exchange reactions[18] ($\pi^+ p \to K^+\Sigma^+$, $\pi^- p \to K^0\Lambda^0$, etc.) gives the intercepts $\alpha_{0q}=0.35$ and $\alpha_{0Q}=0.24$ (with uncertain errors). The intercepts resulting from an analysis of total cross-section data are also consistent with the values of the present analysis provided we postulate[19] that the Pomeranchuk trajectory has a small $I=0$ octet component in addition to the usual $SU(3)$ singlet component. Table I summarizes the situation on the intercepts of the q and Q trajectories.

In conclusion, the following comments may be made: Although the quality of the fits in the present case is not comparable with those which can be made with the Δ-production data, it nevertheless demonstrates that $SU(3)$ symmetry for Regge vertices and Regge behavior are consistent with the data. Further, the same mechanism seems to be operative in the production of these members of the $\frac{3}{2}^+$ decuplet. The q and Q trajectories do not seem to be degenerate,[20] and the values determined from the analysis of the Y_1*(1385)-production reactions are consistent with earlier determinations from other reactions.

ACKNOWLEDGMENTS

The authors wish to acknowledge useful discussions with R. Kraemer and H. E. Fisk. They also appreciate discussions with J. Mucci and R. Edelstein and with J. Mott concerning their data. One of the authors (G. H. R.) wishes to express his appreciation to Carl Kaysen for the hospitality of the Institute for Advanced Study, where this work was completed.

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[19] K. V. L. Sarma and G. H. Renninger, Phys. Rev. Letters **20**, 399 (1969).

[20] K. W. Lai and J. Louie [Nucl. Phys. **B19**, 205 (1970)] have examined reactions (1) and (2) with a view to testing the exchange degeneracy of the K^* and K^{**} exchanges. They find that exchange degeneracy is not indicated in these reactions. D. J. Crennell *et al.* [Phys. Rev. Letters **23**, 1347 (1969)] and P. R. Auvil *et al.* [Phys. Letters **31B**, 303 (1970)] have found that the data on meson-baryon hypercharge exchange reactions similarly do not indicate exchange degeneracy for these exchanges.

PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

Weak Interactions with Lepton-Hadron Symmetry*

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(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

INTRODUCTION

Weak-interaction phenomena are well described by a simple phenomenological model involving a single charged vector boson coupled to an appropriate current. Serious difficulties occur only when this model is considered as a quantum field theory, and is examined in other than lowest-order perturbation theory.[1] These troubles are of two kinds. First, the theory is too singular to be conventionally renormalized. Although our attention is not directed at this problem, the model of weak interactions we propose may readily be extended to a massive Yang-Mills model, which may be amenable to renormalization with modern techniques. The second problem concerns the selection rules and the relationships among coupling constants which are carefully and deliberately incorporated into the original phenomenological Lagrangian. Our principal concern is the fact that these properties are not necessarily maintained by higher-order weak interactions.

Weak-interaction processes, and their higher-order weak corrections, may be classified[2] according to their dependence upon a suitably introduced cutoff momentum Λ. Contributions to the S matrix of the form

$$\sum_{n=1}^{\infty} A_n (G\Lambda^2)^n$$

(where G is the usual Fermi coupling constant and A_n are dimensionless parameters) are called zeroth-order

* Work supported in part by the Office of Naval Research, under Contract No. N00014-67-A-0028, and the U. S. Air Force under Contract No. AF49(638)-1380.

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[1] B. L. Ioffe and E. P. Shabalin, Yadern. Fiz. **6**, 828 (1967) [Soviet J. Nucl. Phys. **6**, 603 (1968)]; Z. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **6**, 978 (1967) [Soviet Phys. JETP Letters **6**, 390 (1967)]; R. N. Mohapatra, J. Subba Rao, and R. E. Marshak, Phys. Rev. Letters **20**, 1081 (1968); Phys. Rev. **171**, 1502 (1968); F. E. Low, Comments Nucl. Particle Phys. **2**, 33 (1968); R. N. Mohapatra and P. Olesen, Phys. Rev. **179**, 1917 (1969).

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weak effects, terms of the form

$$G \sum_{n=0}^{\infty} B_n (G\Lambda^2)^n$$

are called first-order weak effects, and generally, terms of the form

$$G^l \sum_{n=0}^{\infty} C_{ln} (G\Lambda^2)^n$$

are called lth order. (We are disregarding possible logarithmic dependences on the cutoff.) The zeroth-order terms present us with the dangerous possibility of serious violations of parity and hypercharge in strong interactions. First-order terms include the usual lowest-order contributions (order G) to leptonic and semileptonic processes. However, other first-order terms may yield violations of observed selection rules: There can be $\Delta S=2$ amplitudes, yielding a K_1-K_2 mass splitting, beginning at order $G(G\Lambda^2)$, as well as contributions to such unobserved decay modes as $K_2 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., involving neutral lepton pairs, or departures from the leptonic $\Delta S = \Delta Q$ law. We shall say of a model that its divergences are properly ordered if it is true that the zeroth-order terms *do not* yield violations of parity or hypercharge, and if the first-order terms *do* satisfy the observed selection rules of weak-interaction phenomena.

In most conventional formulations of a weak-interaction field theory (say, a vector boson coupled to a quark triplet), the divergences are not properly ordered. Defenders of such theories must argue that there is an effective weak-interaction cutoff which guarantees that the induced higher-order effects are as small as experiment indicates. A remarkably small cutoff,[1] not greater than 3 or 4 GeV, seems necessary. Should such a cutoff be justified, the problem of higher-order departures from known selection rules is solved; all such departures are small.

