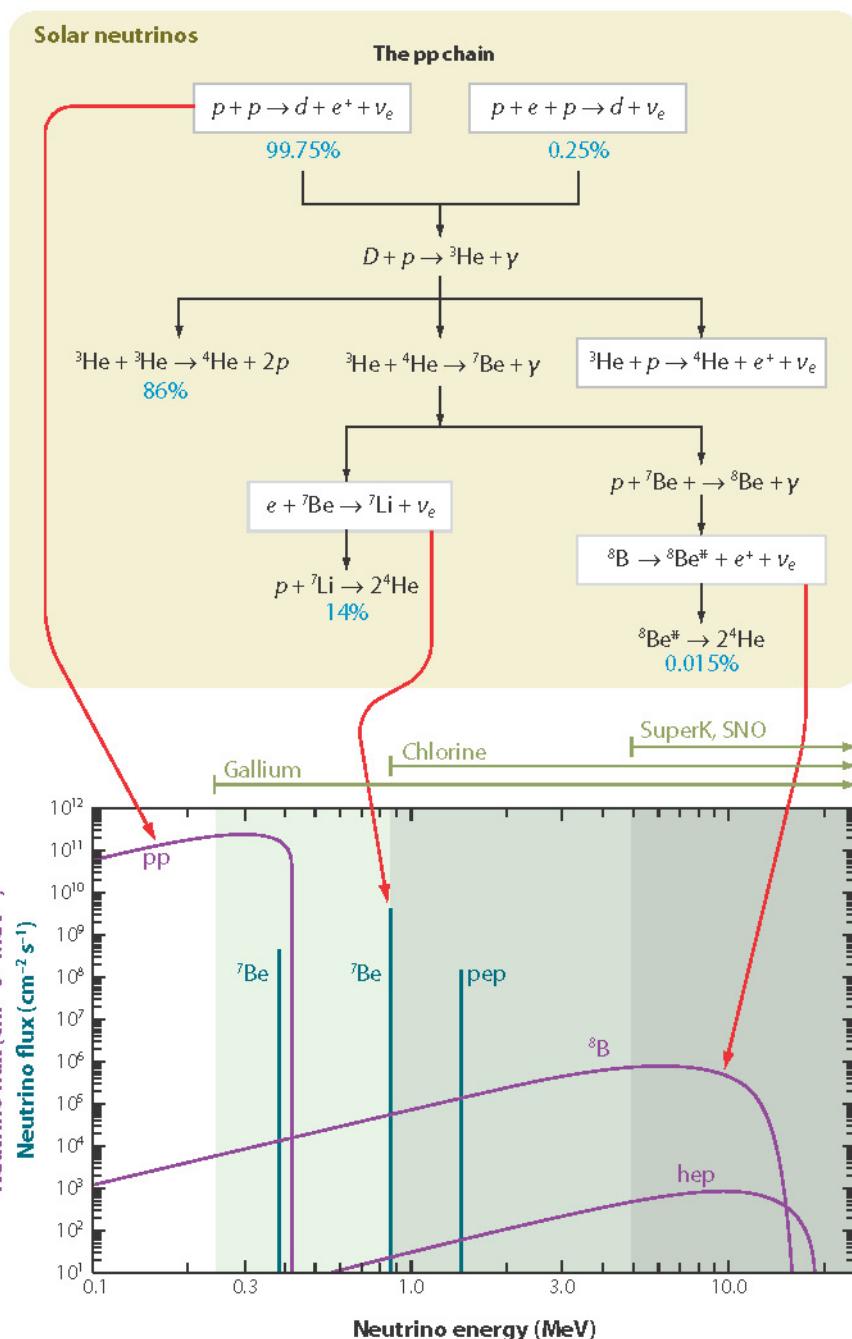
- $E \gtrsim \text{few MeV}$ (matter dominance): $A/\delta m^2 \gtrsim 1$ and $\tilde{\theta}_{12}(x_i) \sim \pi/2$
- $P_{ee} \approx \sin^2 \theta_{12}$
- This is the matter-dominated probability, octant-asymmetric



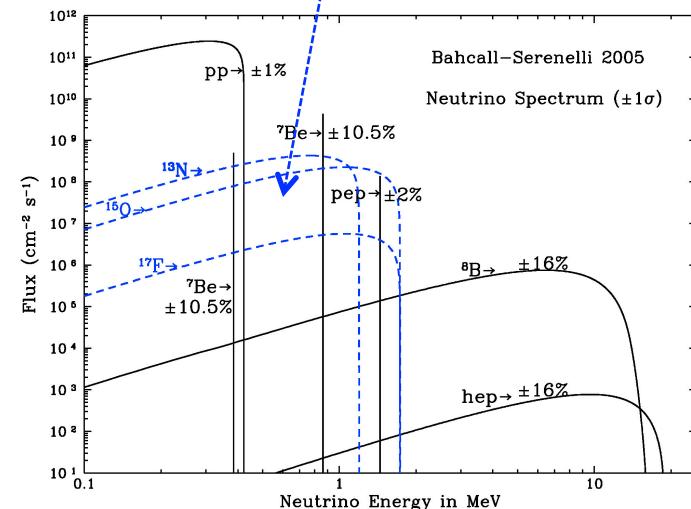
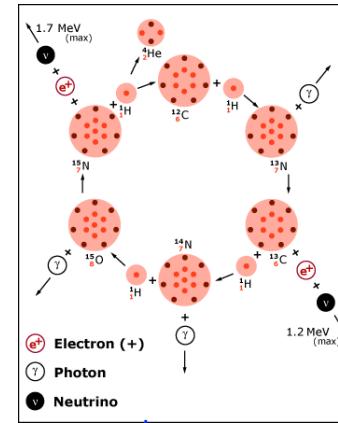
The P_{ee} transition from "low" to "high" E is a signature of matter effects in the Sun.

Thanks to matter effects we can determine the octant of the mixing angle θ_{12} .

Solar neutrinos: Production

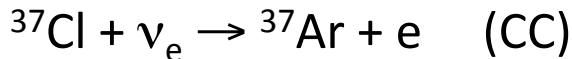


pp (+CNO) cycle

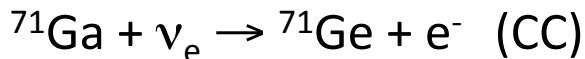


Detection

Radiochemical: count the decays of unstable final-state nuclei.
(low energy threshold, but energy and time info lost/integrated)

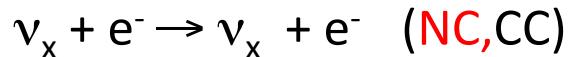


Homestake



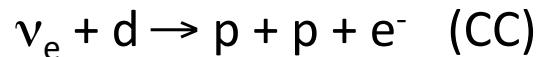
GALLEX/GNO, SAGE

Elastic scattering: events detected in real time with either
“high” threshold (Č, directional) or “low” threshold (Scintillators)

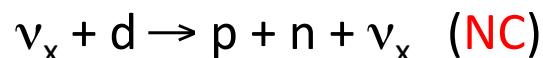


SK, SNO, Borexino

Interactions on Deuterium: CC events detected in real time; NC events separated statistically + using neutron counters.

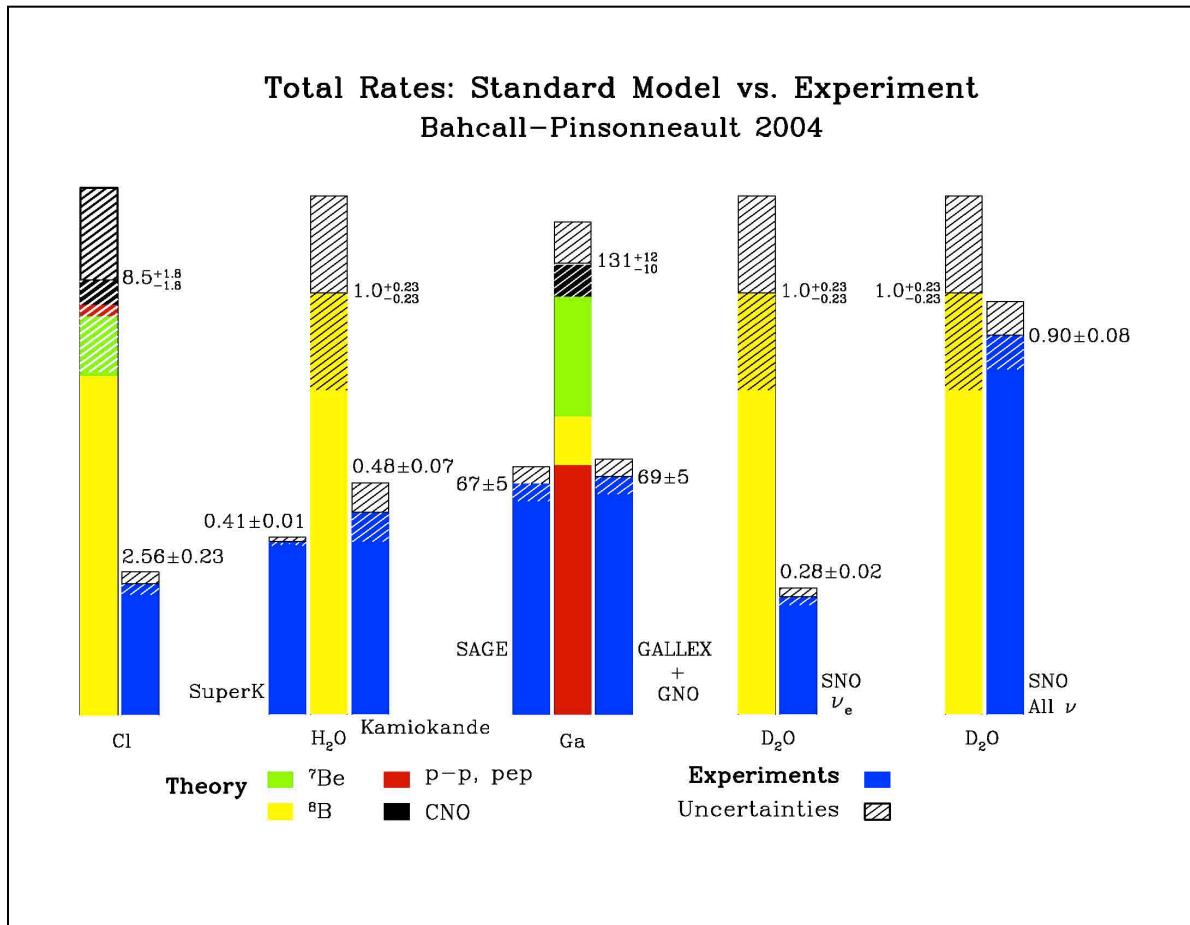


SNO (Sudbury Neutrino Observatory)



Results

All CC-sensitive results indicated a ν_e deficit...

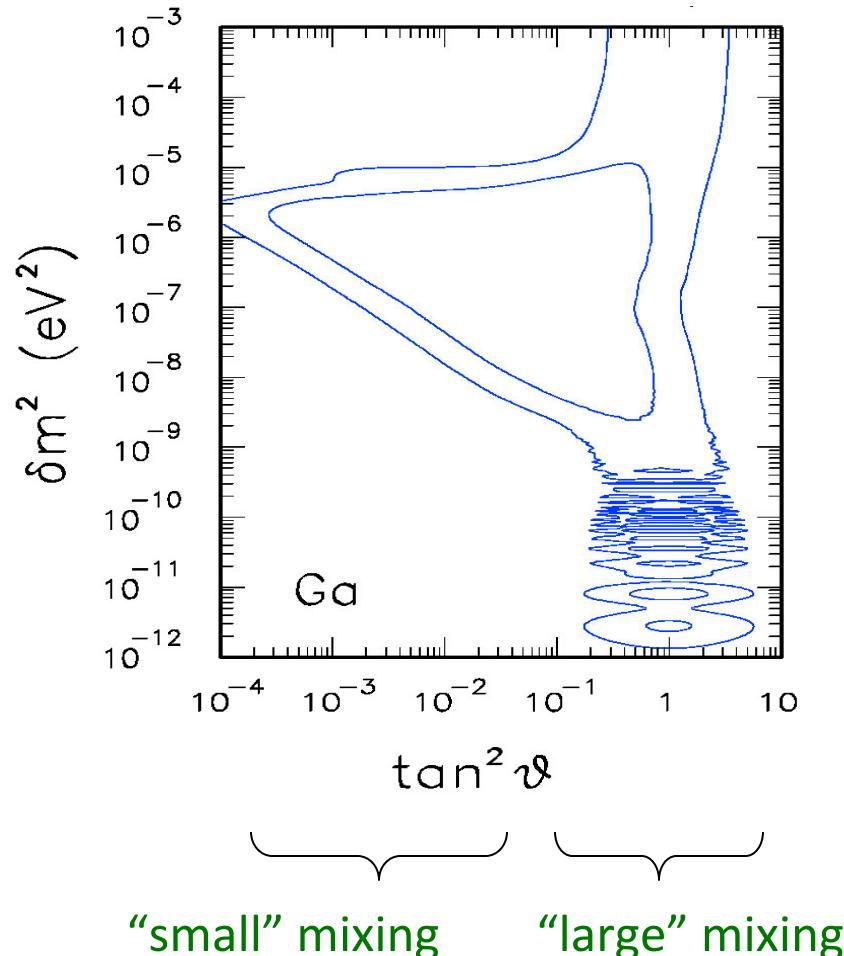


...as compared to solar model expectations

Interpretation

In the “past millennium”: Oscillations? Maybe, but...

- large uncertainties in the parameter space or solar model
- no unmistakable evidence for flavor transitions (“smoking gun”)



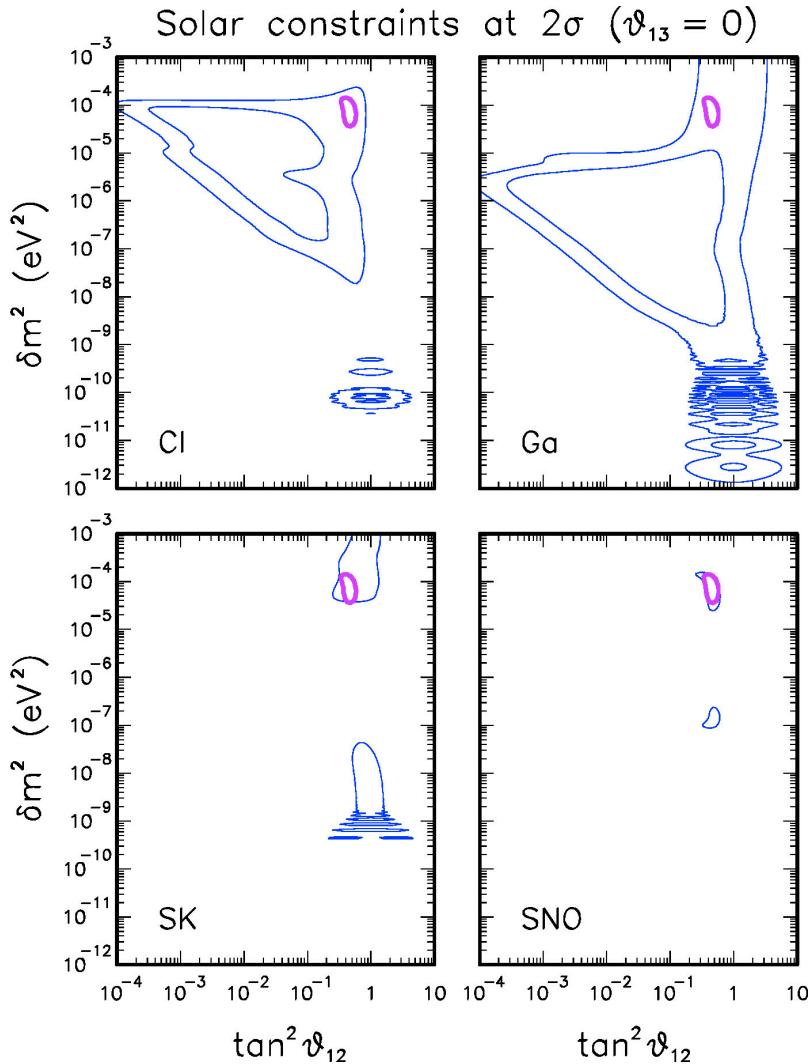
E.g., in Gallium expts:

“matter” (MSW) solutions

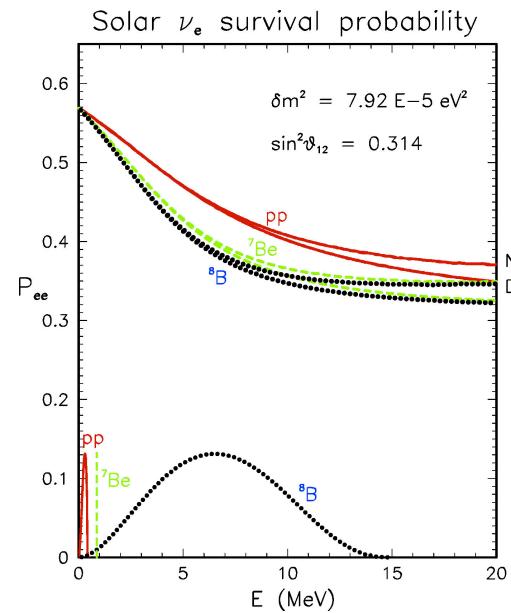
“vacuum” solutions

+ many “exotic”
or non-oscillatory
solutions...

