Flavour symmetries in the I type see-saw model

NBIA PHD School: Neutrinos Undergrounds & in Heavens

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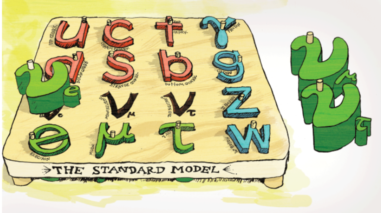
Problems of the Standard Model

1. Huge number of the free parameters
2. Mystery of 3 families
3. Hierarchy of the fermions' masses
4. Neutrino's mass generation
5. The smallness of the neutrino mass
6. Nature of the neutrino: Dirac or Majorana?



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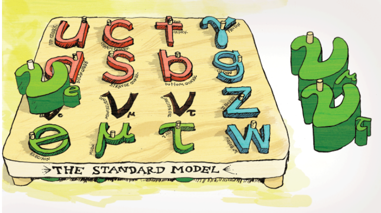

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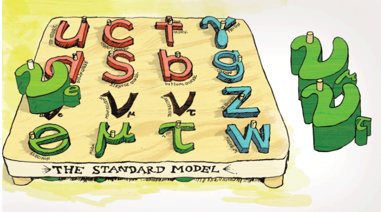

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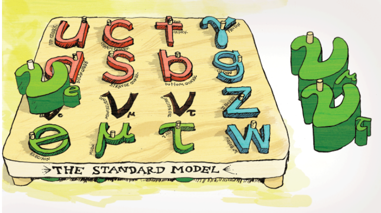

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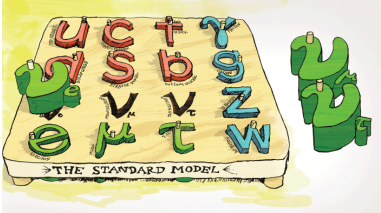

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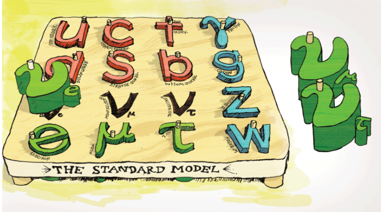

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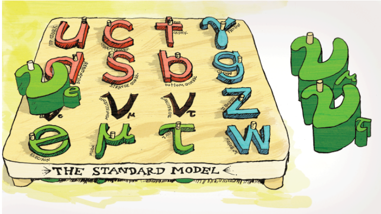

The Yukawa Lagrangian

Mass generation in the framework of SM

Masses of the fermions and gauge bosons

- Masses of the gauge bosons are obtained through the Higgs potential:

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 |\phi|^2 - \lambda |\phi|^4$$

- Masses of the fermions are obtained through the Yukawa Langrangian:

$$\mathcal{L}_Y = - \sum_{\alpha,\beta=e,\mu,\tau} Y'_{\alpha\beta} \overline{L'_{\alpha L}} \phi l'_{\beta R} + Y'^{\nu}_{\alpha\beta} \overline{L'_{\alpha L}} \tilde{\phi} \nu'_{\beta R} + H.c.$$

where: $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $L'_{\alpha L} = \begin{pmatrix} \nu'_{\alpha L} \\ l'_{\alpha L} \end{pmatrix}$, $\tilde{\phi} = i\sigma_2 \phi^*$

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Fermions' masses

Flavour and mass basis

After **S**pontaneous **S**ymmetry **B**reaking (**SSB**):

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

and diagonalization of the Yukawa matrices:

$$\mathbf{V_L^l}^\dagger Y'^l V_R^l = Y^l = diag(m_e, m_\mu, m_\tau)$$

$$\mathbf{V_L^\nu}^\dagger Y'^\nu V_R^\nu = Y^\nu = diag(m_1, m_2, m_3)$$

the mass terms are obtained:

$$\mathcal{L}_Y = - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^l v}{\sqrt{2}} \bar{l}_\alpha l_\alpha - \sum_{k=1,2,3} \frac{y_\nu^k v}{\sqrt{2}} \bar{\nu}_k \nu_k$$

The mixing matrix

Parametrization of the mixing matrix

Mixing matrix

Mixing matrix occurs in the leptonic charged current and is composed of the two diagonalizing matrices:

$$U_{PMNS} = (\mathbf{V}_\mathbf{L}^\mathbf{l})^\dagger \mathbf{V}_\mathbf{L}^\nu$$

The mixing matrix is parametrized by:

1. 3 angles and 1 CP-violating phase (*Dirac neutrino*)
2. 3 angles and 3 CP-violating phases (*Majorana neutrino*)

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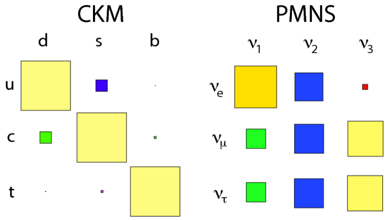
The mixing matrix

$$U^{D}_{PMNS} =$$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

$$U^{M}_{PMNS} = U^{D}_{PMNS}\, diag(1, e^{i\alpha_1/2}, e^{i\alpha_2/2})$$



The flavour symmetry

The tribimaximal mixing

The flavour symmetry

The flavour symmetry is the symmetry $\mathcal{G}_\mathcal{F}$ which added to the symmetry of the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y \times \mathcal{G}_F$ restricts the form of the mixing matrix and masses of the particles.

