Neutrino interactions at high energies

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Neutrinos Underground & in the Heavens II
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Outline

Introduction

Warm–up: Neutrino–lepton cross–section

Quark parton model and Deep Inelastic Scattering (DIS)

Improved quark parton model and DGLAP evolution

Neutrino cross–sections from latest HERA data

Introduction

Why do we need predictions for the ν cross-section?

ν astronomy

- want to measure flux J_ν
- event rate $R \sim J_\nu \, \sigma_\nu$
- even considering the attenuation: $R/J_\nu \propto \sigma_\nu^{0.45}$

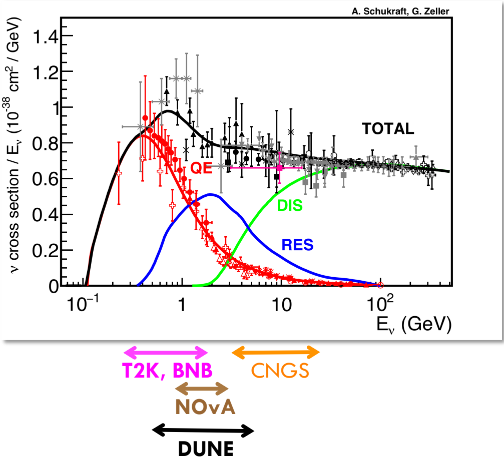
ν oscillations

- measure mass hierarchy
- CP violation

particle physics

- want to measure cross-section σ_ν
- test standard model at c.m. energies up to $\sim 10^3$ TeV: gluon saturation, colour glass condensate, black holes?

The neutrino cross–section at low energies



S. Zeller, JLAB Workshop, May 2015

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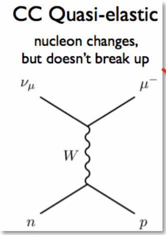

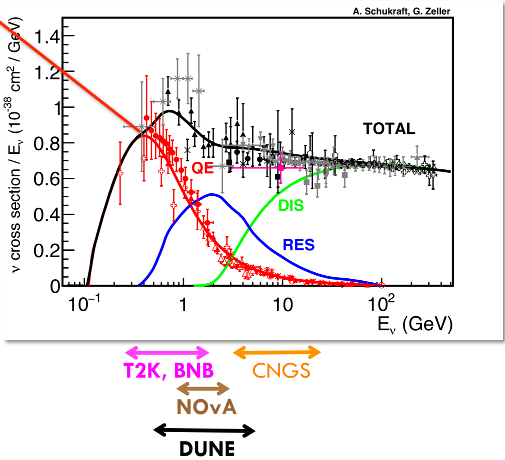

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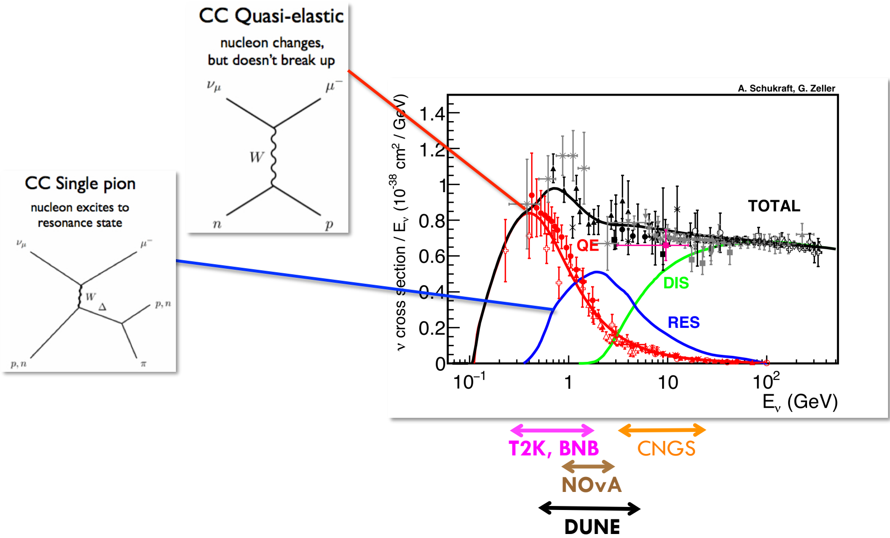
The neutrino cross–section at low energies



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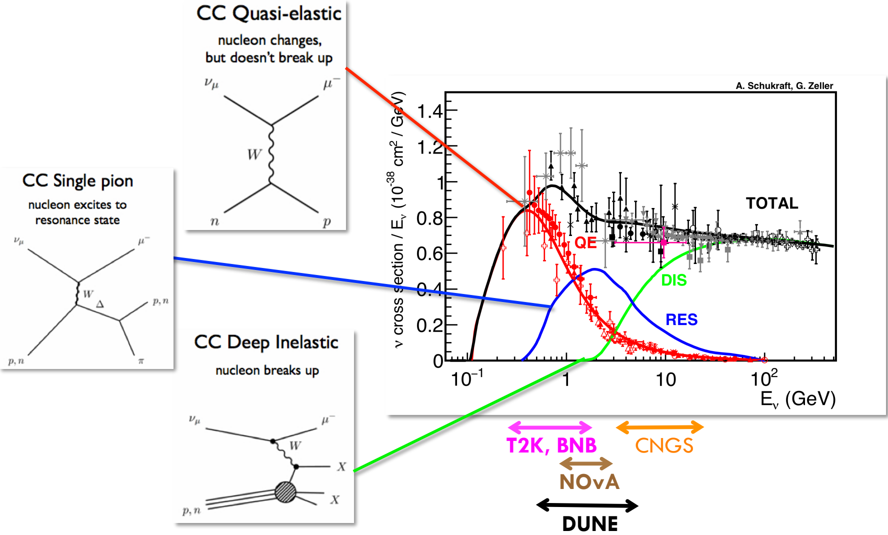
The neutrino cross–section at low energies



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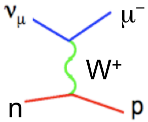
Quasi–elastic scattering



supposedly simple 2–body process

- amplitude can be cast in terms of structure functions
- structure functions (partly) known from e–scattering
- dipole–ansatz for remaining structure function

But . . .

