Neutrino physics with SHiP experiment

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Neutrinos Undergound & in the Heavens-II
NBIA. Copenhagen

Goal of this lecture

Questions we address:

- Do neutrino oscillations predict new particles?
- If yes — what are the properties of these particles?
- How can we find them?

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Outline

Reminder: quantum mechanics and relativity

- Quantum mechanical **correspondence principle**:

$$E = \frac{p^2}{2m} + V(x) \Rightarrow \begin{Bmatrix} E \to i\hbar\frac{\partial}{\partial t} \\ p \to -i\hbar\nabla \end{Bmatrix} \Rightarrow i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2\nabla^2}{2m}\psi + V(x)\psi \qquad (1)$$

- Relativistic dispersion relation $E = \sqrt{p^2c^2 + m^2c^4}$

$$i\hbar\frac{\partial\psi}{\partial t} \stackrel{?}{=} \sqrt{-c^2\hbar^2\vec{\nabla}^2 + m^2c^4}\,\psi \qquad (2)$$

- Non-local operator, not compatible with **causality**!
- Square of (2) leads to the **Klein-Gordon equation:**

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = \left(-\hbar^2c^2\nabla^2 + m^2c^4\right)\psi \Leftrightarrow \left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0 \qquad (3)$$

Problems of quantum mechanics with Klein-Gordon equation

- **Negative energy states** (with $E = -\sqrt{p^2c^2 + m^2c^4}$)
- Probabilistic interpretation is gone

Exercise 1:

a) Show that $\int dx|\psi(x)|^2$ is not conserved for Klein-Gordon equation (3).
*b) Construct a **conserved** "probability current"*
c) Demonstrate that (b) it is not positive-definite

Dirac equation I

- Dirac (1928) proposed to keep the form

$$i\hbar\frac{\partial\psi}{\partial t} = \mathscr{H}\psi \tag{4}$$

Exercise 2: *Show that time evolution (4) with any Hermitian $\mathscr{H}$ automatically conserves* **probability** $P = \int d^3x\,|\psi(x)|^2$

- The function $\mathscr{H}$ is **linear** function of $\hat{\boldsymbol{p}}$ and m

$$\mathscr{H}_{\text{Dirac}} = \boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}} + \beta m \tag{5}$$

where $\boldsymbol{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ and β are Hermitian matrices such that the **square of the Dirac equation** (4) gives the wave equation

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = \left(-\hbar^2c^2\nabla^2 + m^2c^4\right)\psi \Leftrightarrow \left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0$$

Dirac equation II

Exercise 3: *Show that the matrices should obey*

$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\mathbb{1}\delta_{ij} \quad ; \quad \alpha_i\beta + \beta\alpha_i = 0 \quad ; \quad \beta^2 = \mathbb{1} \tag{6}$$

- Roughly speaking Dirac found the way to express the $\sqrt{-c^2\hbar^2\vec{\nabla}^2 + m^2c^4}$ as a linear operator of momentum.

  Exercise 4: *Find matrices α_i, β in* **2** *and* **4** *dimensions. Note, number of α's equals to the number of* **spatial** *dimensions*

- Explicitly Lorentz-covariant form of the Dirac equation is

$$\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0 \tag{7}$$

where

$$\gamma^0 = \beta \quad ; \quad \gamma^i = \beta\alpha_i \tag{8}$$

Gamma-matrices. Exercises

- Gamma-matrices γ^μ:

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \tag{9}$$

- Various (linear combinations of) products of γ-matrices form the basic in the space of all 4×4 complex matrices:
- *Exercise 5*: *How many non-trivial products are there:* γ^μ, $\gamma^\mu\gamma^\nu$, $\gamma^\mu\gamma^\nu\gamma^\lambda$ ... *?*
- *Exercise 6*: *Using property* (9) *demonstrate that the matrices*

$$\Sigma_{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu] \tag{10}$$

realize representation of the Lorentz group

Fermion mass term

First, recall some basics

- Massless fermions can be left and right-chiral

$$\gamma_5 \psi_R = +\psi_R \quad ; \quad \gamma_5 \psi_L = -\psi_L$$

If we pick a particular direction – these are just left and right moving states

- For massless particles the Dirac equation

$$(i\gamma^\mu \partial_\mu - \cancel{m})\psi = 0$$

...preserves chirality

$$\begin{pmatrix} \cancelto{0}{-m} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancelto{0}{-m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

Peskin & Schroeder basis

Chirality

Define operator γ_5 such that

- $\{\gamma_5, \gamma^\mu\} = 0$ and $\gamma_5^2 = 1$
- γ_5 commutes with massless Dirac Hamiltonian

$$\mathscr{H}_{\text{Dirac}} = \boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}}$$

- ...but not with the massive

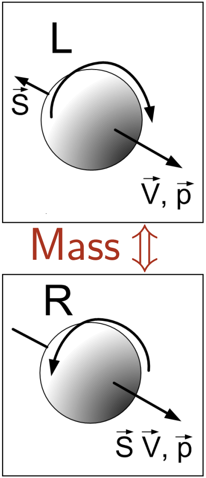
$$\mathscr{H}_{\text{Dirac}} = \boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}} + \beta m$$

- Gauge interactions respects chirality ($D_\mu = \partial_\mu + eA_\mu$)...

$$\begin{pmatrix} 0 & i(D_t + \vec{\sigma}\cdot\vec{D}) \\ i(D_t - \vec{\sigma}\cdot\vec{D}) & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

How to write a mass for neutrino

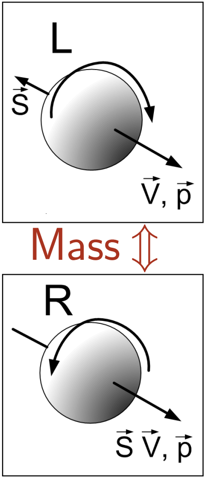
- Neutrinos are **always** left-chiral



How to write a mass for neutrino

- Neutrinos are **always** left-chiral
- Recall:

$$\mathscr{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}}[\bar{p}(x)\gamma_\mu(1-\gamma_5)n(x)][\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)] + \frac{G_F}{\sqrt{2}}[\bar{\nu}_\mu(x)\gamma^\mu(1-\gamma_5)\mu(x)[\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)] + \dots$$



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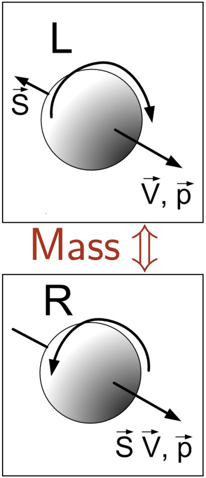
- So Dirac a theory of massive neutrinos should be

$$\mathscr{L} = i\bar{\nu}_L\gamma^\mu\partial_\mu\nu_L - \bar{\nu}_R M\nu_L + \text{h.c}$$



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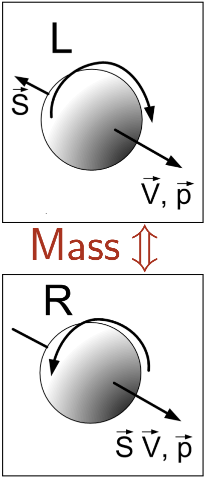
- Recall:

$$\mathscr{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}}[\bar{p}(x)\gamma_\mu(1-\gamma_5)n(x)][\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)] + \frac{G_F}{\sqrt{2}}[\bar{\nu}_\mu(x)\gamma^\mu(1-\gamma_5)\mu(x)[\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)] + \dots$$

- So Dirac a theory of massive neutrinos should be

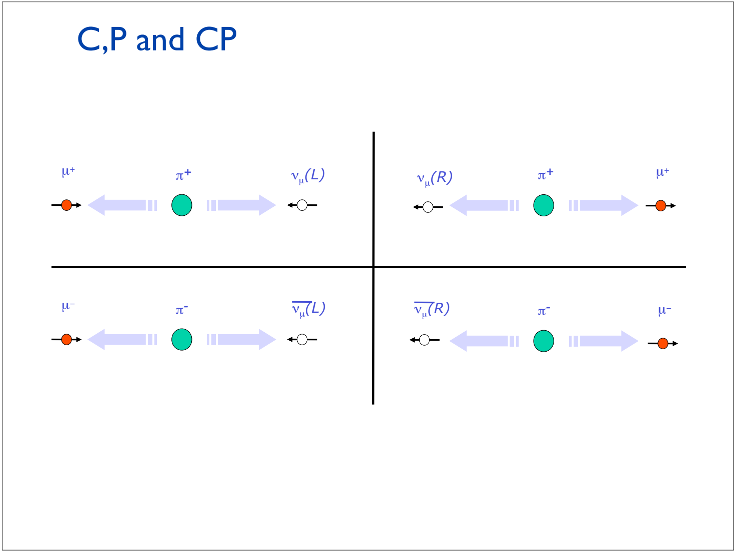
$$\mathscr{L} = i\bar{\nu}_L\gamma^\mu\partial_\mu\nu_L - \bar{\nu}_R M\nu_L + \text{h.c}$$

- …but we do not know "particle" ν_R !



