Theoretical overview of flow

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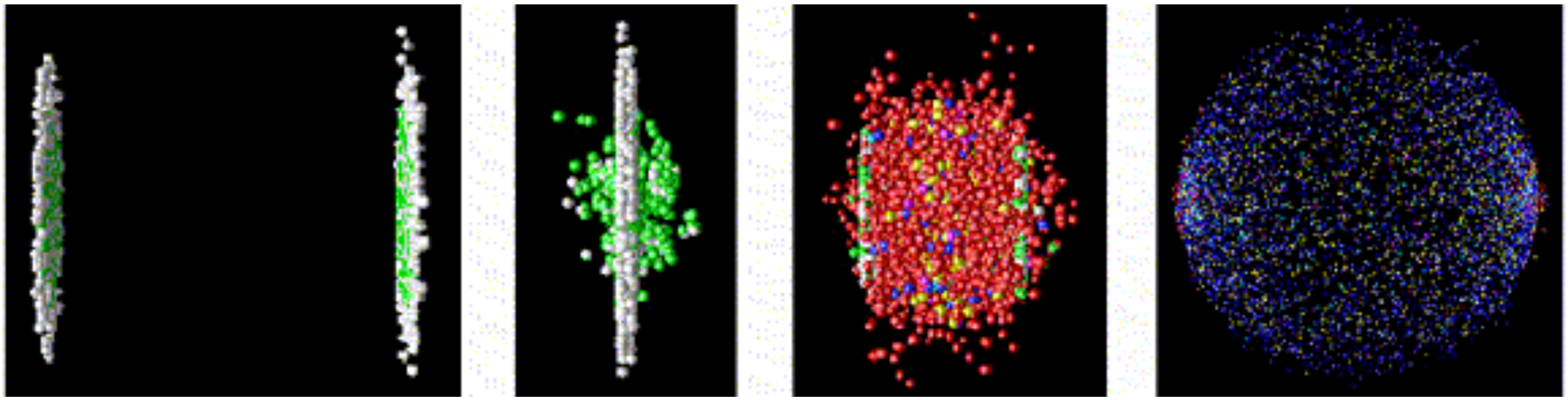
Ultrarelativistic nuclear collisions

- At the LHC: collisions between lead atomic nuclei Pb^{82+} (Pb-Pb runs in 2010, 2011, 2015)
- proton-Pb collisions in 2012 and 2013

The other facility : RHIC

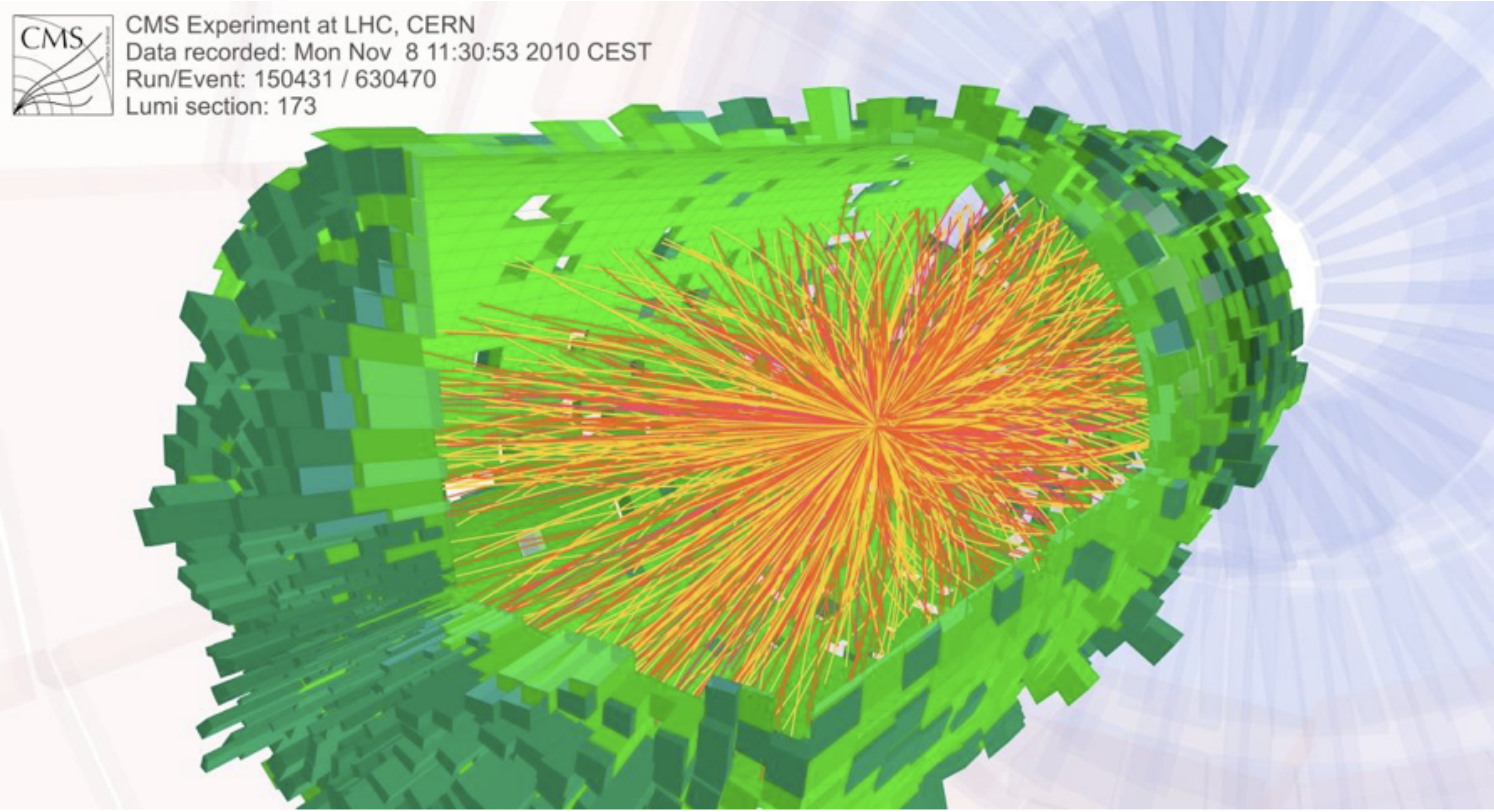
- The only dedicated heavy-ion collider, at Brookhaven (USA), running since 2000.
- Energy per nucleon-nucleon collision:
  RHIC: up to 200 GeV
  LHC: 5.02 TeV

Nuclear collisions at the LHC



- Lorentz contraction factor ~2700: colliding thin pancakes
- *A deconfined quark-gluon plasma* is created and expands into the vacuum
- The best theoretical description of the expansion is a macroscopic one: a small lump of relativistic fluid (v~c), T ~ 200-300 MeV

What we see



Typical Pb-Pb collision "event" at the LHC

The flow paradigm (2010)

- Particles are emitted independently in every event.
- In a hydrodynamic description, the momenta p of outgoing particles are sampled independently from an underlying probability distribution f(p)
- The fluctuations of this single-particle distribution f(p) event to event create the specific correlations seen experimentally.

Alver & Roland arXiv:1003.0194

Consequences of the flow paradigm

- The flow paradigm alone strongly constrains observables, irrespective of any particular hydrodynamic description.

- For instance, the flow paradigm implies that the two-particle correlation is $\langle f(p_1)f(p_2)\rangle - \langle f(p_1)\rangle\langle f(p_2)\rangle$ where $\langle\ldots\rangle$=average over events.

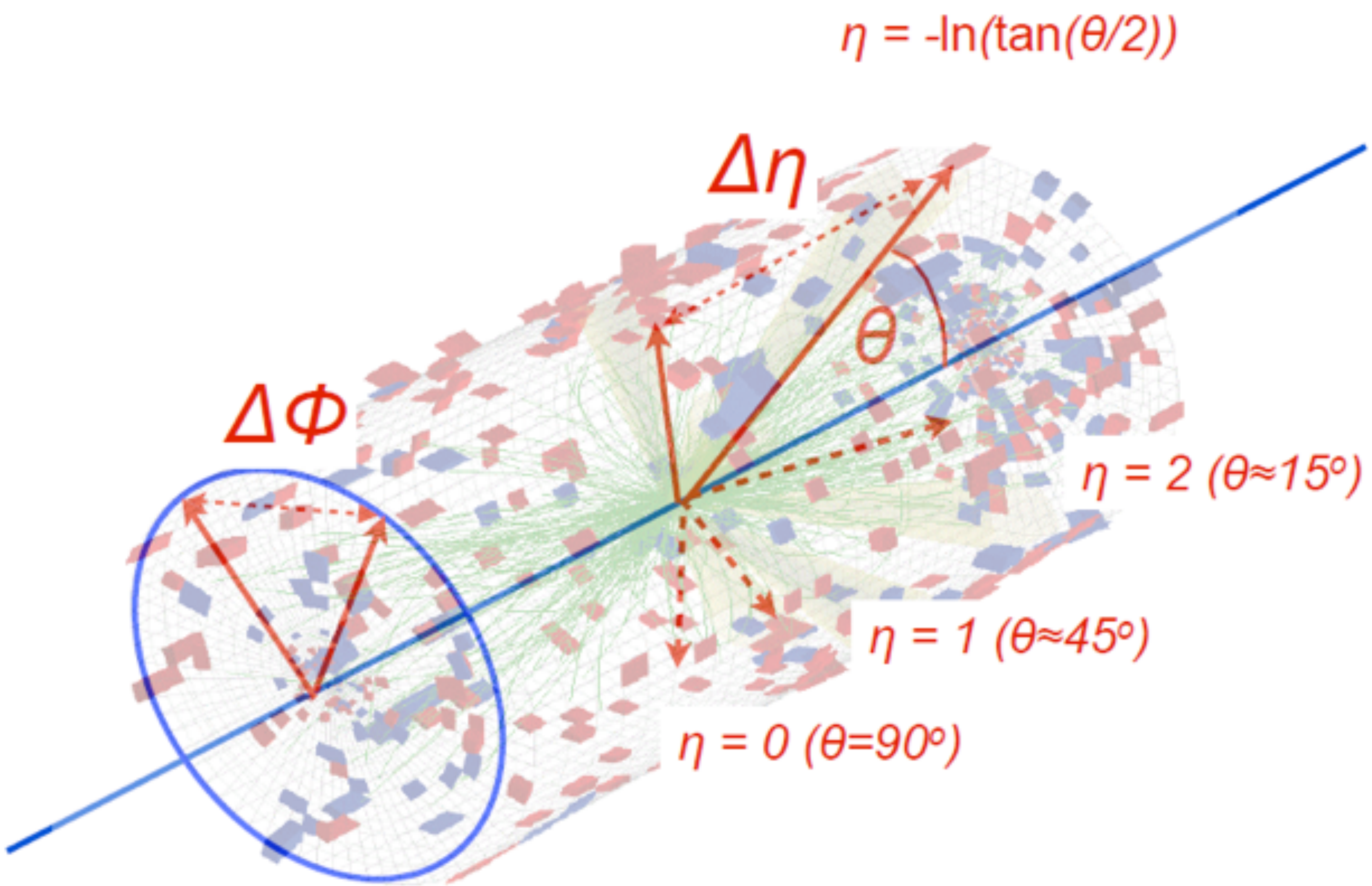
- Viewed as a matrix as a function of p_1 and p_2, it is a covariance matrix, hence its eigenvalues are all positive: a property that can be checked on data.

Bhalerao, JYO, Pal, Teaney, arXiv:1410.7739

More consequences

- Experimentally, higher-order correlations are easily measured (8-particle correlations, for instance) because of the large multiplicity & combinatorics.
- The consequences of the flow paradigm for higher-order correlations are not yet fully explored.
- I will present recent applications of the flow paradigm to higher-order correlations.