- scattering data on Deuterium old and in disagreement
- data on heavier nuclei (e.g. MiniBooNE) also not in agreement with simple picture

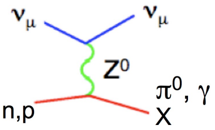
$\Rightarrow$ nuclear effects important

- different effect for ν and $\bar{\nu}$ $\Rightarrow$ need to be understood for CP–violation searches

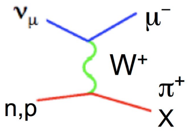
S. Zeller, JLAB Workshop, May 2015

Resonance production

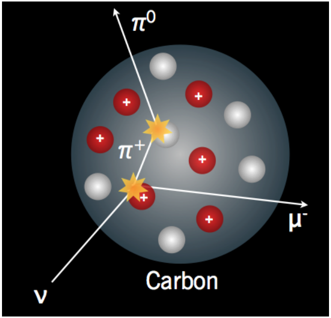
- NC π^0, γ production
  (background for $\nu_\mu \rightarrow \nu_e$ appearance)



- CC π production
  (a complication for $\nu_\mu \rightarrow \cancel{\nu_\mu}$ disappearance)



- γ from ν_μ can be mis–reconstructed as lepton: confusion $\nu_\mu \leftrightarrow \nu_e$
- absorption of π inside nucleus $\rightarrow$ background to CCQE process
- nuclear effects important: transport and interaction of final state particles through nucleus



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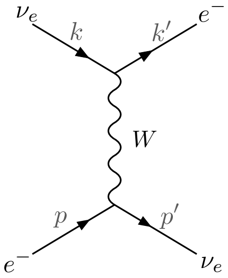
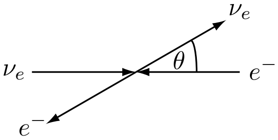
Warm–up: Neutrino–lepton cross–section

Feynman diagrams





Kinematics

Mandelstam variables

$$s = (k+p)^2 = (k'+p')^2\,; \quad t = (k-k')^2 = (p-p')^2\,; \quad u = (k-p')^2 = (p-k')^2$$

inelasticity

$$y = \frac{2p \cdot q}{2p \cdot k} = \frac{u}{s} + 1 = 1 - \cos^2 \frac{\theta}{2}$$

momentum exchange

$$Q^2 = -q^2 = -(k-k')^2 = s\,y$$

Amplitude

Feynman amplitude:

$$
\begin{aligned}
\mathcal{M} &= j^{\mu}\ \Gamma_{\mu\nu}\ j^{\nu} \\
&= \left(-i\frac{g_W}{\sqrt{2}}\bar{u}(k')\frac{1}{2}\gamma^{\mu}(1-\gamma^5)u(k)\right)\left(-i\frac{(g_{\mu\nu}-p_{\mu}q_{\nu}/M_W^2)}{q^2-M_W^2}\right) \\
&\quad\times\left(-i\frac{g_W}{\sqrt{2}}\bar{u}(p')\frac{1}{2}\gamma^{\nu}(1-\gamma^5)u(p)\right)
\end{aligned}
$$

and for $Q^2 = -q^2 \ll M_W^2$

$$
\rightarrow \frac{G_F}{\sqrt{2}}\left(\bar{u}(k')\frac{1}{2}\gamma^{\mu}(1-\gamma^5)u(k)\right)\left(\bar{u}(p')\frac{1}{2}\gamma^{\nu}(1-\gamma^5)u(p)\right)
$$

with

$$
\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}
$$

Cross–section

Squaring, averaging over initial and summing over final spin states:

$$\sum_{\text{spins}} |\mathcal{M}|^2 \propto L^{\mu\nu}_{\text{neutrino}} L^{\text{electron}}_{\mu\nu}$$

with leptonic tensors

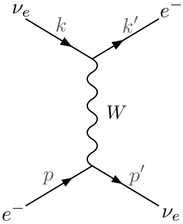
$$\begin{aligned} L^{\mu\nu}_{\text{neutrino}} &= 2(k'^{\mu} k^{\nu} + k'^{\nu} k^{\mu} - (k' \cdot k) g^{\mu\nu} - i\epsilon^{\alpha\beta\mu\nu} k'_{\alpha} k_{\beta}) \\ L^{\text{electron}}_{\mu\nu} &= 2(p'_{\mu} p_{\nu} + p'_{\nu} p_{\mu} - (p' \cdot p) g_{\mu\nu} - i\epsilon_{\alpha\beta\mu\nu} p'^{\alpha} p^{\beta}) \end{aligned}$$

The differential cross–section for νe scattering:

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{|\mathcal{M}|^2}{64\pi^2 s}\,, \\ \Rightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}y} &= \frac{G_F^2}{\pi} s\,. \end{aligned}$$

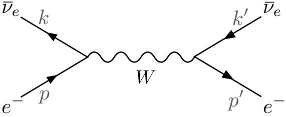
Antineutrino–lepton cross–section

νe scattering



$$\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{G_F^2}{\pi} s \,.$$

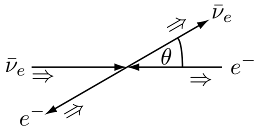
$\bar{\nu}$e scattering

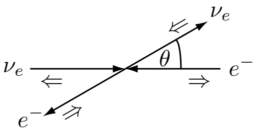


$$\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{G_F^2}{\pi} (1-y)^2 s \,.$$