Others feel that such a small cutoff is implausible and unrealistic, and that one must confront the possibility that $G\Lambda^2$ is large—perhaps obtaining sensible results in the limit $G\Lambda^2 \to \infty$. In this case, one may regard all the first-order terms as having the same general magnitude, that of observed weak phenomena, and nth-order terms as having the magnitude naively expected of nth-order weak interactions.

An elegant solution to the problem of the zeroth-order terms was recently discovered, removing the specter of strong violations of parity and hypercharge.[3,4] One assumes a particular form for the breakdown of chiral $SU(3)$: The symmetry-breaking term must transform like the $(3,\bar{3})+(\bar{3},3)$ representation[5]; in a quark model, like the quark mass term. In this case, the zeroth-order weak interactions may be identified as an object belonging to the same representation as the symmetry-breaking term. After an appropriate $SU(3) \times SU(3)$ transformation, their only effect is to cause a renormalization of the symmetry-breaking terms, giving renormalized quark masses.[4] There is no violation of hypercharge or parity. Indeed, from a speculative stability requirement of the symmetry-breaking term under weak and electromagnetic corrections, the correct value of the Cabibbo angle may be deduced.[4]

Although the zeroth-order terms are controlled with an appropriate model of strong interactions, the first-order terms remain troublesome. Indeed, with a quark model, we immediately encounter strangeness-violating couplings of neutral lepton currents and contributions to the neutral kaon mass splitting to order $G(G\Lambda^2)$.[6] (In such a model, departures from $\Delta S=\Delta Q$ first appear at second order.) For this reason, it appears necessary to depart from the original phenomenological model of weak interactions. One suggestion[7] involves the introduction of a large number of intermediaries of spins one and zero, so coupled that the leading divergences are associated with only the diagonal symmetry-preserving interactions; in this fashion a proper ordering of divergences is readily obtained. But this model is an awkward one involving many intermediaries with different spins but degenerate coupling strengths. Few would concede so much sacrifice of elegance to expediency.[8]

We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are mediated by just one charged vector boson. The weak hadronic current is constructed in precise analogy with the weak lepton current, thereby revealing suggestive lepton-quark symmetry. The extra quark is the simplest modification of the usual model leading to the proper ordering of divergences. Just as importantly, we argue that universality is preserved, in the sense that the

[3] C. Bouchiat, J. Iliopoulos, and J. Prentki, Nuovo Cimento **56A**, 1150 (1968); J. Iliopoulos, *ibid.* **62A**, 209 (1969); R. Gatto, G. Sartori, and M. Tonin, Phys. Letters **28B**, 128 (1968); Nuovo Cimento Letters **1**, 1 (1969).

[4] N. Cabibbo and L. Maiani, Phys. Letters **28B**, 131 (1968); Phys. Rev. D **1**, 707 (1970).

[5] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[6] Of course, one cannot exclude *a priori* the possibility of a cancellation in the sum of the relevant perturbation expansion in the limit $\Lambda \to \infty$.

[7] M. Gell-Mann, M. L. Goldberger, N. M. Kroll, and F. E. Low, Phys. Rev. **179**, 1518 (1969).

[8] For other departures from the conventional theory, see, for example, C. Fronsdal, Phys. Rev. **136B**, 1190 (1964); W. Kummer and G. Segrè, Nucl. Phys. **64**, 585 (1965); G. Segrè, Phys. Rev. **181**, 1996 (1969); L. F. Li and G. Segrè, *ibid.* **186**, 1477 (1969); N. Christ, *ibid.* **176**, 2086 (1968). It should be understood that the ingenious conjecture of T. D. Lee and G. C. Wick [Nucl. Phys. **B9**, 209 (1969)] for removing divergences is logically independent of our analysis. If their hypothesis is correct, the role of the cutoff momentum is played by M_W. Only if M_W is small ($\sim$3–4 GeV) would the problems associated with ordering of divergences be solved; otherwise, a modification of the coupling scheme, such as ours, is still necessary.

leading divergent corrections (i.e., the first-order terms) yield a *common* renormalization to each of the various observed coupling constants.

The new model is discussed in Sec. I. Since Cabibbo's algebraic notion of universality[9] is maintained, that is to say, the entire weak charges generate the algebra of $SU(2)$, we observe in Sec. II that an extension to a three-component Yang-Mills model may be feasible. In contradistinction to the conventional (three-quark) model, the couplings of the neutral intermediary—now hypercharge conserving—cause no embarrassment. The possibility of a synthesis of weak and electromagnetic interactions is also discussed.

In Sec. III we briefly note some of the implications of the existence of a fourth quark, and finally, in Sec. IV we discuss some of the experimental tests of our model of weak interactions.

I. NEW MODEL

We begin by introducing four quark fields.[10] The three quarks $\mathcal{P}$, $\mathfrak{N}$, and λ form an $SU(3)$ triplet, and the fourth, $\mathcal{P}'$, has the same electric charge as $\mathcal{P}$ but differs from the triplet by one unit of a new quantum number $\mathcal{C}$ for charm. The strong-interaction Lagrangian is supposed to be invariant under chiral $SU(4)$, except for a symmetry-breaking term transforming, like the quark masses, according to the $(4,\bar{4})+(\bar{4},4)$ representation. This term may always be put in real diagonal form by a transformation of $SU(4)\times SU(4)$, so that B, Q, Y, $\mathcal{C}$, and parity are necessarily conserved by these strong interactions.

The extra quark completes the symmetry between quarks and the four leptons ν, ν', e^-, and μ^-. Both quadruplets possess unexplained unsymmetric mass spectra, and consist of two pairs separated by one in electric charge.