Pseudorapidity, azimuth



- Trajectories of charged particles:
- polar angle θ (or rapidity η=-ln tan θ/2)
- azimuthal angle φ

Anisotropic flow

- In a single event, azimuthal symmetry is broken. The φ distribution can be written as

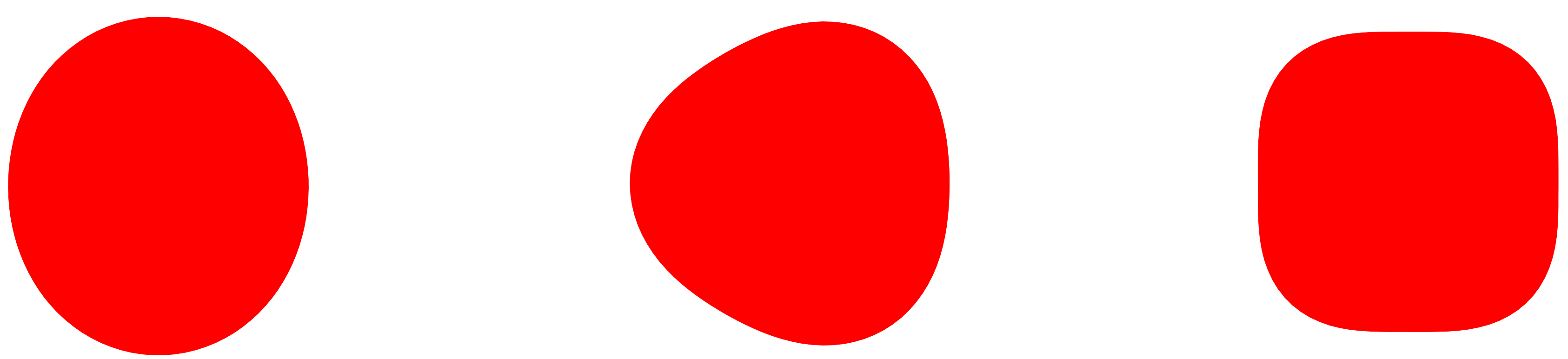
$$f(\varphi)=\sum_n V_n\, e^{-in\varphi}$$

- V_n=Fourier coefficient=anisotropic flow
- $f(\varphi)$ real $\Rightarrow$ $V_{-n}=V_n^*$; normalisation $V_0=1$
- Transformation under rotation

$$\varphi \to \varphi+\alpha$$
$$V_n \to V_n\, e^{in\alpha}$$

Anisotropic flow

- Flow paradigm: implies that all information on correlations is contained in V_n and its event-to-event fluctuations.



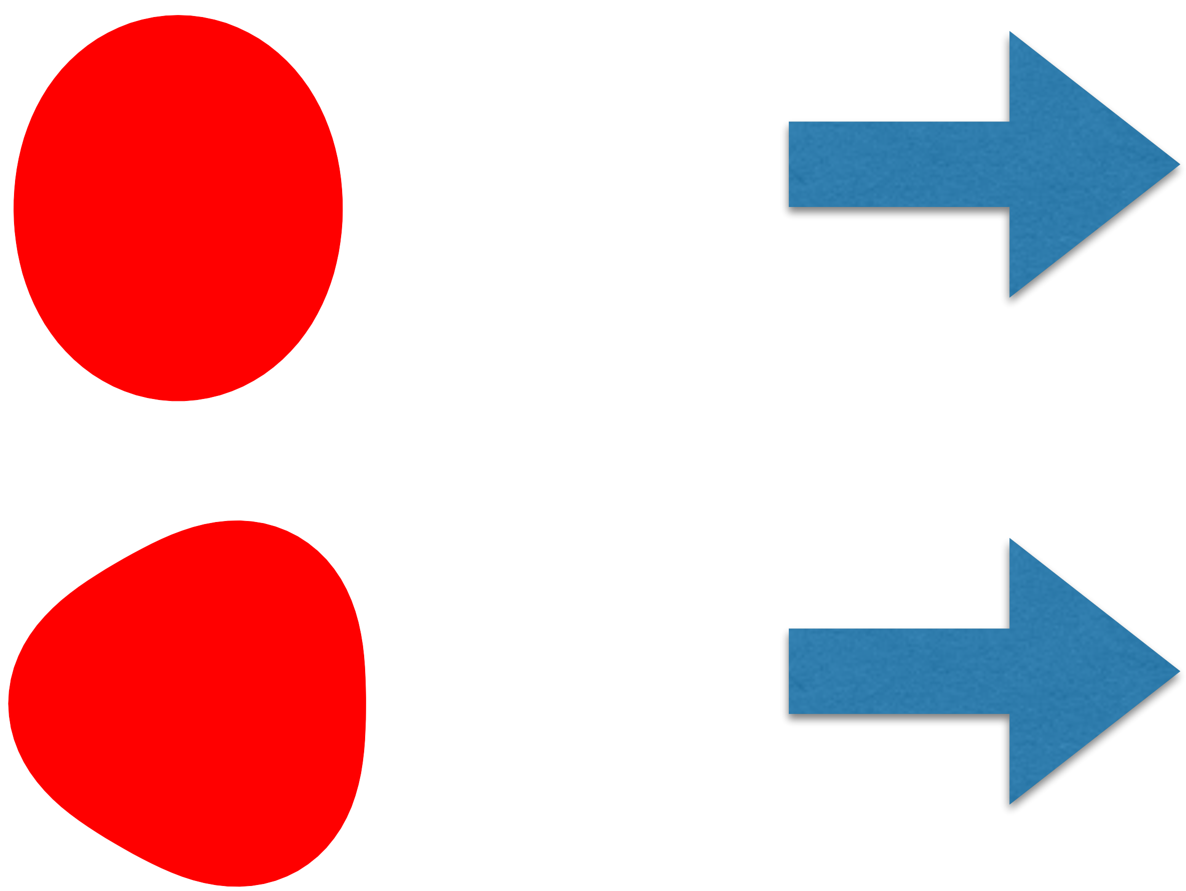
v_2 v_3 v_4

Anisotropic flow and hydrodynamics

Initial transverse density profile

Expansion

Final distribution



Elliptic flow V_2

Triangular flow V_3

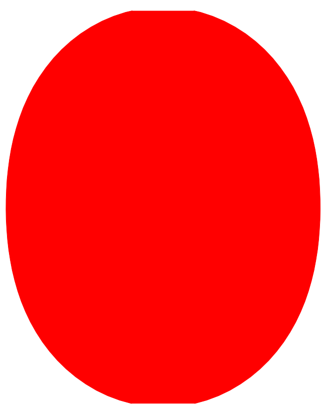
In hydrodynamics, anisotropic flow is a response to the anisotropy of the initial density profile.

Anisotropic flow and hydrodynamics

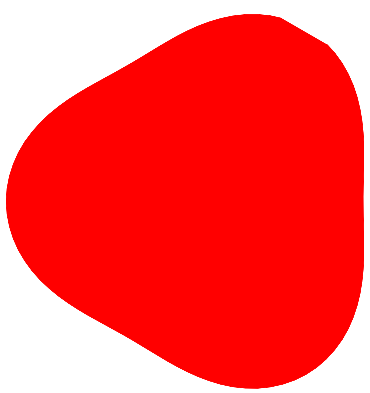
Initial transverse density profile

Expansion

Final distribution



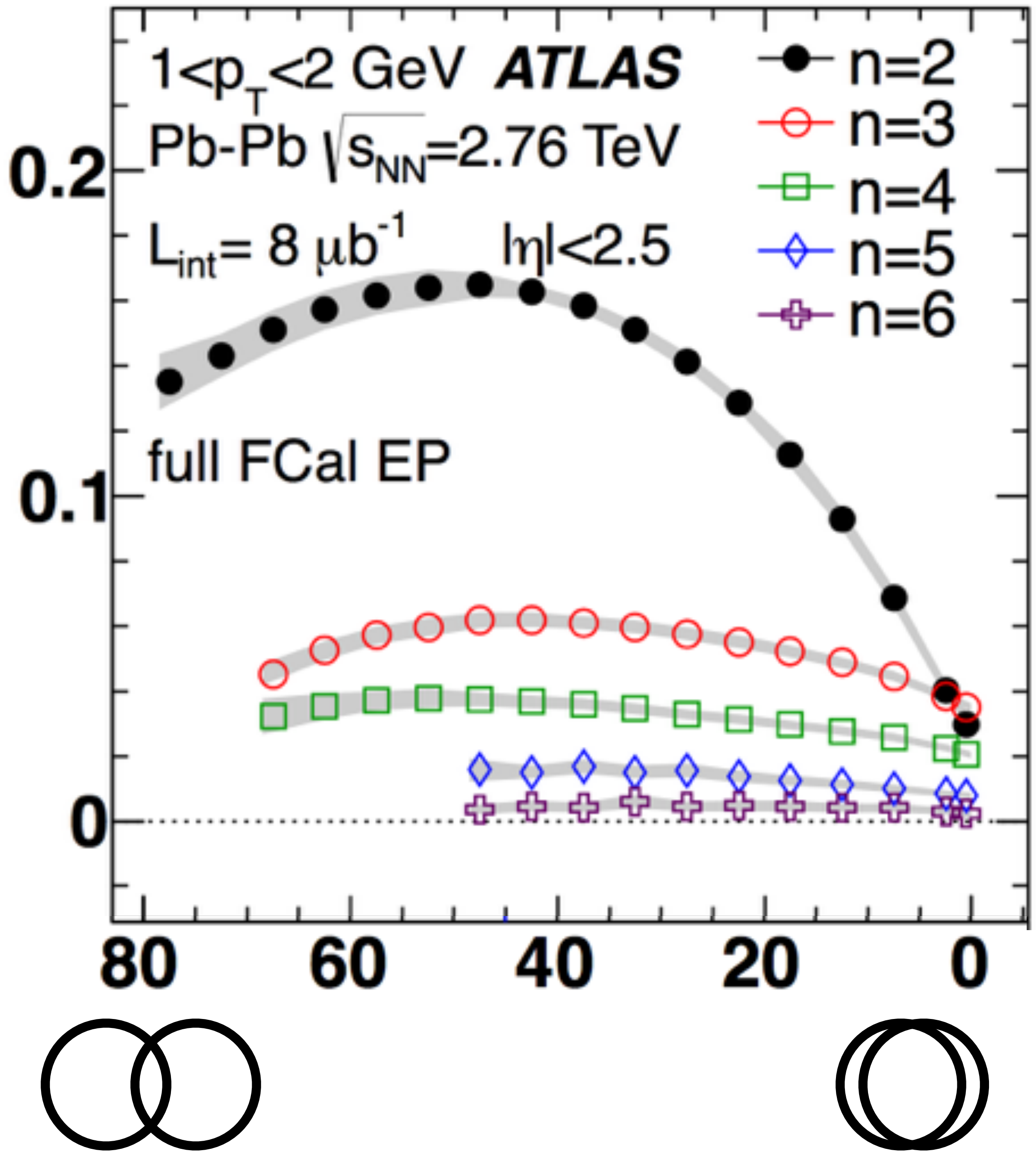
Elliptic flow V_2



Triangular flow V_3

But initial density is poorly constrained theoretically: major uncertainty in hydro calculations

The centrality dependence of v_n



- Root-mean-square values of v_n.
- Largest Fourier harmonic is elliptic flow, v_2
- Steep decrease of v_2 for central collisions: reflects the elliptic geometry of the overlap area
- At the qualitative level, the centrality dependence of v_n is naturally explained by hydrodynamics.