Neutrino **Dirac** mass

Illustrations taken from G. Raven, “CP violation”, CERN Summer student lectures 2010



Prediction

- Neutrino

$$\Psi = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

- Anti-neutrino

$$\bar{\Psi} = \begin{pmatrix} \bar{\nu}_R \\ \bar{\nu}_L \end{pmatrix}$$

Neutrino **Dirac** mass

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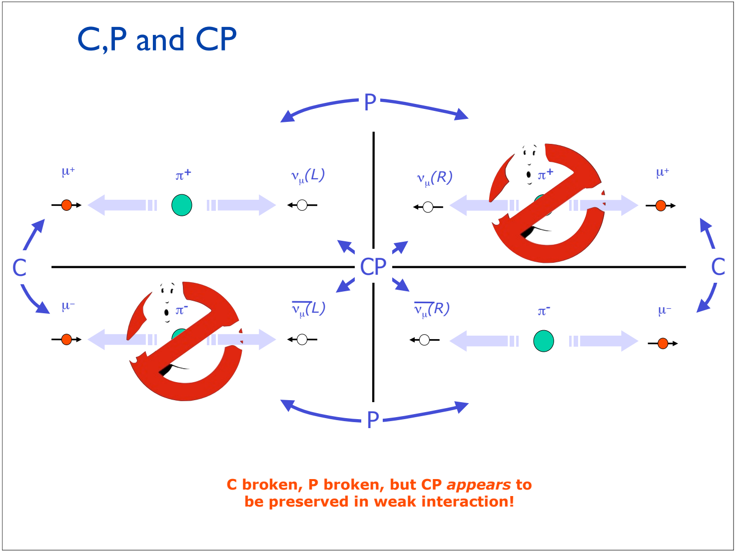


Prediction

- Neutrino

$$\Psi = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

- Anti-neutrino

$$\bar{\Psi} = \begin{pmatrix} \bar{\nu}_R \\ \bar{\nu}_L \end{pmatrix}$$

New particle?

- Have we just predicted a new particle?

New particle?

- Have we just predicted a new particle?

No!

New particle?

- Have we just predicted a new particle?

No!

- All we predicted was a new spin state of an already existing particle

Outline

Other possible neutrino masses I

- Most of you are used to the following form of γ matrices [Peskin & Schroeder book]

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}; \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{11}$$

- Some of you may know that the form (11) is not unique
- Some of you may even know that γ-matrices are simply **a representation of elements of Clifford algebra** and there are infinitely many others
- For example [Bjorken & Drell book]

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}; \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{12}$$

- Any representation of γ-matrices forms a basis in the space of all complex matrices 4×4
- Does this mean that γ-matrices are always **complex**?
- In Eqs. (11)–(12) γ^0, γ^1, γ^3 are **real**, γ^2 is imaginary. Is this always the case?

Majorana representation

See e.g. hep-ph/0605172

- Ettori Majorana noticed that there is a **totally imaginary representation of γ matrices**: $(\gamma^\mu)^* = -\gamma^\mu$

  Exercise 7: *Find this representation explicitly!*

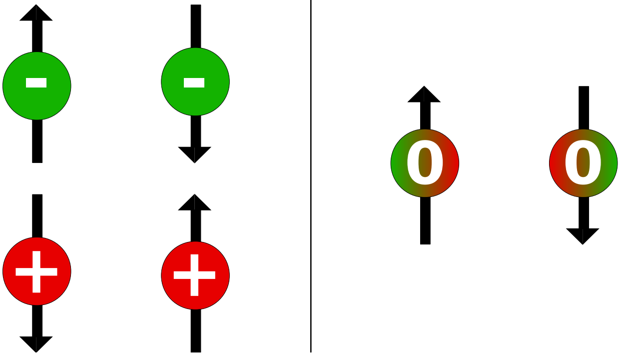
- Therefore the Dirac equation $\left(i\gamma^\mu\partial_\mu - m\right)\chi = 0$ is a differential equation with **real** coefficients
- Hence, it admits **real solutions** $\chi^* = \chi$ – **Majorana fermion**
- Such fermion has **2 degrees of freedom**
- Such fermion can carry **no U(1) charges**

  U(1) transformation $\psi \to e^{i\alpha}\psi$ will rotate any real vector into a complex one unless $q = 0$

Exercise 8: *Write a Lagrangian for Majorana fermion*

Dirac vs. Majorana fermion

Dirac massive particle | Majorana massive particle



4 degrees of freedom

2 degrees of freedom

From [1601.07512]

Detour: particle and anti-particles I

- What is so special about the Majorana solution?
- Recall that Dirac found that the fermion possesses not 2 (spin $\uparrow$ and spin-$\downarrow$) but 4 degrees of freedom
- Additional 2 states are from the negative branch $E = -\sqrt{\boldsymbol{p}^2 + m^2}$

A Theory of Electrons and Protons.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received December 6, 1929.)

§ 1. *Nature of the Negative Energy Difficulty.*

The relativity quantum theory of an electron moving in a given electromagnetic field, although successful in predicting the spin properties of the electron, yet involves one serious difficulty which shows that some fundamental alteration is necessary before we can regard it as an accurate description of nature. This difficulty is connected with the fact that the wave equation,

Detour: particle and anti-particles II

- Consider a state with negative energy ($p_\mu = (-|E|, \boldsymbol{p})$):

$$\begin{aligned}\psi(x) = u(p)e^{-ip\cdot x} = u(p)e^{+i|E|t+i\boldsymbol{p}\cdot\boldsymbol{x}}\\ (p_\mu\gamma^\mu - m)u(p) = 0\end{aligned} \tag{13}$$

- A complex conjugated spinor $\psi^* = u^* e^{+ip\cdot x}$ has **positive** energy:

$$\psi^* = u^* e^{-i|E|t-i\boldsymbol{p}\cdot\boldsymbol{x}} \tag{14}$$

- Does ψ^* obey **the same Dirac equation**?

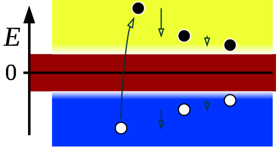
$$\begin{aligned}(\not\partial - m)\psi^* &= e^{+ip\cdot x}(-p_\mu\gamma^\mu - m)u^*\\ &= e^{+ip\cdot x}\Big((-p_\mu(\gamma^\mu)^* - m)u\Big)^* \stackrel{?}{=} 0\end{aligned} \tag{15}$$

Detour: particle and anti-particles III

- If γ-matrices were imaginary $(\gamma^\mu)^* \stackrel{?}{=} -\gamma^\mu$, ψ^* would satisfy Dirac equation whenever ψ does (and this is how Majorana found his representation)

- For a general representation this is not the case. In addition to complex conjugation one should rotate the spinor $u(p)$:

$$\psi_c \equiv C\psi^* = (Cu)e^{+ip\cdot x} \qquad (16)$$



- matrix C is chosen in such a way that

$$-\gamma^\mu C = C(\gamma^\mu)^* \quad ; \quad C^2 = \mathbb{1} \qquad (17)$$

- Spinor ψ^c describes **anti-particle** or **charge-conjugated state**

<u>*Exercise 9*</u>: *Derive Eq. (17). Show that in chiral representation (Peskin & Schroeder)* $C = i\gamma^0\gamma^2$

Detour: particle and anti-particles IV

- Now consider Dirac equation in the external electromagnetic field:

$$\left(\not\partial - e\not A - m\right)\psi = 0 \tag{18}$$

The spinor $\psi_c = C\psi^*$ obeys the equation:

$$\left(\not\partial + e\not A - m\right)\psi_c = 0 \tag{19}$$

– opposite electric charge!

- Majorana fermion: $\psi^c = \psi$
- Is Majorana fermion left or right?
- It is **neither**!

Exercise 10:

– *Demonstrate, that Majorana spinor cannot be chiral. Hint: use Majorana representation and show that* $\gamma_5^* = -\gamma_5$

– *Show that charge-conjugation anti-commutes with chirality: i.e. that if* ψ_L *is left-chiral, than* $(\psi_L)^c$ *is right-chiral*

Neutrino Majorana mass

- For particle that carries no U(1) charge one can write a Majorana mass term
- The only neutral particle in the Standard model is **neutrino**

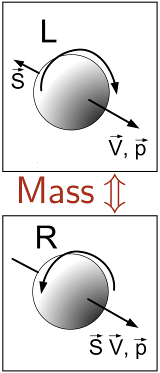
$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\,\bar{\nu}\, M_M\, \nu^c + \text{h.c.} \qquad (20)$$

couples neutrino ν and its anti-particle ν^c.

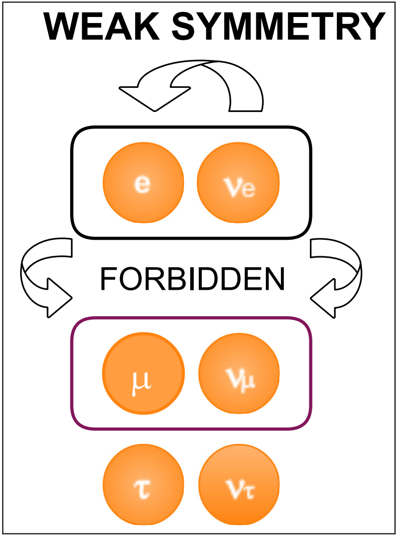
- One can construct a Majorana spinor:

$$\chi = \frac{\nu + \nu^c}{\sqrt{2}}$$

- ...then the mass term (20) is simply: $\mathcal{L}_{\text{Majorana}} = M\bar{\chi}\chi$



Weak interactions



- Weak symmetry rotates electron into neutrino (muon into ν_μ, τ into ν_τ)
- Number of leptons is conserved in each generation
- i.e. we know with high precision that muons μ cannot convert into electrons e.
- By virtue of the electroweak symmetry neutrinos **should not change their types** (i.e. $\nu_e \not\rightleftarrows \nu_\mu$)
- But they **do!**

Neutrino Majorana mass

- Neutrino carries no electric charge, but it is **not** neutral
- ...neutrino is part of the SU(2) doublet $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$
- ...and carries **hypercharge** $Y_L = -1$
- What we call **neutrino** is actually $\nu = (L \cdot \tilde{H})$ (where $\tilde{H}_a = \varepsilon_{ab} H_b^*$)
- Therefore neutrino Majorana mass term is

$$\text{Neutrino Majorana mass} = \frac{\boldsymbol{c}(\bar{L} \cdot \tilde{H}^\dagger)(L^c \cdot \tilde{H})}{\boldsymbol{\Lambda}}$$