The weak lepton current may be expressed as

$$J_\mu{}^L=\bar{l}C_L\gamma_\mu(1+\gamma_5)l, \tag{1}$$

where l is a column vector consisting of the four lepton fields (ν, ν', e^-, μ^-) and the matrix C_L is given by

$$C_L=\begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}. \tag{2}$$

This is a convenient way to rewrite the conventional current. In analogy with this expression, we define the weak hadron current to be

$$J_\mu{}^H=\bar{q}C_H\gamma_\mu(1+\gamma_5)q, \tag{3}$$

where q is the quark column vector $(\mathcal{P}',\mathcal{P},\mathfrak{N},\lambda)$ and the matrix C_H must be of the form

$$C_H=\left[\begin{array}{cc|c}0&0&\\0&0&U\\\hline 0&0&0\quad 0\\0&0&0\quad 0\end{array}\right] \tag{4}$$

in order for $J_\mu{}^H$ to carry unit charge. Pursuing the analogy further, we demand that the 2×2 submatrix U be unitary, so that the matrix C_H is equivalent to C_L under an $SU(4)$ rotation. Of course, it is not convenient to carry out the transformation making C_H and C_L coincide, for this would destroy the diagonalization of the $SU(4)$-breaking term, the quark masses. Nevertheless, suitable redefinitions of the relative phases of the quarks may be performed in order to make U real and orthogonal, so without loss of generality we write

$$U=\begin{bmatrix}-\sin\theta & \cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}. \tag{5}$$

This is just the form of the weak current suggested in an earlier discussion of $SU(4)$ and quark-lepton symmetry.[10] What is new is the observation that this model is consistent with the phenomenological selection rules and with universality even when all divergent first-order terms [i.e., $G(G\Lambda^2)^n$] are considered.

To see this, we proceed diagrammatically in the quark model ignoring the strong $SU(4)$-invariant interactions.[11] Zeroth-order terms occur only in diagrams with only one external quark line, and give contributions to the quark mass operator of the form

$$\delta M(\gamma k)=\sum A_n(G\Lambda^2)^n\bar{q}M_n\gamma\cdot k(1+\gamma_5)q. \tag{6}$$

The A_n are dimensionless parameters, and the matrix M_n is a symmetric homogeneous polynomial of order n in C_H and of order n in $C_H{}^\dagger$. From the definition of C_H, it is seen that M_n must be a multiple of the unit matrix—again in contradistinction to the $SU(3)$ situation. Now, the zeroth-order terms are $SU(4)$ invariant.

There remains an apparent zeroth-order violation of parity, which may be transformed away because of the simple fashion of chiral $SU(4)$ breaking we have assumed. We simply define new quark fields

$$q_i'=(\alpha+\beta\gamma_5)q_i \tag{7}$$

with the real cutoff-dependent parameters α and β chosen so that the entire (bare plus zeroth-order) mass operator, in terms of q_i', is diagonal and parity conserving. The $SU(4)\times SU(4)$-invariant strong interactions are left unchanged. The procedure is analogous

[9] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[10] B. J. Bjorken and S. L. Glashow, Phys. Letters **11**, 255 (1964).

[11] All our results about the zero- and first-order selection rules are trivially extended to the case of an $SU(4)$-invariant strong interaction which consists of a neutral vector boson coupled to quark number, the so-called "gluon" model. The only results of this paper which might be affected by such an interaction are the universality conditions in Eq. (9).

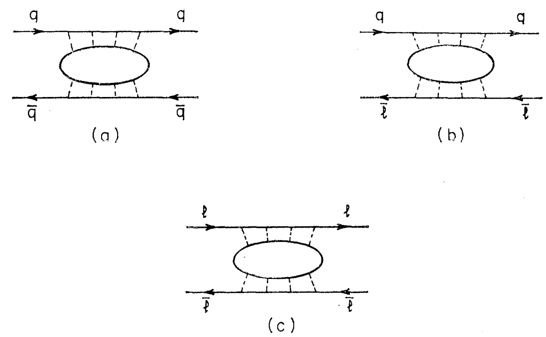

FIG. 1. (a) Connected part of the $q\bar{q}\to q\bar{q}$ amplitude. The crossed (annihilation) channel is also understood. (b) Connected part of the $q\bar{l}\to q\bar{l}$ amplitude. (c) Connected part of the $l\bar{l}\to l\bar{l}$ amplitude.

to that of Ref. 4, with the difference that the transformation (7) is $SU(4)$ invariant and does not change the definition of strangeness (or charm), or of the Cabibbo angle. An important consequence of the fact that M_n does not depend on the Cabibbo angle is that, unlike the situation in Ref. 4, it is impossible in our case to evaluate the Cabibbo angle by imposing a condition on the leading divergences. We conclude that zeroth-order weak effects are not significant.

We now consider the first-order $G(G\Lambda^2)^n$ terms which are of four types: (i) next-to-the-leading contributions to the quark and lepton mass operators, (ii) leading contributions to quark-quark or quark-antiquark scattering, (iii) leading contributions to quark-lepton scattering, and (iv) leading contributions to lepton-lepton scattering. Graphs with more than two external fermion lines yield no larger than second-order effects. Terms of type (i) are harmless: They contribute to observable nonleptonic $\Delta I=\frac{1}{2}$ processes, but since they cannot give $\Delta Y=2$, they do not produce a K_1K_2 mass splitting. On the other hand, type-(ii) diagrams could lead to $\mathfrak{N}\bar{\lambda}\to\bar{\mathfrak{N}}\lambda$, possibly giving rise to first-order contributions to the K_1K_2 mass difference, contrary to experiment. Let us show that they do not.