But, in 2002 (“annus mirabilis”), one global solution was finally singled out by combination of all solar data (“large mixing angle” or **LMA**).

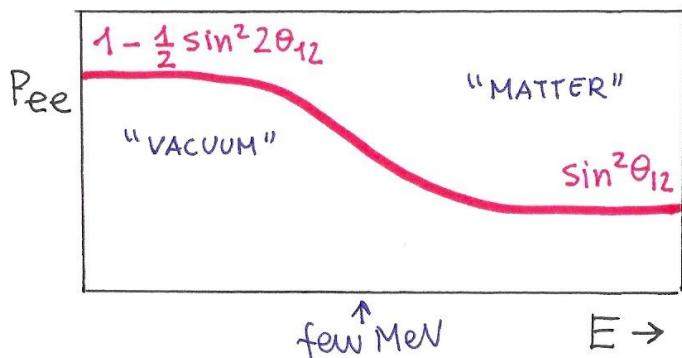
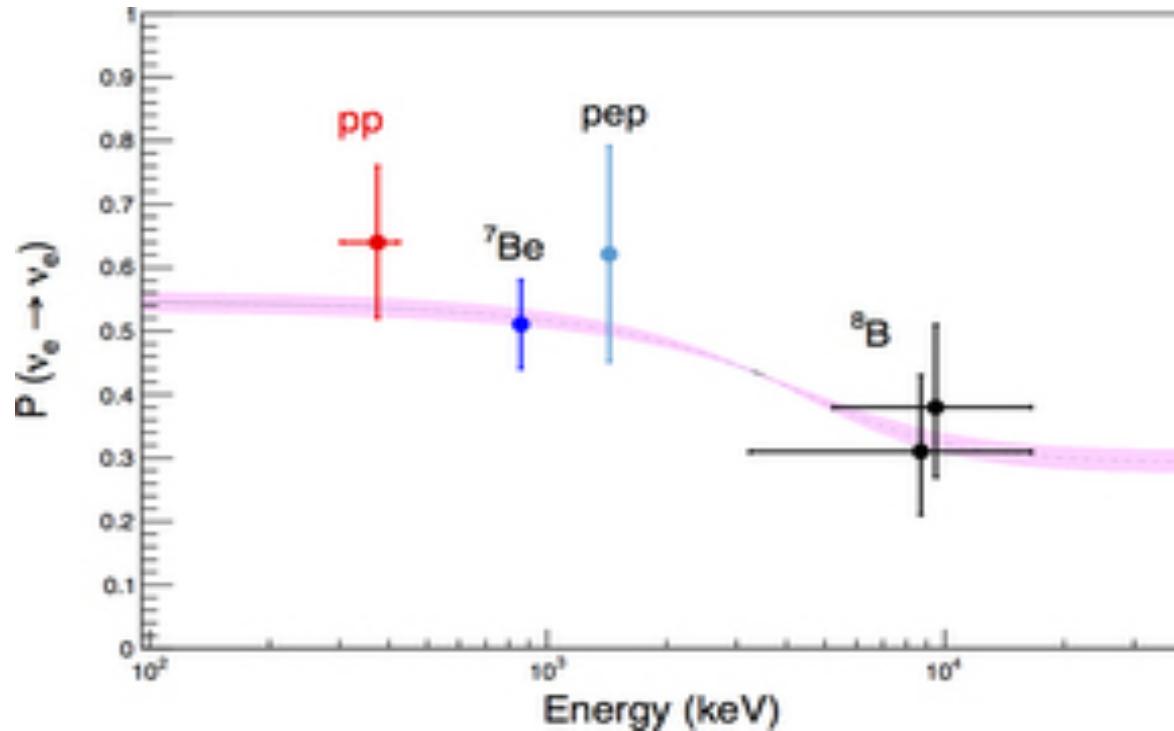


For LMA parameters,
evolution is **adiabatic**
in solar matter.



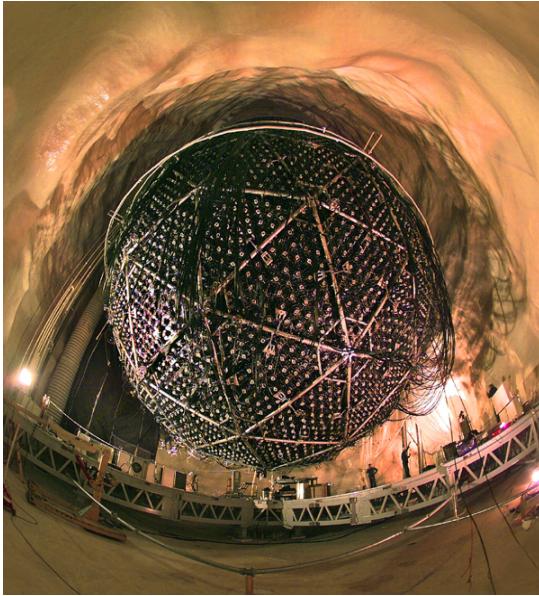
In the Earth: small day/night
(D/N) effects, seen at $\sim 3\sigma$.

Recent test of P_{ee} in Borexino



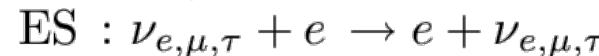
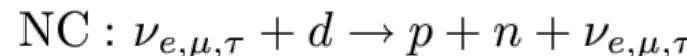
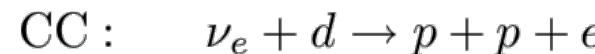
The Pee transition from "low" to "high" E is a signature of matter effects in the Sun.

Thanks to matter effects we can determine the octant of the mixing angle θ_{12} .



Crucial role played by SNO data

In deuterium one can separate CC events (counting only ν_e) from NC events (counting ν_e, ν_μ, ν_τ), and double check via Elast. Scatt. events (due to both NC and CC):



$$\frac{\text{CC}}{\text{NC}} \sim \frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})} \quad \text{thus:}$$

$$\frac{\text{CC}}{\text{NC}} < 1 \Rightarrow \phi(\nu_{\mu,\tau}) > 0 \Rightarrow \nu_e \rightarrow \nu_{\mu,\tau}$$

CC/NC $\sim 1/3 < 1$

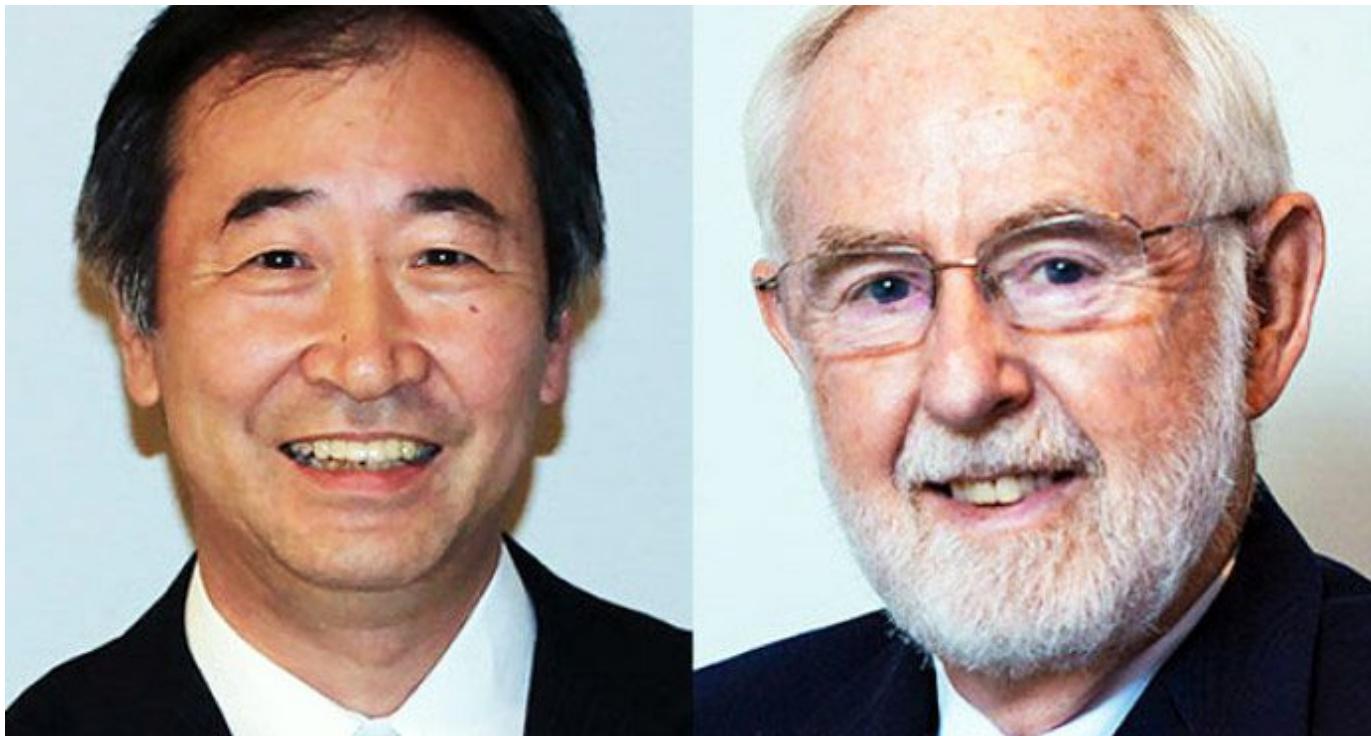
“Smoking gun” proof of flavor change. Solar model OK!

CC/NC $\sim P_{ee} \sim \sin^2 \theta_{12}$ (LMA) $\sim 1/3 < 1/2$

Evidence of: mixing in first octant + matter effects

SK atmospheric + SNO solar = Nobel Prize 2015!

*“...for the discovery of neutrino oscillations,
which shows that neutrinos have mass”*

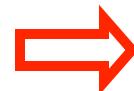


Takaaki Kajita

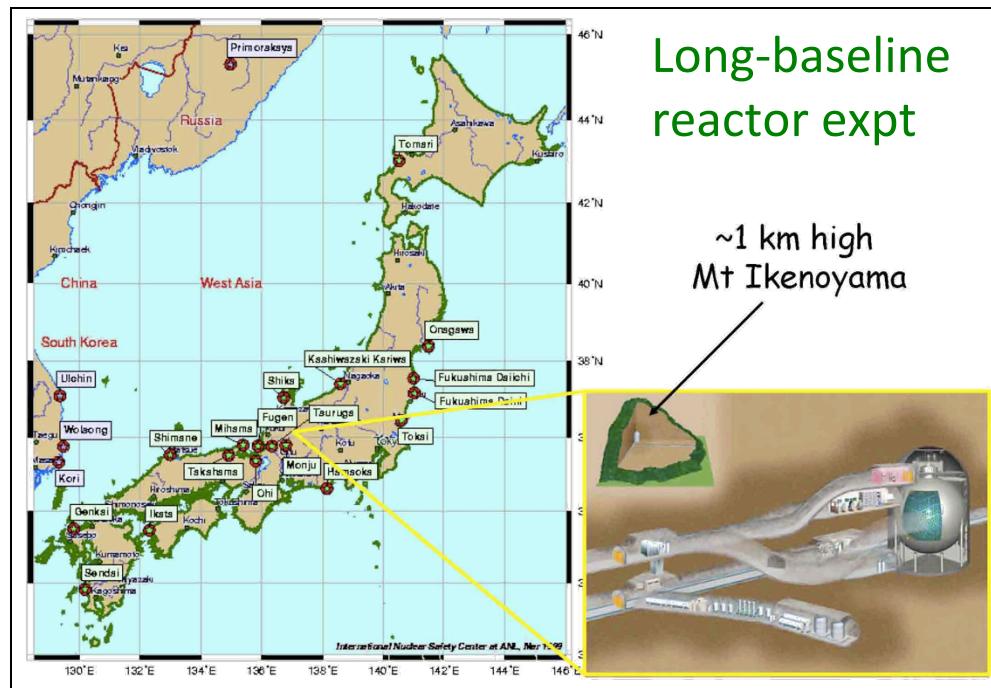
Art McDonald

Also in 2002... KamLAND: 1000 ton mineral oil detector, “surrounded” by nuclear reactors producing anti- ν_e . Characteristics:

$A/\delta m^2 \ll 1$ in Earth crust
(vacuum approxim. OK)
 $L \sim 100-200$ km
 $E_\nu \sim$ few MeV

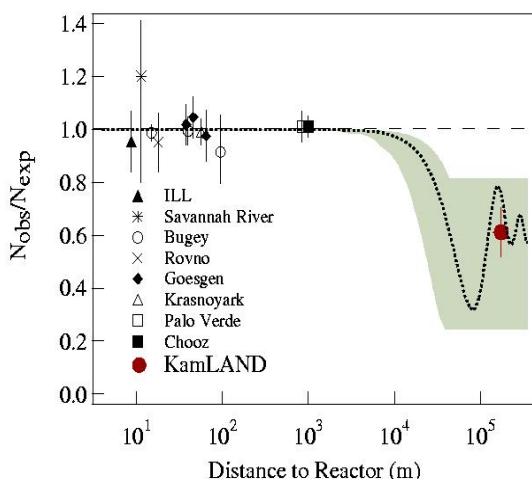


With previous $(\delta m^2, \theta_{12})$ parameters it is $(\delta m^2 L / 4E) \sim O(1)$ and reactor neutrinos should oscillate with large amplitude (large θ_{12})

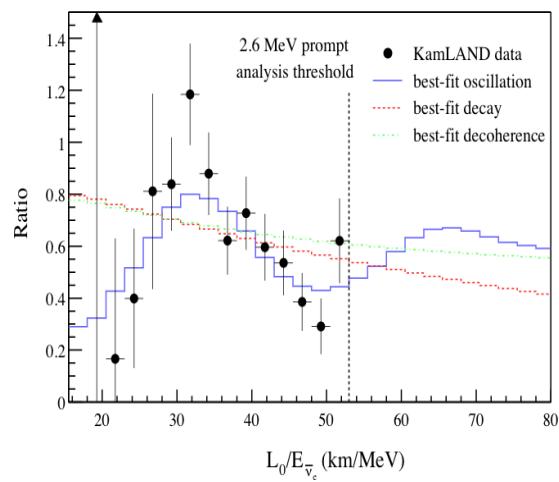


KamLAND results

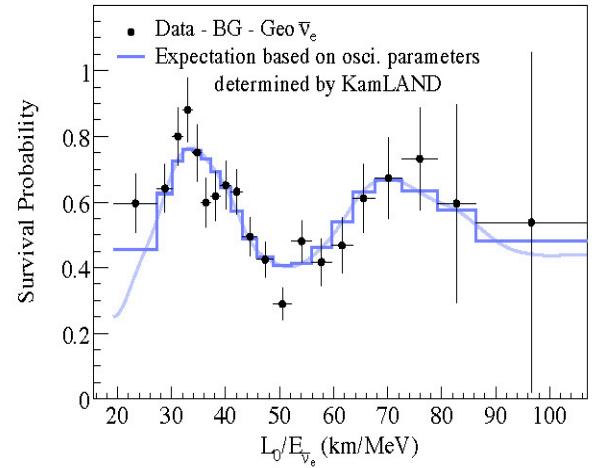
2002: electron flavor disappearance observed



2004: half-period of oscillation observed



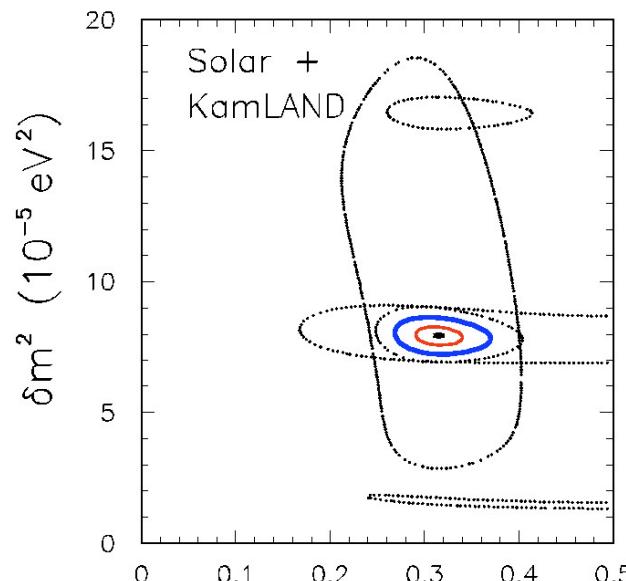
2007+: one period of oscillation observed



Direct observation of δm^2 oscillations!
(get precise δm^2 value from dip/peak position)

