

1 The four mesons

Explain why the following systems are irrelevant to flavor oscillations:

1. $B^+ - B^-$
2. $K - K^*$
3. $T - \bar{T}$ (a T is a meson made out of a t and a \bar{u} quarks.)
4. $K^* - \bar{K}^*$ oscillation

2 Mixing formalism

In this question, you are asked to develop the general formalism of meson mixing.

1. Show that the mass and width differences are given by

$$4(\Delta m)^2 - (\Delta\Gamma)^2 = 4(4|M_{12}|^2 - |\Gamma_{12}|^2), \quad \Delta m\Delta\Gamma = 4\Re(M_{12}\Gamma_{12}^*) \quad (1)$$

and that

$$\left| \frac{q}{p} \right| = \left| \frac{\Delta m - i\Delta\Gamma/2}{2M_{12} - i\Gamma_{12}} \right|. \quad (2)$$

2. When CP is a good symmetry all mass eigenstates must also be CP eigenstates. Show that CP invariance requires $|q/p| = 1$.
3. In the limit $\Gamma_{12} \ll M_{12}$ show that

$$\Delta m = 2|M_{12}|, \quad \Delta\Gamma = 2|\Gamma_{12}| \cos\theta, \quad \left| \frac{q}{p} \right| = 1. \quad (3)$$

4. In class we derived the expressions for $\Gamma_{B^0 \rightarrow f}(t)$ and $\Gamma_{\bar{B}^0 \rightarrow f}(t)$ in the limit $y \ll 1, x$. Find the general formulae for these rates.
5. Show that when $\Delta\Gamma = 0$ and $|q/p| = 1$

$$\begin{aligned} \Gamma_{B \rightarrow X\ell^- \bar{\nu}}(t) &= e^{-\Gamma t} \sin^2(\Delta mt/2), \\ \Gamma_{B \rightarrow X\ell^+ \nu}(t) &= e^{-\Gamma t} \cos^2(\Delta mt/2). \end{aligned} \quad (4)$$

3 Kaon system

1. Explain why $y_K \approx 1$.
2. In a hypothetical world where we could change the mass of the kaon without changing any other masses, how would the value of y_K change if we made m_K smaller or larger.

4 Mixing beyond the SM

Consider a model without a top quark, in which the first two generations are as in the SM, while the left-handed bottom (b_L) and the right-handed bottom (b_R) are $SU(2)$ singlets.

1. Draw a tree-level diagram that contributes to $B - \bar{B}$ mixing in this model.
2. Is there a tree-level diagram that contributes to $K - \bar{K}$ mixing?
3. Is there a tree-level diagram that contributes to $D - \bar{D}$ mixing?