Graphs contributing to type (ii) effects are of the general form shown in Fig. 1(a), where the bubble includes any possible connections among the boson lines, and any number of closed fermion loops. The leading divergent contributions to q-$\bar{q}$ scattering from these graphs have the form

$$T_{HH}=G\sum_{n=2}^{\infty}B_n(G\Lambda^2)^{n-1}[\bar{q}\gamma_\mu(1+\gamma_5)\times B_H{}^{(n)}q\bar{q}\gamma^\mu(1+\gamma_5)B_H{}^{(n)\dagger}q], \quad (8)$$

where the B_n are finite dimensionless parameters independent of masses or momenta. It is clear that these first-order terms are independent of all external momenta. The matrix $B_H{}^{(n)}$ is a polynomial in C_H and $C_H{}^\dagger$ of order k and l, respectively, with $k+l\leq n$. Furthermore, the charge structure of the quark multiplets allows a change of charge no greater than unity, so that $|k-l|$ must be zero or one, and the matrices $B_H{}^{(n)}$ are easily computed (see the Appendix) to be

$$B_H{}^{(n)}=C_H \text{ or } C_H{}^\dagger \quad (k=l\pm1) \quad (8')$$

$$=[C_H,C_H{}^\dagger] \quad (k=l)\,. \quad (8'')$$

Thus, T_{HH} gives rise to contributions with $|\Delta Y|\leq1$ and, in particular, it does not yield a first-order K_1K_2 mass splitting. Of course, the next-to-the-leading divergences of these graphs will give $\Delta Y=2$, and do contribute to a second-order K_1K_2 mass difference, agreeing with experiment.

The leading divergences of types (iii) and (iv) give first-order contributions T_{HL} and T_{LL}, to semileptonic and leptonic processes. There will be a 1-to-1 correspondence among the graphs contributing to T_{LL}, T_{HL} [Figs. 1(b) and 1(c)], and T_{HH}. Because the algebraic properties of C_H and C_L are identical, we construct T_{HL} and T_{LL} from T_{HH} by the appropriate substitutions of $q\to L$ and $C_H\to C_L$.

In processes where the lepton charge changes, no violations of observed selection rules occur, but the first-order terms cause a renormalization of observed coupling constants. It is important to note that these renormalizations are common to leptonic and semileptonic processes, so that the relations

$$G_V(\Delta S=0)=G_\mu\cos\theta\,,\qquad G_V(\Delta S=1)=G_\mu\sin\theta \quad (9)$$

remain true when all first-order terms are included. This renormalization is given by the factor $1+\sum B_n(G\Lambda^2)^{n-1}$. A sufficient condition for these renormalizations to be common is the algebraic version of universality—a condition which is satisfied by our model, as well as by the usual three-quark model.

Next, we turn to the induced first-order couplings of hadrons to neutral lepton currents and self-couplings of neutral lepton currents. The neutral lepton currents are generated by the matrix $C_L{}^0$ and the neutral hadron currents by the matrix $C_H{}^0$, where

$$C_L{}^0=[C_L,C_L{}^\dagger]=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}=[C_H,C_H{}^\dagger]=C_H{}^0. \quad (10)$$

Evidently, there are no induced couplings of neutral lepton currents to strangeness-changing currents. The induced couplings involve the strangeness-conserving current

$$\begin{aligned}J_\mu{}^0&=\bar{q}\gamma_\mu C_H{}^0(1+\gamma_5)q+\bar{l}\gamma_\mu C_L{}^0(1+\gamma_5)l\\&=\bar{\mathcal{P}}'\gamma_\mu(1+\gamma_5)\mathcal{P}'+\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathcal{P}-\bar{\mathfrak{N}}\gamma_\mu(1+\gamma_5)\mathfrak{N}\\&\quad-\bar{\lambda}\gamma_\mu(1+\gamma_5)\lambda+\bar{\nu}'\gamma_\mu(1+\gamma_5)\nu'+\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\\&\quad-\bar{e}\gamma_\mu(1+\gamma_5)e-\bar{\mu}\gamma_\mu(1+\gamma_5)\mu\,. \quad (11)\end{aligned}$$

The coupling constant for this new neutral current-current interaction is a first-order expression of the form

$$G\sum_{n=2}^{\infty} C_n(G\Lambda^2)^{n-1}.$$

We anticipate that its strength should be comparable to the strength of the charged leptonic interactions. The new coupling plays no role in observed decay modes, but is should be detectable in accelerator experiments.

In Sec. II we discuss the possible extension of our model to a Yang-Mills model, where the coupling strength of the neutral W to its current is uniquely determined. These neutral lepton couplings constitute the most characteristic and interesting feature of our model. Relevant experimental evidence is discussed in Sec. IV.

II. YANG-MILLS MODEL OF WEAK INTERACTIONS

Divergences appear in our model of weak interactions, but they are properly ordered; observed selection rules are broken only in order $G^2(G\Lambda^2)^n$. But, the model is certainly not renormalizable. There is at least a possibility that a Yang-Mills model of weak interactions may be less singular.[12] In this section, we show how our model can be extended to include a symmetrically coupled triplet of W's. It is possible that W self-couplings can be introduced to give a complete Yang-Mills theory.