Interpretation in terms of 2ν oscillations

$(\delta m^2, \theta_{12})$ - complementarity of solar/KL neutrinos



← KamLAND

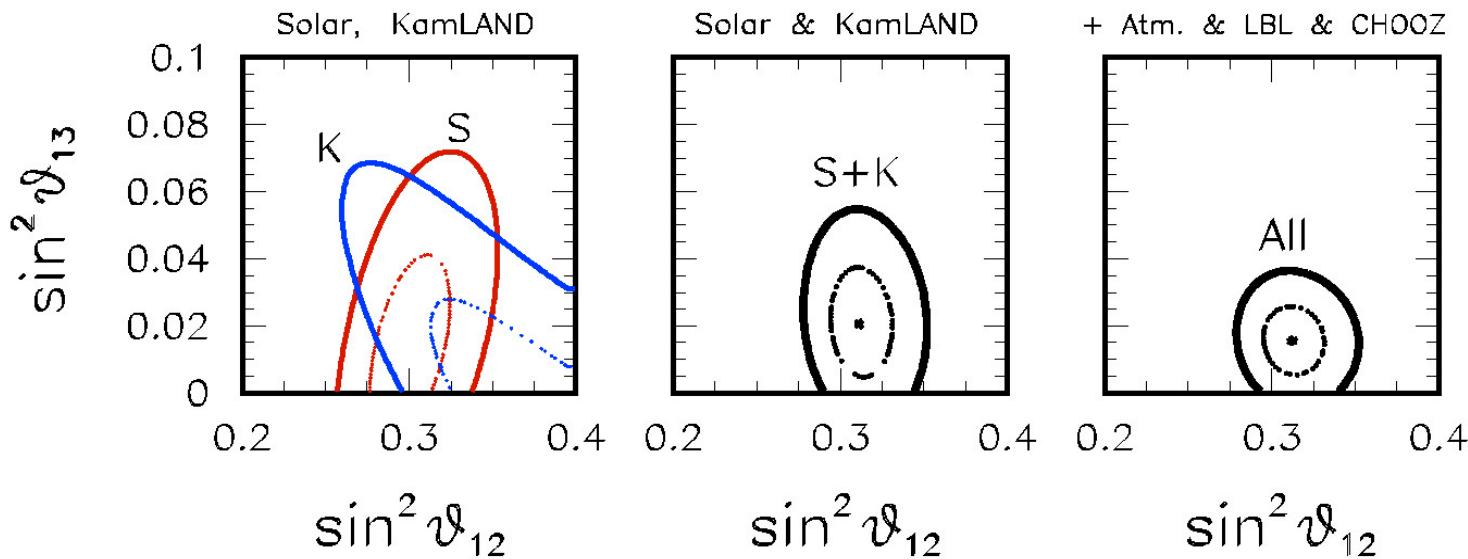


Solar

More refined (3ν) interpretation

Go beyond dominant 3ν oscillations. Include subleading θ_{13} effects in solar+KamLAND combination (as well as other data).

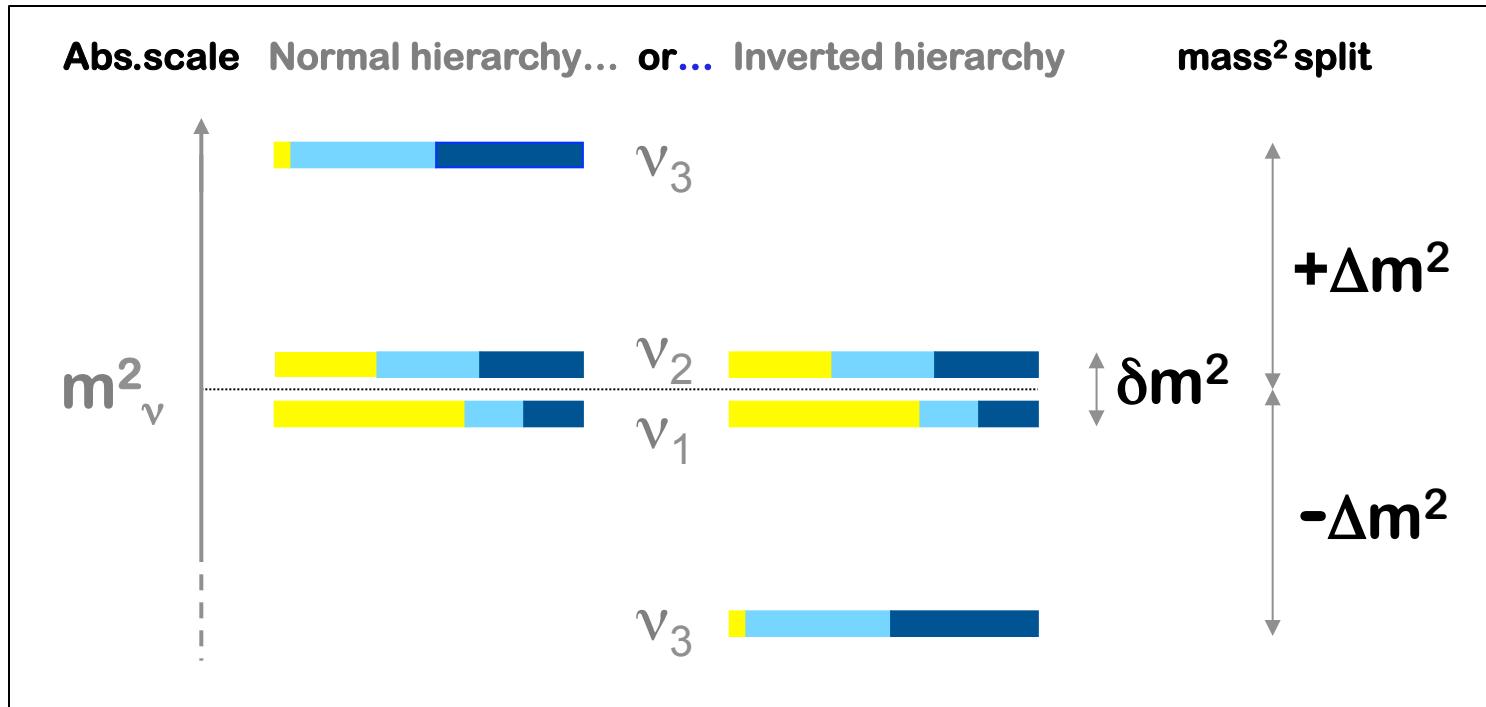
Interesting hints for $\theta_{13} > 0$ emerged as early as 2008... corroborated by T2K in 2011 ... established by reactors in 2012!



$$P_{ee}^{3\nu} = C_{13}^4 P_{ee}^{2\nu}(\delta m^2, \theta_{12}) + S_{13}^4$$

Present 3ν knowledge in one slide (with 1-digit accuracy)

e μ τ



We have seen:

$$\begin{aligned}\delta m^2 &\sim 7 \times 10^{-5} \text{ eV}^2 \\ \Delta m^2 &\sim 2 \times 10^{-3} \text{ eV}^2 \\ \sin^2 \theta_{12} &\sim 0.3 \\ \sin^2 \theta_{23} &\sim 0.5 \\ \sin^2 \theta_{13} &\sim 0.02\end{aligned}$$

We would like to see:

$$\begin{aligned}&\delta (\text{CP}) \\ &\text{sign}(\Delta m^2) \\ &\text{octant}(\theta_{23}) \\ &\text{absolute mass scale} \\ &\text{Dirac/Majorana nature}\end{aligned}$$

+ Physics
beyond 3ν?

(anomalies,
new states or
interactions)

CP and $P_{\alpha\beta}$ for neutrino vs antineutrino (see tutorial)

Exercise : CP(T) properties of $P_{\alpha\beta}$ in vacuum

One of the next frontiers is to investigate CP in the ν sector.
Prove that the general form of $P(\nu_\alpha \rightarrow \nu_\beta)$ is naturally split in a CP-conserving and a CP-violating part, $P = P_{CP} + P_{CP}$:

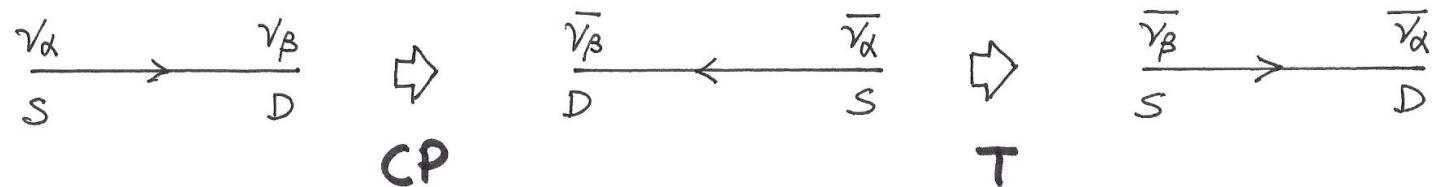
$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re } J_{\alpha\beta}^{ij} \sin^2 \left(\frac{\Delta m_{ij}^2 x}{4E} \right) \quad \leftarrow P_{CP}$$
$$- 2 \sum_{i < j} \text{Im } J_{\alpha\beta}^{ij} \sin \left(\frac{\Delta m_{ij}^2 x}{4E} \right) \quad \leftarrow P_{CP}$$

where CP invariance would imply $U = U^*$ and $P(\nu) = P(\bar{\nu})$.

Prove also that CPT invariance holds, and implies

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha).$$

Action of CP and T transformations on $\nu_\alpha \rightarrow \nu_\beta$ process
from source (S) to detector (D):



$$\text{CP invariance : } P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \quad \Leftarrow (\nu \leftrightarrow \bar{\nu})$$

$$\text{T invariance : } \begin{cases} P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha) \\ P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \end{cases} \quad \Leftarrow (\alpha \leftrightarrow \beta)$$

$$\text{CPT invariance : } P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \quad \Leftarrow (\nu \leftrightarrow \bar{\nu}) \oplus (\alpha \leftrightarrow \beta)$$

For 3ν in vacuum : in the general form of $P_{\alpha\beta}$, it is easy to check that either $(\alpha \leftrightarrow \beta)$ or $(\nu \leftrightarrow \bar{\nu})$ exchange amount to $(U \leftrightarrow U^*)$, only affecting the P_{CP} part. Therefore, CP invariance requires $U = U^*$, while CPT invariance holds in any case.

CP violation as a genuine 3ν effect

Exercise : Conditions to observe CP in vacuum

Consider the general form $P = P_{CP} + P_{\bar{CP}}$. Prove that, in order to have $P_{\bar{CP}} \neq 0$, the following conditions must be satisfied :

- $\delta \neq 0$ or π $\leftarrow U$ must be complex
- $\alpha \neq \beta$ \leftarrow need appearance experiments
- $\theta_{ij} \neq 0$ \leftarrow all mixing angles $\neq 0$
- $\Delta m^2_{ij} \neq 0$ \leftarrow sensitivity to both δm^2 and Δm^2

→ The smallness of θ_{13} and of $\delta m^2 < \Delta m^2$ make it difficult to test CP violation in the neutrino sector !

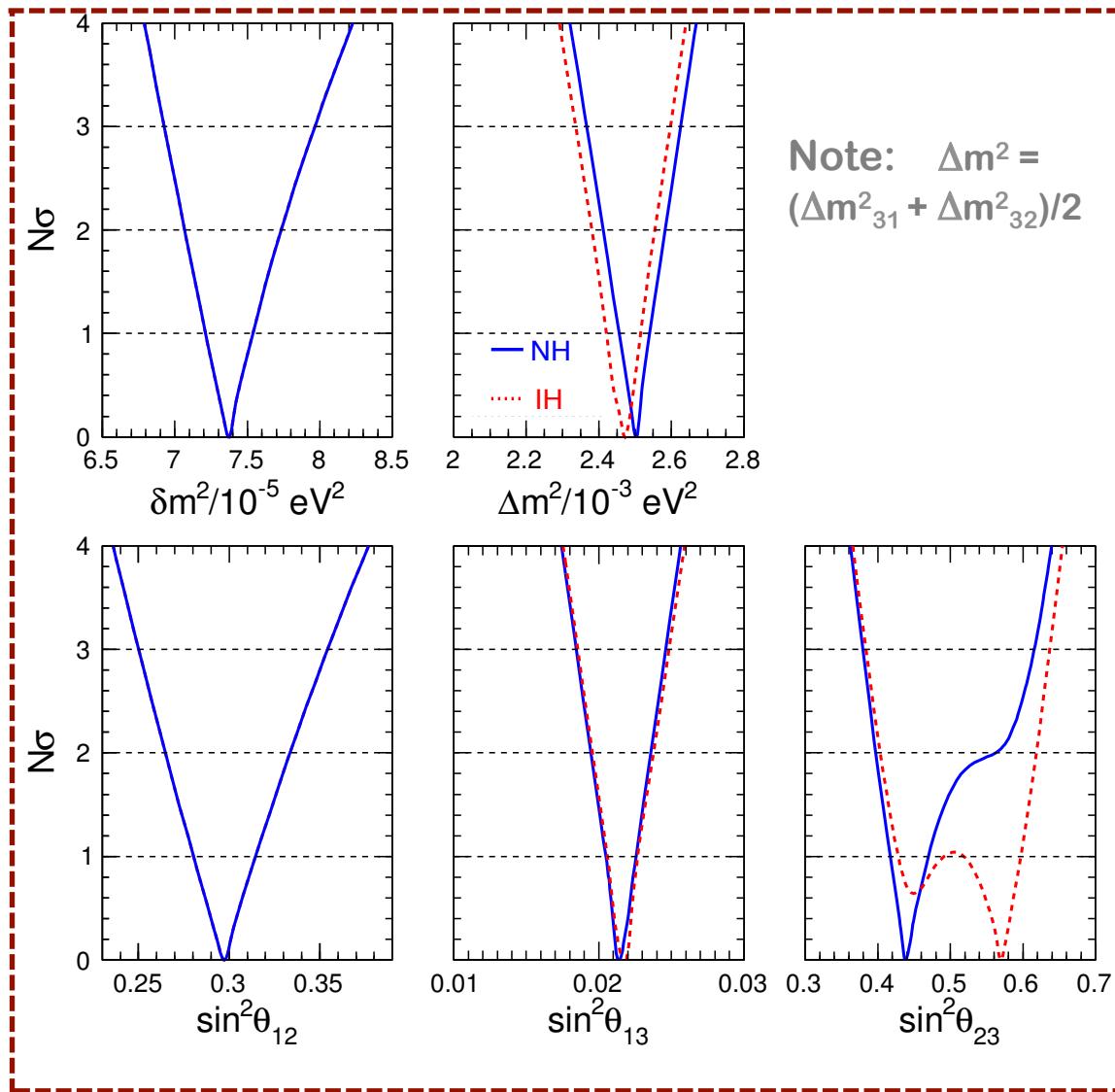
Getting the most by combining all oscillation data....

Global 3 ν analysis 2016

(from arXiv:1601.07777)

Single (known) oscillation parameters

LBL Acc + Solar + KamLAND + SBL Reactors + Atmos



Current 1σ errors
(1/6 of $\pm 3\sigma$ range):

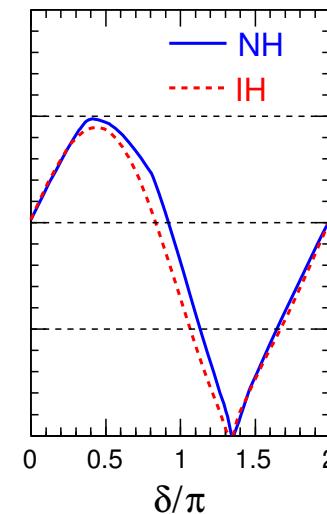
δm^2	2.4 %
Δm^2	1.8 %
$\sin^2 \theta_{12}$	5.8 %
$\sin^2 \theta_{13}$	4.7 %
$\sin^2 \theta_{23}$	~9 %

all < 10%...

Precision Era!