The tribimaximal mixing

Before 2012 it was possible to explain the mixing with the A_4 symmetry which led to the so-called **tribimaximal mixing** pattern:

$$U_{TBM} = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

The flavour symmetry breaking

Neutrino and charged lepton sector

After 2012 there was not plausible to find the symmetry which would restrict appropriately the mixing and Yukawa matrices. There appeared the models in which the symmetry is broken down to the charged lepton and the neutrino sectors:

Flavour Group $\mathcal{G}_F$

Charged Lepton $\mathcal{G}_l$ **Neutrino** $\mathcal{G}_\nu$

I type see-saw mechanism

Majorana, Dirac & Dirac-Majorana mass terms

- Dirac mass term:

$$\mathcal{L}^{D}_{mass} = -m\overline{\nu_R}\nu_L + H.c.$$

- Majorana mass term:

$$\mathcal{L}^{L}_{mass} = -\frac{1}{2}m\overline{\nu^C_L}\nu_L + H.c.$$

- Dirac-Majorana mass term:

$$\mathcal{L}^{D+M}_{mass} = \mathcal{L}^{\mathbf{D}}_{\mathbf{mass}} + \mathcal{L}^{\mathbf{L}}_{\mathbf{mass}} + \mathcal{L}^{\mathbf{R}}_{\mathbf{mass}} = -\sum_{s}\sum_{\alpha=e,\mu,\tau}\overline{\nu_{sR}}\mathbf{M}^{\mathbf{D}}_{\mathbf{s}\alpha}\nu'_{\alpha L}$$

$$+\frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau}\overline{\nu'^{C}_{\alpha L}}\mathbf{M}^{\mathbf{L}}_{\alpha\beta}\nu'_{\beta L} + \frac{1}{2}\sum_{s,s'}\overline{\nu^C_{sR}}\mathbf{M}^{\mathbf{R}}_{\mathbf{ss'}}\nu_{s'R} + H.c.$$

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Dirac-Majorana mass term

I type see-saw mechanism

The Dirac-Majorana mass term can be easily condensed down to:

$$\mathcal{L}^{D+M}_{mass} = \frac{1}{2}\overline{(N'_L)^C} M^{D+M} N'_L + H.c.,$$

where $N'_L = \begin{pmatrix} \nu'_L \\ \nu^C_R \end{pmatrix}$ and $M^{D+M} = \begin{pmatrix} \mathbf{M^L} & \mathbf{M^D} \\ (\mathbf{M^D})^T & \mathbf{M^R} \end{pmatrix}$.

Assuming $M_L = 0$ and $M_R \gg M_D$ the I type see-saw mechanism is obtained:

$$W^T M^{D+M} W = \begin{pmatrix} M^{light} & 0 \\ 0 & M^{heavy} \end{pmatrix}$$

Quadratic see-saw

Symmetries of the mass matrices

The matrices $\mathbf{M_D}$, $\mathbf{M_R}$ and matrix $\mathbf{M^l}$ can be restricted by means of the flavour symmetry group is such a way that:

1. $\mathbf{M_R}$ is restricted by the full symmetry group G_F (the symmetry is conserved at high energy scale) :

$$A_R^T \mathbf{M_R} A_R = \mathbf{M_R} \implies \mathbf{M_R} = \mathcal{M}\mathbf{I}$$

2. $\mathbf{M_D}$ and $\mathbf{M^l}$ are restricted by the residual symmetries of the full flavour group G_F (symmetry is broken down at low energy scale) such that:

$$A_D^\dagger(g)\mathbf{M_D}A_D(g) = M_D,\ g \in G_D$$

$$A_l^\dagger(g)\mathbf{M_l}A_l(g) = M_l,\ g \in G_l$$

The masses and mixing in I type see-saw

- The masses of the neutrino are suppressed by the high energy scale ($\mathcal{M} \approx 10^{15}$ GeV):

$$(m_k)_{light} = \frac{(m_k^D)^2}{\mathcal{M}}$$

- The mixing matrix is 3×6 rectangular matrix parametrized by:
  1. **12** mixing angles
  2. **12** phases- **7** Dirac+ **5** Majorana phases

Further plans

1. All non-abelian subgroups of $U(3)$ until order 100 should be considered as the possible flavour symmetries
2. It is necessary to reject the groups which doesn't allow to fit the charged lepton masses
3. We should try to find the flavour group which allows for the accommodation of the neutrino masses and mixing angles

Questions ?