The Lagrangian with which we work may be written, in the four-quark model, without electromagnetism,

$$\mathcal{L}=\mathcal{L}_{\text{kin}}+\mathcal{L}_s+\mathcal{L}_M+\mathcal{L}_w\,, \qquad (12)$$

where $\mathcal{L}_{\text{kin}}$ is the purely kinematic term

$$\mathcal{L}_{\text{kin}}=\bar{q}\gamma\cdot pq+\bar{l}\gamma\cdot pl+G_{\mu\nu}G^{\mu\nu}+W_{\mu\nu}{}^{\dagger}W^{\mu\nu} \qquad (13)$$

describing four free massless quarks, four leptons, and their strong and weak intermediaries ($X_{\mu\nu}$ denotes the antisymmetric curl of X_μ). $\mathcal{L}_s$ denotes the $SU(4)$-invariant strong interaction, most simply

$$\mathcal{L}_s=fG_\mu\bar{q}\gamma^\mu q\,, \qquad (14)$$

and $\mathcal{L}_w$ is the weak interaction

$$\mathcal{L}_w=gW_\mu{}^{\dagger}[\bar{q}C_H\gamma^\mu(1+\gamma_5)q+\bar{l}C_L\gamma^\mu(1+\gamma_5)l+\text{H.a.}]\,. \qquad (15)$$

The bare-mass term $\mathcal{L}_M$ produces the observed masses of the leptons, the masses of W and G, and gives rise to the observed hierarchy of hadron symmetry,

$$\mathcal{L}_M=\bar{q}M_Hq+\bar{l}M_Ll+m^2G_\mu G^\mu+M^2W_\mu W^\mu\,, \qquad (16)$$

where M_H and M_L are 4×4 matrices. This model gives a complete description of weak-interaction phenomena. The most important new feature is the appearance of neutral currents generated by the most divergent parts of diagrams containing an exchange of W^+, W^- pairs between two fermion lines. The effective coupling strength of these currents is expected to be of order G but, at this stage, we cannot predict its precise numerical value since we are unable to sum the perturbation series. In order to extend this model to a more symmetric one, we introduce an additional weak intermediary W_0 with appropriate couplings.

The couplings of W_0 to hadrons and leptons must be taken to be

$$2^{-1/2}gW_0{}^\mu\{\bar{q}[C_H{}^{\dagger},C_H]\gamma_\mu(1+\gamma_5)q+\bar{l}[C_L{}^{\dagger},C_L]\gamma_\mu(1+\gamma_5)l\}\,. \qquad (17)$$

We emphasize that the introduction of W_0 is by no means necessary in our model; however, we think that it gives a much more symmetric and aesthetically appealing theory.

In the conventional model of weak interactions, the extension to a three-component vector-meson theory cannot be made without contradicting experiment: The neutral boson leads to strangeness-changing decays involving neutral-lepton currents and to $\Delta S=2$ at order G. This is because the commutator of the conventional weak charge with its adjoint yields a strangeness-violating neutral charge. In our case, the corresponding operator is diagonal, and these difficulties are absent.

It is straightforward to show that the introduction of the neutral current does not spoil the proper ordering of divergences: The observed selection rules are preserved by all terms of order $G(G\Lambda^2)^n$. This is shown in the Appendix.

We note that W_0 is coupled to precisely the same neutral current appearing in the last section as an induced coupling. In the symmetric three-W model, its strength is uniquely predicted. Universality now applies to both charged and neutral couplings. That is to say, the leading divergent corrections to each are the same. The bare relationship

$$G_0=\tfrac{1}{2}G \qquad (18)$$

is preserved by the renormalizations, to first order [i.e., including all terms of order $G(G\Lambda^2)^n$]. This assertion is proved in the Appendix.

The introduction of a neutral W opens the possibility of formulating the weak interactions into a Yang-Mills theory. Self-couplings must be introduced among the W triplet in order to ensure the gauge symmetry. This is accomplished if we choose the Lagrangian in a manifestly gauge-invariant fashion:

$$\mathcal{L}=\bar{q}\gamma\Pi_Hq+\bar{l}\gamma\Pi_Ll+\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}+G_{\mu\nu}G^{\mu\nu}+\mathcal{L}_M+\mathcal{L}_s\,, \qquad (19)$$

where

$$\Pi_H{}^\mu=\partial^\mu+ig(\mathbf{C}_H\cdot\mathbf{W}^\mu)(1+\gamma_5)\,, \qquad (19')$$

$$\Pi_L{}^\mu=\partial^\mu+ig(\mathbf{C}_L\cdot\mathbf{W}^\mu)(1+\gamma_5)\,, \qquad (19'')$$

[12] See, for example, S. Mandelstam, Phys. Rev. **175**, 1580 (1969); M. Veltman, Nucl. Phys. **B7**, 637 (1968); H. Reiff and M. Veltman, *ibid.* **B13**, 545 (1969); D. Boulware, Ann. Phys. (N. Y.) **56**, 140 (1970); A. A. Slavnor, University of Kiev Report No. ITP 69/20 (unpublished); E. S. Fradkin and I. V. Tyutin, Phys. Letters **30B**, 562 (1969). Notice, however, that none of these references consider the far more difficult case of vector mesons coupled to nonconserved currents.

TABLE I. Quark quantum numbers.

	Fractional assignment			Integral assignment		
	Q	Y	$\mathcal{C}$	Q	Y	$\mathcal{C}$
$\mathcal{P}'$	$\frac{2}{3}$	$-\frac{2}{3}$	1	0	0	0
$\mathcal{P}$	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	-1
$\mathfrak{N}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	-1	0	-1
λ	$-\frac{1}{3}$	$-\frac{2}{3}$	0	-1	-1	-1

and

$$W^{\mu\nu}=\Pi_W{}^{\mu}W^{\nu}-\Pi_W{}^{\nu}W^{\mu}, \qquad (19''')$$

where

$$(\Pi_W{}^{\mu})_{ij}=\delta_{ij}\partial^{\mu}+ig2^{-1/2}(\mathbf{t}\cdot\mathbf{W}^{\mu})_{ij}. \qquad (19'''')$$

The matrix-valued vectors $\mathbf{C}_H$ and $\mathbf{C}_L$ have components $(C,C^{\dagger},2^{-1/2}[C^{\dagger},C])$ in a basis where charge is diagonal, and $\mathbf{t}$ are the usual 3×3 generators of $O(3)$, with t_3 diagonal. The gauge group thus introduced is an exact symmetry of the entire Lagrangian excepting both $\mathcal{L}_M$ and electromagnetism.