1. Eliminating the sensitivity to the initial state with higher-order harmonics

Yan, JYO, 1502.02502

The origin of higher harmonics

- Rotational symmetry allows a nonlinear coupling with lower-order harmonics

$$V_4 = \chi_4 (V_2)^2 + \ldots$$

- In hydrodynamics, the nonlinear coupling χ_4 is essentially independent of the initial state

Borghini JYO nucl-th/0506045

Teaney Yan 1206.1905

The origin of higher harmonics

- Rotational symmetry allows a nonlinear coupling with lower-order harmonics

$$V_4 = \chi_4 (V_2)^2 + \ldots$$
$$V_5 = \chi_5 V_2 V_3 + \ldots$$
$$V_6 = \chi_{62} (V_2)^3 + \chi_{63} (V_3)^2 + \ldots$$
$$V_7 = \chi_7 (V_2)^2 V_3 + \ldots$$

- All χ_n : independent of initial state
- and can be measured.

Yan JYO 1502.02502

What can we measure?

- V_n cannot be measured on an event-by-event basis: statistical fluctuations way too large.
- Average over events: $\langle V_n \rangle = 0$ by rotational symmetry.
- Measurements of anisotropic flow are extracted from multiparticle correlations.

Example : pair correlation

- Pairs of particles in the same event with azimuthal angles φ_1, φ_2.
- Do a statistical average in the event
  $\{e^{in\varphi 1}\ e^{-in\varphi 2}\} = \{e^{in\varphi 1}\}\ \{e^{-in\varphi 2}\}$ (flow paradigm)
  $= \quad V_n \qquad V_{-n} \quad = |V_n|^2$
- Finally, average over events
  $\langle e^{-in\varphi 1}\ e^{-in\varphi 2}\rangle = \langle |V_n|^2\rangle \quad > 0$

Alver Roland 1003.0194

Generalization

- Now, 3-particles with azimuthal angles $\varphi_1, \varphi_2, \varphi_3$
- $\{e^{4i\varphi 1}\, e^{-2i\varphi 2}\, e^{-2i\varphi 3}\} = \{e^{4i\varphi 1}\}\, \{e^{-2i\varphi 2}\}\, \{e^{-2i\varphi 3}\}$

$$= \quad V_4 \qquad V_{-2} \qquad V_{-2}$$

$$= V_4\, (V_2^*)^2$$

- Finally: $\langle e^{4i\varphi 1}\, e^{-2i\varphi 2}\, e^{-2i\varphi 3}\rangle = \langle V_4\, (V_2^*)^2\rangle$
- In principle, one can measure the average value of any product of V_ns, that is, all *moments*.

Bilandzic 1409.5636

Bhalerao Pal JYO 1411.5160

Measuring nonlinear couplings

Start from definition:

$$V_4 = \chi_4 \, (V_2)^2 + \ldots$$

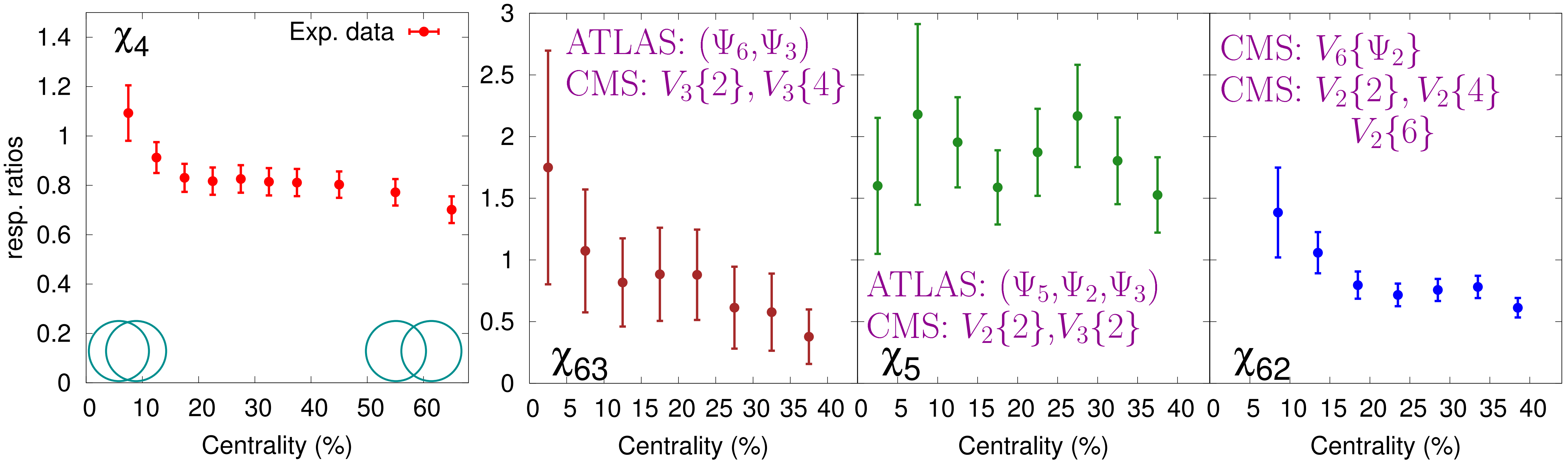
Make both sides invariant under rotations, average over events:

$$\langle V_4 \, (V_2^*)^2 \rangle = \chi_4 \, \langle (V_2)^2 \, (V_2^*)^2 \rangle + \ldots$$

If we neglect the remaining part $+\ldots$, we obtain the nonlinear response χ_4 in terms of moments, which are measured.

Note: $\langle V_4 \, (V_2^*)^2 \rangle$ is measured with better relative accuracy than $\langle |V_4|^2 \rangle$.

Nonlinear couplings at the LHC



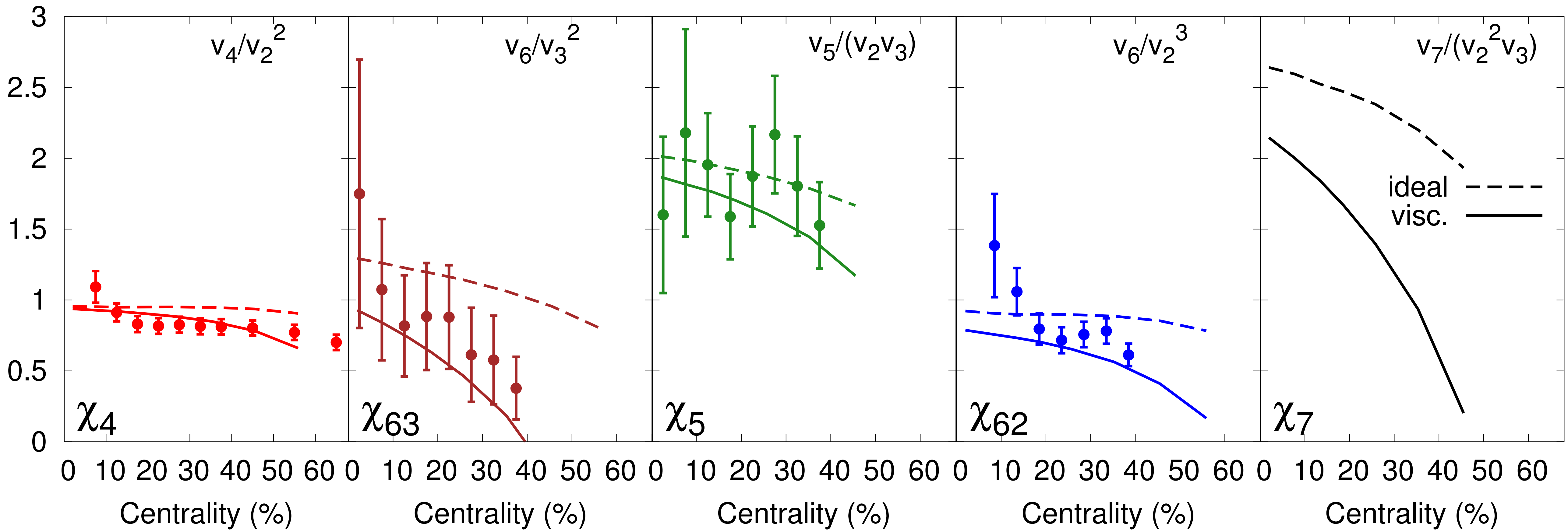
The χ_n are of order unity and vary mildly with centrality, unlike V_n itself.

Yan JYO 1502.02502

Nonlinear couplings and hydro

- Hydro ≠ experiment: we calculate V_n and χ_n in a single « hydro event ».
- Since χ_n are independent of the initial density profile, we choose a simple smooth profile: e.g., Gaussian for χ_4 and χ_{62}
- Solve relativistic ideal (or viscous) hydrodynamics with an EOS from lattice QCD.
- Transform the fluid into hadrons when it cools down to T_f=150 MeV

Nonlinear couplings and hydro



Both ideal and viscous ($\eta/s=1/4\pi$) results are in the ballpark for all coefficients, all centralities.

Viscous marginally better than ideal.

Yan JYO 1502.02502

2. Symmetric cumulants

Giacalone, Yan, Noronha-Hostler, JYO, 1605.08303

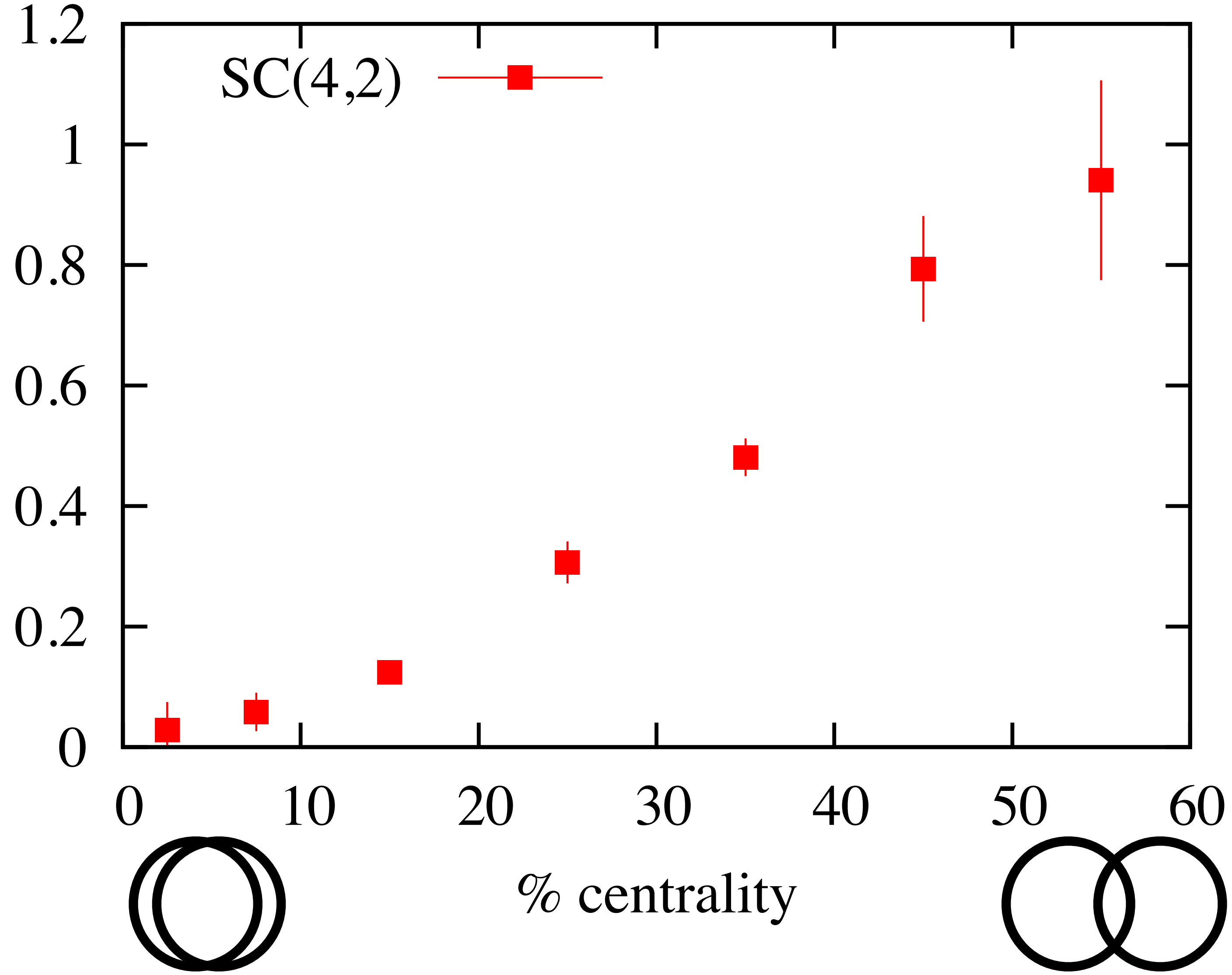
New data from ALICE

- « symmetric cumulants » : specific 4 particle correlations.
- Using the flow paradigm, the symmetric cumulant can be recast as a correlation between the magnitudes of 2 different Fourier harmonics:

$$SC(4,2) \equiv \frac{\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle}{\langle v_4^2 \rangle \langle v_2^2 \rangle}$$