Helicity determines angular dependence

- can work out cross–section from helicity and rotation of spins $d^J_{J_z J'_z}(\theta)$
- neutrinos and the electrons they interact with are always left–handed, i.e. spin opposite to momentum
- antineutrinos are right–handed



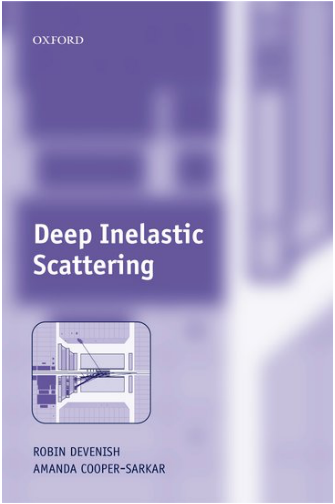


- neutrino and electron have same spins
- overall spin $J_z = 1$
- amplitude for scattering by θ given by $d^1_{11}(\theta) = \frac{1}{2}(1 + \cos\theta) = (1 - y)$
- cross–section $\propto (1 - y)^2$

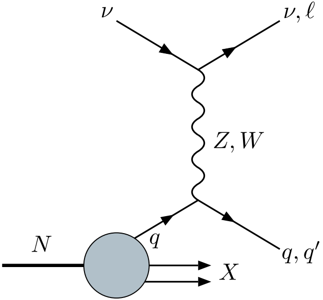
- neutrino and electron have opposite spins
- overall spin $J_z = 0$
- amplitude for scattering by θ given by $d^0_{00}(\theta) = 1$
- cross–section $\propto 1$

Quark parton model and Deep Inelastic Scattering (DIS)

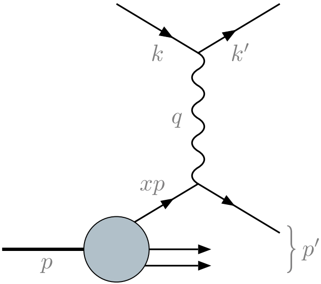
This is how I learned about DIS. . .



DIS



DIS



four Lorentz invariants:

- centre of mass energy $\sqrt{s}$
  $s = (p + k)^2$
- momentum transfer
  $Q^2 = -q^2 = -(k - k')^2$
- Bjorken scaling variable
  $x = Q^2/(2p \cdot q)$
- inelasticity
  $y = p \cdot q/(p \cdot k)$

Hadronic tensor

The hadron is strongly coupled thus in principle, one cannot compute cross–section perturbatively.

Ansatz:

$$\sum_{\text{spins}} |\mathcal{M}|^2 \propto L^{\mu\nu}_{\text{neutrino}} W_{\mu\nu}$$

with hadronic tensor:

$$\begin{aligned} W_{\mu\nu} = &-W_1 g^{\mu\nu} + \frac{W_2}{m^2} p^\mu p^\nu - i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \frac{W_3}{2m^2} \\ &+ q^\mu q^\nu \frac{W_4}{m^2} + (p^\mu q^\nu + q^\mu p^\nu) \frac{W_5}{m^2} + i\,(p^\mu q^\nu - q^\mu p^\nu) \frac{W_6}{2m^2} \end{aligned}$$

The **structure functions** W_i are real functions of x, Q^2.

Due to symmetry ($\mu \leftrightarrow \nu$) and current conservation, not all W_i contribute and recasting $F_2 = \nu W_2/m^2$, $F_3 = \nu W_3/m^2$, $F_L = F_2 - 2xW_1$ (where $\nu = p \cdot q$) one finds

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{G_F^2 M_W^4}{4\pi(Q^2 + M_W^2)^2 x} \left[Y_+ F_2(x, Q^2) + y^2 F_L(x, Q^2) \pm Y_- F_3(x, Q^2) \right]$$

with

$$Y_\pm = (1 \pm (1-y)^2)\,.$$

The upper (lower) sign is for neutrinos (antineutrinos).

Proton content

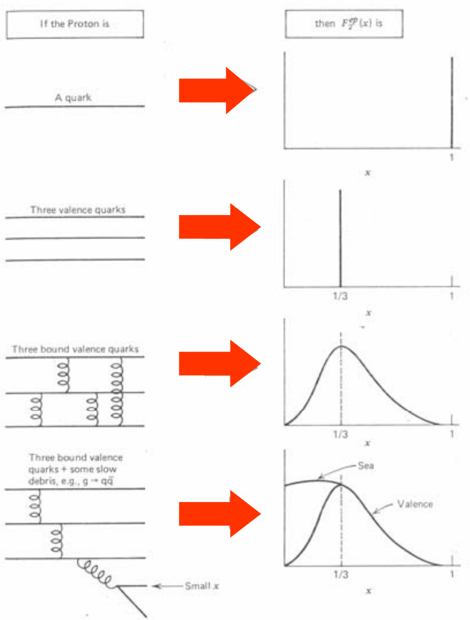


Image credit: Ulrich Uwer

Quark parton model

- Consider scattering of neutrinos off point–like spin–$\frac{1}{2}$ partons
- quarks and anti–quarks have the same weak currents as, e.g. electrons

$\Rightarrow$ cross–sections for quark and antiquark scattering:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{G_F^2 M_W^4}{\pi(Q^2 + M_W^2)^2} \quad \text{and} \quad \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{G_F^2 M_W^4}{\pi(Q^2 + M_W^2)^2}(1-y)^2$$