The Yang-Mills model is undoubtedly the most attractive way to couple a triplet of vector mesons and the only one for which people have expressed some hope of constructing a renormalizable theory. The massless case has been proved to be renormalizable[12]; however, very little is known about the physically more interesting massive theory. In fact, the naive power counting shows that the highest divergence in a Yang-Mills theory is $g^{2n}\Lambda^N$ with $N=6n$. Notice that in the absence of the self-couplings the corresponding divergences are given, as we have already seen, by $N=2n$. So, at first sight, the Yang-Mills theory seems to be much more divergent than the ordinary coupling of the vector mesons with the currents. However, one can show that the naive limit $N=6n$ can be considerably lowered. We have already been able to show that $N\leq 3n$ and we believe that one can still lower this limit to at least $N=2n$. In other words, we believe that the introduction of the self-couplings does not make the theory more divergent.

Let us briefly consider a more daring speculation. It has long been suspected[13] that there may be a fundamental unity of weak and electromagnetic interactions, reflected phenomenologically by the common vectorial character of their couplings. For this reason, it may have been wrong for us to introduce a gauge symmetry for the weak interactions not shared by electromagnetism. As a more speculative alternative, consider the possibility of a four-parameter gauge group involving $\mathbf{W}$, and an additional Abelian singlet W_S, broken only by the mass term $\mathcal{L}_M$. Suppose, however, that a one-parameter gauge symmetry, corresponding to a linear combination A of W_0 and W_S remains unbroken. Then A must be massless, and may be identified as the photon. The orthogonal neutral combination B is massive, and acts as an intermediary of weak interactions along with $W^{\pm}$. This model could be correct only if the weak bosons are very massive (100 GeV) so that the weak and electromagnetic coupling constants could be comparable. With this model, the relation (18) would not persist, and the weak neutral current would involve $(1-\gamma_5)$ as well as $(1+\gamma_5)$ currents. The precise form of the model would depend on what linear combination of W_0 and W_S is the photon.

[13] J. Schwinger, Ann. Phys. (N. Y.) **2**, 407 (1957); S. L. Glashow, Nucl. Phys. **10**, 107 (1959); **22**, 579 (1961).

III. ANOTHER QUARK MAKES $SU(4)$

Having introduced four quarks, we must consider strong interactions which admit the algebra of chiral $SU(4)$. Does this mean we should expect $SU(4)$ to be an approximate symmetry of nature? Nothing in our argument depends on how much $SU(4)$ is broken; the divergences are necessarily properly ordered. However, for the higher-order nonleading divergences to be as small as they must be, the breaking of $SU(4)$ cannot be too great: The limit on the cutoff Λ is replaced by a limit on Δ, a parameter measuring $SU(4)$ breaking; and from the observed K_1K_2 mass difference we now conclude that Δ must be not larger than 3–4 GeV. Thus, some residue of $SU(4)$ symmetry should persist.

We expect the appearance of charmed hadron states.[10] Meson multiplets, made up of a quark-antiquark pair, must belong to the 15-dimensional adjoint representation of $SU(4)$, consisting of an uncharmed $SU(3)$ singlet and octet, as well as two $SU(3)$ triplets of charm ±1. The structure of baryons depends on the quantum numbers assigned to the quarks. The two simplest possibilities are shown in Table I. For the more conventional fractional charge assignment, the baryons are made up of three quarks, and must belong to one of the representations contained in $4\times4\times4$. The only possibility is a 20-dimensional representation, which contains, besides the baryon octet, a triplet and sextet of charmed states and a doubly charmed triplet. The $j=\frac{3}{2}^+$ baryon decuplet belongs to another 20-dimensional representation with a charmed sectet, a doubly charmed triplet, and a triply charmed singlet.

With the integral-charge assignment, the baryon octet must be made of two quarks and an antiquark, the decuplet of three quarks and two antiquarks. The lepton and quark charged spectra now coincide, and the synthesis of weak and electromagnetic interactions appears more plausible. Moreover, there is no difficulty in obtaining the correct value for the π^0 lifetime.

Why have none of these charmed particles been seen? Suppose they are all relatively heavy, say ~2 GeV. Although some of the states must be stable under strong (charm-conserving) interactions, these will decay rapidly ($\sim10^{+13}$ sec^{-1}) by weak interactions into a very wide variety of uncharmed final states (there are about a hundred distinct decay channels). Since the charmed particles are copiously produced only in associated production, such events will necessarily be of very complex topology, involving the plentiful decay products of both charmed states. Charmed particles could easily have escaped notice.

Finally, we briefly comment on the leptonic decay rates of ρ, ω, and ϕ (Γ_ρ, Γ_ω, and Γ_ϕ). Our electric current contains $SU(3)$ singlet as well as octet terms, so that the inequality

$$m_\omega \Gamma_\omega + m_\phi \Gamma_\phi \geq \tfrac{1}{3} m_\rho \Gamma_\rho \qquad (20)$$

may be deduced from the Weinberg spectral function sum rules and ω, ϕ, ρ dominance.[14] A stronger result is obtained if we extend Weinberg's Schwinger-term hypothesis to the vector currents of $SU(4)$:

$$m_\omega \Gamma_\omega + m_\phi \Gamma_\phi \geq m_\rho \Gamma_\rho . \qquad (21)$$

This result is in poor agreement with experiment, which favors the equality in (20). A resolution of this difficulty that does not abandon the Schwinger-term symmetry requires the introduction of a third $Y=T=0$ vector meson, another partner of ω and ϕ, corresponding to the $SU(4)$ singlet vector current.