- ALICE has recently measured SC(4,2) as a function of centrality

New data from ALICE



ALICE Collaboration, arXiv:1604.07663

ATLAS « event-plane correlation »

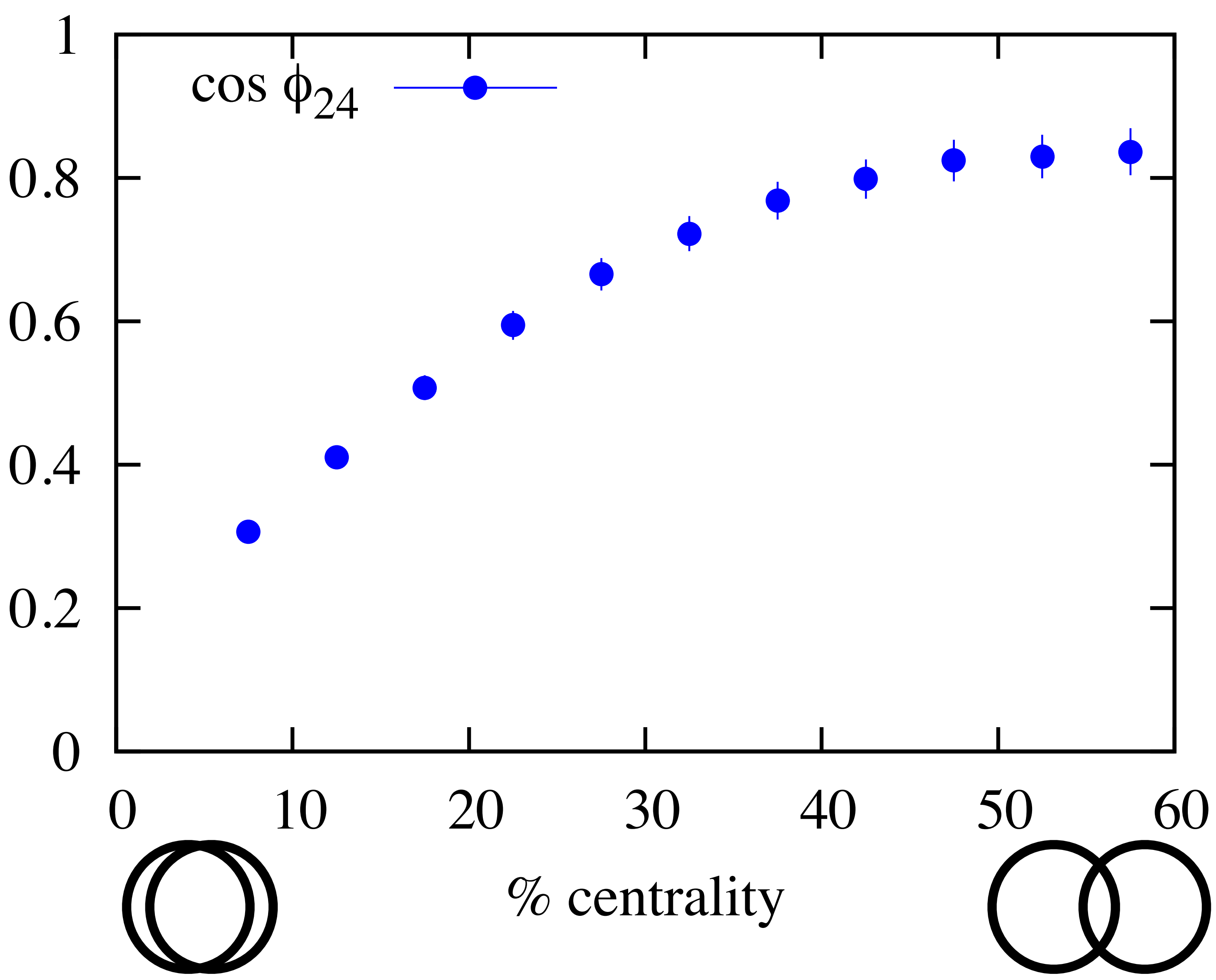
- The *event-plane correlation* measured by ATLAS is in fact a linear (Pearson) correlation between the complex flow coefficients V_4 and $(V_2)^2$

$$\cos\Phi_{24} \equiv \frac{\mathrm{Re}\langle V_4 (V_2^*)^2\rangle}{\sqrt{\langle v_4^2\rangle\langle v_2^4\rangle}}$$

- It is also a measure of the correlation between V_4 and V_2, which involves the relative angle *and* the magnitudes.

Luzum JYO arXiv:1209.2323

ATLAS « event-plane correlation »



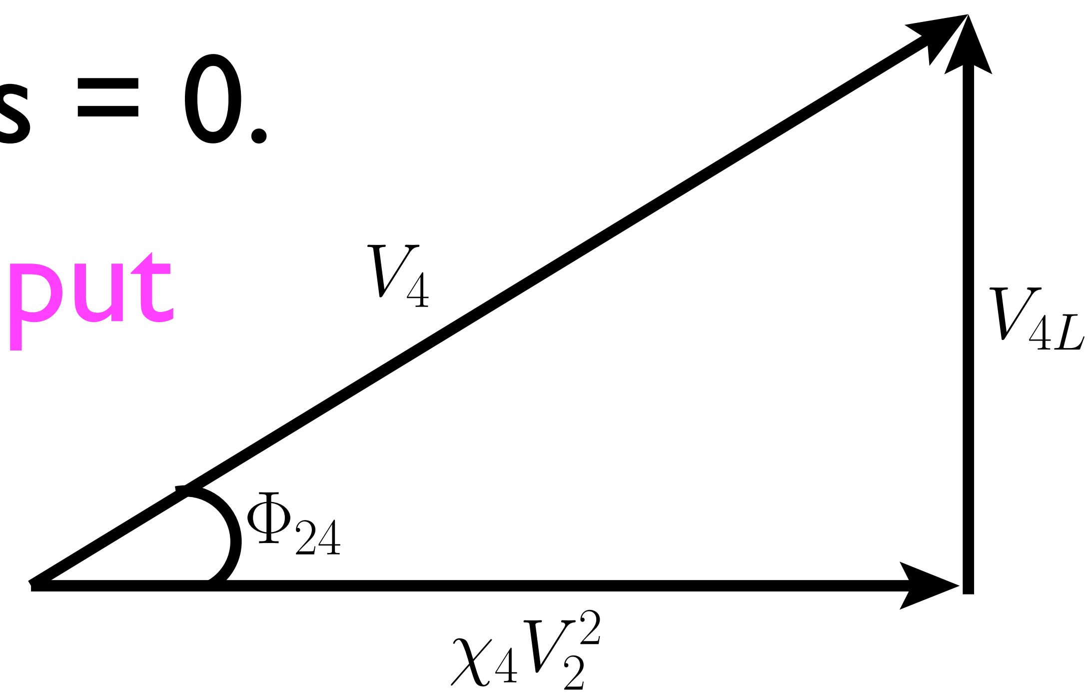
ATLAS Collaboration, arXiv:1403.0489

Can we compare ALICE and ATLAS?

- We have two measures of the correlation between V_4 and V_2, the *symmetric cumulant (ALICE)* and the *event-plane correlation (ATLAS).*
- I derive a *quantitative relation* between these two measures, test it on hydro calculations and then on data.

Modeling the correlation

- Decompose $V_4 = V_{4L} + \chi_4 (V_2)^2$, with χ_4 = constant fixed so that linear correlation between the two terms = 0.
- Just math, no physics input



- Then Φ_{24} measures the relative magnitude of the 2 terms: $\chi_4^2 \langle v_2^4 \rangle = \langle v_4^2 \rangle \cos^2 \Phi_{24}$

Modeling the correlation

- $V_4 = V_{4L} + \chi_4 (V_2)^2$
- We assume that the two terms are *independent* (stronger than *uncorrelated*)
- Then: correlation between $(v_4)^2$ and $(v_2)^2$ is only from the nonlinear part:

$$\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle = \chi_4^2 \left(\langle v_2^6 \rangle - \langle v_2^4 \rangle \langle v_2^2 \rangle \right)$$

Result

Expressing χ_4 as a function of the event-plane correlation, we obtain:

$$SC(4,2) = \left(\frac{\langle v_2^6 \rangle}{\langle v_2^4 \rangle \langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{24}$$

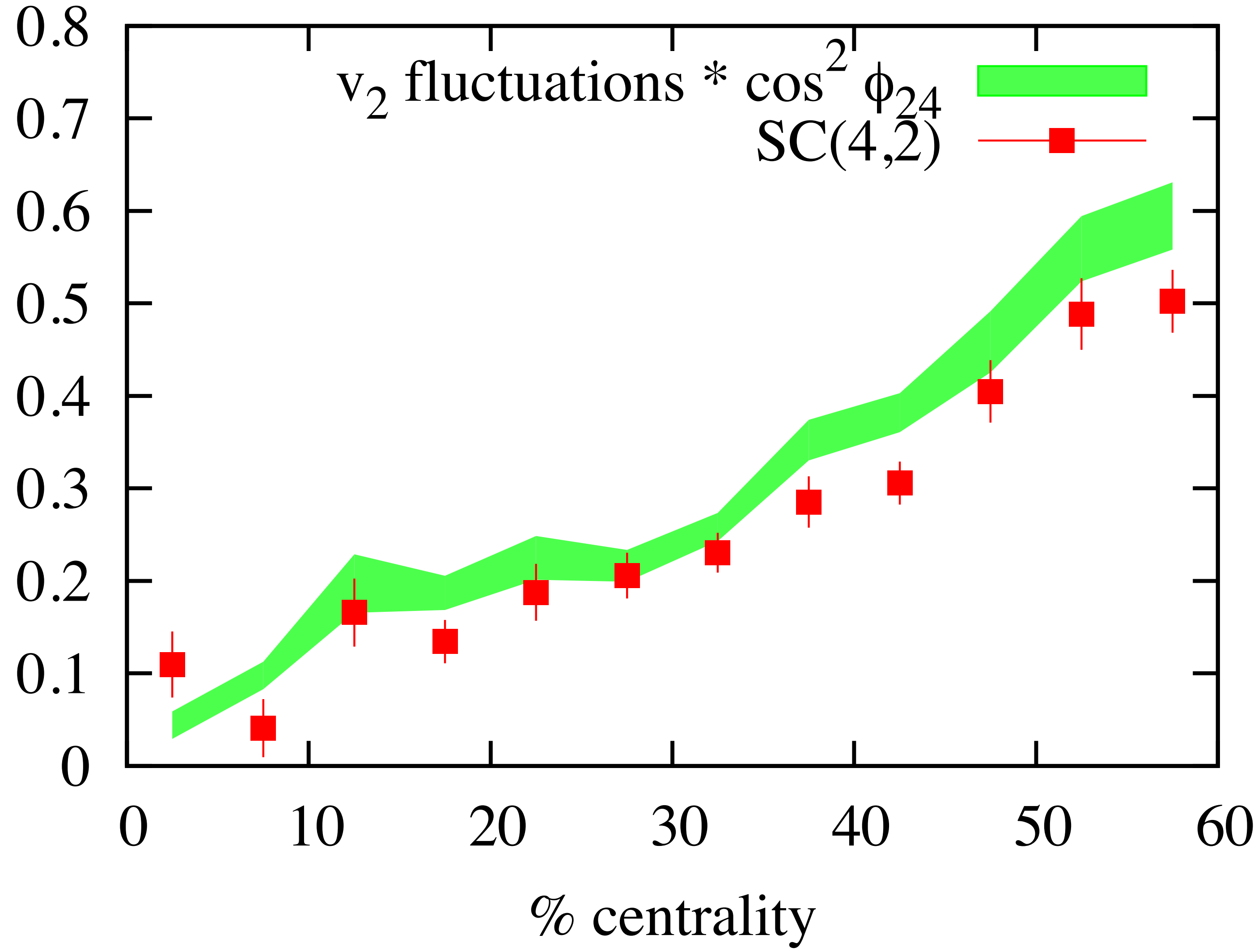
symmetric cumulant

elliptic flow fluctuations

event-plane correlation

Event-by-event hydrodynamics

We compute both sides of the equation independently in event-by-event viscous hydro with Glauber initial conditions