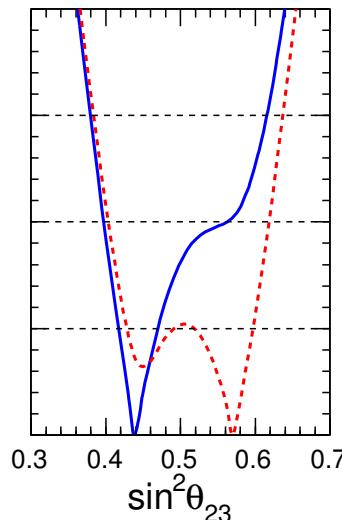
Single (unknown) oscillation parameters

LBL Acc + Solar + KamLAND + SBL Reactors + Atmos



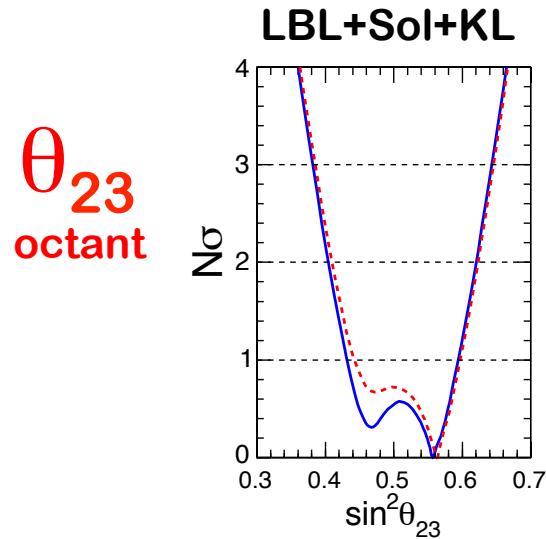
δ_{CP}

NH or IH

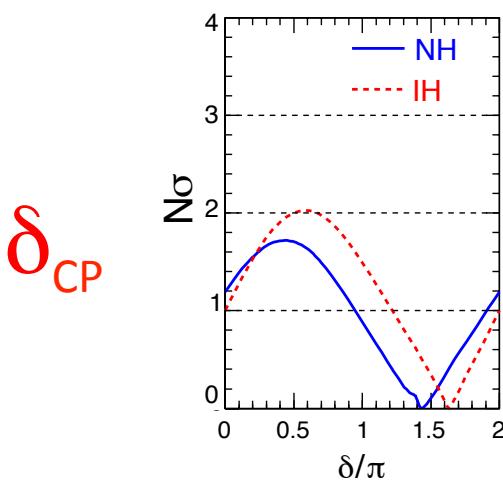


θ_{23} octant

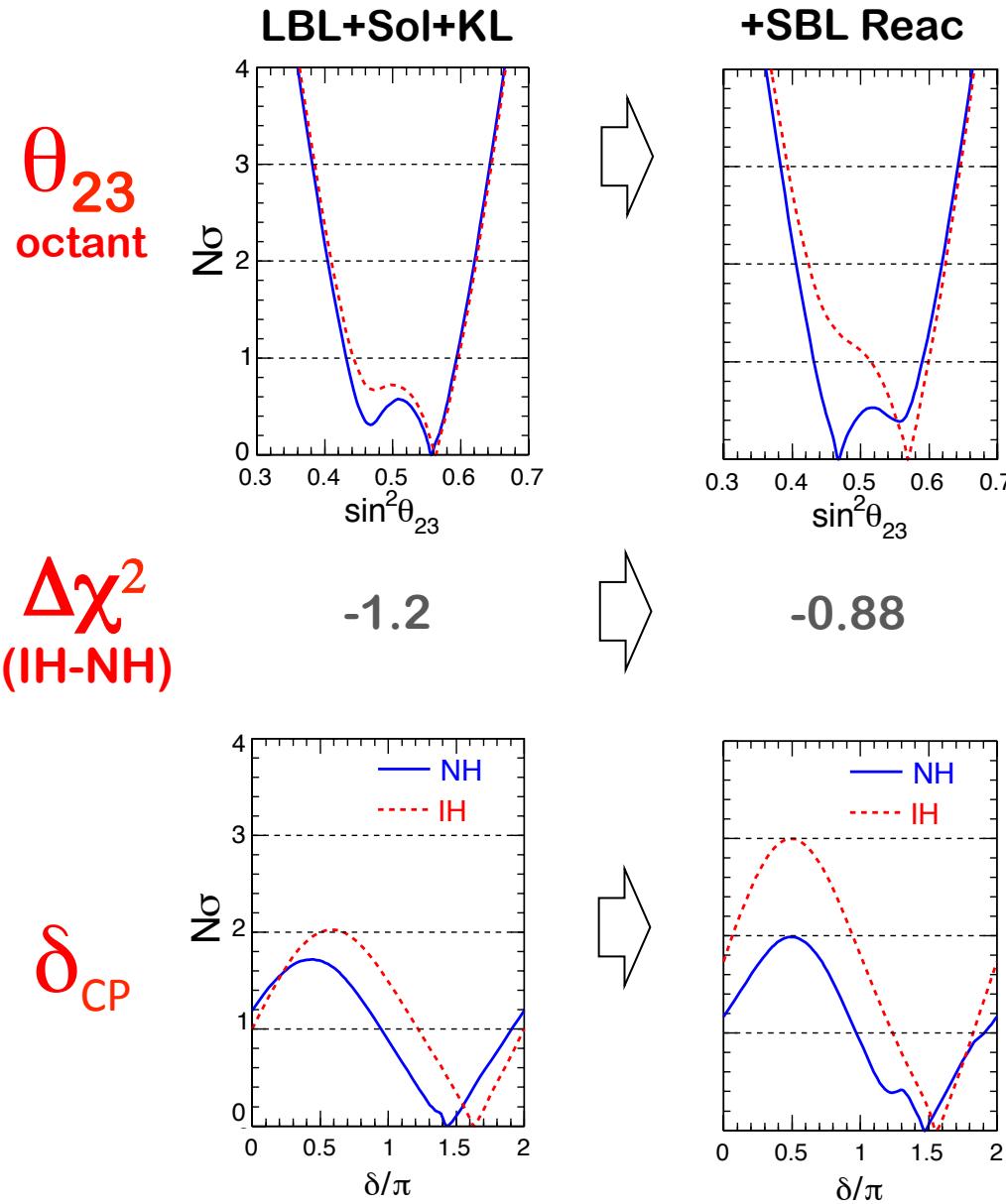
More on single (unknown) parameters:



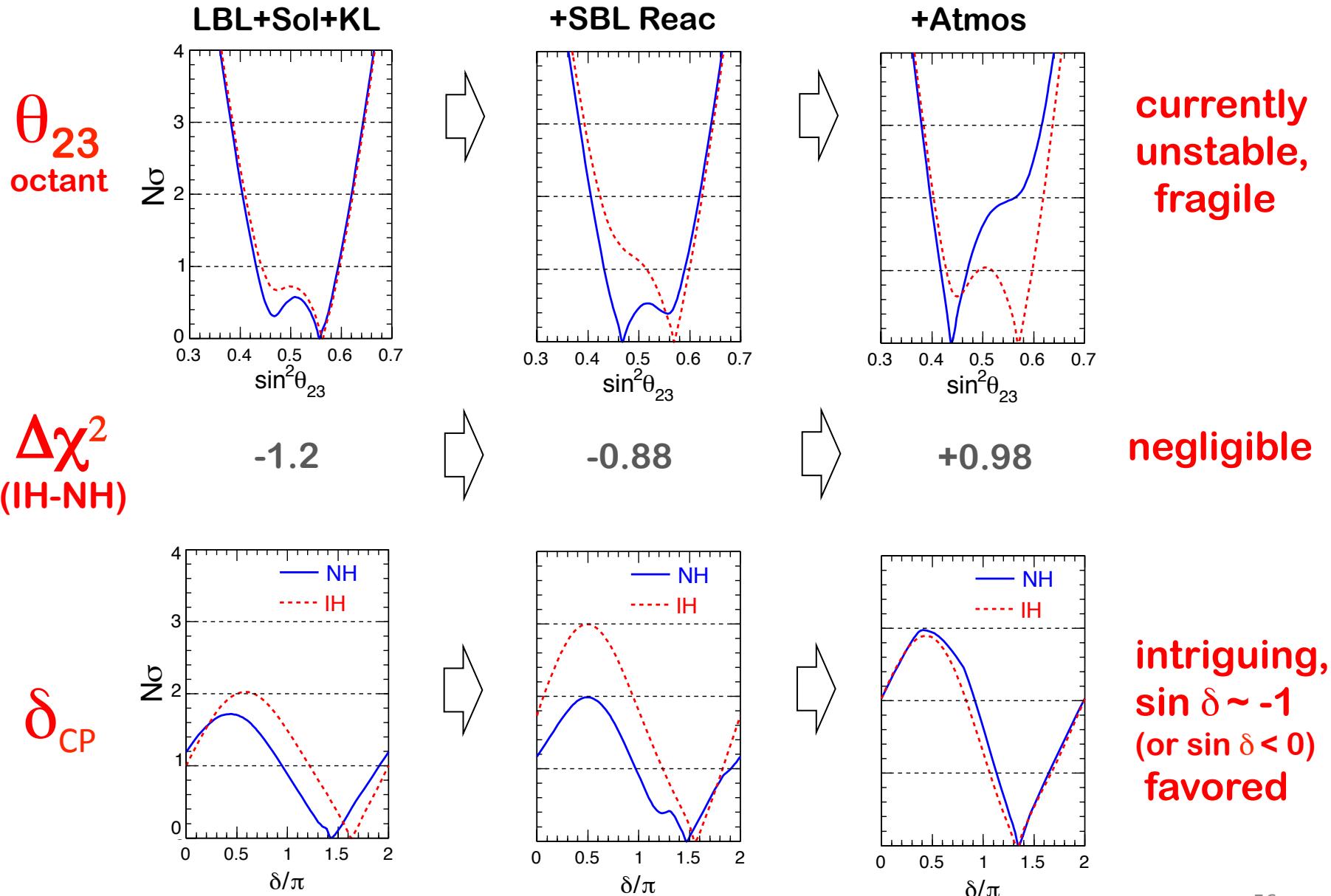
$\Delta\chi^2$
(IH-NH)
-1.2



More on single (unknown) parameters:

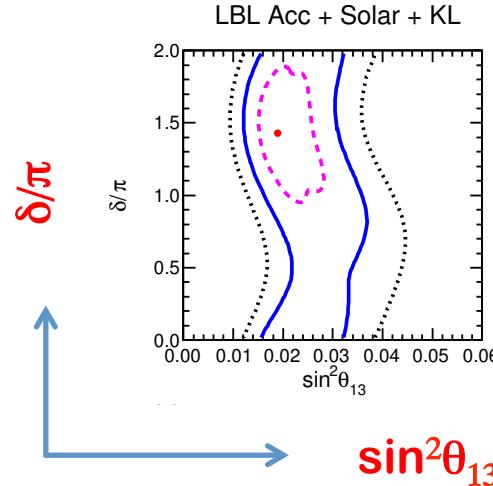


More on single (unknown) parameters:



More on CPV phase

From variances to covariances: analysis of a 2D plot

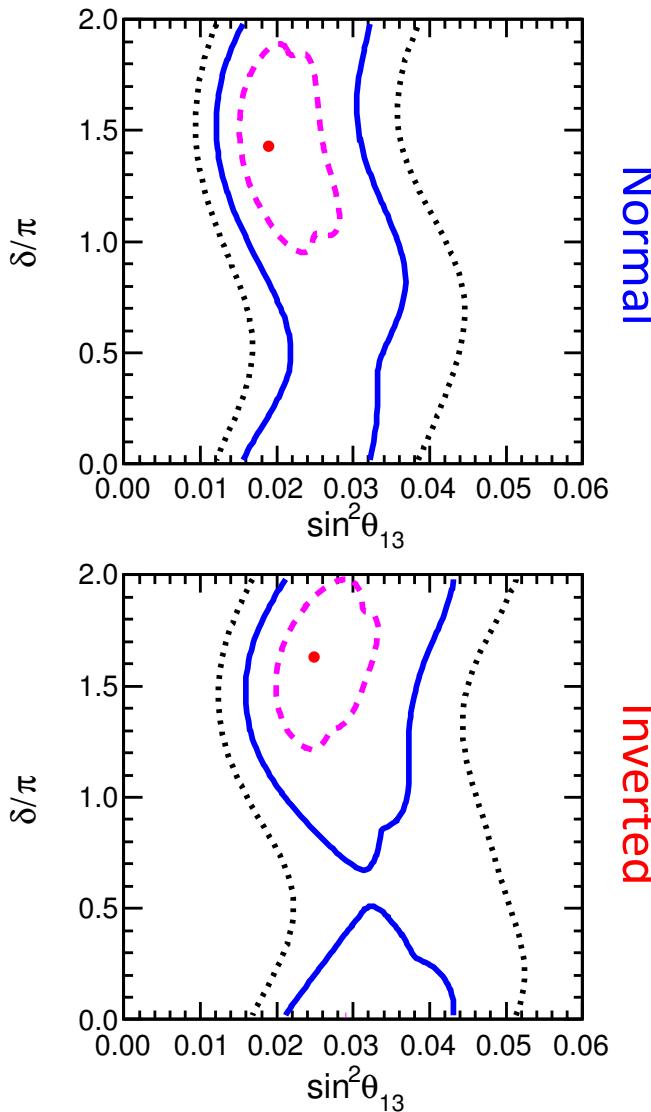


Leading appearance amplitude at LBL Acc. $\sim \sin^2 \theta_{23} \sin^2(2\theta_{13})$
 \rightarrow uncertainty on θ_{23} somewhat affects subleading terms

Subleading CPV appearance amplitude for ν	$\sim -\sin\delta$
Subleading CPV appearance amplitude for anti-ν	$\sim +\sin\delta$
\rightarrow T2K & NOvA ν signal maximized for	$\sin\delta \sim -1$ ($\delta \sim 1.5\pi$)
\rightarrow T2K anti- ν signal minimized for	$\sin\delta \sim -1$ ($\delta \sim 1.5\pi$)

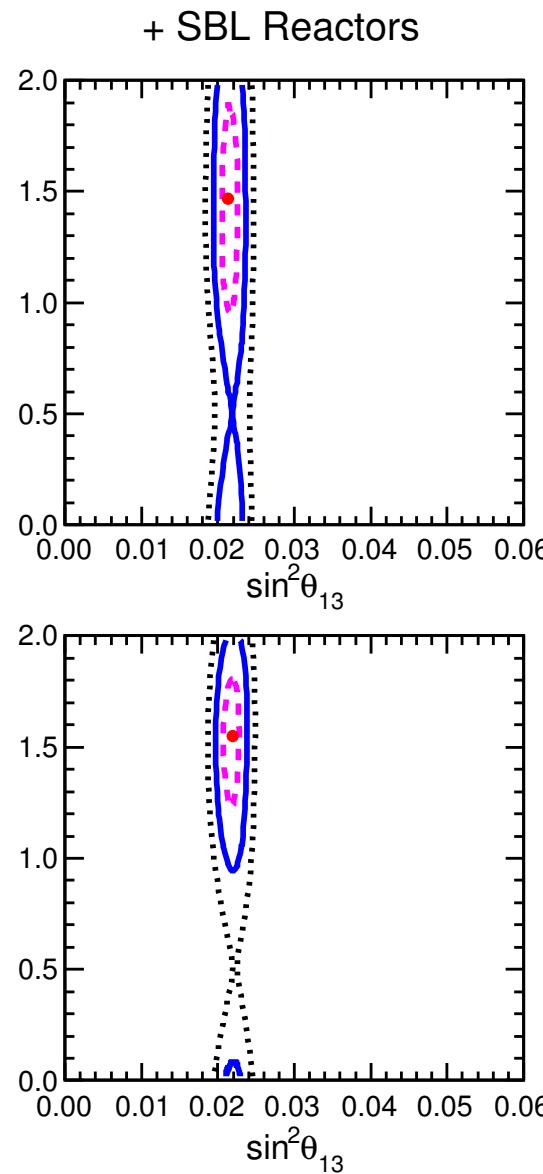
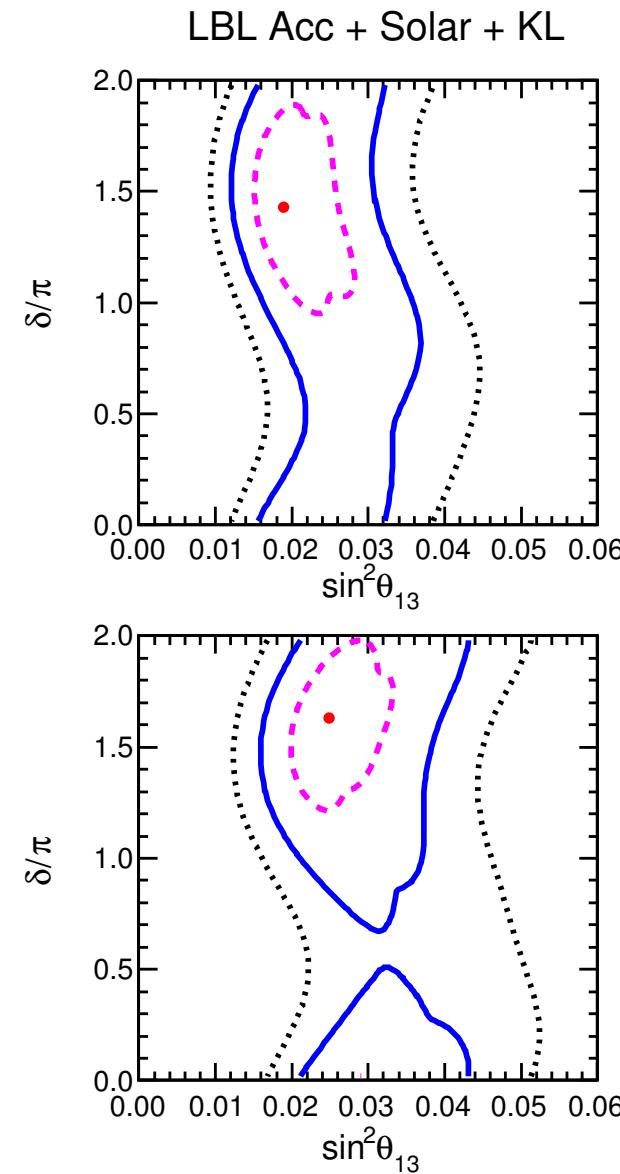
Note: subleading $\sin\delta$ dependence worked out in last, longest exercise of tutorial