IV. EXPERIMENTAL SUGGESTIONS

In this section, we discuss some of the observable effects characteristic of our picture of strong and weak interactions. Firstly, consider the experimental implications of the existence of a new quantum number—charm—broken only by weak interactions. The charmed particles, because they are heavy, are too short lived to give visible tracks. However, they should be copiously produced in hardonic collisions at accelerator energies:

$$(\text{hadron or } \gamma) + (\text{hadron}) \to X^{(+)} + X^{(-)} + \cdots ,$$

where $X^{(\pm)}$ are oppositely charmed particles, each rapidly decaying into uncharmed hadrons with or without a charged lepton pair. The purely hadronic decay modes could provide illusory violations of hypercharge conservation in strong interactions. The leptonic decay modes provide a mechanism for the seemingly direct production of one or two charged leptons in hadron-hadron collisions.[15] Conceivably, muons thus produced may be responsible for the anomalous observed angular distribution of cosmic-ray muons in the 10^{12}-eV range,[16] where these directly produced muons may dominate the sea-level muon flux.

Should this last speculation about cosmic rays be correct, we need to revise radically estimates of the flux of ν and $\bar{\nu}$ in this energy range. We expect the charmed particle decays to yield equal numbers of each lepton variety; this gives a flux of electron neutrinos and antineutrinos equal to the muon flux, and 10–100 times greater than other estimates. This fact is of crucial importance to the possible detection of the resonance scattering[17]

$$\bar{\nu} + e^- \to \bar{\nu}' + \mu^- .$$

Charmed particles may be produced singly by neutrinos in such reactions as

$$\nu' + N \to \mu^- + X , \quad \bar{\nu}' + N \to \mu^+ + X ,$$

where the charmed particle X would have a variety of decay modes, including leptonic ones. With the fractional charge assignment, the neutrino processes are suppressed by $\sin^2\theta$ and the antineutrino processes are forbidden. On the other hand, with the integral-charge assignment, the neutrino processes are again proportional to $\sin^2\theta$ while the antineutrino processes are proportional to $\cos^2\theta$.

The second new feature of our model is the appearance of neutral leptonic and semileptonic couplings involving a specified ($Y=0$) current and with a coupling constant comparable with the Fermi constant. Without the introduction of a W_0, we may say only $G_0 \sim G$. To be more definite, we shall phrase our arguments in terms of the value $G_0 = \frac{1}{2} G$ of Eq. (18).

Let us summarize the presently available data about these interactions.[18] Consider the following three reactions induced by muon neutrinos:

$$\text{(i)} \quad \nu' + e^- \to \nu' + e^- ,$$
$$\text{(ii)} \quad \nu' + p \to \nu' + p ,$$
$$\text{(iii)} \quad \nu' + p \to \nu' + \pi^+ + n .$$

None of these neutral couplings have been observed; experimentally, we can only quote limits. From the absence of observed forward energetic electrons in the CERN bubble-chamber experiments, we may conclude

$$G_0 \leq G ,$$

a limit which is close to but consistent with our prediction.

For reaction (ii), it is found that

$$R = \sigma(\nu' p \to \nu' p) / \sigma(\bar{\nu}' p \to \mu^+ n) \leq 0.5 .$$

Because our neutral current contains both $I=0$ and $I=1$ parts, we cannot unambiguously predict this ratio. In a naive quark model, where the proton consists of only $\mathfrak{N}$ and $\mathcal{P}$ quarks, we find $R = \frac{1}{4}$, again close but consistent.

Finally, we quote the experimental limit on reaction (iii):

$$R' = \sigma(\nu' + p \to \pi^+ + n + \nu') / \sigma(\nu' + p \to \pi^+ + p + \mu^-) \leq 0.08 .$$

[14] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. Mathur, and S. Okubo, *ibid.* **19**, 470 (1967).

[15] In a recent experiment, P. J. Wanderer *et al.* [Phys. Rev. Letters **23**, 729 (1969)] have performed a search for W's by measuring the intensity and polarization of prompt energetic muons from the interaction of 28-GeV protons with nuclei. Their results are compatible with the assumption that all 25-GeV prompt muons have electromagnetic origin. There is no indication of the existence of W's. However, the published evidence does not seem to be relevant to the existence of charmed particles, which are produced in pairs, decay into many final states, and are not expected to produce many very energetic muons.

[16] H. E. Bergeson *et al.*, Phys. Rev. Letters **21**, 1089 (1968).

[17] M. G. K. Menon *et al.*, Proc. Roy. Soc. (London) **A301**, 137 (1967).

[18] See D. H. Perkins, in Proceedings of the Topical Conference in Weak Interactions, CERN, 1969 [CERN Report No. 69-7], pp. 1–42 (unpublished).

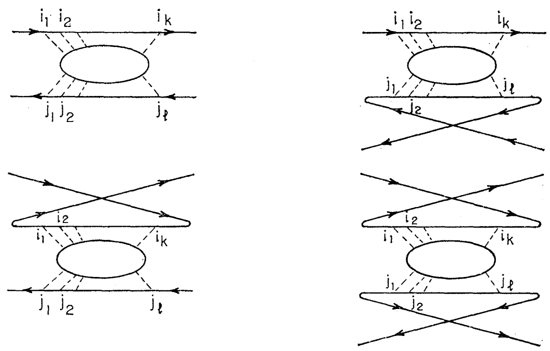

FIG. 2. Decomposition of the $q\bar{q}\rightarrow q\bar{q}$ connected amplitude by crossing the external fermion lines.

Because this transition is $\Delta I=1$, we unambiguously predict $R'=\frac{1}{9}$ under the hypothesis of $\Delta(1238)$ dominance. In each of these three reactions, experiment is very close to a decisive test of our model.

In our model, the parity-violating nonleptonic interaction is also changed. In particular, the parity-violating one-pion-exchange nuclear force is no longer suppressed by $\sin^2\theta$.

