

Problem 10/11/16 :

(1)

We have

$$\Gamma(B^{\circ} \rightarrow f) \propto \left| \frac{p}{q} g_{-}(t) \right|^2$$

$$\Gamma(B^{\circ} \rightarrow f) \propto |g_{+}(t)|^2$$

Inserting the expressions for g_{\pm}

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[\cosh \frac{\Delta \Gamma t}{2} \pm \cos \omega t \right]$$

$$\approx \frac{e^{-\Gamma t}}{2} (1 \pm \cos \omega t)$$

or $1/\Delta \Gamma \gg 1/\Gamma = \text{sim}$

So $N_f(t) \propto \frac{e^{-\Gamma t}}{2} \left[\left(1 + \left|\frac{p}{q}\right|^2\right) + \left(1 - \left|\frac{p}{q}\right|^2\right) \cos \omega t \right] \quad *1$

Similar for $N_{\bar{f}}$ but with p and q interchanged

$$N_{\bar{f}}(t) \propto \frac{e^{-\Gamma t}}{2} \left[\left(1 + \left|\frac{q}{p}\right|^2\right) + \left(1 - \left|\frac{q}{p}\right|^2\right) \cos \omega t \right] \quad *2$$

Now we have

$$\left| \frac{q}{p} \right|^4 = \frac{1 - a_{11}^d}{1 + a_{31}^d}$$

and as $a_{11}^d \ll 1$ ($|a_{11}^d| \ll 1$) we get

$$\left| \frac{q}{p} \right|^4 \approx (1 - a_{11}^d)^2 \Rightarrow \left| \frac{q}{p} \right|^2 \approx 1 - a_{11}^d$$

Inserting this in *1 and *2 gives

$$N_f(t) \propto e^{-\Gamma t} \left[1 + \frac{a_{sl}^d}{2} - \frac{a_{sl}^d}{2} \cos \Delta m t \right]$$

$$N_{\bar{f}}(t) \propto e^{-\Gamma t} \left[1 - \frac{a_{sl}^d}{2} + \frac{a_{sl}^d}{2} \cos \Delta m t \right]$$

which can be rewritten to the answer by using definition of ξ .

b) The oscillating term has a magnitude that is tiny compared to the overall decay rate. Thus the efficiency of μ^+ vs μ^- needs to be extremely well controlled as a function of decay time. Also production asymmetry needs good control.

c) We have
$$a_{sl}^d = \text{Im} \left(\frac{\Gamma_{12}}{M_{12}} \right)$$

Inserting definition of φ gives

$$a_{sl}^d = \frac{|\Gamma_{12}|}{|M_{12}|} \text{Im}(e^{i\varphi}) = \frac{|\Gamma_{12}|}{|M_{12}|} \sin \varphi \quad *3$$

If we have $\Delta m = M_2 - M_1$, $\Delta \Gamma = \Gamma_1 - \Gamma_2$ we have

$$\Delta m + \frac{i\Delta\Gamma}{2} = -2 \frac{\varphi}{p} \left(M_{12} - \frac{i\Gamma_{12}}{2} \right)$$

Now square this.

$$\left(\Delta m + \frac{i\Delta\Gamma}{2}\right)^2 = 4\left(\frac{q}{p}\right)^2 \left(M_{12} - \frac{i\Gamma_{12}}{2}\right)^2$$

$$\begin{aligned} (\Delta m)^2 + \frac{\Delta\Gamma^2}{4} + i\Delta m\Delta\Gamma &= 4\left(M_{12}^* - \frac{i\Gamma_{12}^*}{2}\right)\left(M_{12} - \frac{i\Gamma_{12}}{2}\right) \\ &= 4|M_{12}|^2 - |\Gamma_{12}|^2 - 4i\operatorname{Re}(M_{12}^*\Gamma_{12}) \end{aligned}$$

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Compare part $\Delta m\Delta\Gamma = -4\operatorname{Re}(M_{12}^*\Gamma_{12})$

$$\begin{aligned} \Delta m^2 - \frac{\Delta\Gamma^2}{4} &= 4|M_{12}|^2 - |\Gamma_{12}|^2 = 4|M_{12}|^2 \\ \parallel & \\ \Delta m^2 &\text{ (as } \Delta\Gamma \ll \Delta m) \end{aligned}$$

↖ As $\Gamma_{12} \ll M_{12}$

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$$\frac{\Delta\Gamma}{\Delta m} = -\frac{|\Gamma_{12}|}{|M_{12}|} \operatorname{Re}(e^{i\varphi}) = -\frac{|\Gamma_{12}|}{|M_{12}|} \cos\varphi$$

Inserts this into *3 finally gives

$$a_{SL}^d = -\frac{\Delta\Gamma}{\Delta m} \frac{\sin\varphi}{\cos\varphi} = -\frac{\Delta\Gamma}{\Delta m} \tan\varphi$$