- probability that struck (anti)quark has momentum fraction x: $q(x)$, $\bar{q}(x)$ **Parton Density Functions (PDFs)**
- in the quark parton model, we **incoherently** sum over the probabilities

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{G_F^2 M_W^4}{\pi(Q^2 + M_W^2)^2} \sum_i \left[x q_i(x) + (1-y)^2 x \bar{q}_i(x)\right]$$

- so we can identify

$$\begin{aligned} F_2(x, Q^2) &= 2\sum_i x(q_i(x) + \bar{q}_i(x))\,, \\ F_L(x, Q^2) &= 0\,, \\ xF_3(x, Q^2) &= 2\sum_i x(q_i(x) - \bar{q}_i(x)) \end{aligned}$$

Quark parton model

confinement vs. perturbativity

- consider proton–W/Z scattering
- at high Q^2 and in the center–of–mass frame, the proton has almost infinite longitudinal momentum and no transverse momentum

$\Rightarrow$ the interactions of the partons inside the proton are time–dilated

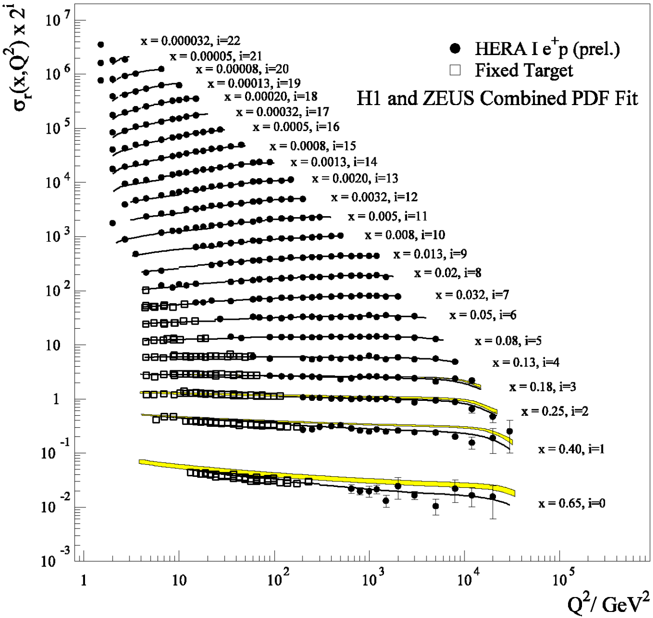
- for high Q^2, the interaction time is very short

$\Rightarrow$ the W/Z 'sees' free partons

Bjorken scaling

- structure functions depend on x only, not on Q^2
- observed at intermediate x
- but violations for small and large x

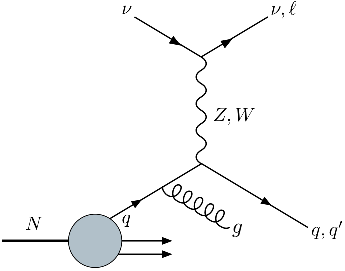
Bjorken scaling

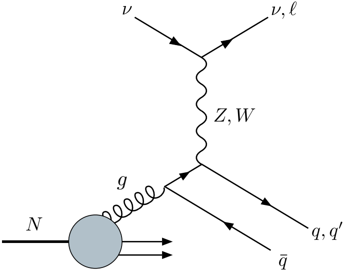


Courtesy PDG

Improved quark parton model and DGLAP evolution

Improved quark parton model





- gluonic radiative corrections and gluon boson fusion
- renormalisation of parton densities

$\Rightarrow$ $q(x) \to q(x, Q^2)$, i.e. violation of Bjorken scaling

- origin of scale–violation:

$$\int_{\kappa^2}^{p_t^2(\max)} \frac{\mathrm{d}p_t^2}{p_t^2} \to \ln\left(\frac{Q^2}{\kappa^2}\right) + \ldots$$

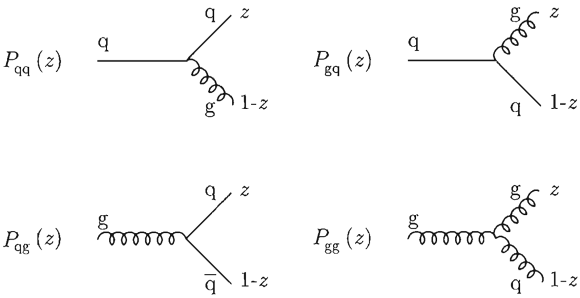
- Q^2–dependence can however be computed perturbatively: DGLAP evolution

DGLAP evolution

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_x^1 \frac{\mathrm{d}\xi}{\xi}$$

$$\times \begin{pmatrix} P_{q_i q_j}(\frac{x}{\xi},\alpha_s(Q^2)) & P_{q_i g}(\frac{x}{\xi},\alpha_s(Q^2)) \\ P_{g q_j}(\frac{x}{\xi},\alpha_s(Q^2)) & P_{gg}(\frac{x}{\xi},\alpha_s(Q^2)) \end{pmatrix} \begin{pmatrix} q_j(\xi,Q^2) \\ g(\xi,Q^2) \end{pmatrix}$$

- 'splitting functions' $P_{q_i q_j}(z,\alpha_s)$ give the probabilities for $q_i \to q_j(z) g(1-z)$.
- similar for $P_{qg}(z,\alpha_s)$, $P_{gq}(z,\alpha_s)$ and $P_{gg}(z,\alpha_s)$
- calculated perturbatively



DGLAP evolution

structure functions

- not simply sum of PDF's:

$$F_2 = \sum_i \int_x^1 \frac{\mathrm{d}\xi}{\xi} \left[xq_i(\xi, Q^2) C_q\left(\frac{x}{\xi}, \alpha_s\right) + \; xg(\xi, Q^2) C_g\left(\frac{x}{\xi}, \alpha_s\right) \right]$$

- coefficient functions C_q, C_g can also be calculated perturbatively
- similarly for F_L and xF_3

PDF fitting: idea

problem

- ideally, would like to calculate PDFs from first principles
- however, interactions of partons are soft ($Q^2 \lesssim \Lambda^2_{\mathrm{QCD}}$)

$\rightarrow$ non-perturbative regime

- lattice?