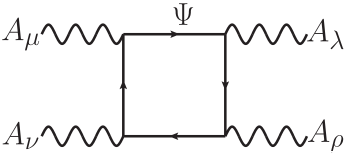
- Notice that this operator violates **lepton number**
- Assuming $\boldsymbol{c} \sim \mathcal{O}(1)$ one gets

$$\boldsymbol{\Lambda} \sim \frac{v^2}{m_{\text{atm}}} \sim 10^{15}\ \text{GeV}$$

- This is **Weinberg operator** or "dimension-5 operator"

Neutrino masses and effective field theory

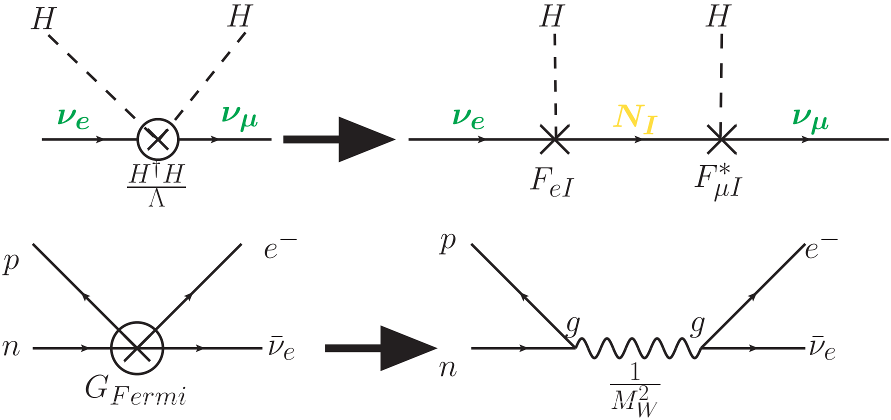
- In the logic of EFT one expects that some "heavy" particles had mediated this type of interaction and that at energies $E \sim \Lambda$ new particles should appear
- Example, at energies $E < m$ light-on-light scattering is mediated by virtual fermions, leading to **Heisenber-Euler Lagrangian**



$$\mathscr{L}_{H-E} = \frac{\alpha^2}{45 m^4}\left((\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2\right)$$

- *Exercise 11*:
  a) Count mass dimension of $(\bar{L}\cdot\tilde{H}^\dagger)(L^c\cdot\tilde{H})$ and convince yourself that Λ in Weinberg's operator has the dimension of mass
  b) Count mass dimension of the Heisenberg-Euler term $(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2$
  Hint: any Lagrangian has mass-dimension 4 and this determines the canonical dimension of all fields

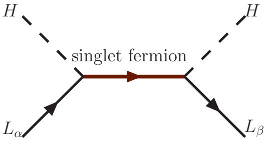
"Resolving" neutrino mass term I



Neutrino oscillations mean that there exist new particles!

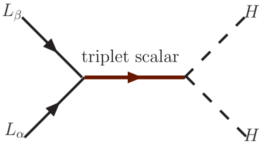
Seesaw mechanisms

There are many ways to "resolve" the Weinberg's operator, i.e. to couple to left doublets *L* and the Higgs double *H*



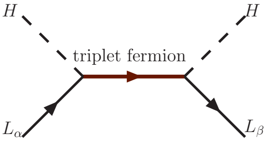
Type I see-saw

extra singlet fermion



Type II see-saw

extra SU(2) triplet scalar with hypercharge 1



Type III see-saw

extra SU(2) triplet fermion with hypercharge 0

Also models with "loop mediated neutrino masses", split-seesaw, etc.

Strumia & Vissani "Neutrino masses and mixings and…" [hep-ph/0606054v3]

Type I seesaw mechanism. I

Below I will concentrate on the Type-I seesaw mechanism

- Let us add a right-chiral fermion N to the Standard Model:

$$\mathscr{L}_{\text{Seesaw Type I}} = \mathscr{L}_{\text{SM}} + i\bar{N}\partial\!\!\!/ N + F\bar{N}(\tilde{H}\cdot L) + \mathscr{L}_{\text{Majorana}}(N) \qquad (21)$$

- State $\nu = (\tilde{H}\cdot L)$ is a $SU(3)\times SU(2)\times U(1)$ gauge singlet
  - $\Rightarrow$ N carries no $U(1)$ charges
  - $\Rightarrow$ Majorana mass term $\mathscr{L}_{\text{Majorana}}(N) = \frac{1}{2}\bar{N}\boldsymbol{M}N^c + \text{h.c}$ is possible for N

Type I seesaw mechanism. II

- In terms of ν and N we get:

$$\mathscr{L}_{\text{Seesaw Type I}} = \mathscr{L}_{\text{SM}} + i\bar{N}\partial\!\!\!/ N + \frac{1}{2}\begin{pmatrix}\bar{\nu} \\ \bar{N}^c\end{pmatrix}\begin{pmatrix}0 & m_{\text{Dirac}} \\ m_{\text{Dirac}} & \boldsymbol{M}\end{pmatrix}\begin{pmatrix}\nu^c \\ N\end{pmatrix} \qquad (22)$$

- here **Dirac mass** $m_{\text{Dirac}} = F\langle H\rangle$, F is a **active-sterile Yukawa coupling**
- The mass term in (22) is diagonalized via rotation by the angle ϑ

$$\begin{aligned} \boldsymbol{\nu} &= \cos\vartheta\, \nu - \sin\vartheta\, N^c \approx \quad \nu - \vartheta \times N^c \\ \boldsymbol{N} &= \sin\vartheta\, \nu^c + \cos\vartheta\, N \approx \quad N + \vartheta \times \nu^c \end{aligned} \qquad (23)$$

assuming $\vartheta \ll 1$ and neglecting $\mathscr{O}(\vartheta^2)$ terms

- The **mixing angle** ϑ is defined via

$$\vartheta \simeq \frac{m_{\text{Dirac}}}{\boldsymbol{M}} \qquad (24)$$

Type I seesaw mechanism. III

- Both $\boldsymbol{\nu}$ and $\boldsymbol{N}$ have Majorana mass terms:

$$\mathscr{L}_{\text{Seesaw Type I}} = \mathscr{L}_{\text{SM}} + i\bar{\boldsymbol{N}}\partial\!\!\!/ \boldsymbol{N} + \frac{1}{2}\bar{\boldsymbol{\nu}} m_\nu \boldsymbol{\nu}^c + \frac{1}{2}\bar{\boldsymbol{N}} M_N \boldsymbol{N}^c \qquad (25)$$

where

$$m_\nu \simeq \frac{(m_{\text{Dirac}})^2}{\boldsymbol{M}} \quad \text{and} \quad M_N \simeq \boldsymbol{M}$$

- Type-I seesaw model describes two particles
  - $\boldsymbol{\nu}$ – massive neutrino (mass eigenstate) with the mass m_ν
  - $\boldsymbol{N}$ – a new particle with mass M_N

We call this new particle

"Sterile neutrino"

or

"right-handed neutrino"

or

"heavy neutral lepton" (**HNL**)

Properties of sterile neutrinos

- Recall that we defined the particle $\boldsymbol{N}$ as rotation fro m the basic (ν, N)

$$\left.\begin{aligned} \boldsymbol{\nu} &= \nu - \vartheta \times N^c \\ \boldsymbol{N} &= N + \vartheta \times \nu^c \end{aligned}\right\} \Rightarrow \nu = \boldsymbol{\nu} + \vartheta \times \boldsymbol{N}^c$$

- Neutrino ν has the following interaction types:

$$\begin{aligned} \text{Charged currents:} \quad & \mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \bar{e} \gamma^\mu (1 - \gamma_5) \nu W_\mu \\ \text{Neutral currents:} \quad & \mathscr{L}_{\rm NC} = \frac{g}{2\cos\theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu \end{aligned} \tag{26}$$

- Sterile neutrino $\boldsymbol{N}$ thus **inherits** the interactions

$$\begin{aligned} \text{Charged current-like:} \quad & \tilde{\mathscr{L}}_{\rm CC} = \frac{g\,\vartheta}{\sqrt{2}} \bar{e} \gamma^\mu (1 - \gamma_5) \boldsymbol{N}^c W_\mu \\ \text{Neutral current-like:} \quad & \tilde{\mathscr{L}}_{\rm NC} = \frac{g\,\vartheta}{\cos\theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \boldsymbol{N}^c Z_\mu \end{aligned} \tag{27}$$

Lepton number conservation in the Standard Model

- If there is a conserved charge, the **lightest carrier of this charge** is stable
- Lepton sector: 3 conserved quantities **lepton flavour number**

Particle	L_e	L_μ	L_τ	L_{tot}
e^-	1	0	0	1
ν_e	1	0	0	1
μ^-	0	1	0	1
ν_μ	0	1	0	1
τ^-	0	0	1	1
ν_τ	0	0	1	1

Prohibited decays:

- $\mu \to e\gamma$
- $\mu \to eee$
- $\tau \to \mu\mu\mu$

Exercise 12: *What conservation laws make stable*
– *Proton?*
– *Electron?*
– *What decay modes would be available if any of these conservation laws were gone?*

Exercise 13: *Determine neutrino flavours and neutrino/anti-neutrino type in the following reaction:*

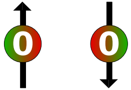
$$\tau^- \to \mu^- + {}^{?}\nu + {}^{?}\nu$$

A quantum effect, known as **quantum anomaly** violates these symmetries. Not important for particle physics processes at zero temperature

Total lepton number

- Type-I seesaw Lagrangian means that neutrinos have **Majorana mass**

$$\mathscr{L}_{\nu\ \text{mass}} = \frac{1}{2} m_\nu \bar{\boldsymbol{\nu}} \boldsymbol{\nu}^c \Rightarrow \frac{1}{2} m_\nu \bar{\nu} \nu^c + \mathscr{O}(\vartheta)$$



- Total lepton number is violated
- We killed one the global symmetries of the Standard Model
- There are consequences for killing it

Mass and charge eigenstates

Some useful slang

- **Mass eigenstates** (or "*propagation basis*") – state that obeys a free wave equation $(\Box + m^2)\psi = 0$
- **Charge eigenstate** – eigenstate of a charge operator

- ν, N are (weak) **charge** eigenstates
- $\boldsymbol{\nu}, \boldsymbol{N}$ are **mass** eigenstates
- Neutrinos ν_e, ν_μ, ν_τ are charge eigenstates

  often called *flavour eigenstates* or *active neutrinos* or *gauge eigenstates*
- All charges of N are equal to **zero**. Therefore it is **sterile neutrino**. Mass state $\boldsymbol{N}$ interacts with W and Z bosons.