LBL Acc + Solar + KL

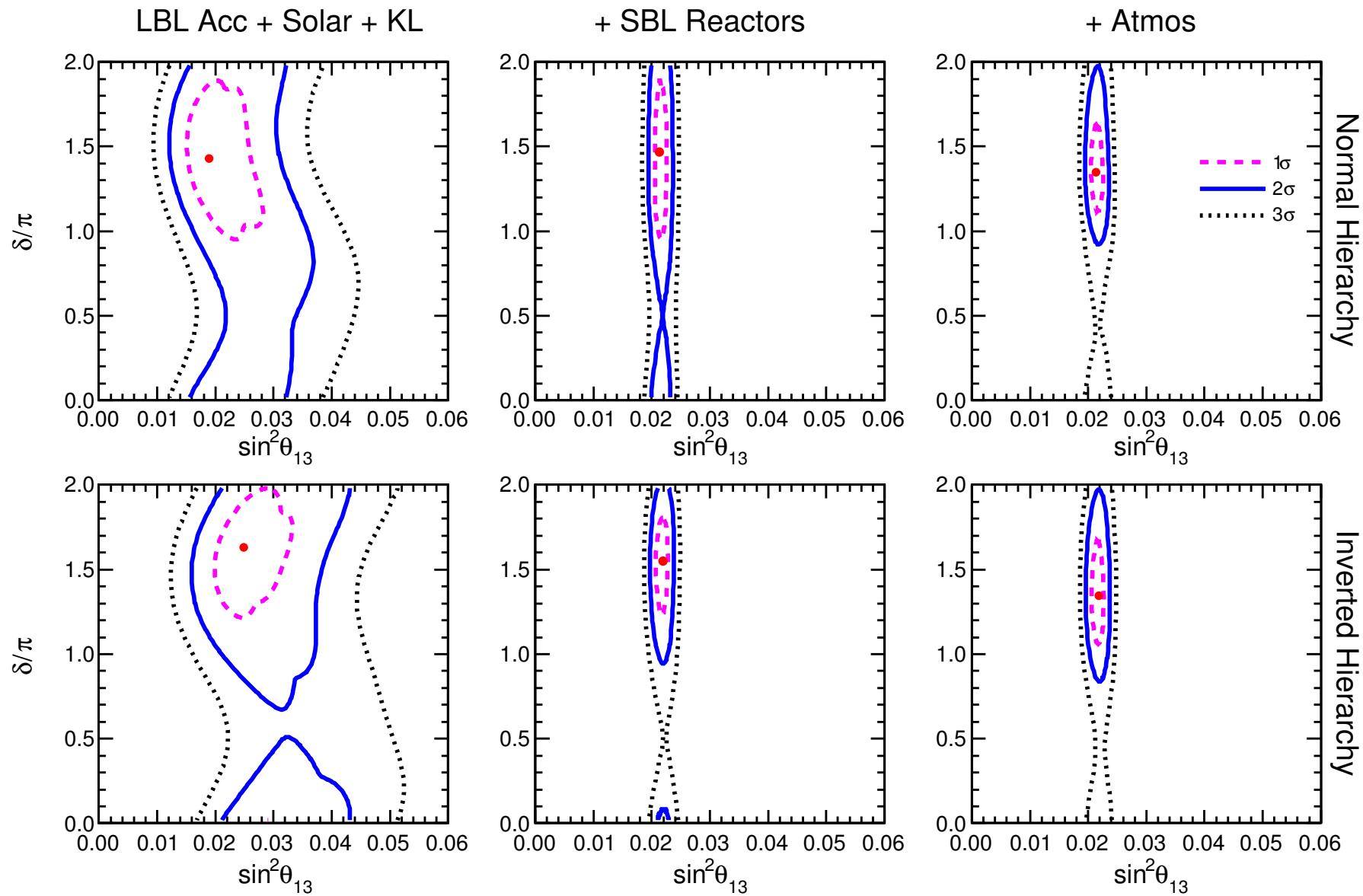


Interesting facts (still statistically limited) about
LBL accelerator + Solar + KamLAND data set:

- (1) By themselves, these data have almost the same $\sin^2\theta_{13}$ best fit (~ 0.02) as SBL reactors [also Solar + KL data alone: “old” hint for $\theta_{13} > 0$]
- (2) For such best fit, $\nu_\mu \rightarrow \nu_e$ appearance event rates in T2K and NOvA are “large” $\rightarrow \sin\delta \sim -1$
- (3) Conversely, anti($\nu_\mu \rightarrow \nu_e$) appearance event rate in T2K is “small” $\rightarrow \sin\delta \sim -1$ again!
- (4) Large uncertainty in $\sin^2\theta_{13}$ partly due to degeneracy with $\sin^2\theta_{23}$



SBL reactor data:
strong constraints
on $\sin^2\theta_{13}$
improved bounds
on $\sin\delta$



Indications for $\sin\delta < 0$ corroborated by **atmospheric neutrino data**

Latest T2K and NOvA results @ Neutrino'16 provide further hints of $\sin\delta \sim -1$

PARTICLE PHYSICS

Neutrinos Hint of Matter-Antimatter Rift

An early sign that neutrinos behave differently than antineutrinos suggests an answer to one of the biggest questions in physics.



Olena Shmahalo/Quanta Magazine

As neutrinos and antineutrinos change flavors they may illuminate the differences between matter and antimatter.

By Natalie Wolchover

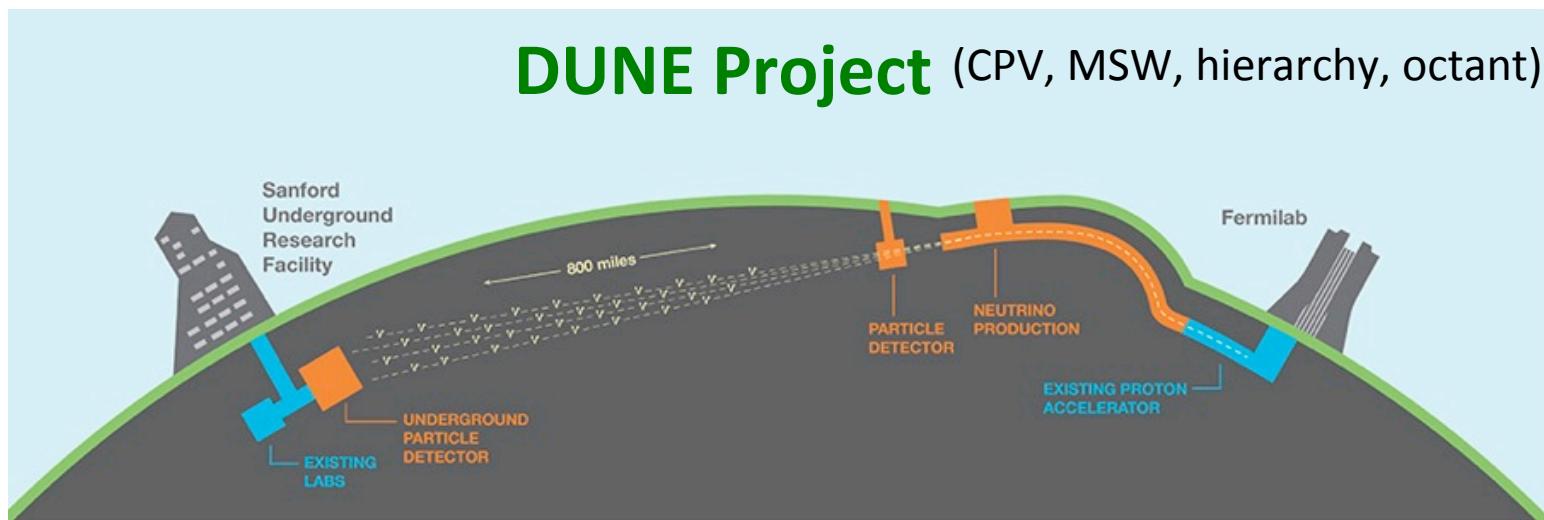
July 28, 2016



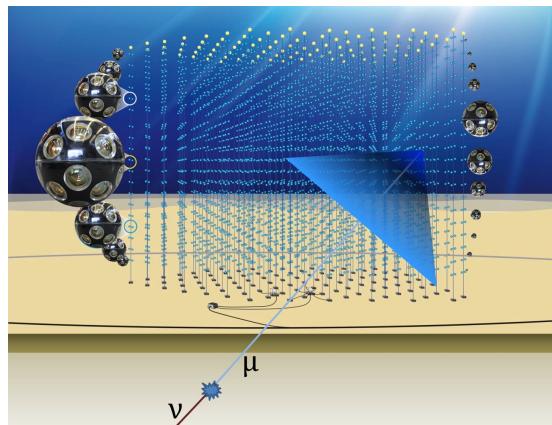
MOST VIEWED • RECENT

The search for CPV, hierarchy, octant, and other subleading (non)standard effects in vacuum and in matter is motivating new big experimental projects, both **underground** and **underwater/ice**, e.g.,

DUNE Project (CPV, MSW, hierarchy, octant)

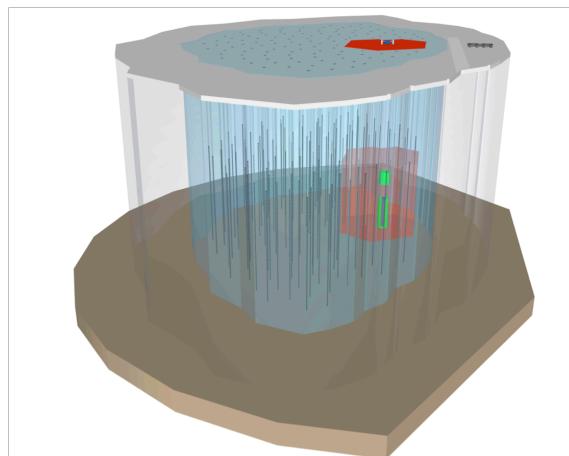


Km3/ORCA

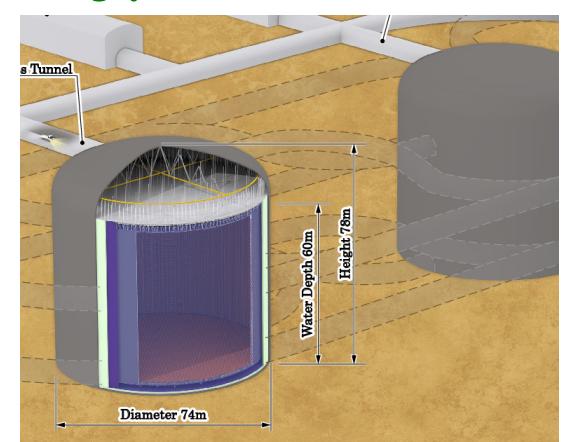


(MSW, hierarchy, octant)

IceCube/PINGU



Hyper-Kamioka



(multipurpose)

... whose results might provide new guidance for theor. models

Underlying symmetries? A vast spectrum of possibilities...

No organizing principle
("anarchy")



Discrete family symmetries
("geometry")

linear relations between
 $\theta_{13}\cos\delta$ and θ_{12}, θ_{23}

Continuous flavor symmetries
("dynamics")

links between neutrino
masses/angles/phases

Common quark-lepton features
("complementarity")

links between
 θ_{13} and θ_c

Additional material...

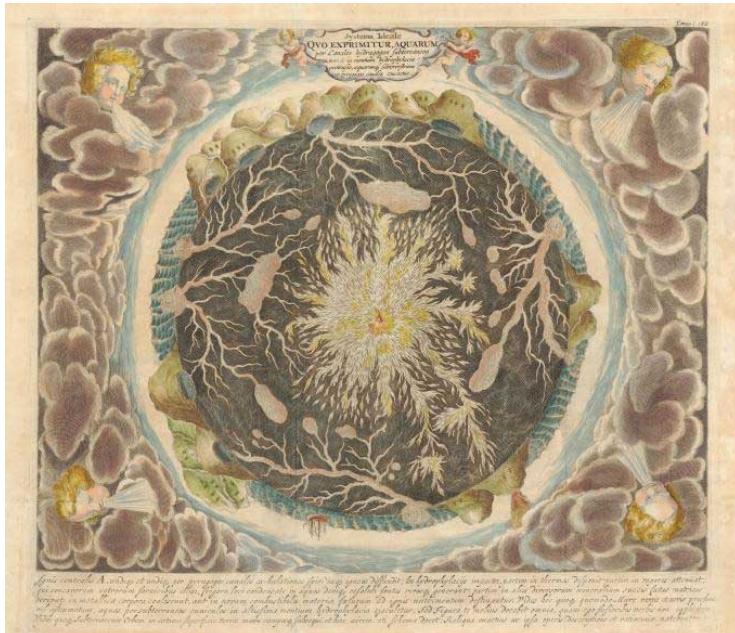
- Geoneutrinos
- Supernova neutrinos
- Comments on mass hierarchy
- Comments on LBL appearance prob.

Thank you for your attention!

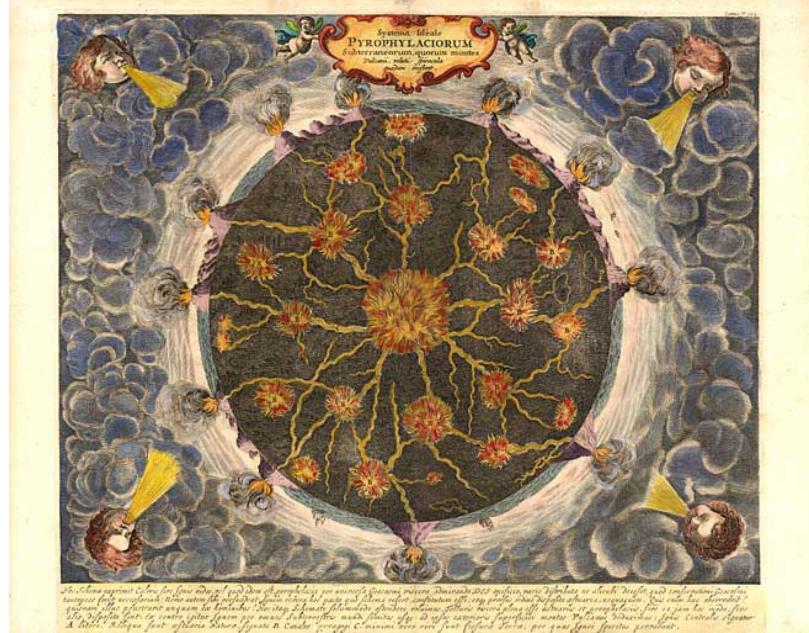
Interesting side results from low-energy neutrino oscillation searches: **Geoneutrinos**

Cartoons of the Earth's interior, circa 1700

...Internal waters

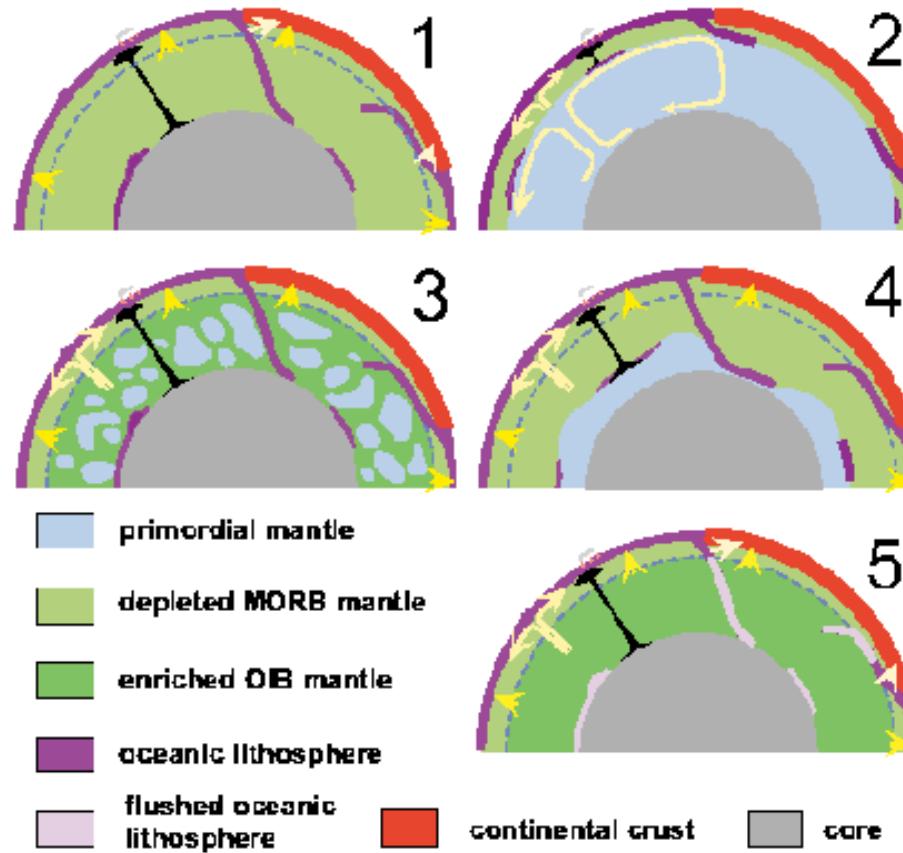


...Internal fires



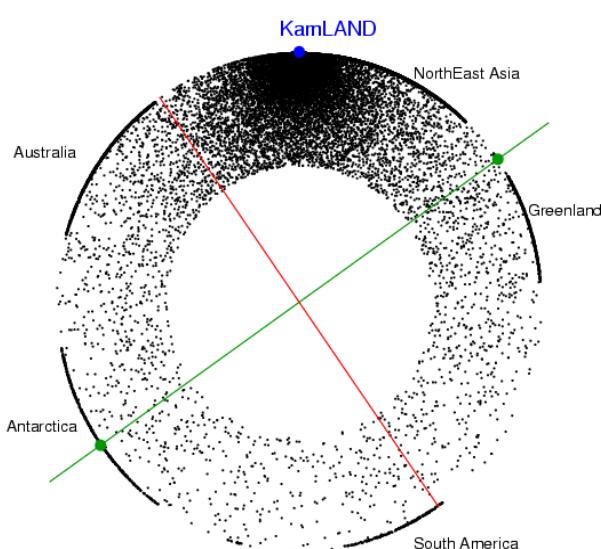
Athanasius Kircher, "Mundus Subterraneus" (Amsterdam, 1665)

Cartoons of the Earth's interior, 300 ys later...