DGLAP evolution

- however, can calculate the evolution of PDFs in the perturbative regime ($Q^2 \gg \Lambda^2_{\mathrm{QCD}}$)
- assume parametric form at input scale and evolve to other scale

PDF fitting: procedure

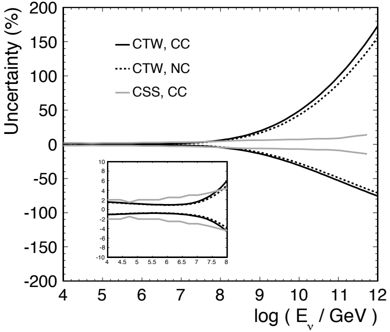
- chose parametrisation at input scale Q_0^2, e.g.

$$
\begin{aligned}
xg &= x^{\lambda_g}(1-x)^{\eta_g} P_g(x) \\
xS &= x^{\lambda_S}(1-x)^{\eta_S} P_S(x) \\
&\dots
\end{aligned}
$$

  where S is a convenient linear combination of quark PDFs
- evolve to scale of measurement: $Q_0^2 \to Q^2$
- calculate F_2, F_L and xF_3 functions and (differential) cross sections
- determine parameters λ_i, η_i, $P_i(x)$ by fitting to data

Neutrino cross–sections from latest HERA data

How accurately can we predict the ν cross-section?



CSS: Cooper-Sarkar and Sarkar, JHEP **01** (2008) 075

CTW: Connolly *et al.*, arXiv:1102.0691

Does the uncertainty **really** blow up to $\mathcal{O}(1)$?

How well do we know the proton?

high–energy neutrino cross–sections are probing high Q^2 and low x

- energy–momentum conservation

$$Q^2 = s\,x\,y$$

- for $y \sim 1$, $x = Q^2/s$
- nucleon at rest:

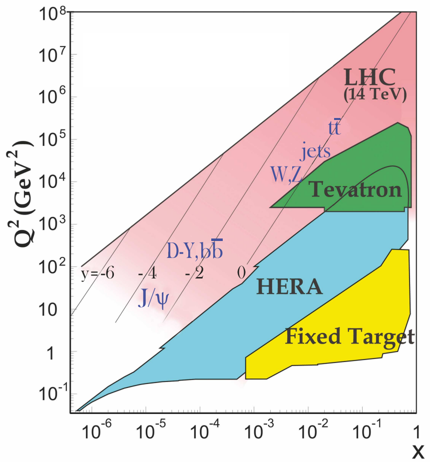
$$s \simeq 2 m_N E_\nu$$

- from propagator:

$$Q^2 \sim M_W^2, M_Z^2$$

- with $M_W^2, M_Z^2 \approx 10^4\ \mathrm{GeV}^2$:

$$x = 5 \cdot 10^{-3} \left(\frac{E_\nu}{1\,\mathrm{PeV}} \right)^{-1}$$



Courtesy PDG

PDF errors

experimental uncertainties

- many experimental errors correlated
- correlation matrix diagonalised

$\rightarrow$ linearly independent eigenvectors = variations of best-fit PDF

- can add errors from eigenvectors in quadrature

model/parameter uncertainities

- some parameters/model assumptions get fixed before fit
- vary these parameters within c.l. interval

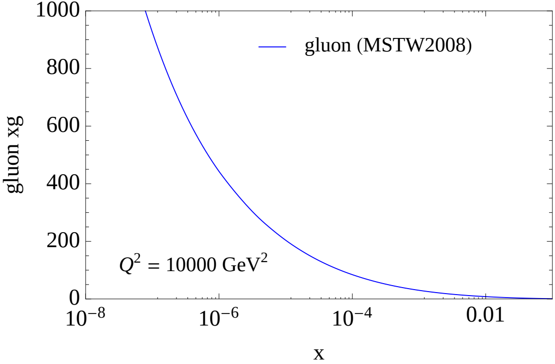
$\rightarrow$ variations of best-fit PDF

α_s uncertainties

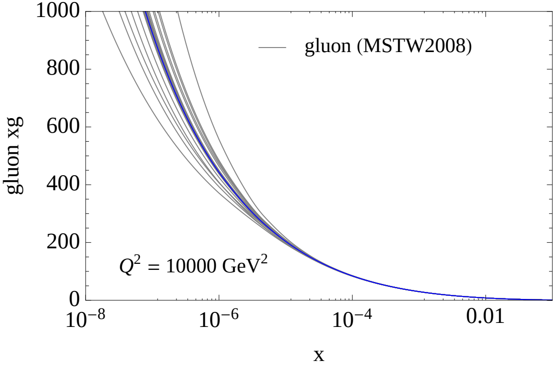
- α_s determines how quickly PDFs rise at low x

$\rightarrow$ possibly large effect

PDF errors


gluon (MSTW2008)
Q^2 = 10000 GeV^2
gluon xg
x
1000
800
600
400
200
0
10^{-8}
10^{-6}
10^{-4}
0.01