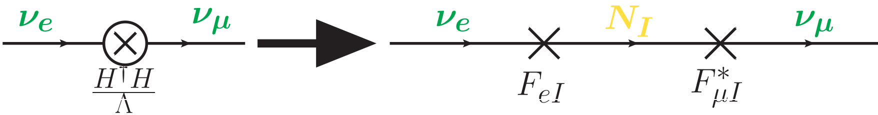
  often this distinction is not made

Outline

From mass term to oscillations

- Mix ν_e and ν_μ with the right-chiral state N:

$$\mathscr{L}_{\text{Dirac}} = (m_{\text{Dirac}})_e \bar{\nu}_e N + (m_{\text{Dirac}})_\mu \bar{\nu}_\mu N \tag{28}$$



- There is a **massive state**

$$\nu_1 = \frac{(m_{\text{Dirac}})_e \nu_e + (m_{\text{Dirac}})_\mu \nu_\mu}{\sqrt{(m_{\text{Dirac}})_e^2 + (m_{\text{Dirac}})_\mu^2}}$$

and orthogonal to it massless state ν_2

Exercise 14:

- *Work out oscillation probability for Lagrangian* (28) *in case when N is right-chiral component of the Dirac neutrino*
- *In case when N has a Majorana mass term $M_N \bar{N} N^c$*

Number of free parameters?

Active neutrino masses $\mathscr{M}_{\text{active}} = -m_{\text{Dirac}} \frac{1}{M_{\text{Majorana}}} m_{\text{Dirac}}^T$

- Rank of the active neutrino mass matrix $\leq \mathcal{N}$ – the number of sterile neutrinos.
- At least **two** sterile neutrinos are required to explain two mass splittings (in which case $\sum \mathscr{M}_i \approx (1 \text{ or } 2) m_{\text{atm}}$
- Number of new parameters for $\mathcal{N}$ sterile neutrinos:

> $\mathcal{N}$ real Majorana masses $+\, 3 \times \mathcal{N}$ complex Yukawas (Dirac masses) $-\, 3$ phases absorbed in redefinitions of ν_e, ν_μ, ν_τ.

- In total this brings us $7 \times \mathcal{N} - 3$ new parameters with $\mathcal{N}$ sterile neutrinos.

Number of unknown parameters?

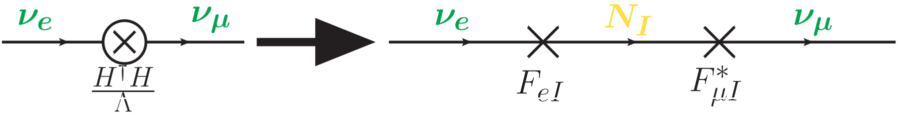
- Number of new parameters

$$7\times\mathcal{N}-3=\begin{cases}11, & \mathcal{N}=2\\ 18, & \mathcal{N}=3\end{cases}$$

- Neutrino oscillation experiment may determine **9** parameters (**3** masses, **3** mixing angles, **2** Majorana phases and **1** Dirac CP phase)
- Undetermined parameters are: $\mathcal{N}$ Majorana masses $+$ some ratios of Yukawas (for example, one replace $F_{\alpha I}\leftrightarrow F_{\alpha J}(M_I/M_J)^{1/2}$ for some pairs $I\neq J$.)
- With the **full knowledge** of PMNS and active neutrino masses/phases we will be able to determine

$$\begin{array}{ll}\mathbf{7}\text{ out of 11 parameters} & \mathcal{N}=2\\ \mathbf{9}\text{ out of 18 parameters} & \mathcal{N}=3\\ \dots & \end{array}$$

Type I see-saw with 3 sterile neutrinos



- The full neutrino mass matrix (3 active and $\mathcal{N} = 3$ sterile)

$$\text{Neutrino mass term} = \begin{pmatrix} \bar\nu_e \\ \bar\nu_\mu \\ \bar\nu_\tau \\ \bar N_1 \\ \bar N_2 \\ \bar N_3 \end{pmatrix} \begin{pmatrix} 0 & \text{Dirac mass} \\ \text{Dirac mass} & \text{Majorana mass} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

- Neutrino masses are given by the **see-saw formula**:

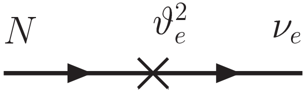
$$\textbf{Neutrino masses}\ \mathcal{M}_{\text{active}} = -m_{\text{Dirac}} \frac{1}{M_{\text{Majorana}}} m^T_{\text{Dirac}}$$

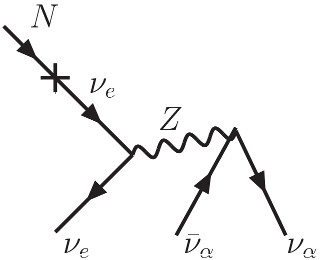
Outline

Properties of sterile neutrinos I

- Sterile neutrino ***N*** behaves as **super**weakly interacting heavy neutrino
- At energies $E \ll M_W$ they have "Fermi-like interactions" with a smaller Fermi constant

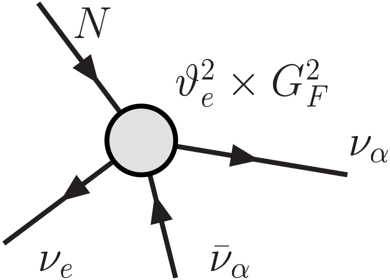
$$G_F \longrightarrow \vartheta \times G_F$$





→

Quadratic mixing $N \leftrightarrow \nu$ of sterile neutrinos N to ν_e



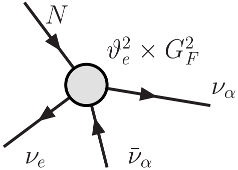
Properties of sterile neutrinos II

$$\text{Yukawa coupling} \sim \left(\frac{M_{\text{sterile}} m_\nu}{v^2}\right)^{1/2} \approx 4\times 10^{-8}\left(\frac{M_{\text{sterile}}}{1\ \text{GeV}}\right)^{1/2}$$

$$\text{Mixing angles} \quad \vartheta^2 = \frac{m_\nu}{M_N} \approx 5\times 10^{-11}\left(\frac{1\ \text{GeV}}{M_{\text{sterile}}}\right)$$

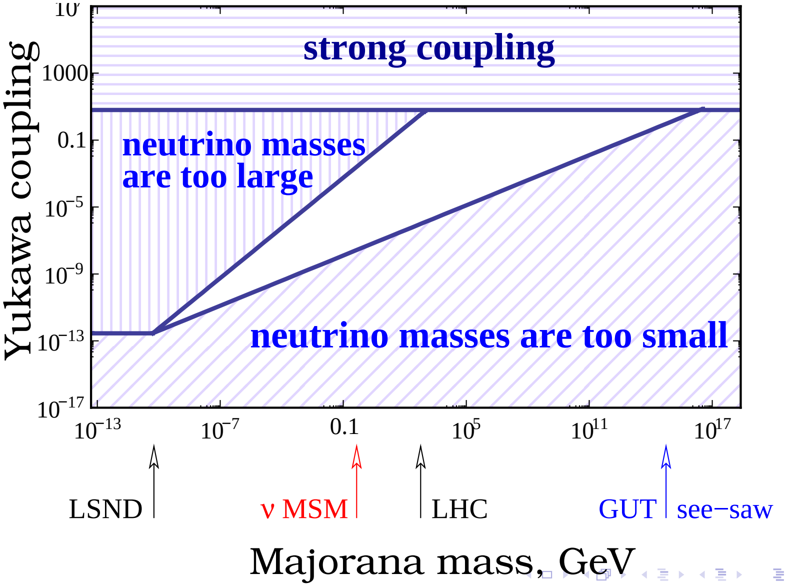
Mass M_N	Yukawa coupling	ϑ^2
1 eV	1.3×10^{-12}	5×10^{-2}
1 keV	4.1×10^{-11}	5×10^{-5}
m_e	9.2×10^{-10}	1×10^{-7}
1 MeV	1.3×10^{-9}	5×10^{-8}
m_π	1.5×10^{-8}	4×10^{-10}
m_K	3×10^{-8}	1×10^{-10}
1 GeV	4.1×10^{-8}	5×10^{-11}
m_t	5.3×10^{-7}	3×10^{-13}
1 TeV	1.3×10^{-6}	5×10^{-14}
10^{15} GeV	1.3	5×10^{-26}

$$G_F \longrightarrow \vartheta \times G_F$$



How heavy can sterile neutrinos be?

How heavy can sterile neutrinos be?