(Albarède and van der Hilst, 1999)

Geoneutrino data can help to constrain Earth models



(from Sanshiro Enomoto)

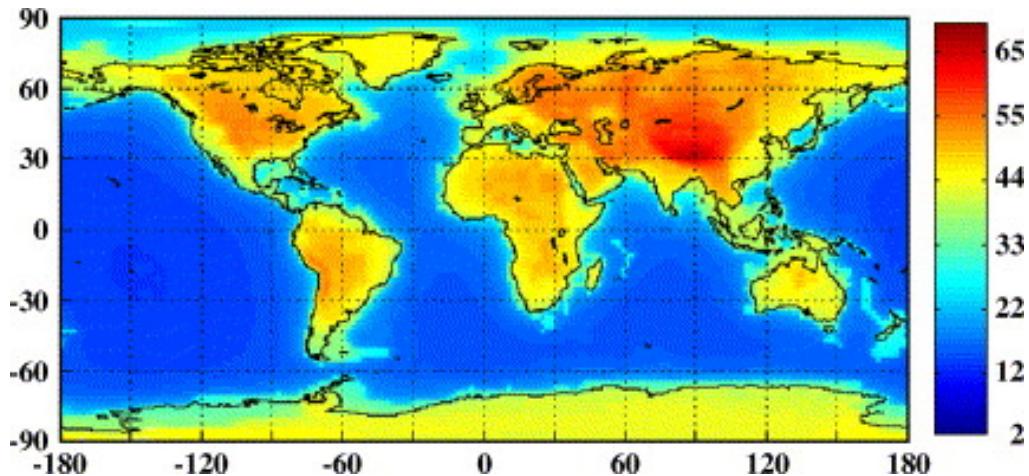
Crustal signal reasonably constrained: more interest in unknown mantle signal

(from G. Fiorentini et al.)

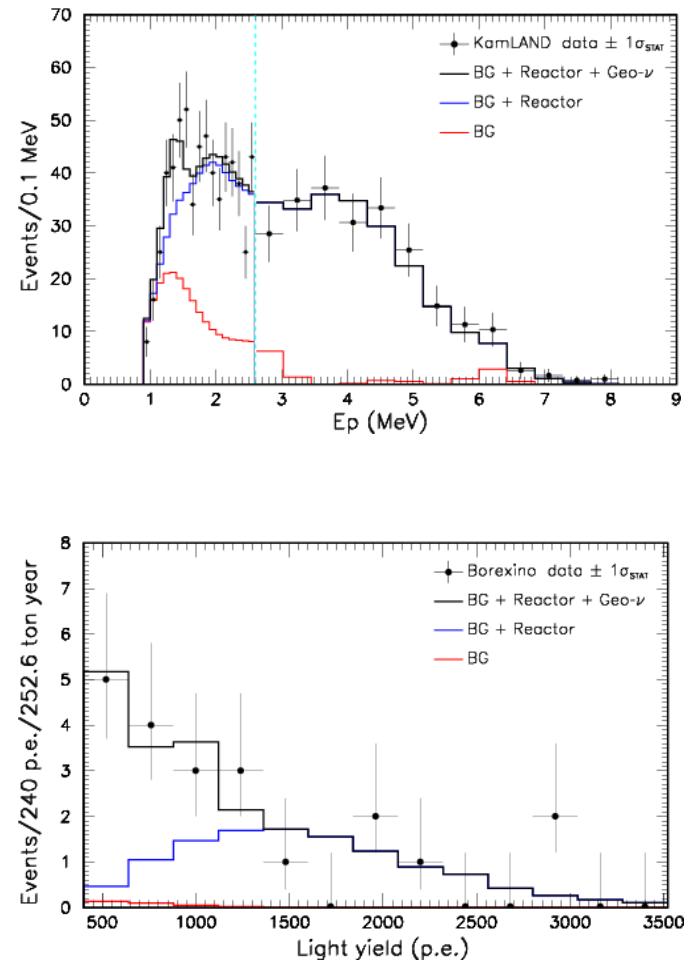
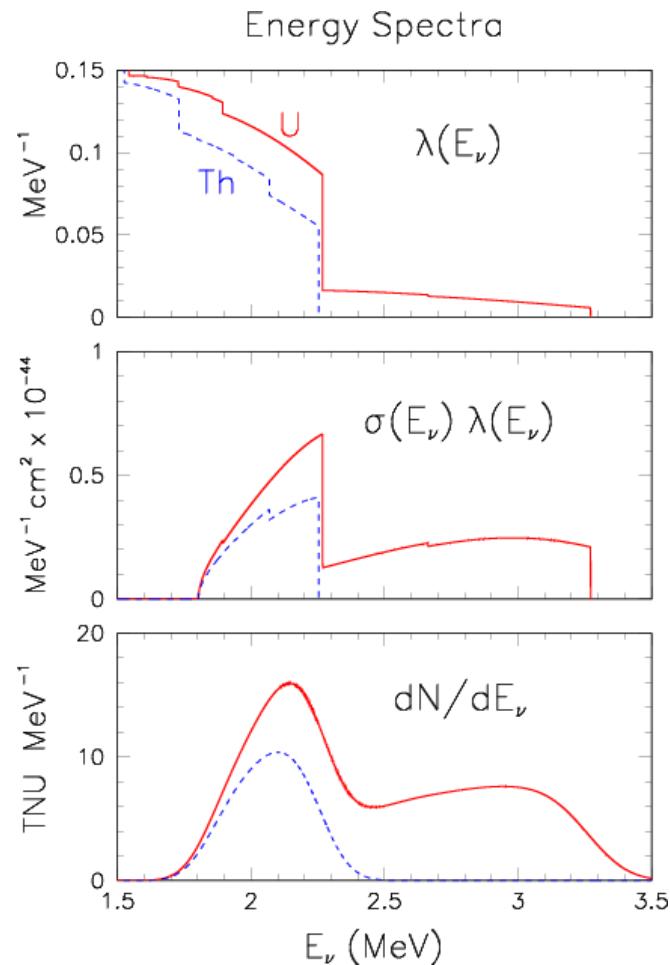
U, Th and K decay produce both heat and geo-neutrinos

U, Th, and K are (thought to be) highly differentiated
(abundant in the crust, diluted in the mantle, absent in the core)

Therefore, geonu fluxes can probe Heat + Structure (weighted by $1/L^2$)

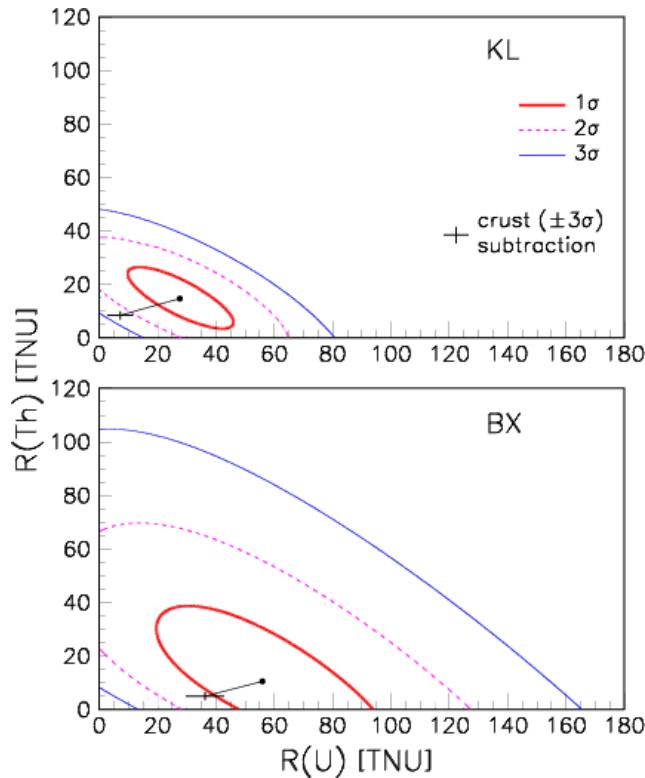


Theoretical spectra vs KamLAND and Borexino data



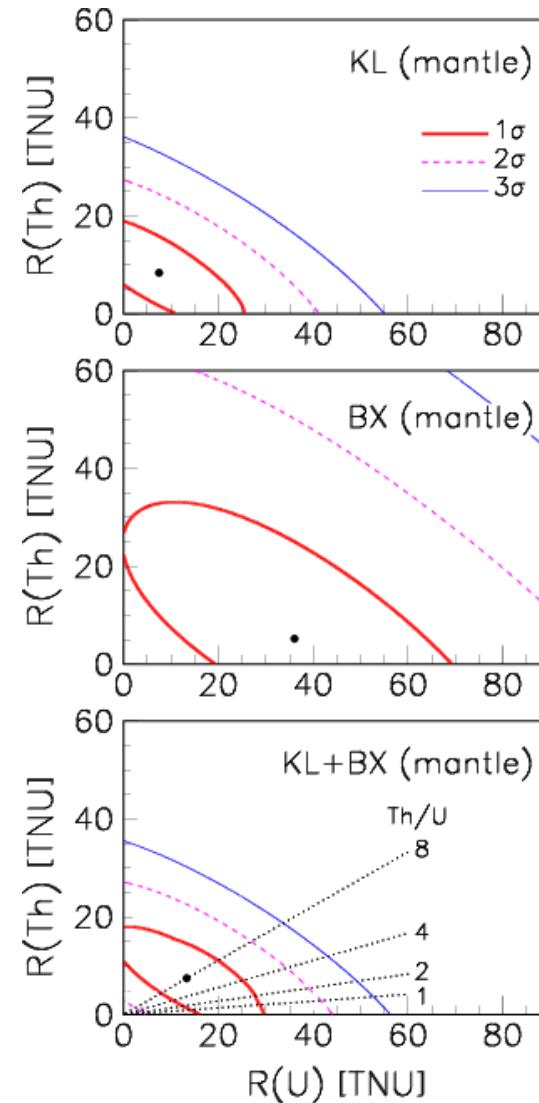
(Not the latest data, but OK for the discussion)

Overall KL, BX geo-n
signals: >4s evidence

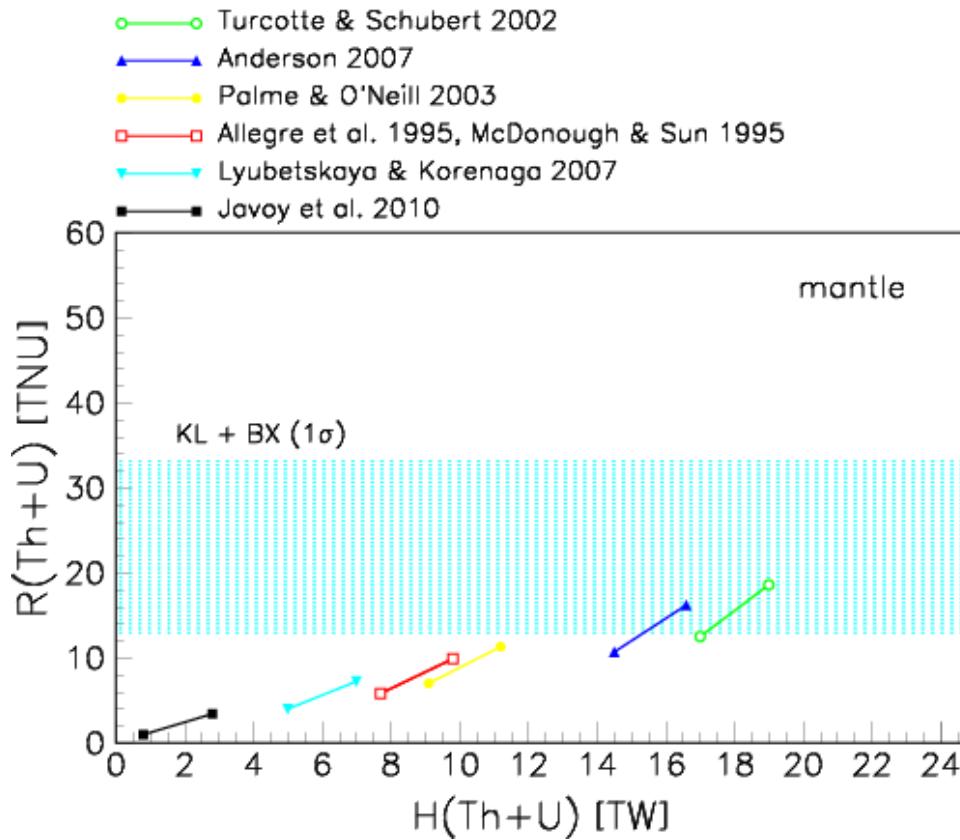


If estimated crust
signal is subtracted...

...hint of mantle
signal a ~2sigma (KL+BX)



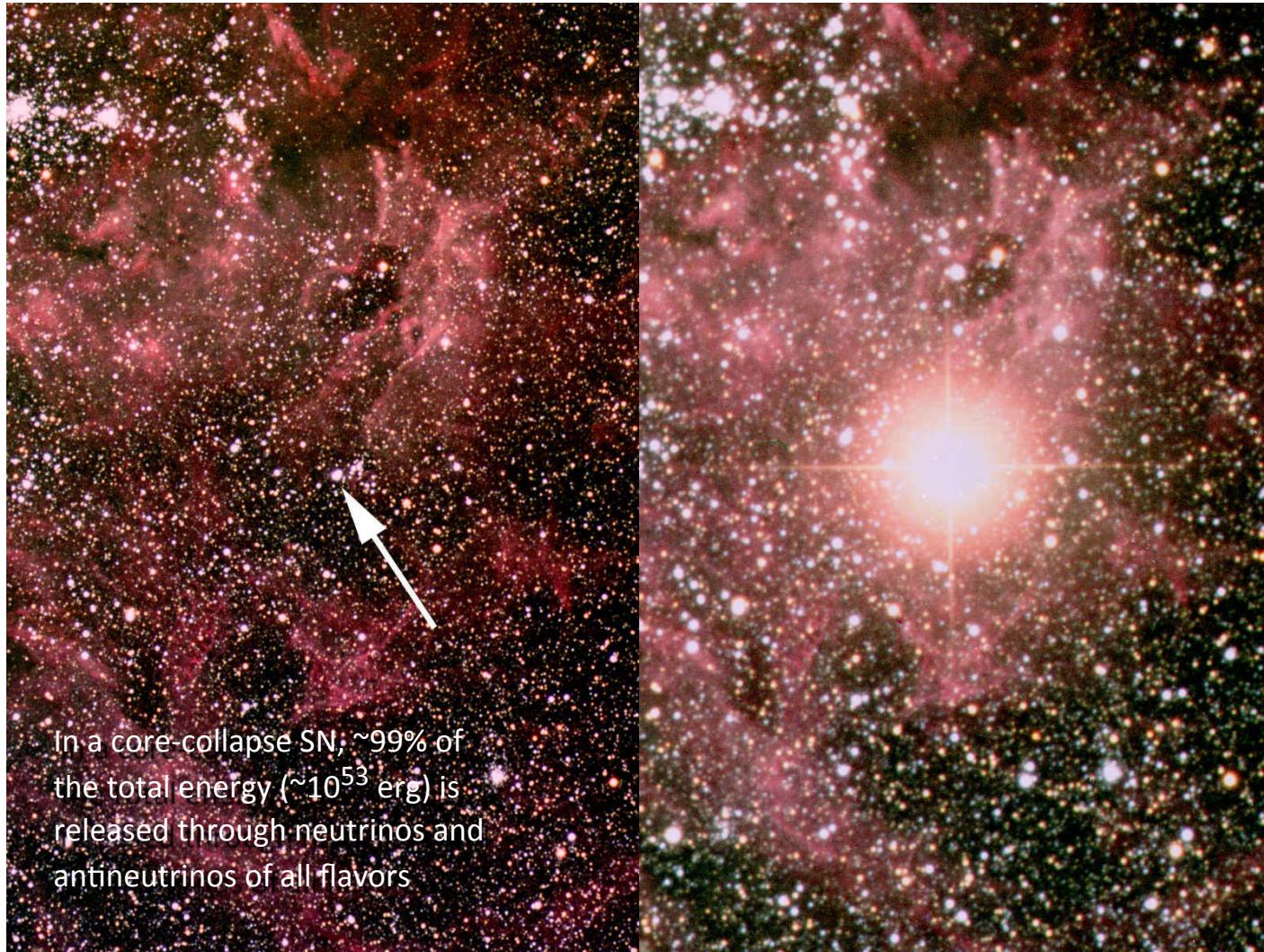
Results prefer models with relatively high mantle heat, but with large errors (no model excluded at $>2\sigma$)



We are just making first steps in a long-term research program which will provide a unique understanding of the Earth's interior.