PDF errors



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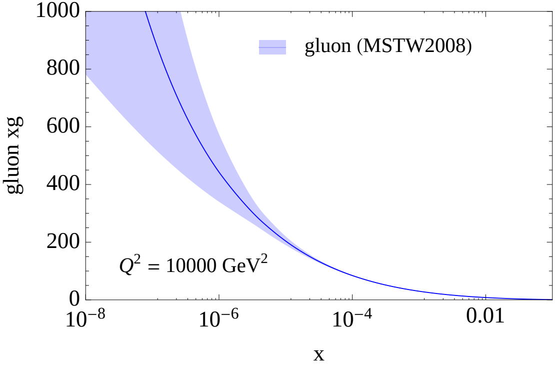

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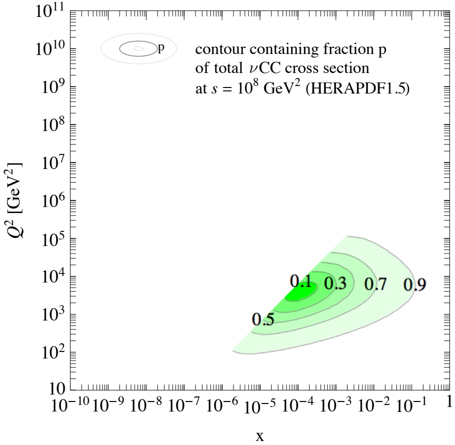
10^{-6}
10^{-4}
0.01
x

A detailed comparison

Cooper-Sarkar, Mertsch, Sarkar. JHEP08 (2011) 042; arXiv:1106.3723 [astro-ph.HE]

- use only up-to-date PDFs:
  - HERAPDF1.5 ✓
  - CT10 ✓
  - MSTW2008 ✗ (does not include combined HERA data)
- work **consistently** at NLO
- use only publicly available tools (e.g. LHAPDF)
- highlight different contributions to uncertainty **within DGLAP**:
  - experimental
  - parameters
  - model

The kinematic range



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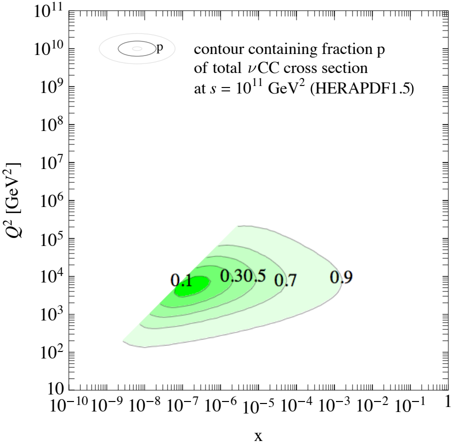


Pitfalls

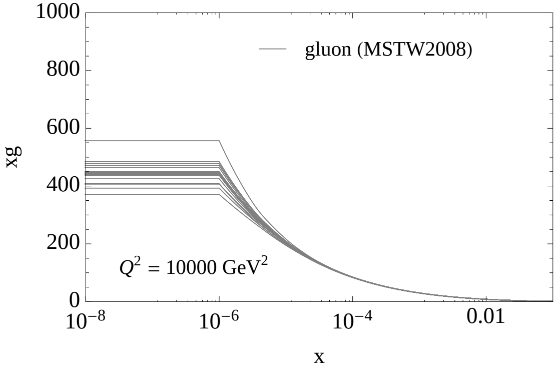
event generators, e.g. PYTHIA

- are for the most part LO
- using NLO PDFs: inconsistent ✗

LHAPDF

- PDFs provided on a limited grid of points (x, Q^2)
- going beyond this grid: PDFs "freeze" ✗

Pitfalls



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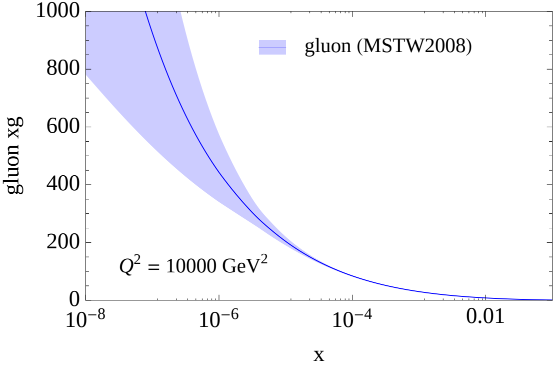
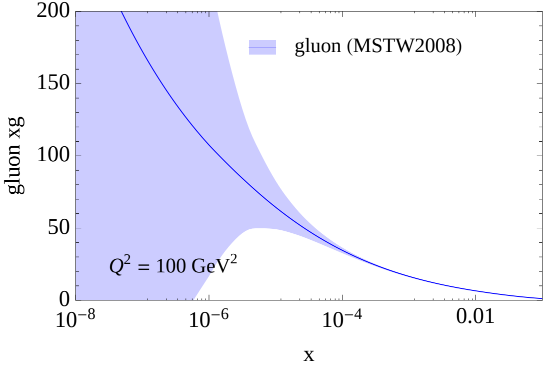
gluon parametrisation

- some groups choose a general parametrisation
- gluon PDF can go negative: meaning?

Example: MSTW2008 gluon momentum distribution



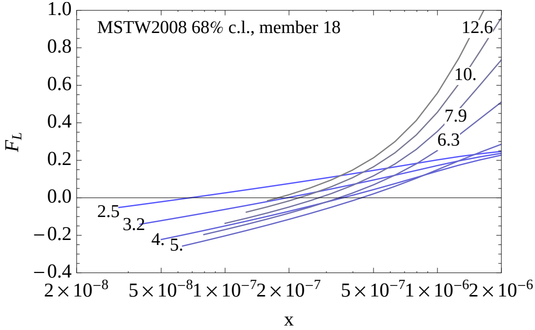
Example: MSTW2008 gluon momentum distribution



Could the gluon become negative?

at NLO, the gluon **could** become negative

however longitudinal structure function F_L **must** stay positive



With MSTW2008, F_L **does** go negative!