Next we consider some experiments which could discover the existence of W_0. A simple and attractive possibility is the search for muon tridents in the semiweak reaction[19]

$$\mu^- + Z \rightarrow \mu^- + W_0 + Z\,,$$

with the subsequent muonic decay of W_0. Another possibility is the reaction[20]

$$e^+e^- \rightarrow \mu^+\mu^-\,.$$

The interference between the W^0 and photon contributions causes an asymmetry of the μ^+ angular distribution relative to the momentum of the incident e^+ given by

$$\delta=\frac{N_F-N_B}{N_F+N_B}=\frac{3M_W{}^2}{16\sqrt{2}}\,\frac{G}{\alpha\pi}\,\frac{s}{s-M_W{}^2}\,,$$

where

$$G=10^{-5}M_p{}^{-2}\,,\quad \alpha=1/137\,,\quad \text{and}\quad s=4E_e{}^2\,.$$

Away from the W^0 pole, the effect is rather small (less than 1% for $E_e=3.5$ GeV) and it is masked by a similar effect due to the two-photon contribution. However, the factor $s/(s-M_W{}^2)$ makes the asymmetry increase sharply and change sign near M_W. Therefore, this reaction is an excellent tool to sweep a substantial mass range looking for W's. Another effect, much harder to detect, would be the direct observation of parity violation in $e^+e^-\rightarrow\mu^+\mu^-$. This requires the measurement of μ polarization.

Finally, we recall from Sec. III that the $SU(4)$ description of leptonic decays of vector mesons suggested the existence of another strongly coupled neutral $I=0$ vector meson with considerable coupling to lepton pairs. Evidence for its existence could come from colliding beam experiments.

[19] M. Tannenbaum (private communication).

[20] N. Cabibbo and R. Gatto, Phys. Rev. **129**, 1577 (1961).

APPENDIX

In this appendix we determine the form of the leading weak corrections to the q-$\bar{q}$, q-$\bar{l}$, and l-$\bar{l}$ amplitudes.

We have already shown that the wave-function renormalization of spinors is the same for both quarks and leptons and contributes a common factor to T_{HH}, T_{HL}, and T_{LL}. Therefore we need consider only the q-$\bar{q}$ amplitude T_{HH}. The other amplitudes T_{HL} and T_{LL} can be obtained from T_{HH} by appropriate substitutions. In the following, we shall omit the common wave-function renormalization factors.

For the sake of clarity, let us first consider our model of weak interactions, where we have three vector bosons symmetrically coupled.

The graphs of Fig. 1(a) can be decomposed into four classes of terms, obtained by keeping the boson lines fixed and reversing the fermion lines, as shown in Fig. 2. We then obtain for the contribution to T_{HH} corresponding to these four classes of diagrams

$$\begin{aligned}T_{HH}{}^{(n,k,l)}&=\bar{q}\gamma_\mu(1+\gamma_5)[C_{i_1}C_{i_2}\cdots C_{i_k}-(-1)^kC_{i_k}C_{i_{k-1}}\cdots C_{i_1}]q\\&\quad\times P_{j_1\cdots j_l;\,i_1\cdots i_k}\bar{q}^\mu(1+\gamma_5)\\&\qquad\times[C_{j_1}C_{j_2}\cdots C_{j_l}-(-1)^lC_{j_l}C_{j_{l-1}}\cdots C_{j_1}]q\,,\\&\qquad k+l\leq n\,,\quad k\,,l\geq 1\,.\end{aligned}\qquad\text{(A1)}$$

All the i's and j's go from 1 to 3 and the sum over all indices is understood. $P_{j_1\cdots j_l;\,i_1\cdots i_k}$ is a tensor made out of the invariant tensors δ_{ij} and ϵ_{ijk}.

It is easy to show that for any k

$$\mathrm{Tr}[C_{i_1}C_{i_2}\cdots C_{i_k}-(-)^kC_{i_k}C_{i_{k-1}}\cdots C_{i_1}]=0\,.\qquad\text{(A2)}$$

Therefore, since the interaction is $O(3)$ invariant, the connected part of T_{HH} has the form

$$\begin{aligned}T_{HH}&=G\sum_{n=0}^{\infty}b_n(G\Lambda^2)^n\\&\quad\times(\bar{q}\gamma_\mu(1+\gamma_5)\mathbf{C}_Hq)\cdot(\bar{q}\gamma^\mu(1+\gamma_5)\mathbf{C}_Hq)\,.\end{aligned}\qquad\text{(A3)}$$

In the case where we have only charged bosons, the argument is even simpler. Each of the indices $i_1\cdots i_k$, $j_1\cdots j_l$ appearing in Eq. (A1) takes only two possible values. With the relations

$$\begin{aligned}(C_H)^2&=(C_H{}^\dagger)^2=0\,,\\(C_HC_H{}^\dagger)^2&=C_HC_H{}^\dagger\,,\\(C_H{}^\dagger C_H)^2&=C_H{}^\dagger C_H\,,\end{aligned}\qquad\text{(A4)}$$

Eq. (A1) explicitly reads

$$\begin{aligned}T_{HH}{}^{(n;k,l)}&=(\bar{q}\gamma_\mu(1+\gamma_5)[C_H,C_H{}^\dagger]q)\\&\qquad\times(\bar{q}\gamma^\mu(1+\gamma_5)[C_H,C_H{}^\dagger]q)\,,\quad k=l\\T_{HH}{}^{(n,k,l)}&=(\bar{q}\gamma_\mu(1+\gamma_5)C_Hq)(\bar{q}\gamma^\mu(1+\gamma_5)C_H{}^\dagger q)\,,\\&\qquad k=l+1\,.\quad \text{Q.E.D.}\end{aligned}$$