Supernova neutrinos and self-interactions

It is worth reminding that the only two known, localized sources in neutrino astronomy (so far) are the Sun and the SN 1987A.



In a core-collapse SN, ~99% of the total energy ($\sim 10^{53}$ erg) is released through neutrinos and antineutrinos of all flavors

Flavor changes induced by “usual MSW” effects: studied for >20 y.

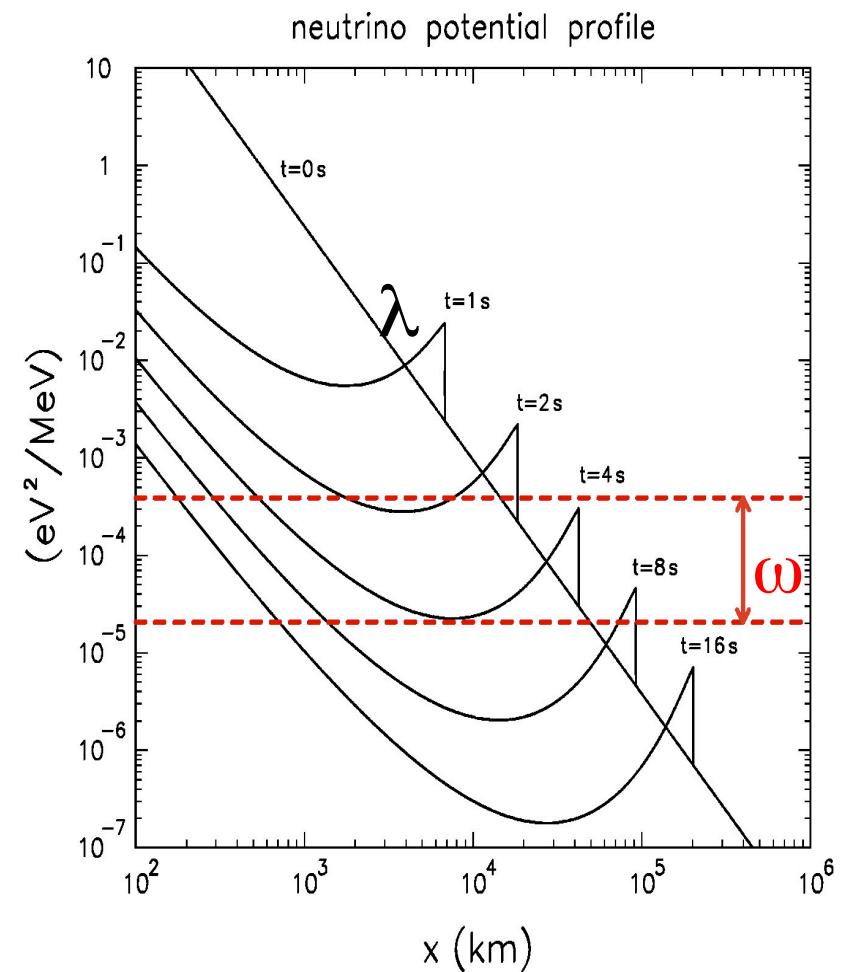
Well-known MSW effects can occur in a SN envelope when the ν potential $\lambda = \sqrt{2} G_F N_e$ is close to osc. frequency $\omega = \Delta m^2 / 2E$ ($\Delta m^2 = |m_3^2 - m_{1,2}^2|$, $\theta_{13} \neq 0$).

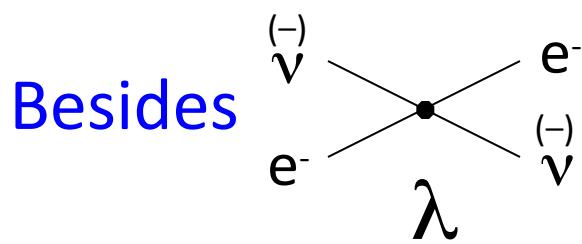
For $t \sim$ few sec after bounce,
 $\lambda \sim \omega$ at $x \gg 10^2$ km (large radii).

What about small radii?

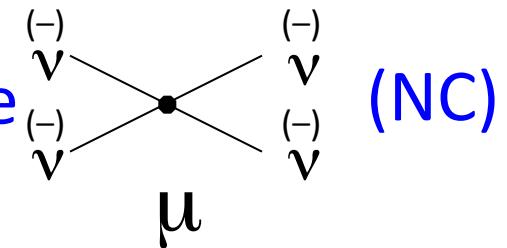
Popular wisdom:

$\lambda \gg \omega$ at $x < O(10^2)$ km,
thus flavor transitions suppressed.
Incorrect!





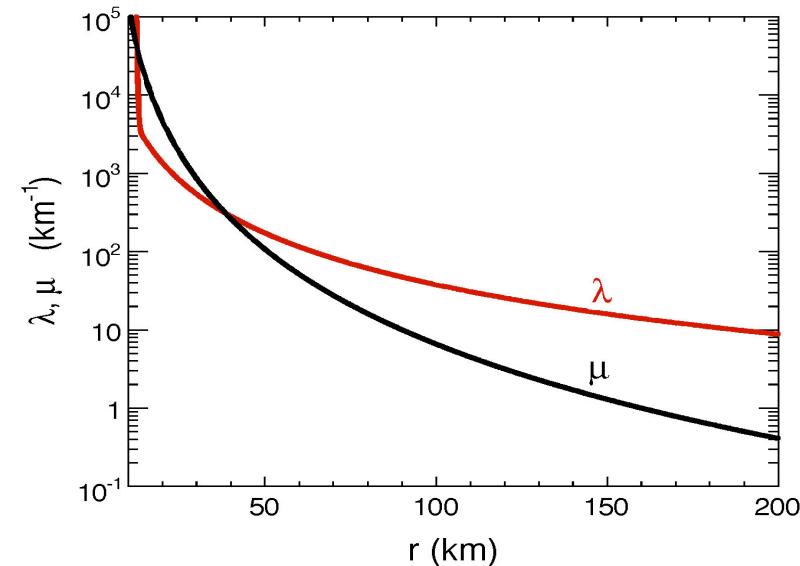
(CC) one has to include



At small r , neutrino and antineutrino density (ν and $\bar{\nu}$) high enough to make self-interactions important. Strength:

$$\mu = \sqrt{2} G_F (N_\nu + N_{\bar{\nu}})$$

Angular modulation factor: $(1 - \cos \Theta_{ij})$
 If averaged: “single-angle” approxim.
 Otherwise : “multi-angle” (difficult)



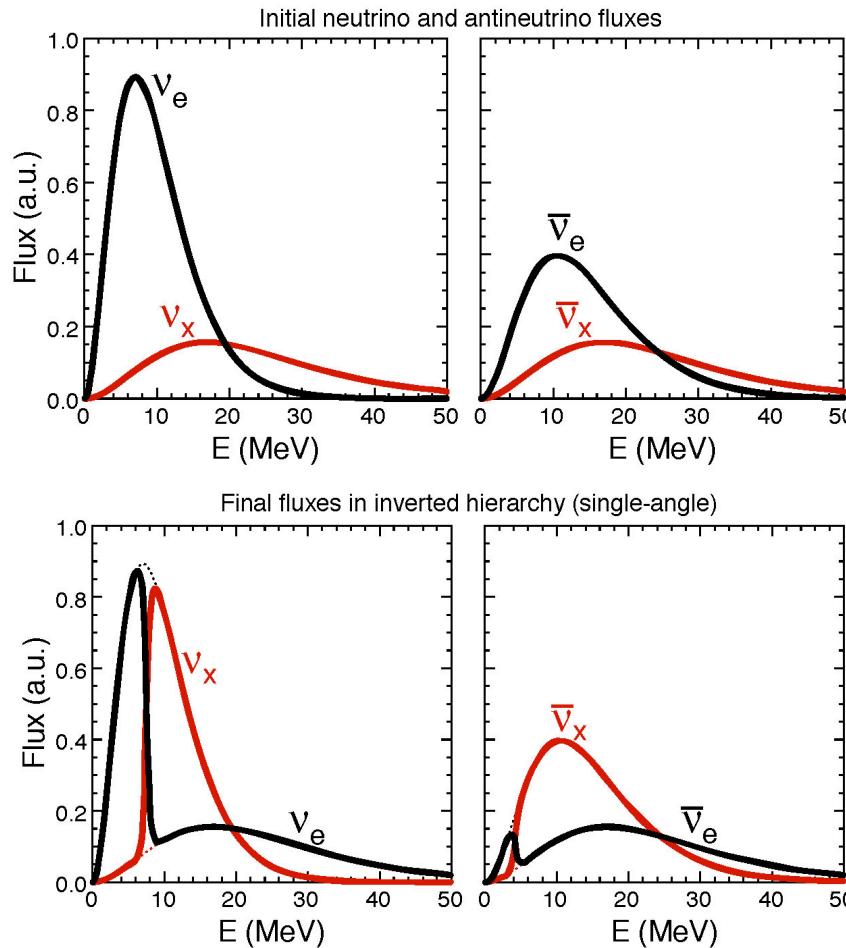
Self-interaction effects known for ~25 y in SN. But, recent boost of interest after new crucial results, first obtained numerically and then analitically.

Lesson: self-interactions (μ) can induce large, nonlinear, non-MSW flavor changes at small radii, despite large matter density λ

It turns out that a dense neutrino gas behaves as a system of coupled spins, with beautiful examples of synchronized and collective phenomena

$$\begin{aligned}\dot{\mathbf{P}} &= \left[+\omega \mathbf{B} + \lambda \mathbf{z} + \mu \sum_E (\mathbf{P} - \bar{\mathbf{P}}) \right] \times \mathbf{P} \\ \dot{\bar{\mathbf{P}}} &= \left[-\omega \mathbf{B} + \lambda \mathbf{z} + \mu \sum_E (\mathbf{P} - \bar{\mathbf{P}}) \right] \times \bar{\mathbf{P}}\end{aligned}$$

E.g., due to self-interaction effects, flavor may be swapped abruptly in certain energy ranges for inverted hierarchy (“spectral split”)



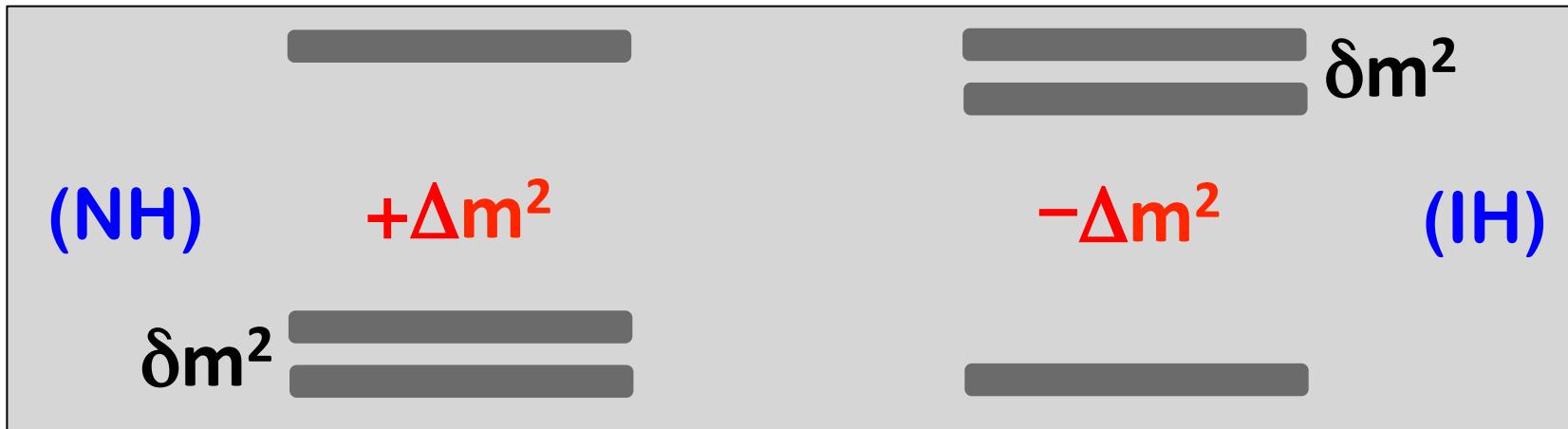
Initial fluxes at the neutrinosphere ($r \sim 10$ km)

Final fluxes at the end of collective effects ($r \sim 200$ km)

But, recent works revealed further layers of complexity – no obvious picture of SN neutrino flavor oscillations emerged so far ...

Comments on mass hierarchy

No hints so far, but we'll get there via oscillations...



... if we can observe **interference** of oscill. driven by $\pm \Delta m^2$ with oscill. driven by another quantity **Q** with known sign. Three options:

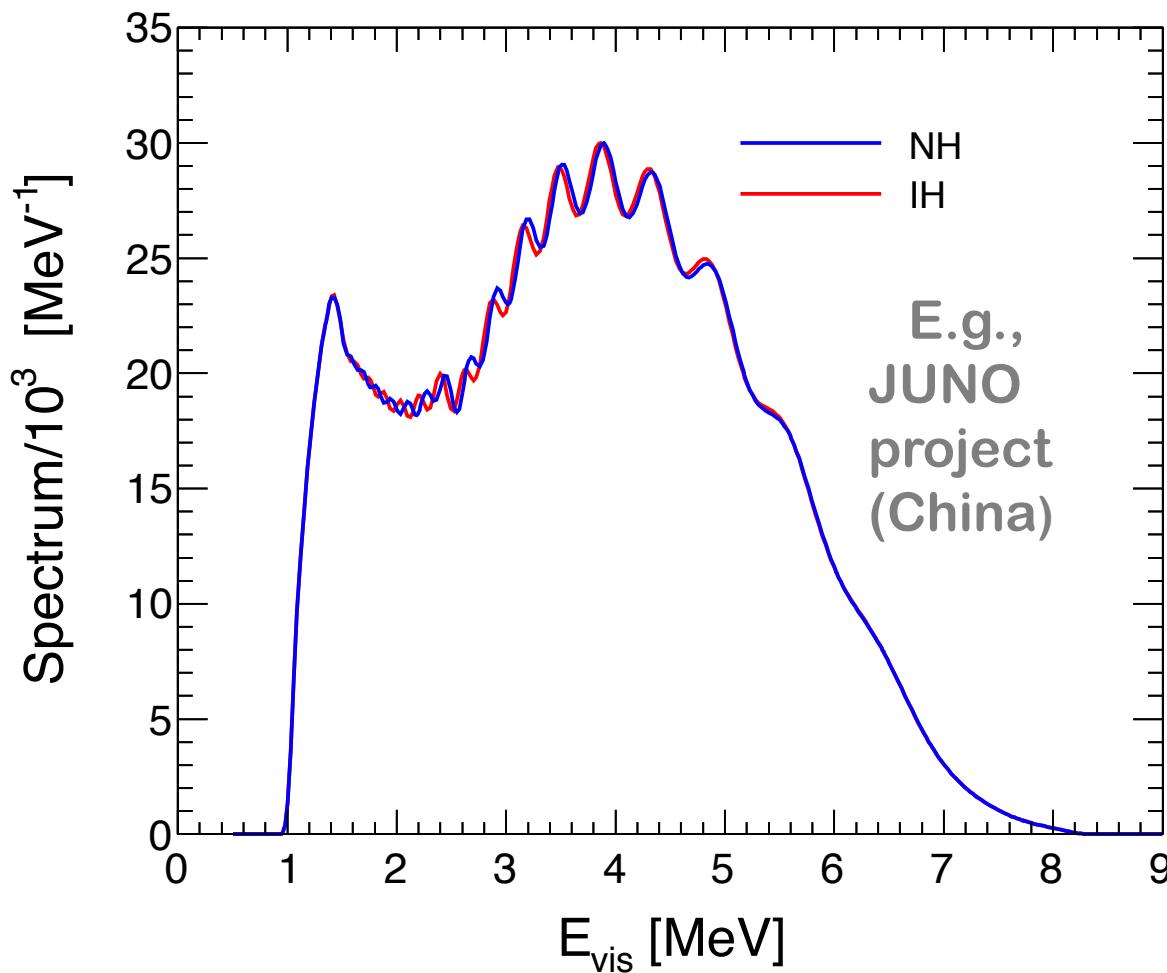
$$Q = \delta m^2 \quad (\text{medium-baseline reactors})$$

$$Q = 2\sqrt{2} G_F N_e E \quad (\text{matter effects in accel./atmosph. } v)$$

$$Q = 2\sqrt{2} G_F N_\nu E \quad (\text{collective effects in supernovae})$$

[Nonoscillation searches may provide further handles]

Make $\pm \Delta m^2$ interfere with δm^2 at medium-baseline reactors
Very challenging!