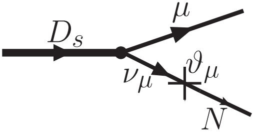


Outline

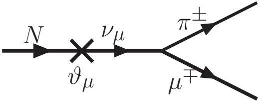
Interactions of sterile neutrinos

- Sterile neutrino is produced "as neutrino" if the mass of the parent particle is larger than M_N

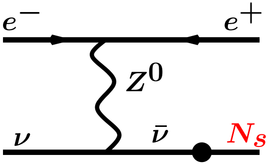
- $\pi \to e + \nu \ \Rightarrow \pi \to e + N$ if $M_N < m_\pi - m_e$;
  $K \to e + \nu \ \Rightarrow K \to e + N$ if $M_N < m_K - m_e$;
  $K \to \mu + \nu \ \Rightarrow K \to \mu + N$ if $M_N < m_K - m_\mu$, etc.



- Sterile neutrino **decay** through **charge current** and **neutral current** interactions



- Interaction cross-section of sterile neutrinos with matter is ϑ^2 times **smaller** than that of ordinary neutrino
- Even if the kinematics is right, for each X neutrinos that you produce you also get $\vartheta^2 X$ sterile neutrinos (recall $\vartheta^2 \sim 10^{-11}$ for $M_N \sim 1$ GeV!)



Outline

Kink searches in the spectrum of β-decay

- Tritium β-decay

$$T \to {}^3\text{He} + e + \cancel{\bar{\nu}} + \boldsymbol{N}$$

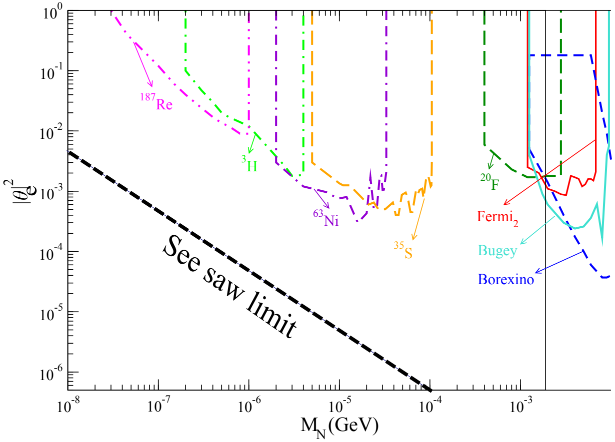
- If sterile neutrno $\boldsymbol{N}$ mixes with electron anti-neutrino $\bar{\nu}_e$ with the mixing angle ϑ_e^2 the amount of electrons with energy E is proportional to:

$$N_e(E)dE \propto (1-\vartheta^2)(E_0 - E) + \vartheta^2\sqrt{(E_0 - E)^2 - M_N^2}$$

where E_0 is β-spectrum endpoint

- for $E_e < E_0 - M_N$ there is a **kink** in the β-decay spectrum due to additional contribution of the second term

Kink searches based bounds on ϑ_e^2



Adopted from **Atre et al. "The Search for Heavy Majorana Neutrinos" [0901.3589]**

Outline

Peak searches?

- Accumulate a lot of pions. They decay through $\pi \to e + \nu$ and $\pi \to \mu + \nu$
- Pion decay width

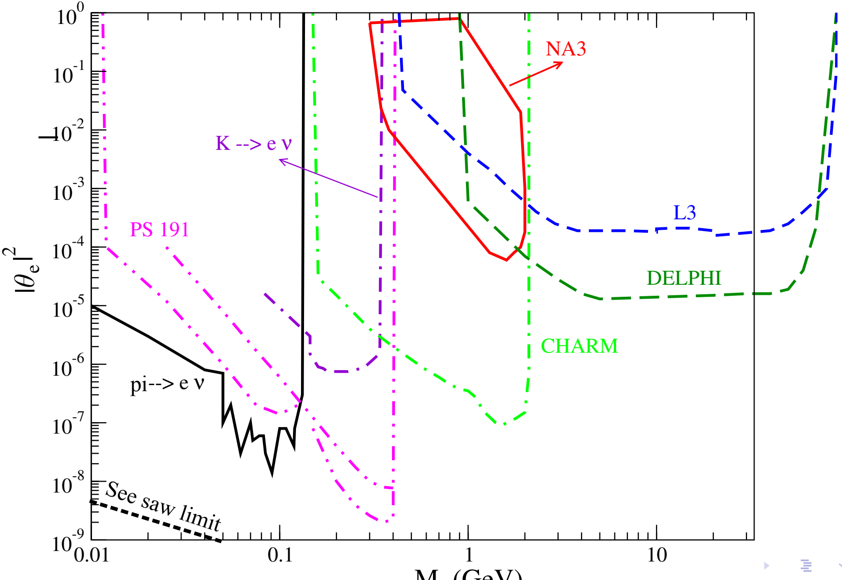
$$\Gamma_{\pi \to e\nu} = \frac{G_F^2 f_\pi^2 \cos^2\theta_c m_\pi^3}{8\pi} \left(\frac{m_e}{m_\pi}\right)^2 \left(1 - \frac{m_e^2}{m_\pi^2}\right)$$

See Okun "Leptons & quar

- Factor $\left(\frac{m_e}{m_\pi}\right)^2$ is due to the **helicity suppression** (because charge currents couple to left particles only and pion is a pseudoscalar)
- Sterile neutrino are Majorana particles. Both left and right components couple. Therefore

$$\Gamma_{\pi \to eN} = \Gamma_{\pi \to e\nu} \times \vartheta_e^2 \times \left(\frac{M_N}{m_e}\right)^2$$

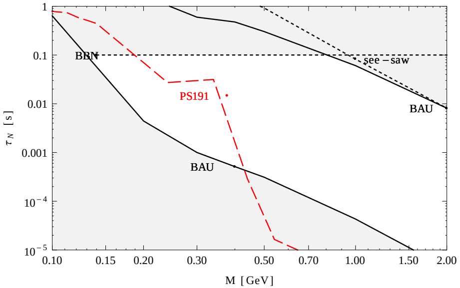
Peak searches for larger masses



Outline

Challenge

[Canetti & Shaposhnikov (2011)]



- Production: $N_{\text{produced}} \propto |\vartheta|^2 \times N_\nu$
- Decay lifetime:

$$l_N = c\tau_N \propto \frac{1}{G_F^2 M_N^5 |\vartheta|^2}$$

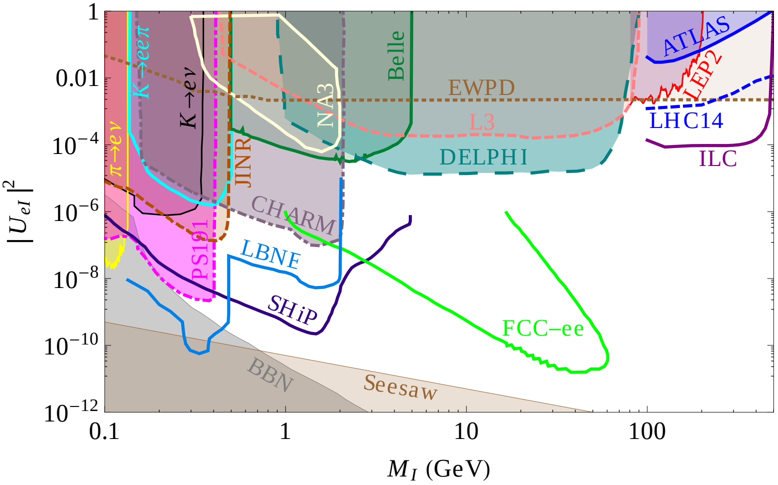
- Probability to decay over distance L: $p(L) = 1 - \exp(-L/l_N)$
- Number of events in the detector with length L_{det}

$$N_{\text{events}} \propto \frac{L_{\text{det}}}{l_N} \propto |\vartheta|^2$$

- Probability $\propto |\vartheta|^4$ unless the particle decays $\sim 100\%$ inside the detector

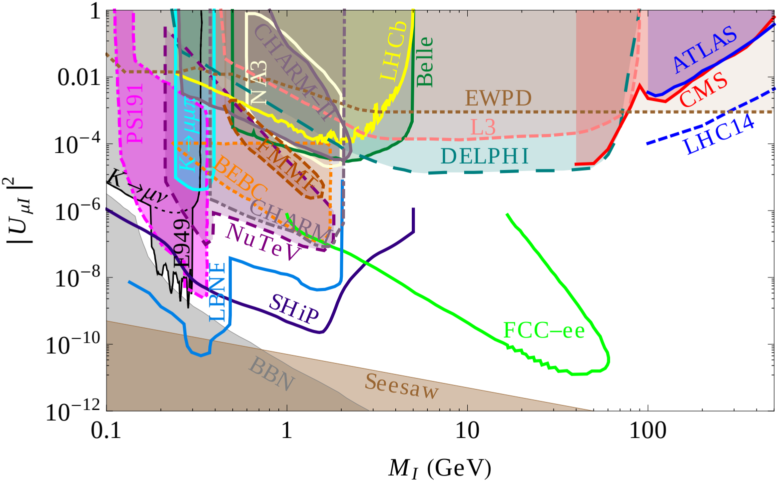
Bounds on sterile neutrino coupling ϑ_e^2

From "SHiP Physics Paper" [1504.04855]



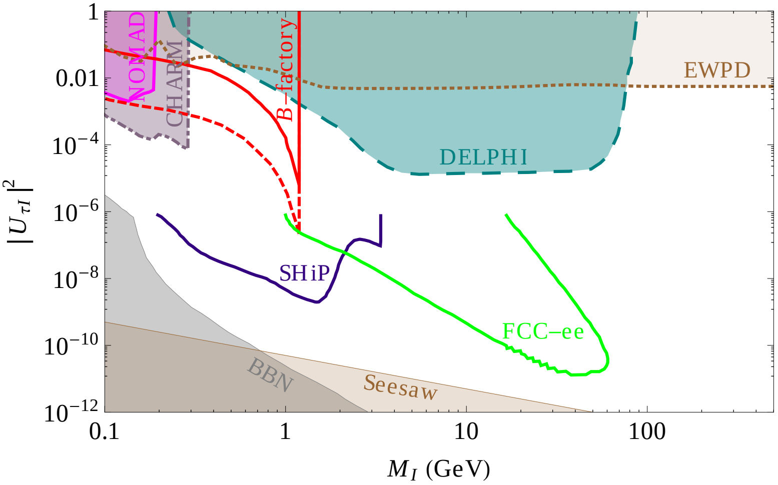
Bounds on sterile neutrino coupling ϑ_μ^2