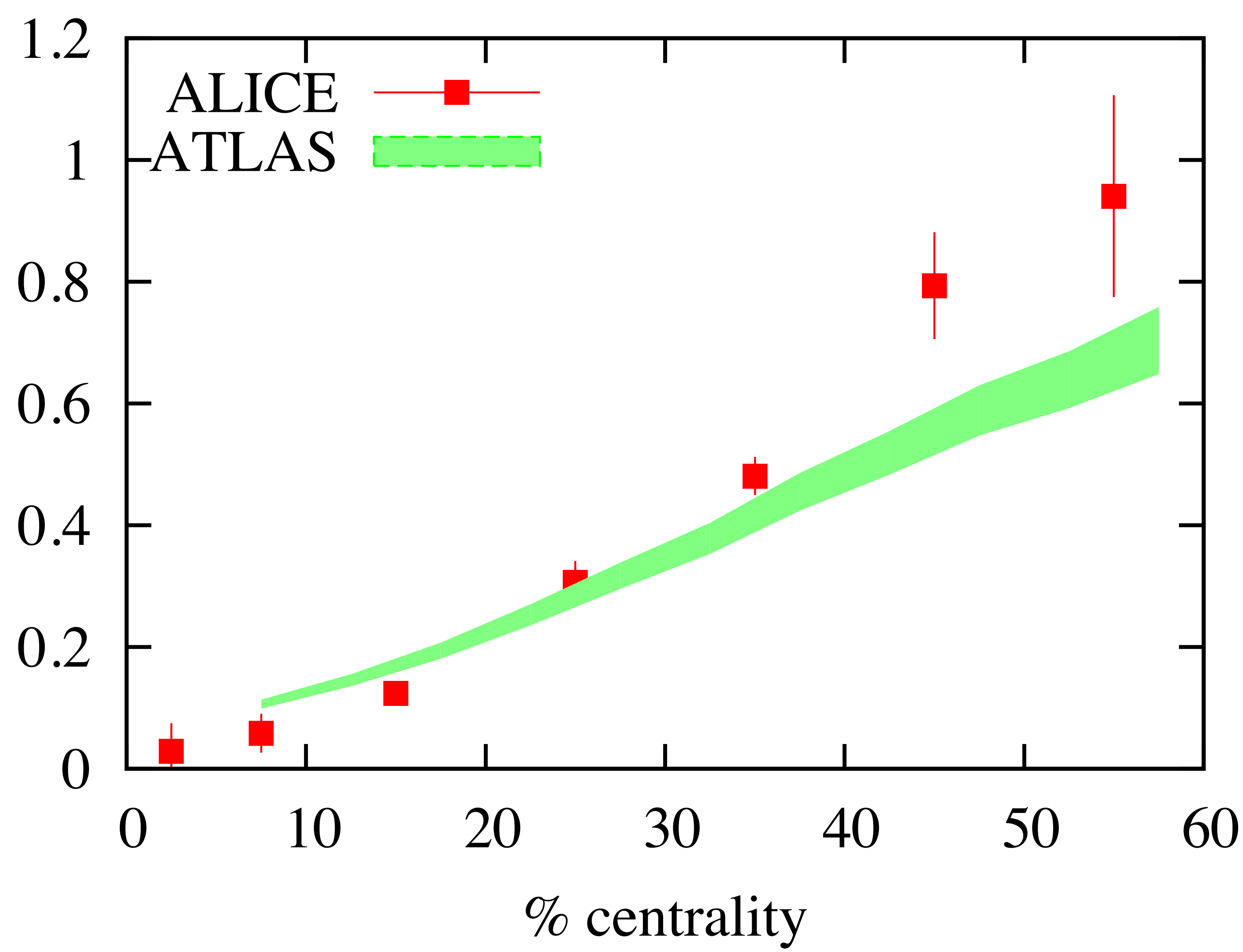


The relation is satisfied to a good approximation for all centralities

ALICE versus ATLAS

Using elliptic flow fluctuations (cumulants) and event plane correlations from ATLAS:

SC(4,2)



ALICE versus ATLAS

- Agreement not as good as in hydro. why?
- ATLAS event-plane correlations are measured with a large pseudorapidity gap and over a wide interval -4.8 to 4.8
- ALICE SC(4,2) is measured without any gap and over the interval -0.8 to 0.8
- Longitudinal flow fluctuations induce a decoherence which may explain why the ATLAS result is smaller.

Predictions

Same methodology applied to different orders:

$$SC(4,3) = \left(\frac{\langle v_2^4 v_3^2\rangle}{\langle v_2^4\rangle\langle v_3^2\rangle} - 1\right)\cos^2\Phi_{24}$$

$$SC(5,2) = \left(\frac{\langle v_2^4 v_3^2\rangle}{\langle v_2^2 v_3^2\rangle\langle v_2^2\rangle} - 1\right)\cos^2\Phi_{235}$$

$$SC(5,3) = \left(\frac{\langle v_2^2 v_3^4\rangle}{\langle v_2^2 v_3^2\rangle\langle v_3^2\rangle} - 1\right)\cos^2\Phi_{235}$$

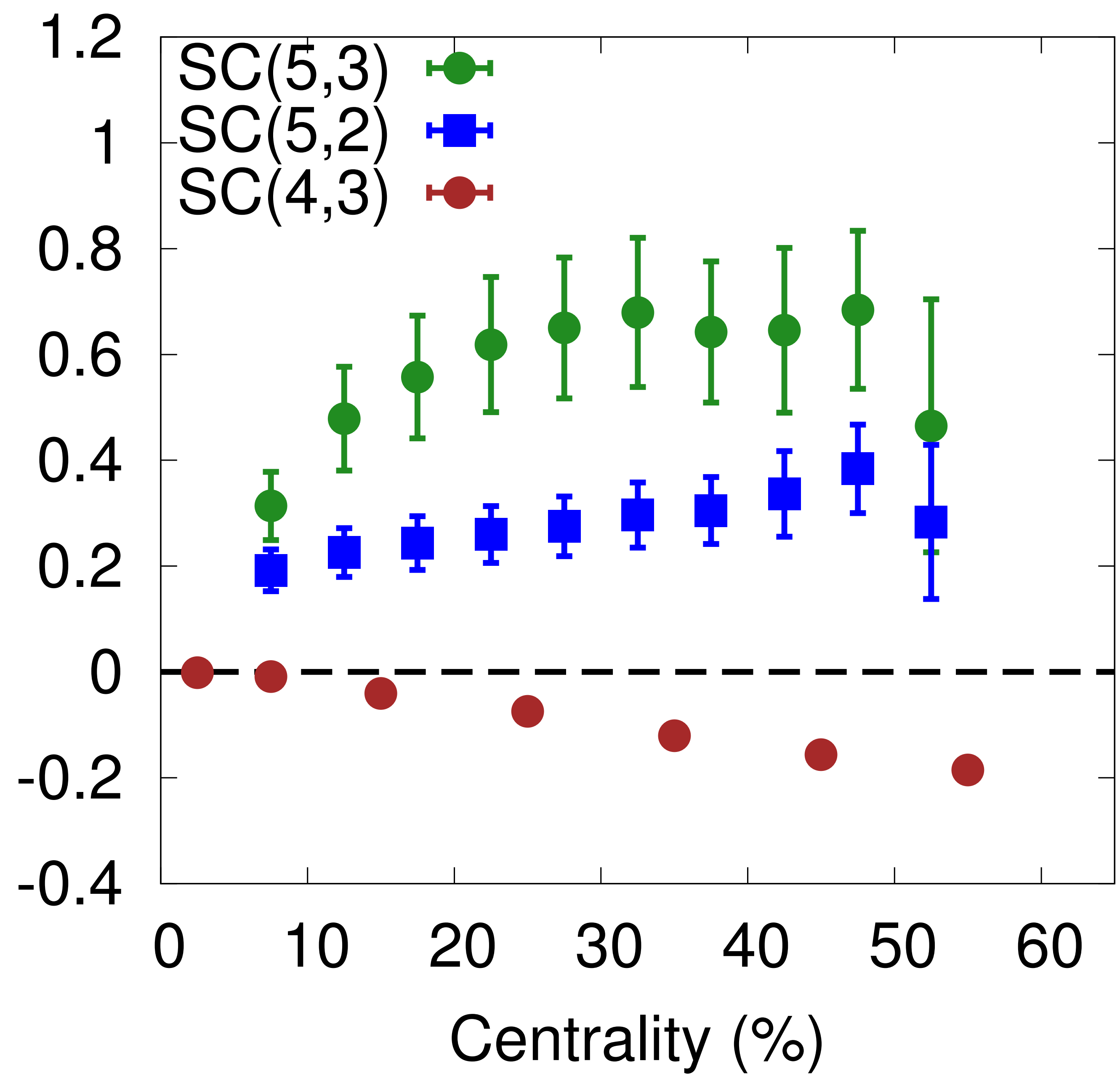
symmetric cumulants

flow fluctuations

event-plane correlations

Predictions

Data-driven predictions (no hydro calculation!) using ATLAS results on v_n fluctuations and event-plane correlations.