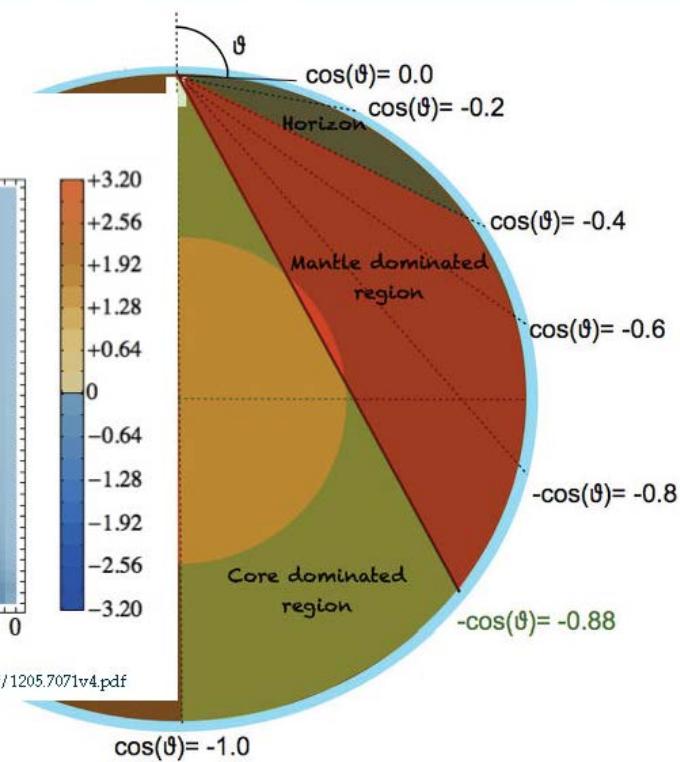
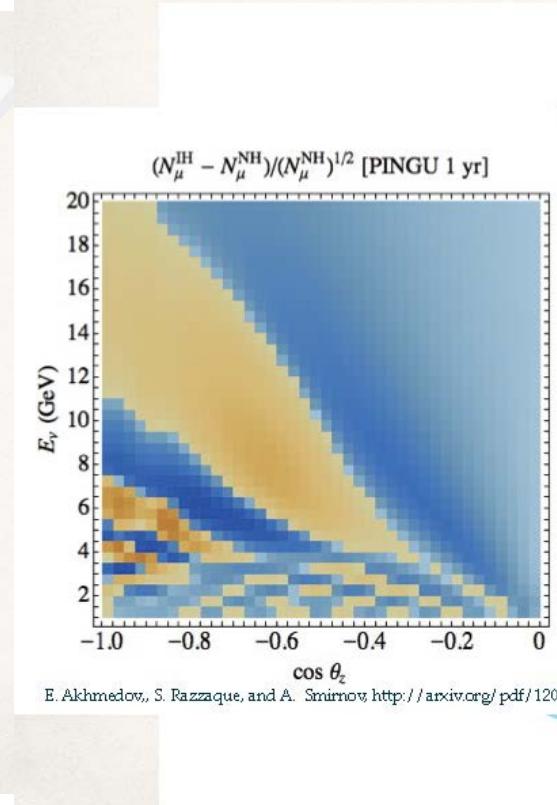
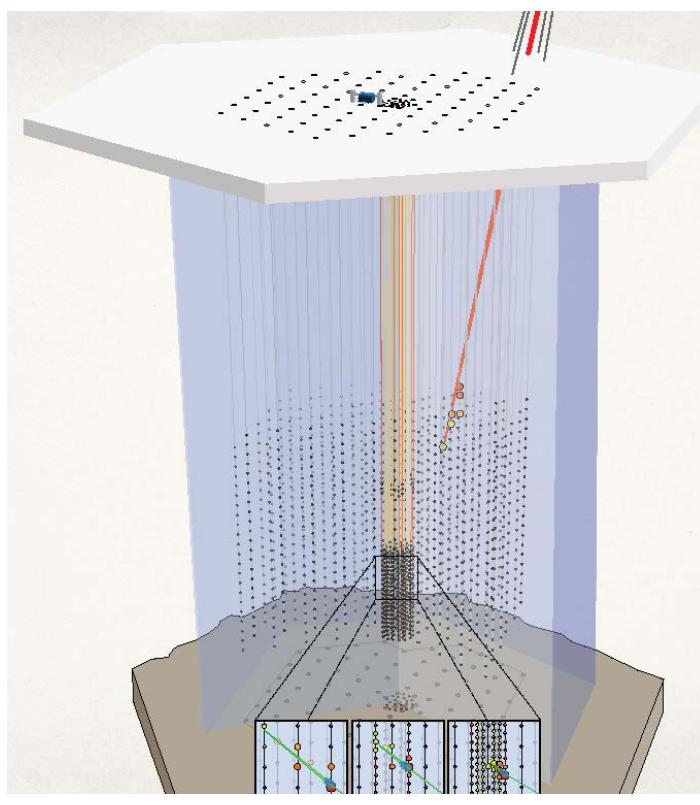


Will also improve δm^2 and θ_{12} accuracy by O(10)

Make $\pm \Delta m^2$ interfere with $G_F N_e E$ in atmospheric expts

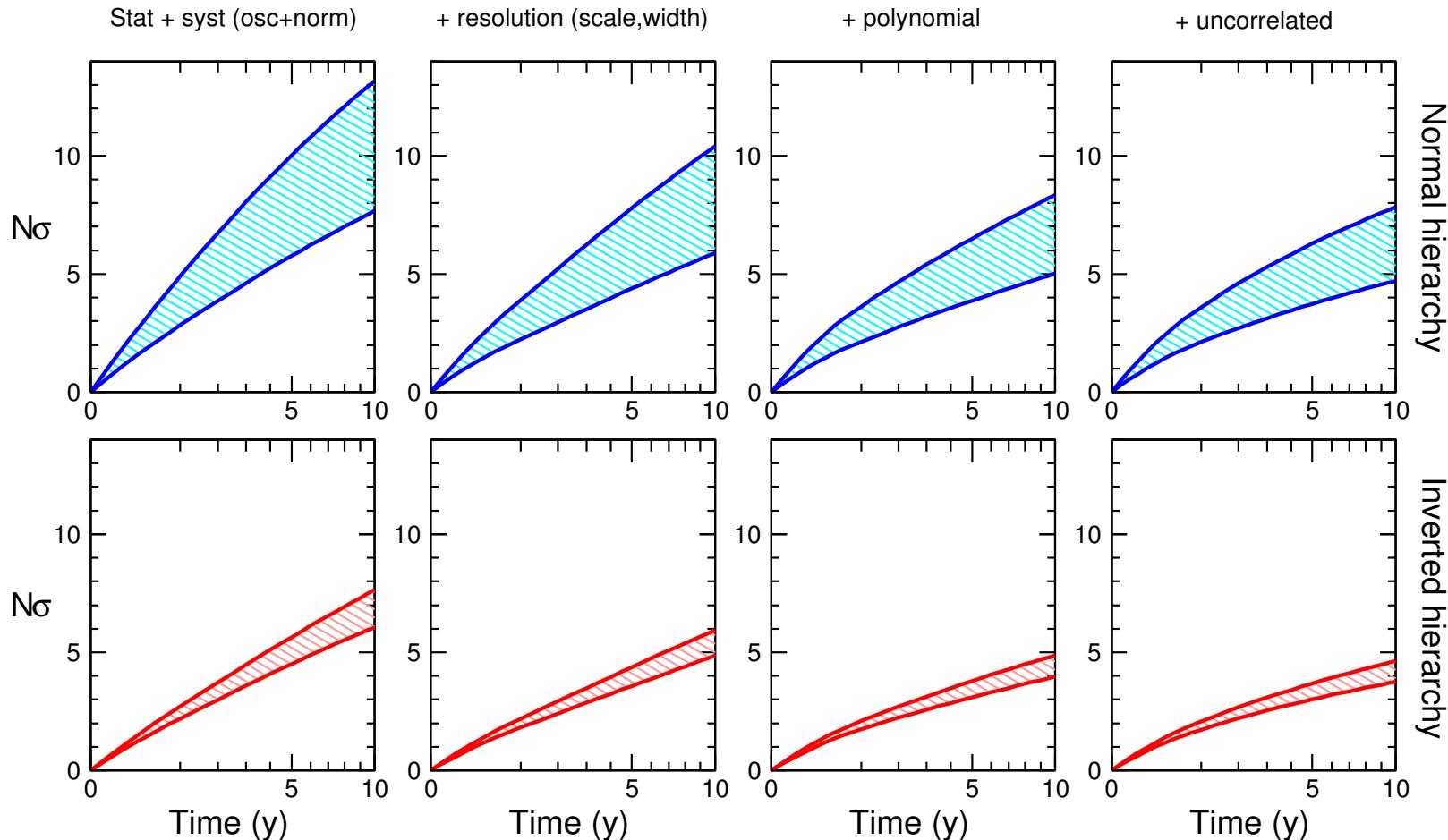
Possible in large-volume detectors (Hyper-Kamiokande, INO...) via **atmospheric neutrinos**. With very high statistics, they might be sensitive to matter effects, which are different for neutrinos and antineutrinos and for normal and inverted hierarchy

Recent studies on PINGU in IceCube and ORCA in KM3-Net



Make $\pm \Delta m^2$ interfere with $G_F N_e E$ in atmospheric expts

NH/IH atm. oscillation analyses will face new systematics challenges



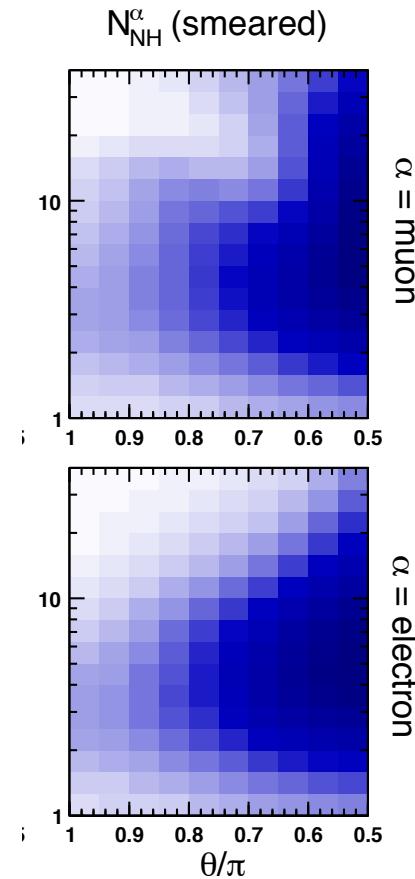
An example of hierarchy sensitivity study for PINGU, arXiv:1503.01999
Must account for “shape” syst’s of energy-angle atmospheric ν spectra

This is what we can observe:
energy-angle spectra of
 μ -like and e-like events

By eye, you would not notice
any difference from NH to IH
(IH figure not shown)

Typically, few % variations in each
bin, smaller than color ladder step!

Crucial to control systematic
errors at (few) percent level.



Oscillations
are largely
smeared out!

Comments on LBL appearance probability

In this context one often refers to the following appearance probability...

$$\begin{aligned}
 P_{\text{app}} &\simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2} \\
 &\pm \alpha \sin 2\theta_{13} \xi \sin \delta_{\text{CP}} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \\
 &+ \alpha \sin 2\theta_{13} \xi \cos \delta_{\text{CP}} \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \\
 &+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2},
 \end{aligned}$$

$$\alpha \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \simeq \pm 0.03, \quad \Delta \equiv \frac{\Delta m_{31}^2 L}{4E}, \quad \xi \equiv \sin 2\theta_{12} \sin 2\theta_{23}, \quad \hat{A} \equiv \pm \frac{2\sqrt{2}G_F n_e E}{\Delta m_{31}^2}$$

(Cervera et al. 2000; Freund, Huber, Lindner, 2000; Freund, 2001)

- Complicated, but all interesting information there: θ_{13} , δ_{CP} , mass hierarchy (via A)

... in various different versions, e.g.,

$$\begin{aligned}
 P_{\nu_\mu \rightarrow \nu_e} = & 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \frac{\Delta m_{13}^2 L}{4E} \times \left[1 \pm \frac{2a}{\Delta m_{13}^2} (1 - 2s_{13}^2) \right] & \theta_{13} \text{ driven} \\
 & + 8c_{13}^2 s_{12} s_{13} s_{23} (c_{12} c_{23} \cos \delta - s_{12} s_{13} s_{23}) \cos \frac{\Delta m_{23}^2 L}{4E} \sin \frac{\Delta m_{13}^2 L}{4E} \sin \frac{\Delta m_{12}^2 L}{4E} & \text{CP even} \\
 & \mp 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta \sin \frac{\Delta m_{23}^2 L}{4E} \sin \frac{\Delta m_{13}^2 L}{4E} \sin \frac{\Delta m_{12}^2 L}{4E} & \text{CP odd} \\
 & + 4s_{12}^2 c_{13}^2 \{ c_{13}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12} c_{23} s_{12} s_{23} s_{13} \cos \delta \} \sin^2 \frac{\Delta m_{12}^2 L}{4E} & \text{solar driven} \\
 & \mp 8c_{12}^2 s_{13}^2 s_{23}^2 \cos \frac{\Delta m_{23}^2 L}{4E} \sin \frac{\Delta m_{13}^2 L}{4E} \frac{aL}{4E} (1 - 2s_{13}^2) & \text{matter effect (CP odd)}
 \end{aligned}$$

Note that matter effects are trivially “CP-violating” (i.e., induce a difference between neutrinos and antineutrinos) since ordinary matter contains electrons but not positrons.

Future LBL experiments seeking to measure genuine CP effects due to the phase δ must “disentangle” matter effects.

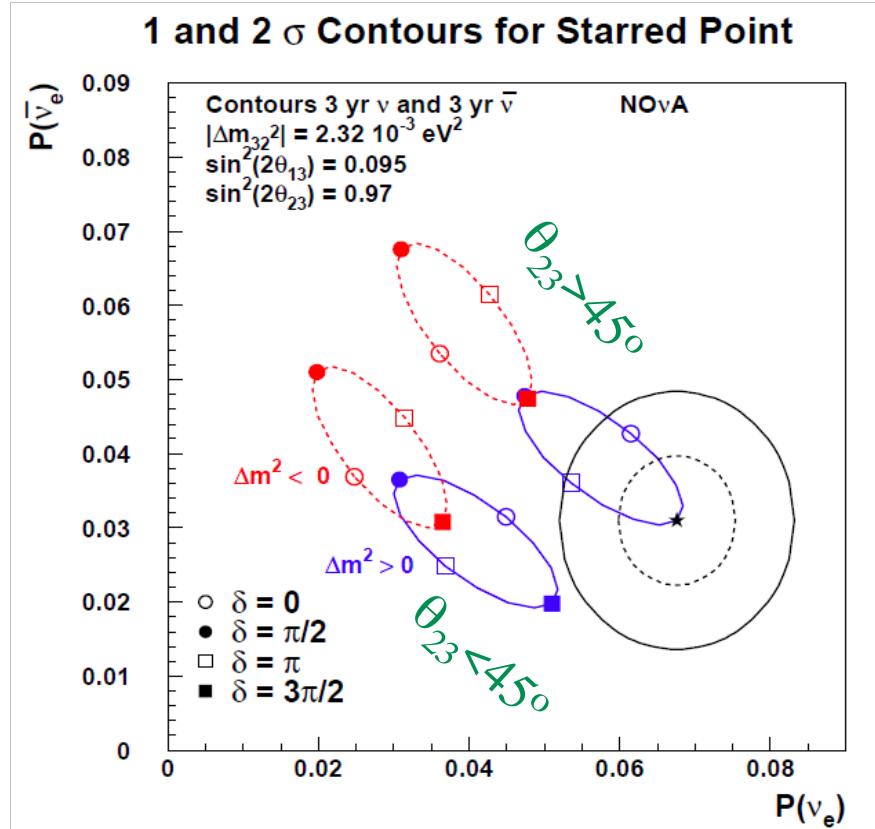
A phenomenological analysis of this equation, in the context of current and future oscillation searches seeking effects of hierarchy, octant, CP-violation, and Earth matter, would require dedicated lectures.

There are many excellent talks/lectures/reviews devoted to such studies, and to the optimization of prospective facilities in order to observe subleading effects.

However, it's quite difficult to find a pedagogical derivation of the previous (approximate) formula, which is at the basis of bi-probability plots (\rightarrow) and of various optimizations.

You can find it in the tutorials (last and longest exercise).

Discussion of present and future LBL experiments often refers to **bi-probability plots** for electron flavor appearance in neutrino and antineutrino channels, e.g.:



At fixed L , E and N_e , the ellipses are parametric curves as a function of the CP-violating phase δ (\rightarrow measurable in principle via nu-antineutrino comparison)