Flavor physics — Problem Set #3

1 Direct CP violation

Consider the decays $B^\pm \to f^\pm$. Direct CP violation probes interference between two decay amplitudes. Denote

$$\begin{aligned} A_f &= |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}\,, \\ \overline{A}_{\bar f} &= |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}\,, \end{aligned} \tag{1}$$

where $\phi_{1,2}$ are CP violating (weak) phases, and $\delta_{1,2}$ are CP conserving (strong) phases. It is further useful to define

$$\phi_f = \phi_1 - \phi_2\,, \qquad \delta_f = \delta_1 - \delta_2\,, \qquad r_f = |a_2/a_1|\,. \tag{2}$$

Commonly, $r_f \ll 1$. Show that in order to observe direct CP violation, $\Gamma(B \to f) \neq \Gamma(\bar B \to \bar f)$, we need two amplitudes with different weak and strong phases.

2 CPV in mixing in the Kaon system

In class we saw that the semi-leptonic asymmetry in the B system equals

$$A_{\rm SL} = \frac{\Gamma_{B^0\to\ell^+X}(t) - \Gamma_{\overline{B^0}\to\ell^-X}(t)}{\Gamma_{B^0\to\ell^+X}(t) = \Gamma_{\overline{B^0}\to\ell^-X}(t)} = \frac{1-|q/p|^4}{1+|q/p|^4}\,. \tag{3}$$

Show that for Kaons

$$A_{\rm SL} = \frac{\Gamma_{K_L\to\ell^+\nu\pi^-}(t) - \Gamma_{K_L\to\ell^-\nu\pi^+}(t)}{\Gamma_{K_L\to\ell^+\nu\pi^-}(t) + \Gamma_{K_L\to\ell^-\nu\pi^+}(t)} = \frac{1-|q/p|^2}{1+|q/p|^2}\,. \tag{4}$$

Can you explain the difference?

3 $B \to \pi^+\pi^-$ decays

Show that, if the $B \to \pi\pi$ decays were dominated by tree diagrams, then $S_{\pi\pi} = \sin 2\alpha$.

4 CP violation in B decays 1

The following table summarizes the different $b \to qqq'$ modes with $q' = s$ or d. The second and third columns give examples of hadronic final states (usually those which are experimentally most convenient to study). The fourth column gives the CKM dependence of the decay amplitude A_f, using the notation of Eq. (86) of your notes, with the dominant term first and the subdominant second. The suppression factor of the second term compared to the first is given in the last column. "Loop" refers to a penguin versus tree-suppression factor (it is mode-dependent and roughly $\mathcal{O}(0.2 - 0.3)$) and $\lambda \simeq 0.23$ is the expansion parameter in the Wolfenstein parametrization of the CKM matrix.

1. Fill in the missing blocks in the table.

2. Within the SM, which of these process measures $\sin 2\beta$? List them by the accuracy of their theoretical interpretation.

$\bar{b} \to \bar{q}q\bar{q}'$	$B^0 \to f$	$B_s^0 \to f$	CKM dependence of A_f	Suppression
$\bar{b} \to \bar{c}c\bar{s}$		$\psi\phi$	$(V_{cb}^* V_{cs})\, T + (V_{ub}^* V_{us})\, P^u$	loop$\times\lambda^2$
$\bar{b} \to \bar{s}s\bar{s}$		$\phi\phi$		λ^2
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-		λ^2/loop
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-		$(V_{cb}^* V_{cd})\, T + (V_{tb}^* V_{td})\, P^t$	
$\bar{b} \to \bar{s}s\bar{d}$	$K_S K_S$		$(V_{tb}^* V_{td})\, P^t + (V_{cb}^* V_{cd})\, P^c$	≤ 1
$\bar{b} \to \bar{u}u\bar{d}$		$\rho^0 K_S$	$(V_{ub}^* V_{ud})\, T + (V_{tb}^* V_{td})\, P^t$	
$\bar{b} \to \bar{c}u\bar{d}$	$D_{CP}\pi^0$	$D_{CP}K_S$		λ^2
$\bar{b} \to \bar{c}u\bar{s}$	$D_{CP}K_S$	$D_{CP}\phi$	$(V_{cb}^* V_{us})\, T + (V_{ub}^* V_{cs})\, T'$	

Answer:

The complete table can be found in the PDG review: "CP violation in the quark sector".

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Consider the decays $\bar{B}^0 \to \psi K_S$ and $\bar{B}^0 \to \phi K_S$. Unless explicitly noted, we always work within the framework of the standard model.

1. $\bar{B}^0 \to \psi K_S$ is a tree-level process. Write down the underlying quark decay. Draw the tree level diagram. What is the CKM dependence of this diagram? In the Wolfenstein parametrization, what is the weak phase of this diagram?

2. Write down the underlying quark decay for $B^0 \to \phi K_S$. Explain why there is no tree level diagram for $B^0 \to \phi K_S$.

3. The leading one loop diagram for $B^0 \to \phi K_S$ is a gluonic penguin diagram. There are several diagrams and only their sum is finite. Draw a representative diagram with an internal top quark. What is the CKM dependence of the diagram? In the Wolfenstein parametrization, what is the weak phase of the diagram?

4. Next we consider the time dependent CP asymmetries. We define as usual

$$\lambda_f \equiv \frac{\bar{A}_f}{A_f}\frac{q}{p}, \qquad A_f \equiv A(B^0 \to f), \qquad \bar{A}_f \equiv A(\bar{B}^0 \to f). \tag{5}$$

   In our case we neglect subleading diagrams and then we have $|\lambda| = 1$ and thus

$$a_f \equiv \frac{\Gamma(\overline{B^0}(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\overline{B}^0(t) \to f) + \Gamma(B^0(t) \to f)} = -\Im\lambda_f \, \sin(\Delta m_B \, t) \tag{6}$$

   Both $a_{\psi K_S}$ and $a_{\phi K_S}$ measure the same angle of the unitarity triangle. That is, in both cases, $\Im\lambda_f = \sin 2x$ where x is one of the angles of the unitarity triangle. What is x? Explain.

5. Experimentally,

$$\Im\lambda_{\psi K_S} = 0.59(14), \qquad \Im\lambda_{\phi K_S} = 0.47(19). \tag{7}$$

   Do you think these two results are in disagreement?

6. Assume that in the future we will find

$$\Im\lambda_{\psi K_S} = 0.59(14), \qquad \Im\lambda_{\phi K_S} = 0.22(3). \tag{8}$$

   That is, that the two results are not the same. Below are three possible "solutions". For each solution explain if you think it could work or not. If you think it can work, show how. If you think it cannot, explain why.

(a) There are standard model corrections that we neglected.

(b) There is a new contribution to $B^0 - \bar{B}^0$ mixing with a weak phase that is different from the SM one.

(c) There is a new contribution to the gluonic penguin with a weak phase that is different from the SM one.

Answer:

(a) In the SM the correction are very small. The small parameter is $|V_{ub}V_{us}|V_{cb}V_{cs}| < 10^{-2}$, so it is very unlikely to make significant change of the naive prediction.

(b) New contribution to $B - \bar{B}$ mixing would affect both asymmetries the same, so in that case we will still have the equal.

(c) New contribution to the penguin decay would affect only $a_{\phi K_S}$ and could cause the difference.