3. The statistics of v_2 fluctuations

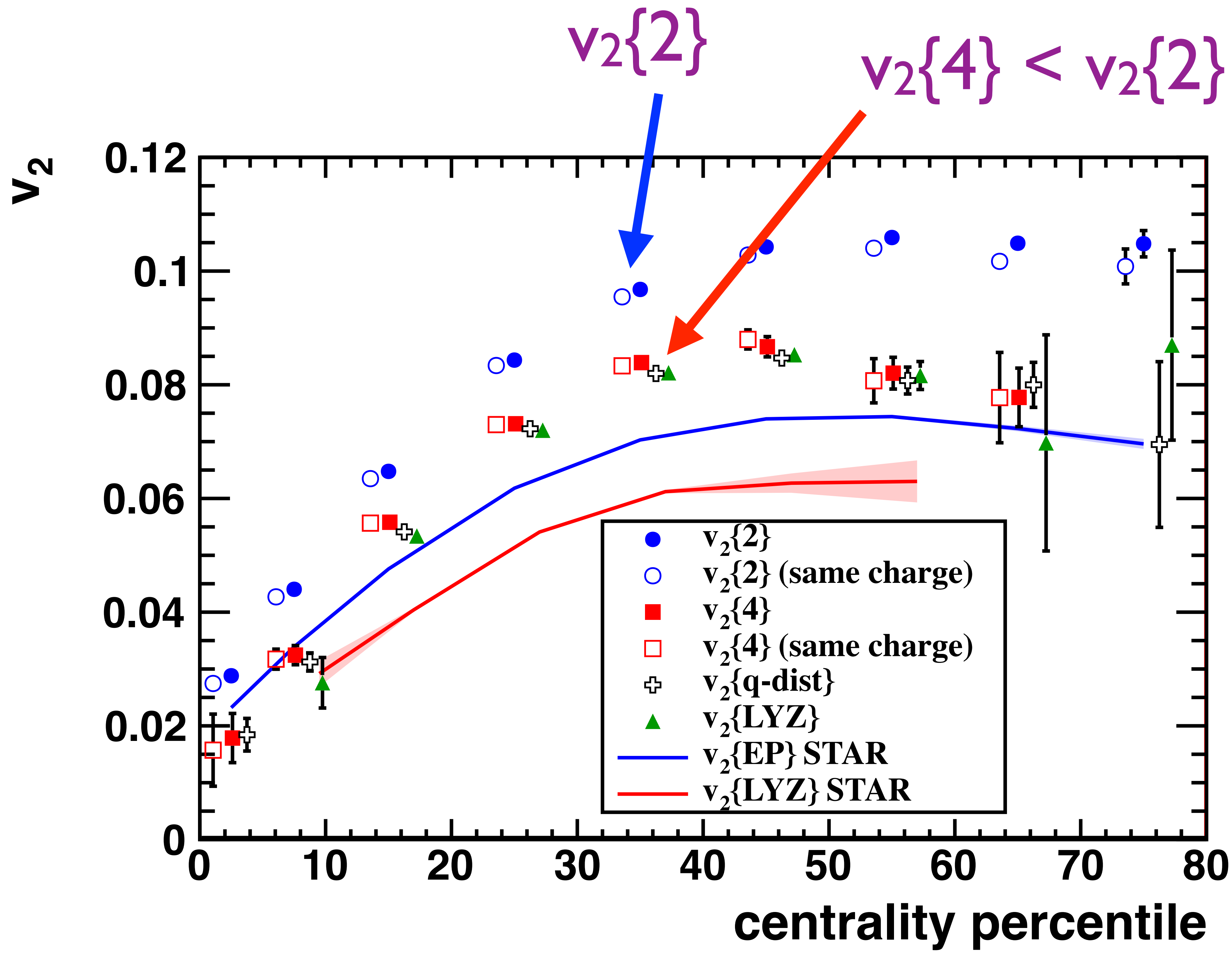
Giacalone, Yan, Noronha-Hostler, JYO, 1608.01823

The fluctuations of elliptic flow

- One can measure much more than the rms value of v_2. Also higher order moments and cumulants

$$v_2\{2\}=(\langle v_2{}^2\rangle)^{1/2}$$
$$v_2\{4\}=(2\langle v_2{}^2\rangle^2-\langle v_2{}^4\rangle)^{1/4}$$
$$v_2\{6\}=((\langle v_2{}^6\rangle-9\langle v_2{}^4\rangle\langle v_2{}^2\rangle+12\langle v_2{}^2\rangle^3)/4)^{1/6}$$

- $v_2\{4\} < v_2\{2\}$: implied by flow paradigm alone.
- $v_2\{4\}=v_2\{6\}$ if fluctuations are 2-dim. Gaussian.



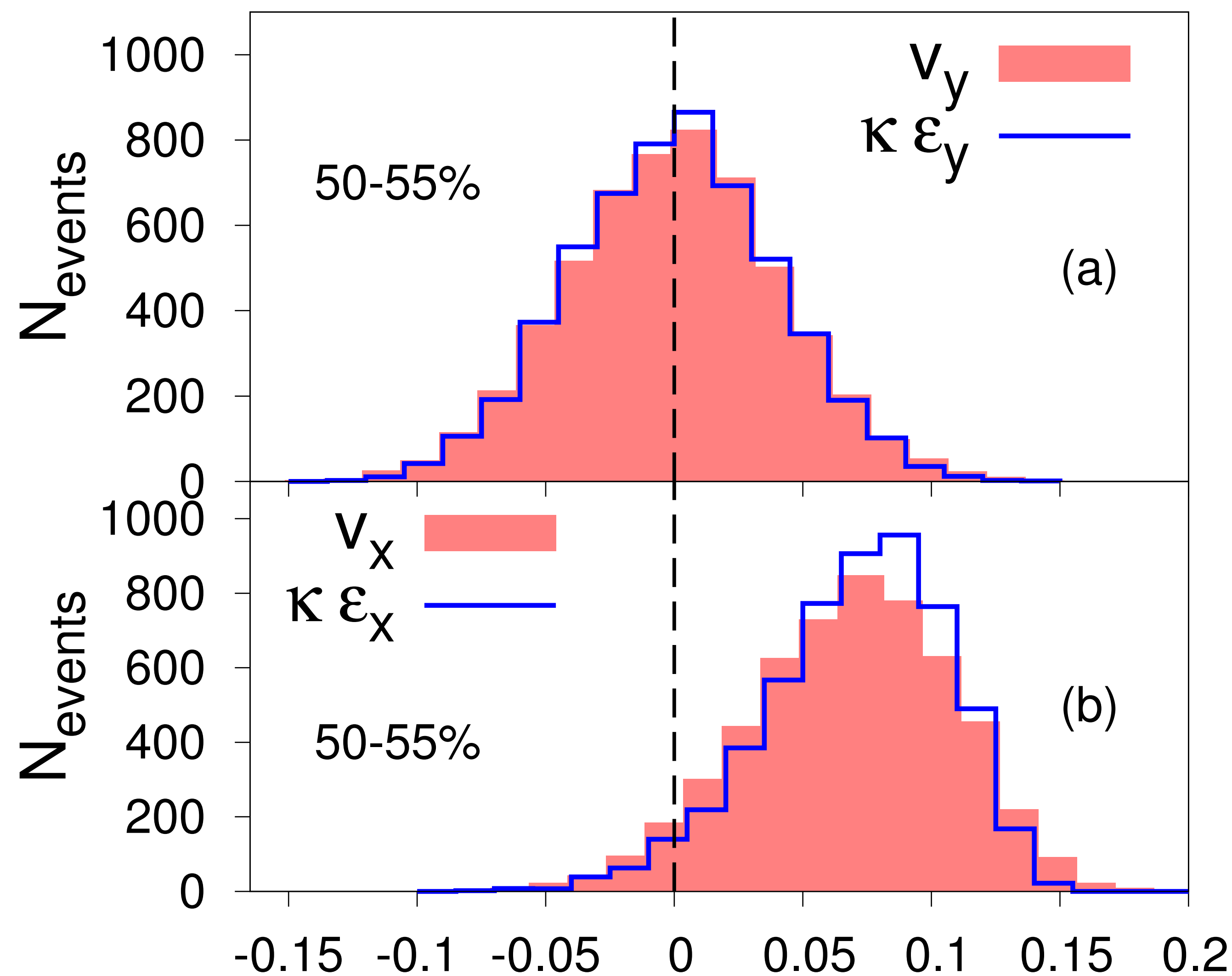
ALICE 1011.3916

Fluctuations of v_2 in hydrodynamics

Decompose V_2 into real (cos) and imaginary (sin) parts

$$V_2 = v_x + i\, v_y$$

Probability distribution in a hydrodynamic calculation:



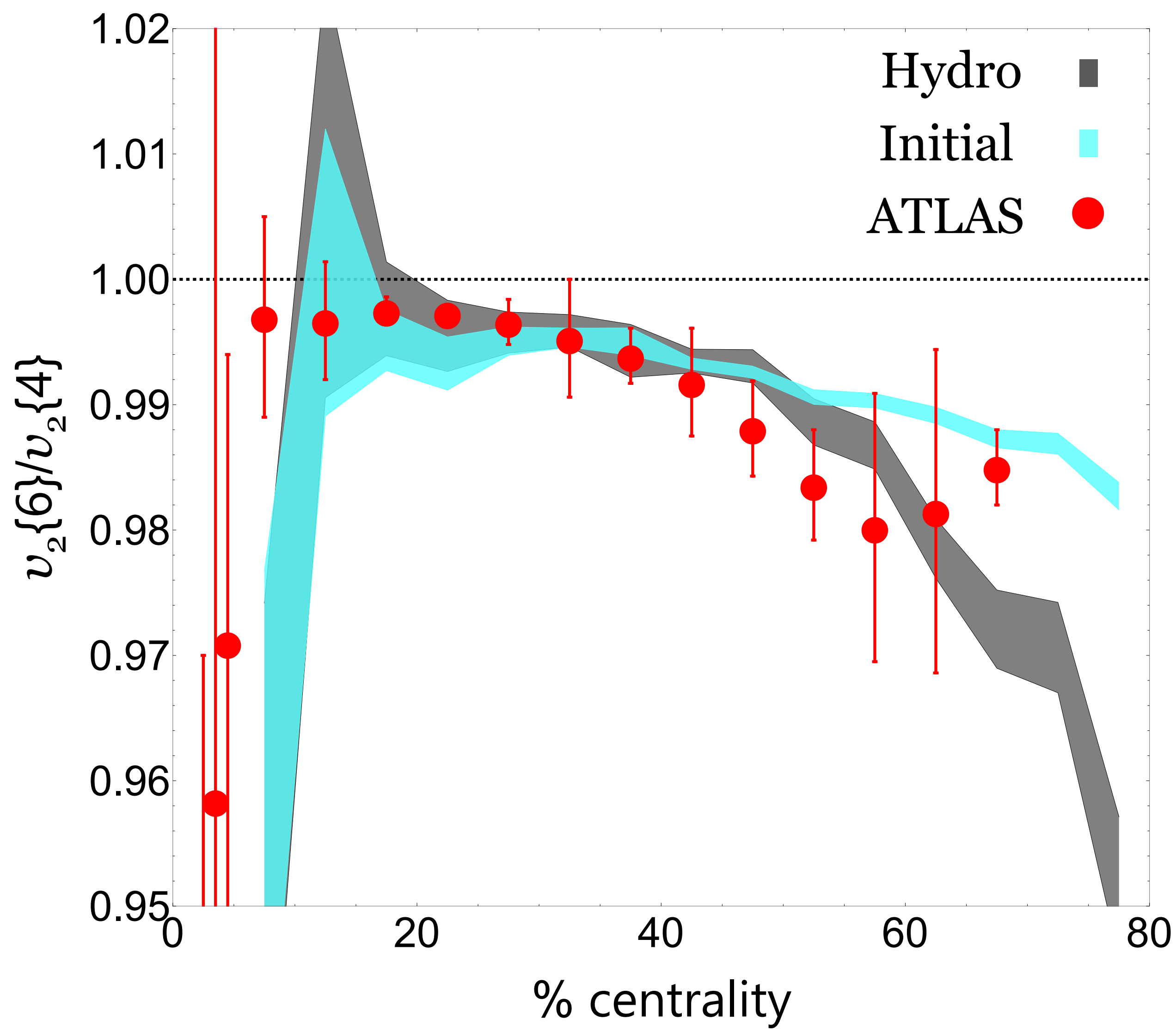
Fluctuations of v_x are not symmetric: they have negative skew
skewness=3rd centered moment : $s \equiv \langle (v_x - \langle v_x \rangle)^3 \rangle$

Non-Gaussian fluctuations

We have shown by an expansion in powers of the fluctuations that

$v_2\{6\}-v_2\{4\} = s/(3\langle v_x\rangle^2)$

Negative skewness, $s<0$, naturally explains the small lifting of degeneracy seen by ATLAS:



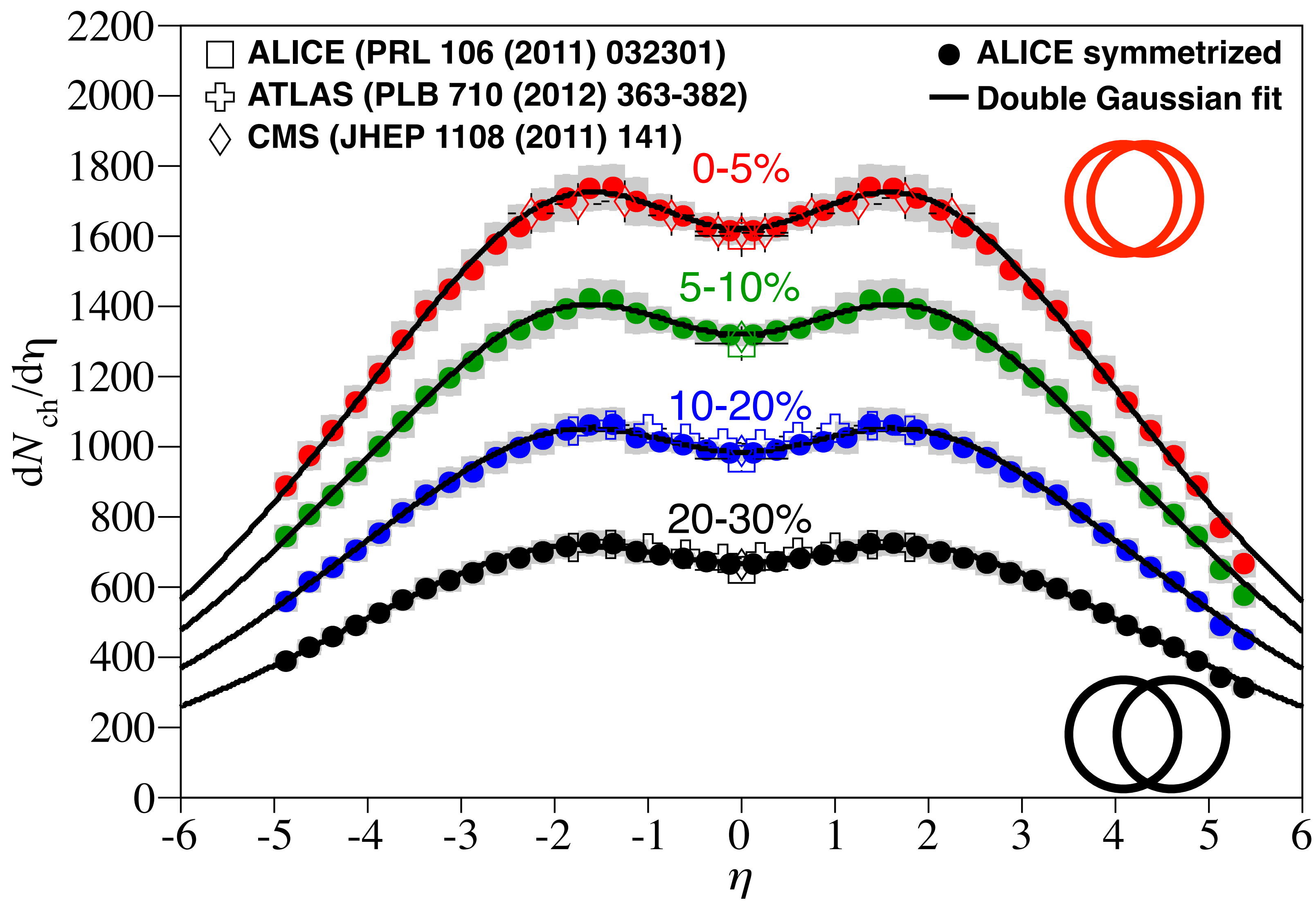
ATLAS arXiv:1408.4342

Conclusions

- Bulk (soft particle) correlations of arbitrary order are, at the qualitative level, naturally explained by the flow paradigm and minimal assumptions from hydrodynamics (e.g., eccentricity scaling of v_2).
- The consequences of the flow paradigm for higher-order correlations have not yet been fully explored.
- In order to go beyond semi-quantitative tests of hydrodynamics, it is important to eliminate the uncertainty from the initial state (e.g. measure nonlinear couplings)

Backup slides

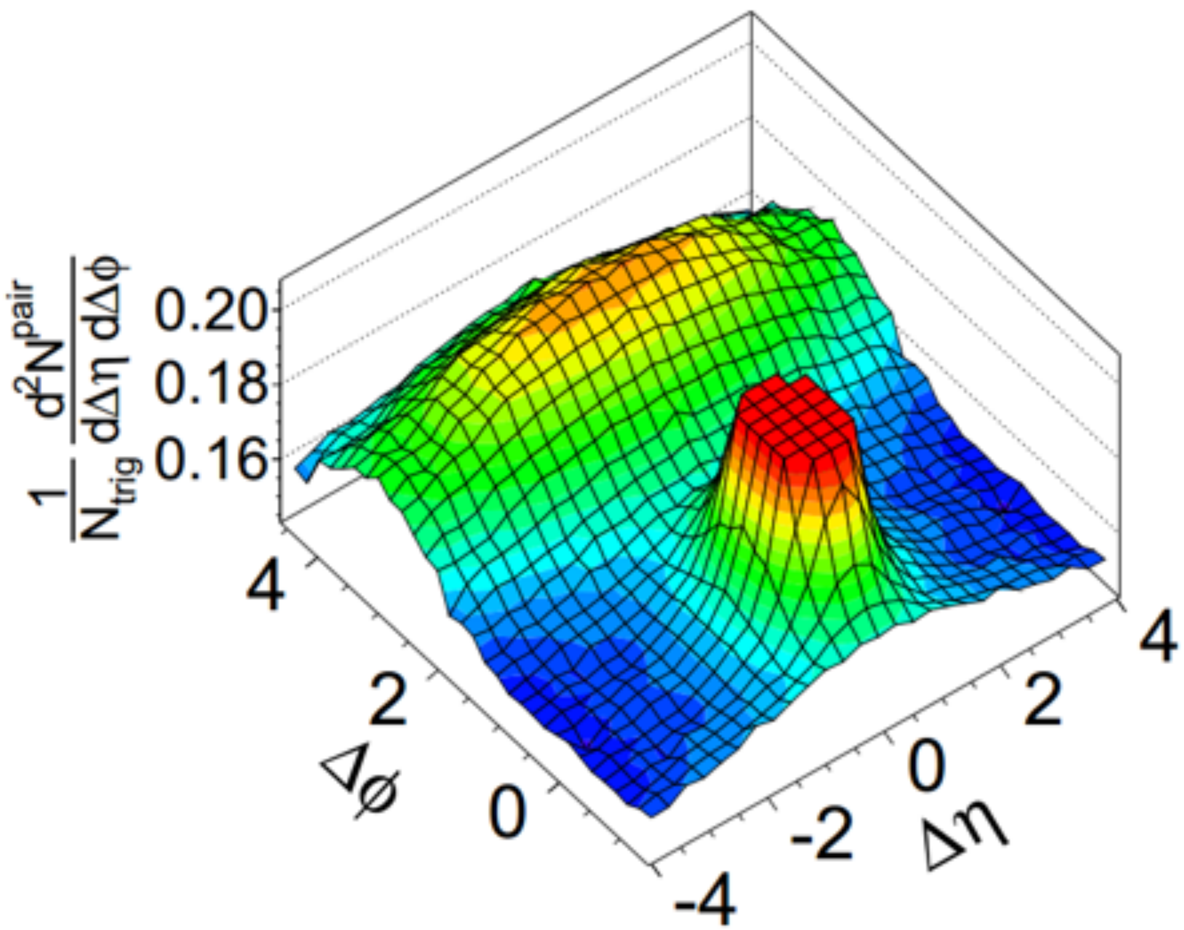
Multiplicity and centrality



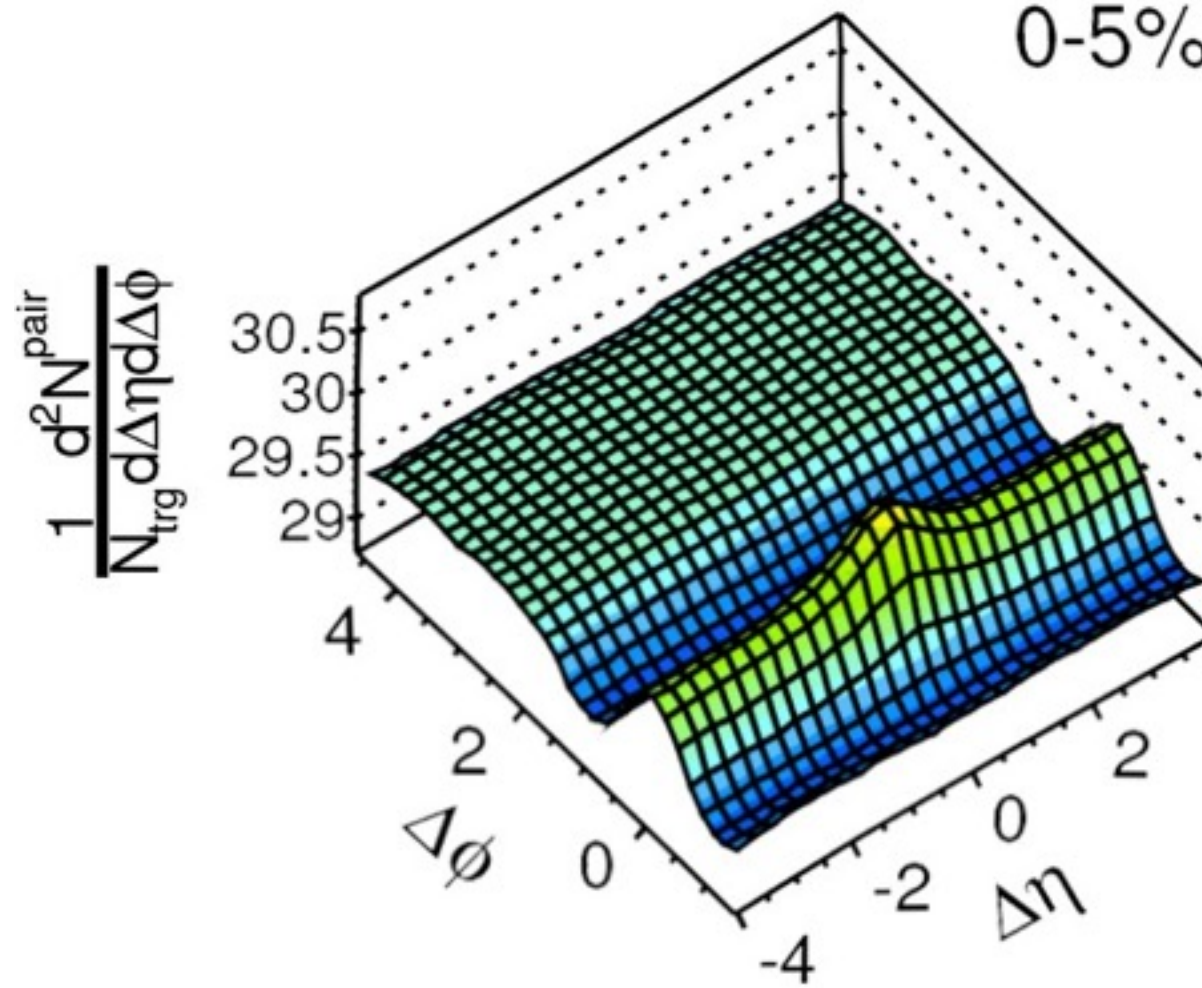
ALICE arxiv:1304.0347

- Collisions classified from more central to less central in 5% bins
- More central creates more particles
- A central collision (b=0) typically produces 25000 particles.