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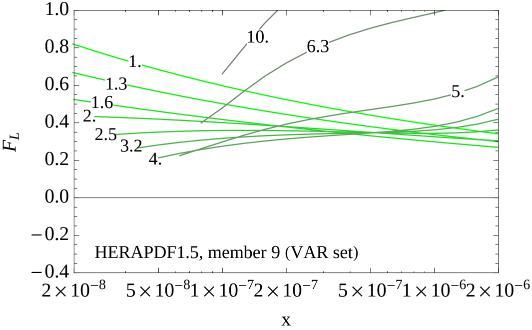
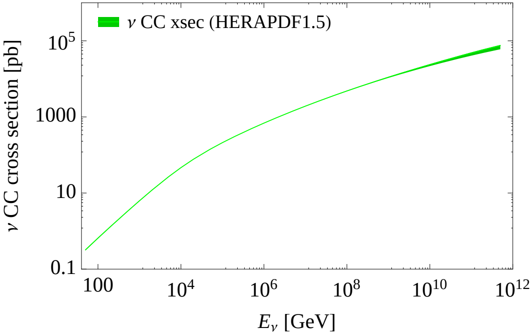
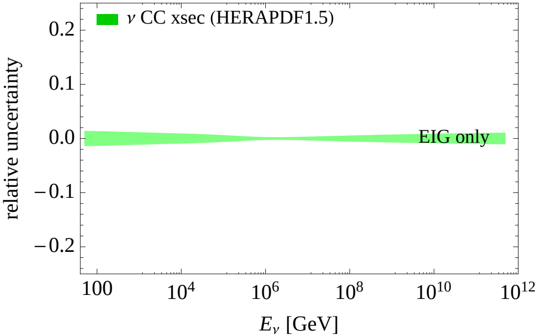


With HERAPDF1.5, F_L **does** stay positive!

Total ν CC cross-section (HERAPDF1.5)



ν CC cross-section uncertainty (HERAPDF1.5)



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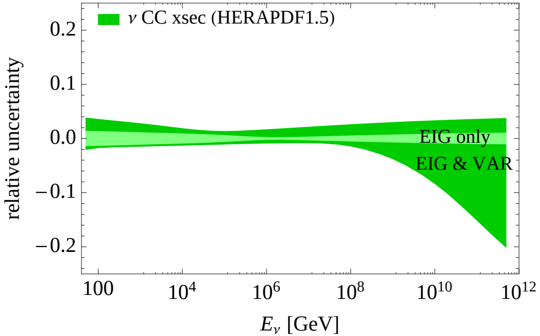

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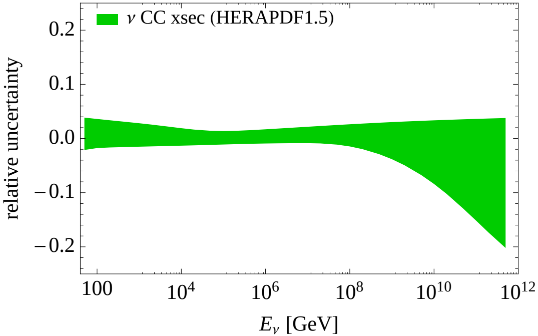

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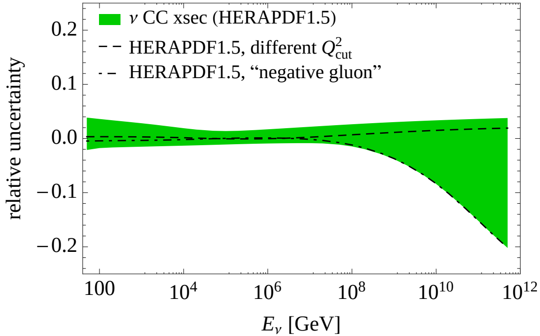

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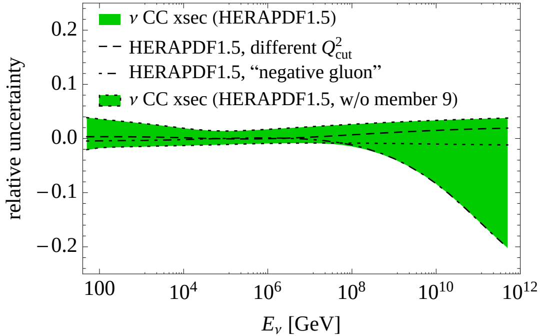
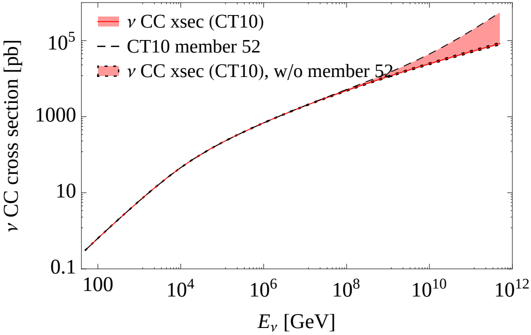
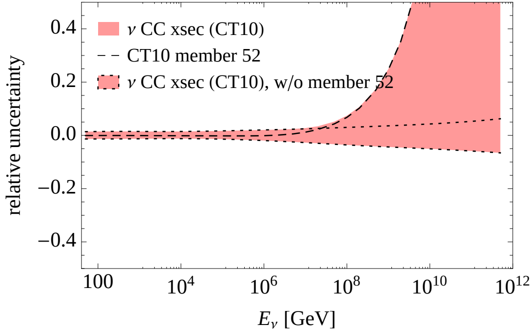