From “SHiP Physics Paper” [1504.04855]



Bounds on sterile neutrino coupling ϑ_τ^2

From "SHiP Physics Paper" [1504.04855]



Outline

Finding superweakly interacting particles in lab

Proposal to Search for Heavy Neutral Leptons at the SPS

W. Bonivento (INFN, Cagliari & CERN) , A. Boyarsky (Leiden U.) , H. Dijkstra (CERN) , U. Egede (Imperial Coll., London) , M. Ferro-Luzzi, B. Goddard (CERN) , A. Golutvin (Imperial Coll., London) , D. Gorbunov (Moscow, INR) , R. Jacobsson, J. Panman (CERN) M. Patel (Imperial Coll., London) , O. Ruchayskiy (LPHE, Lausanne) , T. Ruf (CERN) , N. Serra (Zurich U.) , M. Shaposhnikov (LPHE, Lausanne) , D. Treille (CERN) Hide

Discovery

Oct 7, 2013 - 21 pages

CERN-SPSC-2013-024, SPSC-EOI-010
e-Print: arXiv:1310.1762 [hep-ex] | PDF

Abstract (arXiv)
A new fixed-target experiment at the CERN SPS accelerator is proposed that will use decays of charm mesons to search for Heavy Neutral Leptons (HNLs), which are right-handed partners of the Standard Model neutrinos. The existence of such particles is strongly motivated by theory, as they can simultaneously explain the baryon asymmetry of the Universe, account for the pattern of neutrino masses and oscillations and provide a Dark Matter candidate. Cosmological constraints on the properties of HNLs now indicate that the

Several years ago an idea of a new dedicated experiment to search for steril neutrinos (aka "heavy neutral leptons") got crystallized

SHiP : Search for Hidden particles

Search for rare particles becomes **official CERN theme**

It took then 1 year to create a collaboration

A facility to Search for Hidden Particles (SHiP) at the CERN SPS

SHiP Collaboration (M. Anelli *et al.*) Show all 235 authors

Apr 20, 2015 · 234 pages

CERN-SPSC-2015-016, SPSC-P-350
e-Print: arXiv:1504.04956 [physics.ins-det] | PDF
Experiment: CERN-SPS-SHIP

SHiP
Search for Hidden Particles

- About **250** members of the SHiP collaboration from **44** institutions worldwide
- SHiP is now an official CERN project

Timeline

- Approval by CERN **2019**
- Data taking **2024**

Designing an experiment (very schematic)

1. Need a lot of particles, decaying to neutrinos
   - Muons? ($\mu \to e + \bar{\nu}_e + \nu_\mu$) – light
   - Pions? ($\pi \to e + \bar{\nu}_e$, $\pi \to \mu + \bar{\nu}_\mu$) – Yes! Below 140 MeV
   - Kaons? ($K \to e + \bar{\nu}_e$, $K \to \mu + \bar{\nu}_\mu$) – Yes! Below 490 MeV
   - D-mesons ($D^+ = |c\bar{d}\rangle$, $D_s^+ = |c\bar{s}\rangle$, $D^0 = |c\bar{u}\rangle$) – Yes! Below 1.8 GeV
   - B-mesons – ...
2. To produce D-mesons we need to produce charmed quarks. $M_c \simeq 2$ GeV
3. Need $10^{??}$ mesons
4. Take a proton beam and hit into the target

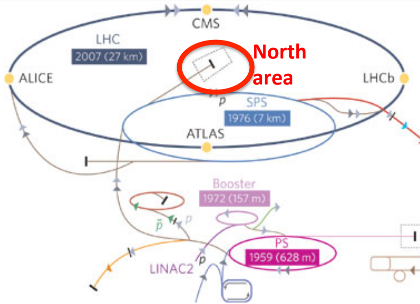
$$N_{\text{mesons}} = 2 \times X_{q\bar{q}} \times N_{PoT}$$

5. Want to increase N_{PoT} – high intensity proton beam
6. Want to increase $X_{q\bar{q}}$ – high energy beam

Super Proton Synchrotron (SPS)

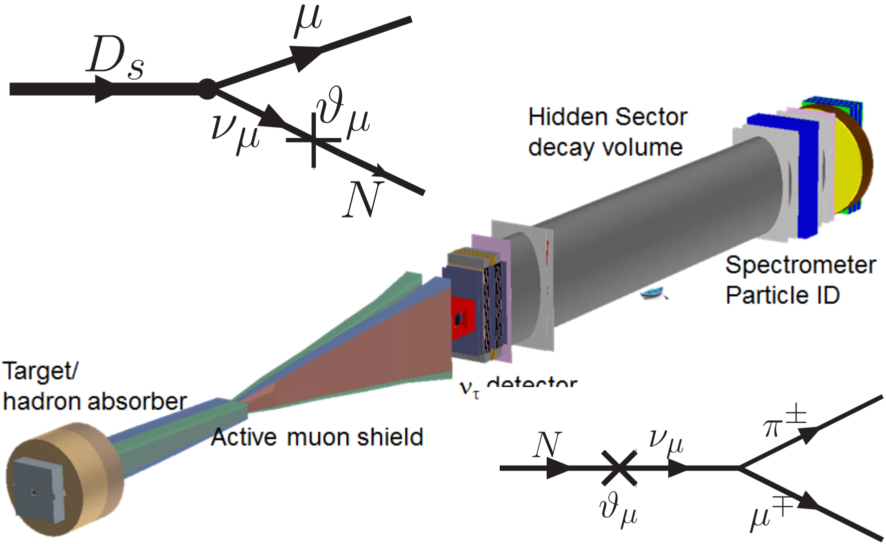
$N_{D-\text{mesons}} = 2 \times X_{c\bar{c}} \times N_{PoT}$

- High energy proton beam – 400 GeV
- 4×10^{19} PoT (protons on target per year). 2×10^{20} PoT over 5 years
- Beam intensity: 4×10^{13} protons/sec
- Produces a lot of c-quarks: $X_{c\bar{c}} \sim 10^{-3}$



SHiP (*Search for Hidden Particles*) experiment

Step by step overview



SHiP (*Search for Hidden Particles*) experiment

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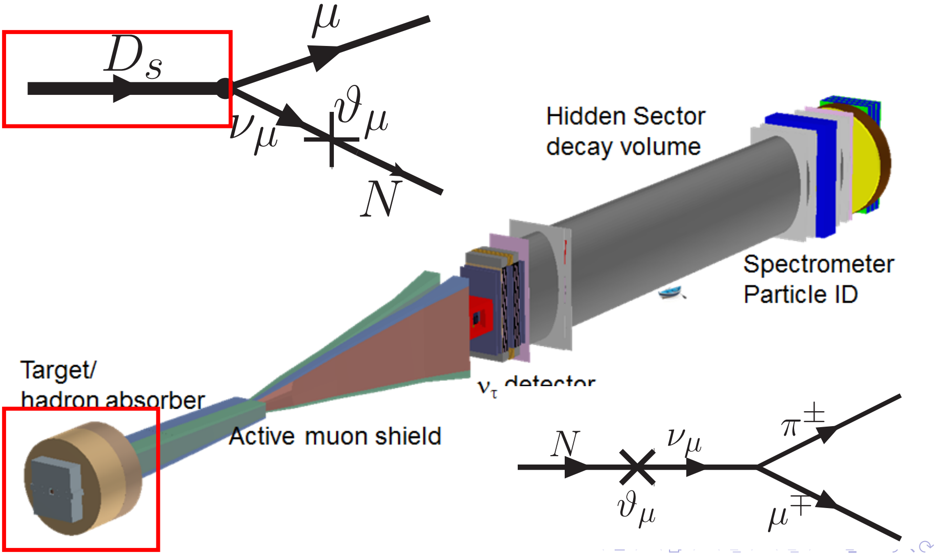
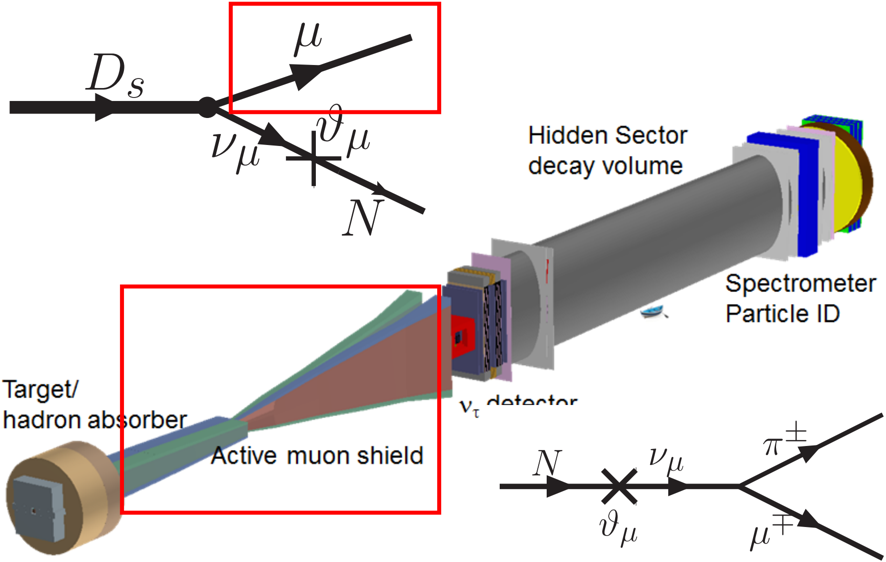


