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Number of light and heavy neutrinos in Seesaw type I

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Seesaw mass matrix and diagonalization

Mass matrix

$$\mathcal{L}^{D+M} = \frac{1}{2}\tilde{N}^T \mathcal{C}^\dagger M \tilde{N} + H.c.$$

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

$$\tilde{N} = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$$

two mass scales

$M_R \gg M_D$

Block diagonalization

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

$$\lambda(M_R) \gg |M_D|$$

$$W^T M W \simeq \begin{pmatrix} M_{light} & 0 \\ 0 & M_{heavy} \end{pmatrix}$$

$$\left.\begin{aligned} M_{light} &\simeq -M_D^T M_R^{-1} M_D \\ M_{heavy} &\simeq M_R \end{aligned}\right\} \textit{Seesaw}$$

e.g.

$$M_R \sim 10^{15}\ [GeV],\ \ M_D \sim 10^{2}\ [GeV] \Rightarrow\ M_{light} \sim 10^{-2}\ [eV]$$

$$LHC\ [wishful\ thinking?]\ M_R \sim 10^{3}\ [GeV],\ \ M_D \sim 10^{-4}\ [GeV] \Rightarrow\ M_{light} \sim 10^{-2}\ [eV]$$

Problems

Why $\lambda(M_R) \gg |M_D|$?

Example

$$M_R = \begin{pmatrix} 100 & 100 & 100 \\ 100 & 100 & 100 \\ 100 & 100 & 100 \end{pmatrix} \Longrightarrow \textit{rank one matrix} \Rightarrow \lambda(A) = (300, 0, 0)$$

$$M = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 100 & 100 & 100 \\ 1 & 1 & 1 & 100 & 100 & 100 \\ 1 & 1 & 1 & 100 & 100 & 100 \end{pmatrix} \Longrightarrow$$

M_R, M_D rank one
M has four zero eigenvalues!!!

Czakon, Gluza and Zralek [2] gave a formal proof that for $M_R \gg M_D$ we can not get a fourth light sterile neutrino.

Problems

In general a full mass matrix M is complex and symmetric

Not Hermitian, not even normal!!!

Consequences:

1. A complex symmetric matrix may not even be diagonalizable

$$A = \begin{pmatrix} 2i & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{array}{c} A^\dagger \neq A\,,\ A^\dagger A \neq AA^\dagger \\ \textit{algebraic multiplicity} \neq \textit{geometric multiplicity} \Rightarrow \textit{not diagonalizable} \end{array}$$

2. Even if it is, we have $M_{heavy} \simeq M_R \Rightarrow$ small differences can dramatically change eigenvalues

$$A = \begin{pmatrix} \mu & 1 & 0 \\ 0 & \mu & 1 \\ 0 & 0 & \mu \end{pmatrix}, \qquad E = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \epsilon & 0 & 0 \end{pmatrix}.$$

If μ — eigenvalue of A, λ — (any) eigenvalue of $A + E$, then

$$|\lambda - \mu| = \epsilon^{1/3}$$

If $\epsilon = 10^{-15}$, then $|\lambda - \mu| = 10^{-5}$ — spectrum shifted by a number 10 orders larger than the parameter ϵ!

My first findings

Number of nutrinos in CP invariant case

In the CP invariant seesaw scenario with two mass scales $M_R \gg M_D$, $M_D \in M_{3\times n},\ M_R \in M_{n\times n}$ and $\lambda(M_R) \gg |m_D|$ exactly 3 light neutrinos are present.

Matrix Decomposition

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & M_R \end{pmatrix} + \begin{pmatrix} 0 & M_D \\ M_D^T & 0 \end{pmatrix} \equiv \hat{M}_R + \hat{M}_D$$

Proof scheme

From Weyl's inequalities

$$\alpha_j + \beta_k \le \gamma_{j+k-1} \quad \textit{and} \quad \gamma_{n-j-k} \le \alpha_{n-j+1} + \beta_{n-k+1}$$

$$|\lambda_i(M) - \lambda_i(\hat{M}_R)| \le \rho(\hat{M}_D) \le \|\hat{M}_D\|_F = \sqrt{\sum_{i,j} m_{ij}^2} \;,\; \rho(\hat{M}_D) = \max\{|\lambda(\hat{M}_D)|\}$$

But

$$\sqrt{\sum_{i,j} m_{ij}^2} \sim |m_D| \Rightarrow \rho(\hat{M}_D) \le |m_D| \;,\; m_D \in M_D$$

On the other hand matrix $\hat{M}_R$ has at least 3 eigenvalues equal 0.

Thus $$|\lambda_i(M) - 0| \le \rho(\hat{M}_D) \quad \textit{for}\ \lambda_i(\hat{M}_R) = 0$$

Hence, three eigenvalues of M must be smaller than $|m_D|$.

Conclusions

Conclusion (I): At least 3 light neutrinos exist.

Similar steps with assumption that $\lambda(M_R) \gg |m_D|$ gives
Conclusion (II): Remaining masses must be heavy.

Conclusions (I) and (II) implies Conclusion (III):
Heavy masses $N_{1,\ldots,n}$ are maximally shifted by $|m_D|$ from eigenvalues of M_R.

e.g.

$$\lambda_1(M_R) = 100,\ |m_D| \sim 1 \Rightarrow \lambda_1(M) \simeq 100 \pm 1$$

Bibliography

[1] Rejandra Bhatia. *Matrix Analysis*. Springer, 1996.
[2] M. Czakon, J. Gluza, M. Zralek. *Seesaw mechanism and four light neutrino states*. Phys. Rev., D64: 117304, 2001