Total ν CC cross-section (CT10)

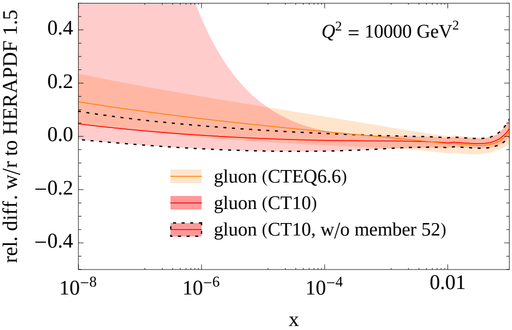


cross-section for member 52 rises $\propto E_\nu^{0.7}$; central member $\propto E_\nu^{0.3}$

ν CC cross-section uncertainty (CT10)

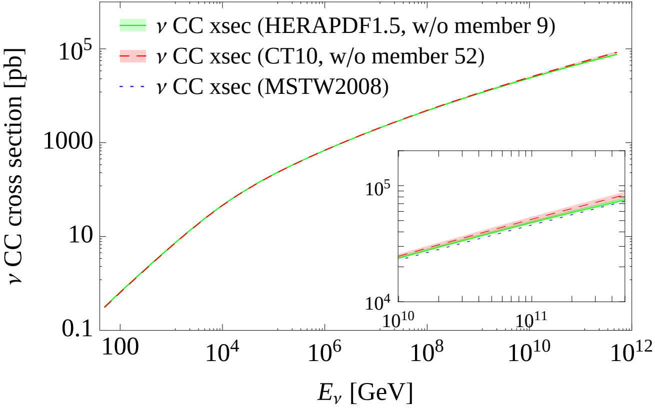


member 52 of CT10



member 52 put in by hand

ν CC cross-section (excluding rogue members)



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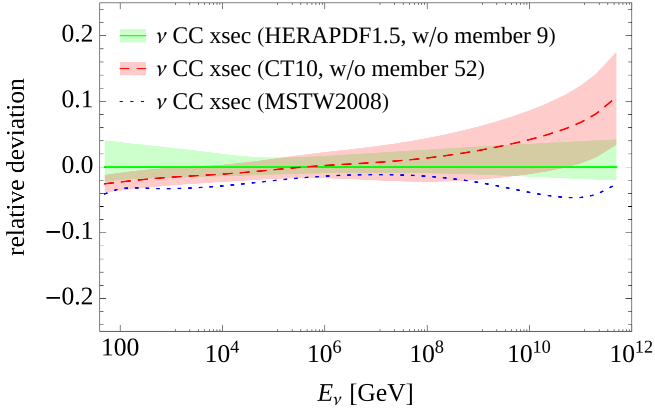


Using these results . . .

event generators

- cross–sections implemented in event generators used, e.g. by IceCube: `ANIS` and `NuGen`
- currently working with `GENIE` developpers

total cross–sections

tables available in paper and online:
http://www-pnp.physics.ox.ac.uk/∼cooper/neutrino/

`DISPred` code

- need differential distributions?
- want to generate look–up tables?

⇒ run the code yourself:
http://dispred.hepforge.org/

- requirements:
  - `ROOT`
  - Gnu Scientific Library (`GSL`)
  - PDF sets: `LHAPDF`

gluon at low x

$\ln 1/x$ resummation

- DGLAP contains terms $\sim (\alpha_s \ln x_0/x)^n$
- at low x this becomes larger than 1

$\rightarrow$ need to resumm $\ln 1/x$ terms

Froissart bound

- DGLAP predicts $xg \propto x^{-\delta}$ at low x

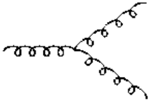
$\rightarrow$ $\sigma \propto s^{\delta}$ at large s

- however, unitarity demands s, $\sigma \propto (\ln s/s_0)^2$ at most

non–linear effects

- DGLAP eqns. are linear
- however, in DGLAP gluon and sea quark density large at small x

$\rightarrow$ gluon saturation? gluon recombination?



P_{gg}

$\rightarrow$ would tame the rise of σ

example

colour glass condensate

non–linear effects

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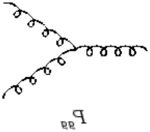


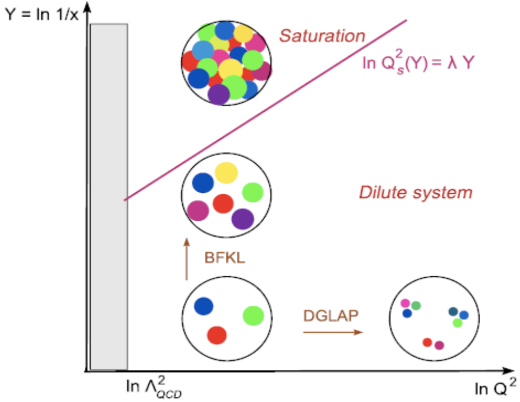
$\rightarrow$ would tame the rise of σ

example

colour glass condensate

non–linear effects

‘Phase diagram’ of QCD



modified from CERN courrier

Summary

- ν e scattering
- DIS and the quark parton model: Bjorken scaling
- improved quark parton model: DGLAP evolution

- cross-sections central values for
  - HERAPDF1.5
  - CT10
  - MSTW2008

  agree very well
- for HERAPDF1.5 and CT10 (under moderate assumptions) uncertainty is $\lesssim 10\,\%$, even at $E_\nu \sim 10^{20}$ GeV
- many pitfalls... e.g tabulated PDFs in LHAPDF "freeze" below some x value etc. $\rightarrow$ Don't try to do this at home!
- Any measured deviation from these cross-sections would signal the need for new physics!