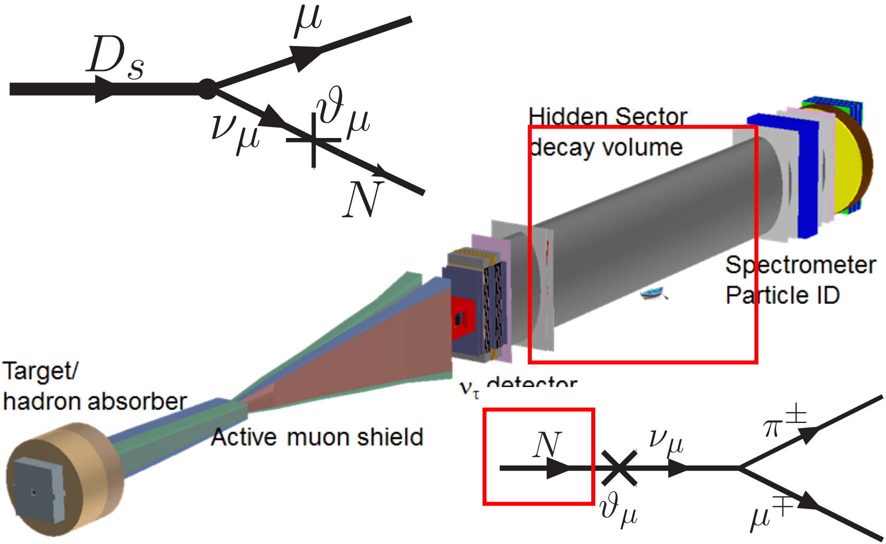
SHiP (*Search for Hidden Particles*) experiment

Step by step overview



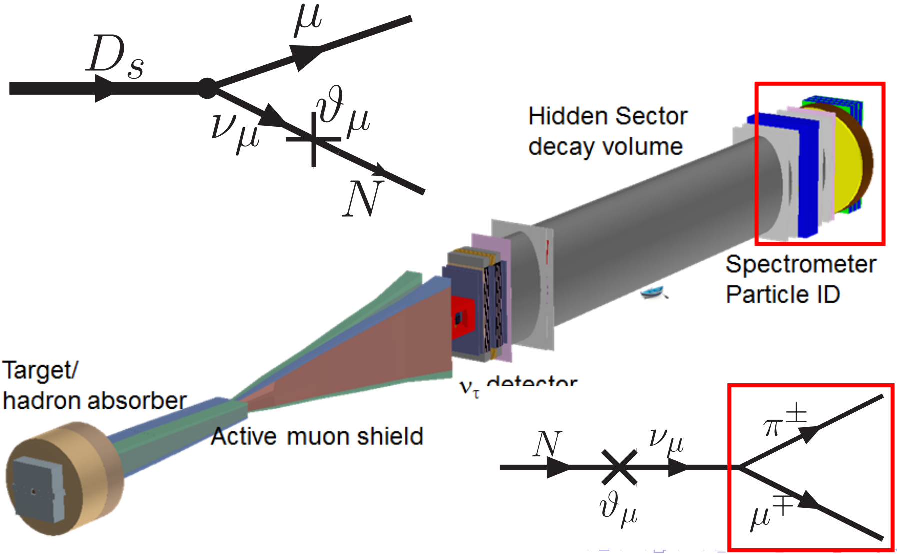
SHiP (*Search for Hidden Particles*) experiment

Step by step overview



SHiP (*Search for Hidden Particles*) experiment

Step by step overview



A facility to Search for Hidden Particles at the CERN SPS: **the SHiP physics case**

85 authors from 65 countries

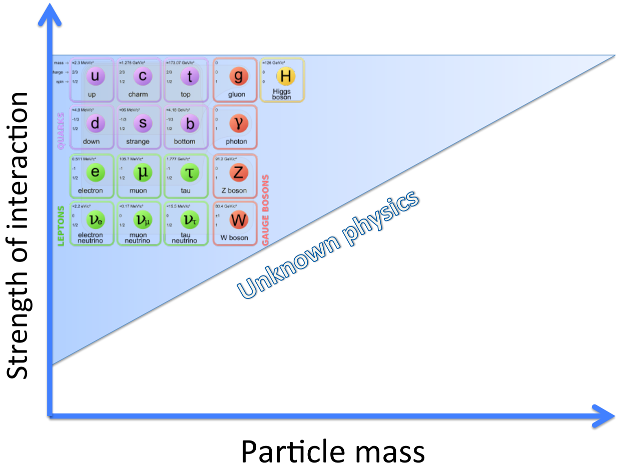
Physics reach beyond the original idea

SHiP physics case paper

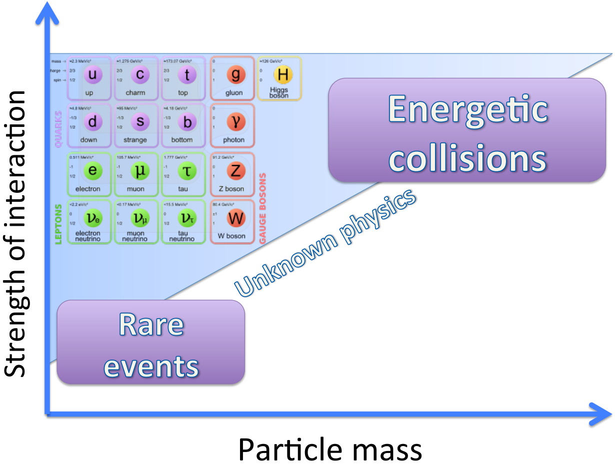
From: *A facility to Search for Hidden Particles at the CERN SPS: the SHiP* **physics case**

Classification of **vector portals** ; Kinetic mixing ; Anomaly-free gauge groups ($B-L$, $L_\mu - L_\tau$ etc) ; Other froms of vector portals. ; **Chern-Simons portal** ; Matter states charged under new $U(1)$; Higgs mechanism in the **dark sector** ; **Supersymmetric** $U(1)'$ models ; Self-intereaction of **dark matter via light mediators** ; Production and detection of **kinetically mixed dark photons and baryonic vectors**. **Scalar portal** ; **Hidden Valleys** ; Light scalars in supersymmetry ; **Singlet extensions** ; **Additional Abelian gauge groups**; Models with R**-parity violation** ; Linear scalar portals: Higgs-scalar mixing ; Existing experimental limits ; Probing Exotic Higgs Decays at SHiP ; Hidden sector scalars ; Hidden sector fermions and vectors ;**Pseudoscalar portals** ; Scalar portals and Dark Matter ; **Scalar as a mediator between DM and the SM** ; **Scalar as a DM candidate** ; **Dark pions** ; **Light inflatons** ; **Neutrino portal** ; **Heavy neutral leptons** ; **Left-right symmetric models** ; Left-right symmetric models with GeV-scale HNLs ; **Inverse seesaw and GeV scale singlet fermions** ; **ALPs and other PNGBs at SHiP** ; Connection to Dark Matter ; ALPs coupled to two gauge bosons ; ALPs coupled to SM fermions ; **SUSY**; **A Very Light Supersymmetric Neutralino and R-Parity Violation** ; **Light particles from the SUSY breaking sector ; Origin of light sgoldstinos** Light Dirac **gauginos** ; SUSY vector portal I: **Hidden Photinos** ; **R-parity conserving photinos** ; . . .

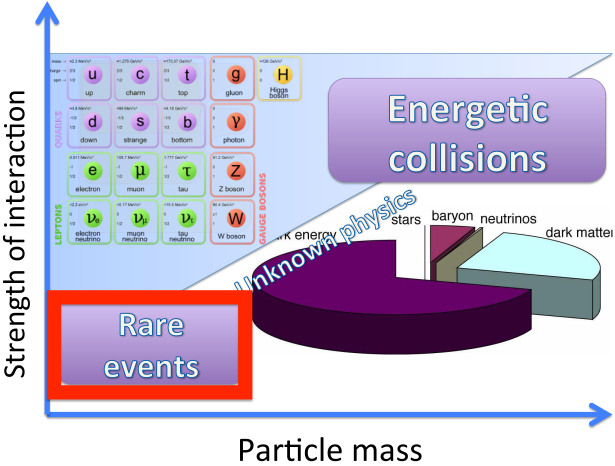
New particles?



New particles?



New particles?

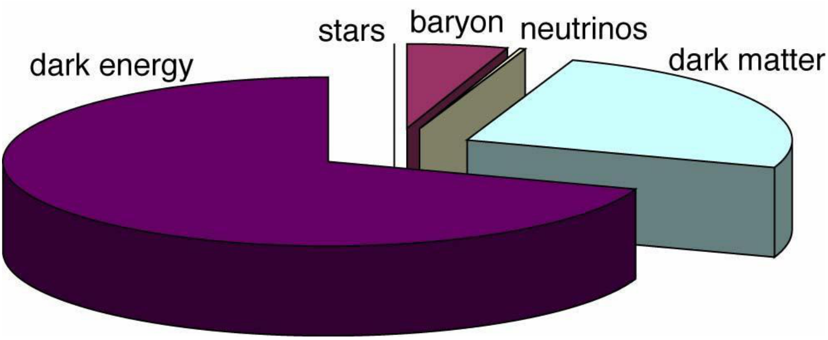


Outline

1. Relativity and quantum mechanics
2. Majorana mass
3. Neutrino oscillations
4. Properties of sterile neutrinos
5. How to search for sterile neutrinos
6. Kink searches
7. Peak searches
8. Sterile neutrinos heaver than kaon
9. Next step
10. Why this is interesting?

We describe the Universe with a handful of numbers. . .

. . . but what particles are behind these numbers?



We describe the Universe with a handful of numbers. . .

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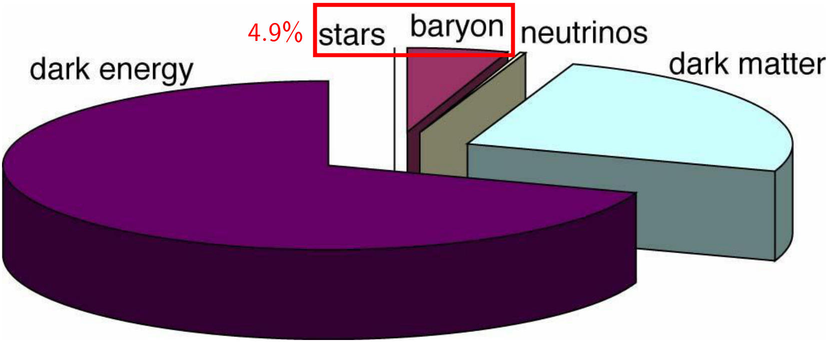
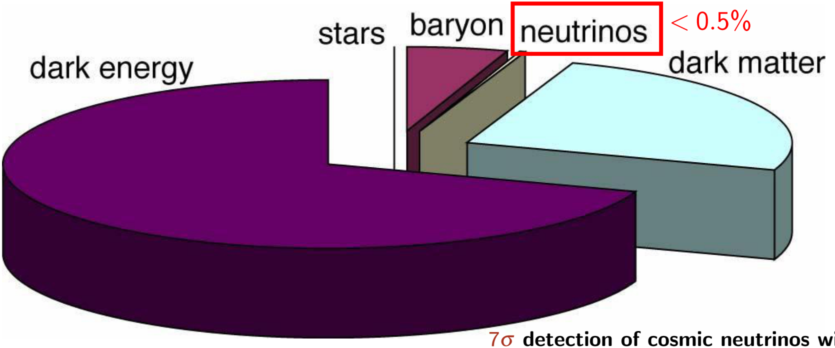
What broke the symmetry between particles and antiparticles in the early Universe?



We describe the Universe with a handful of numbers. . .

. . . but what particles are behind these numbers?

We know (indirectly) **that neutrinos have masses** (Nobel prize 2015) **but we do not know their values**

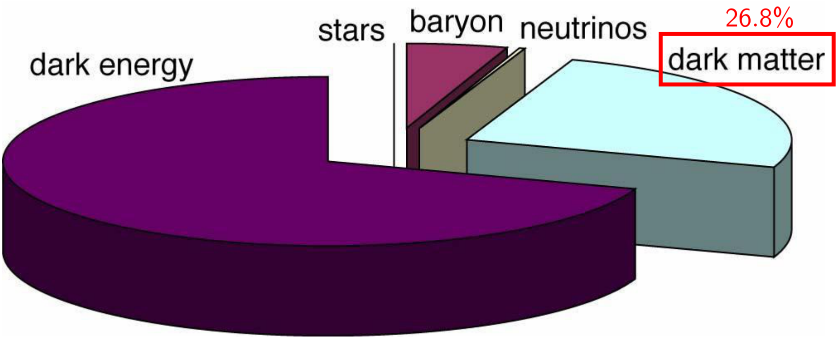


7σ **detection of cosmic neutrinos with Planck**

We describe the Universe with a handful of numbers. . .

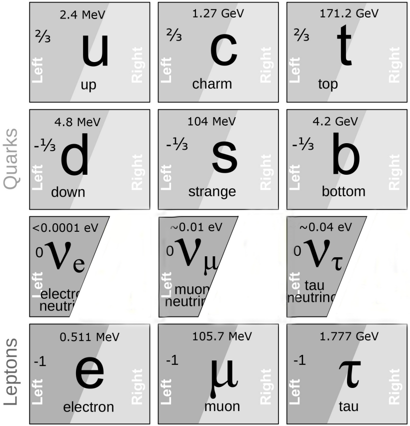
. . . but what particles are behind these numbers?

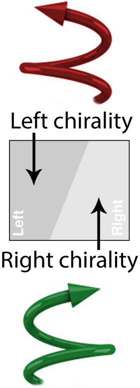
We measure the gravity pull of a substance that fills the Universe, gives mass to galaxies, but does not emit any light



Resolving cosmological puzzles with just 3 particles

Boyarsky, Ruchayskiy, Shaposhnikov *"The Role of sterile neutrinos in cosmology and astrophysics"* Ann.Rev.Nucl.Part.Sci. (2009)





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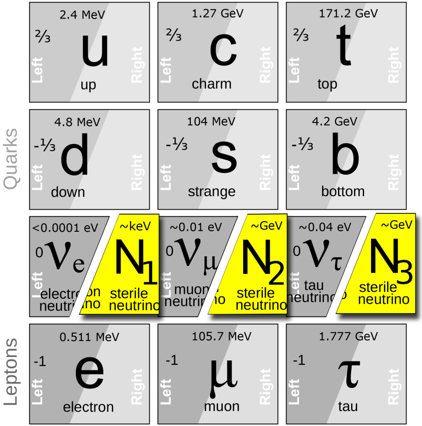

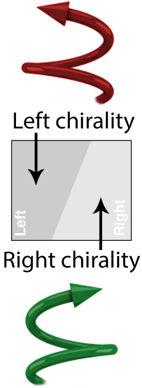

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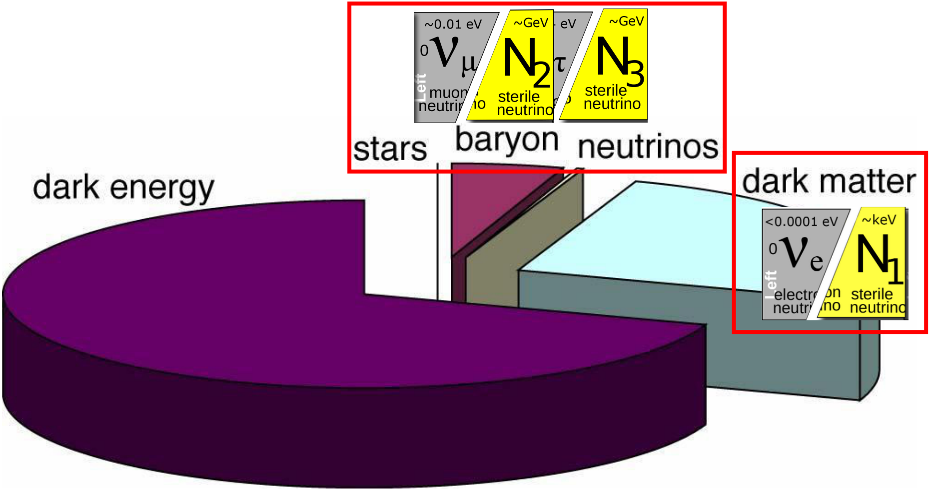

Questions?

List of abbreviations/notations I

If there is some professional jargon that I use but do not explain – please, tell me, I'll put it here

SM: Standard Model of elementary particles; particles in the Standard Model Lagrangian.

Active neutrinos: in the context of these lectures the SM neutrinos (ν_e, ν_μ, ν_τ) will be called sometimes "active" (or "ordinary" neutrinos). We will also mean *charge* (rather than *mass*) eigenstates

BSM: (= "beyond the Standard Model") – phenomena (puzzles) that cannot be explained by the conventional particle physics (Standard Model) coupled to the Einstein gravity

BAU: baryon asymmetry of the Universe: absence of primordial anti-matter in the visible part of the Universe

BBN: (= "Big Bang Nucleosynthesis") primordial synthesis of light elements (Deuterium, Helium, Lithium). Abundances of these elements, predicted by the hot Big Bang theory have been confirmed experimentally which serves as the **most distant** clue about the history of the Universe

List of abbreviations/notations II

CMB: cosmic microwave background – relic radiation from recombination of protons and electrons into hydrogen when the Universe was about $380,000$ years old

DM: dark matter – at galactic scales and above the motion of tracers of gravitational potential are not described by Newtonian gravity sourced by the observed matter

Ordinary matter: (also sometimes "**baryons**") in the cosmological context by this name one calls all matter that exists in the form of gas, stars, etc. and is made of the ordinary particles (baryons + electrons).

Sterile neutrinos: (denoted by N or N_I, where $I = 1, 2, \ldots$) the right-handed counterparts of the active neutrinos inert with respect to the SM interactions

PMNS matrix: Pontecorvo-Maki-Nakagawa-Sakata matrix

Planck mass/scale: for particle of such mass Compton wave length equals to its Schwarzschild radius, $M_{\rm Pl} = 1.2 \times 10^{19}$ GeV.

Indexes: $\alpha, \beta = \{e, \mu, \tau\}$ – flavour indexes; $I = 1, 2, \ldots \mathcal{N}$ – index numbering right-handed fermions.

List of abbreviations/notations III

$\mathscr{M}$: Mass matrix of active neutrinos (size 3×3)

m_D: (sometimes, $m_{\rm Dirac}$) – Dirac matrix, mixing active and sterile neutrinos generated by the Yukawa interaction with the Higgs boson (size $3\times \mathscr{N}$)

M_N: (sometimes M_I, where $I=1,2,\dots$) – Majorana mass of sterile neutrino, with good precision coinciding with its propagation mass ($p^2=M_N^2$)

Higgs vev: $v=174$ GeV. This means that Dirac mass $m_D=Fv$ (rather than $m_D=\frac{1}{\sqrt{2}}Fv$ used e.g. in Peskin & Schroeder)

M_*: Reduced Planck mass used in cosmology ($M_*\equiv\sqrt{\frac{3}{8\pi g_{\rm EFF}}}M_{\rm Planck}$)

Unless otherwise stated $\hbar=c=k_{\rm Boltzmann}=1$.