Two-particle correlations



proton-proton collision



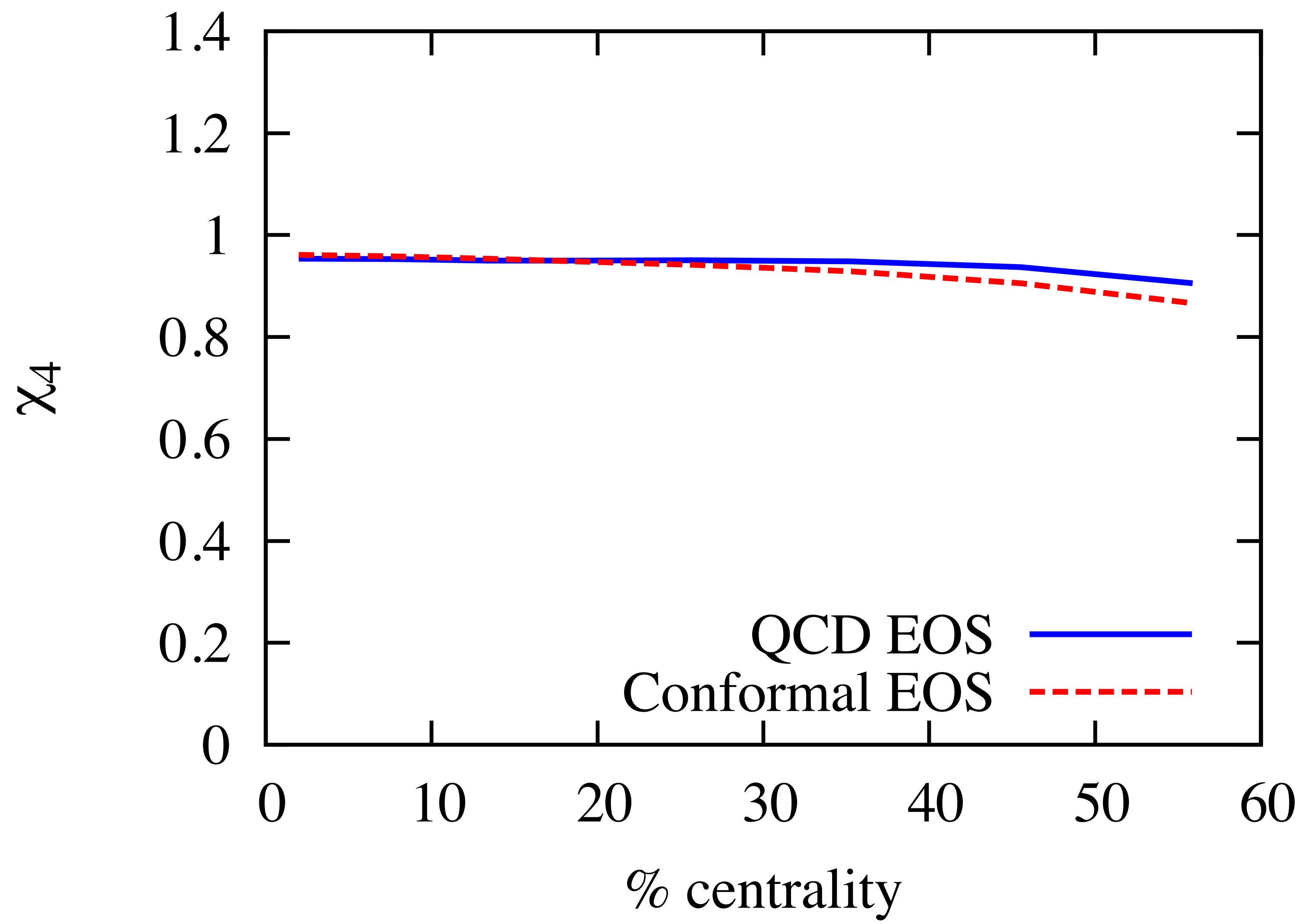
central Pb+Pb collision

CMS 1201.3158

What do nonlinear couplings tell us?

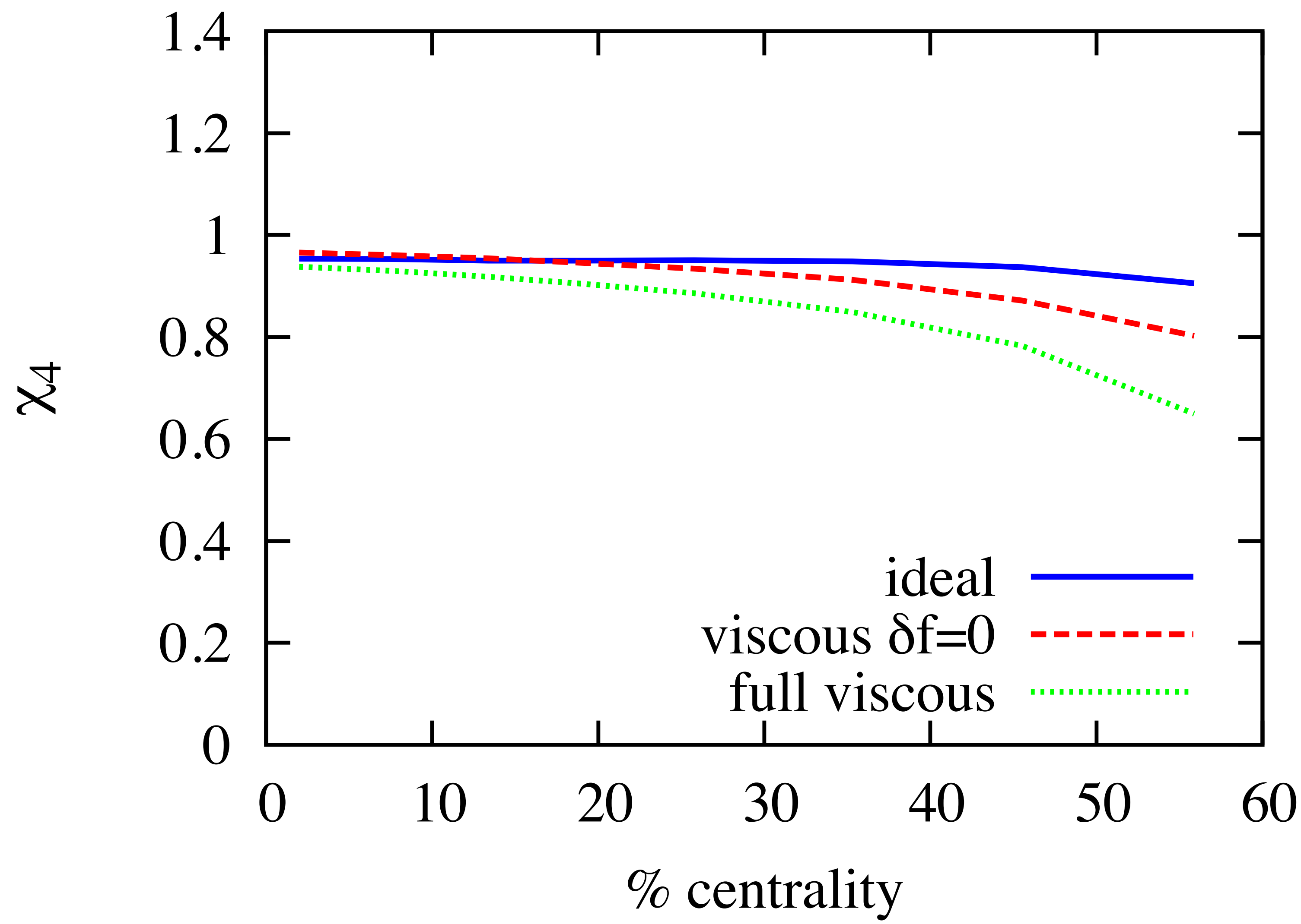
- They are insensitive to the initial state, but what about other parameters?
- Equation of state
- Viscosity of the quark-gluon plasma
- Freeze-out temperature
- Viscous « corrections » at freeze-out
  [note that viscosity enters in 2 different places in hydrodynamic calculations]

Sensitivity to equation of state



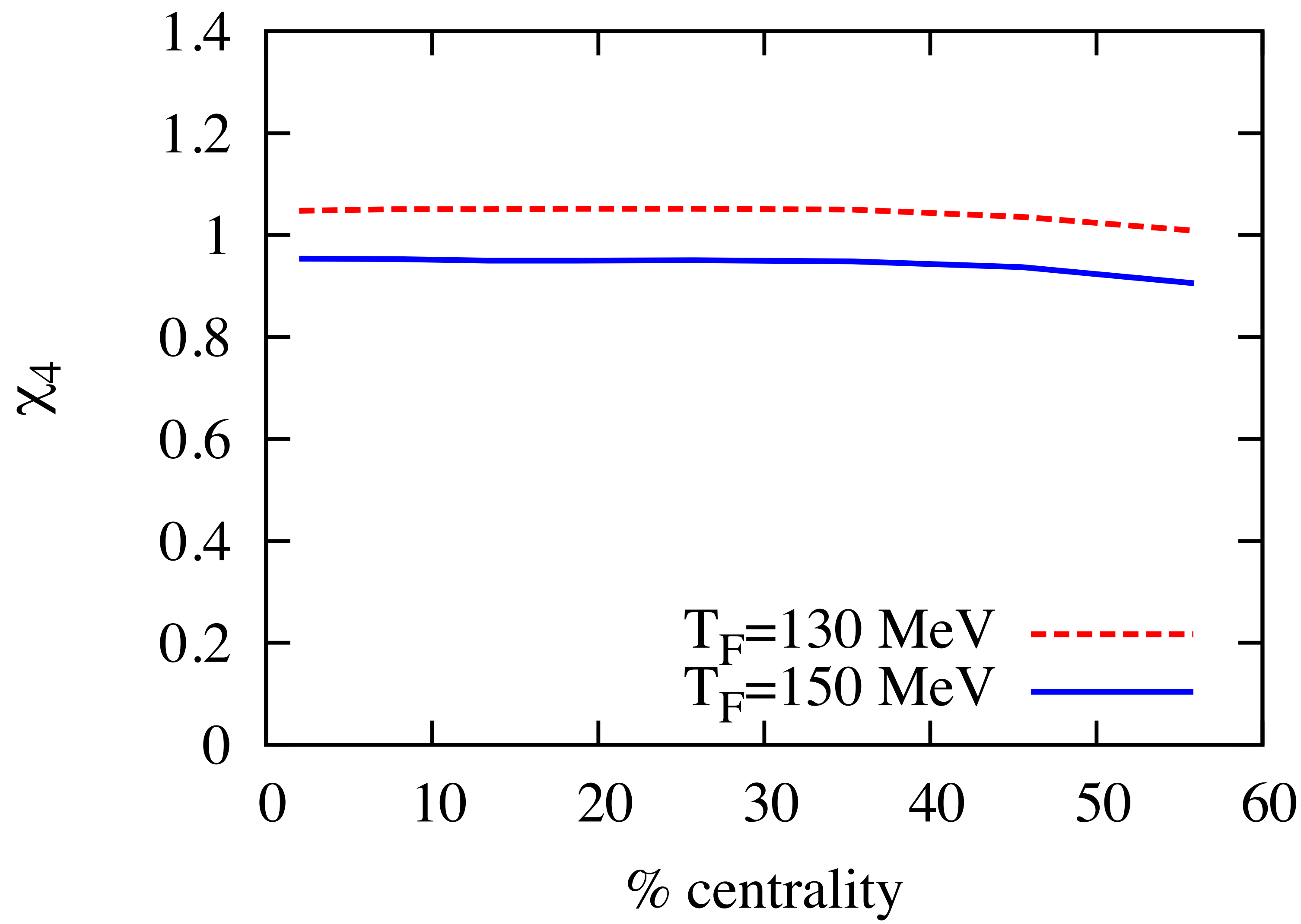
EOS from lattice QCD versus conformal ε=3P: nonlinear coupling surprisingly insensitive to EOS.

Sensitivity to viscosity



Moderate effect of viscosity, mostly through the viscous correction to phase-space distribution at freeze-out

Sensitivity to freeze-out temperature

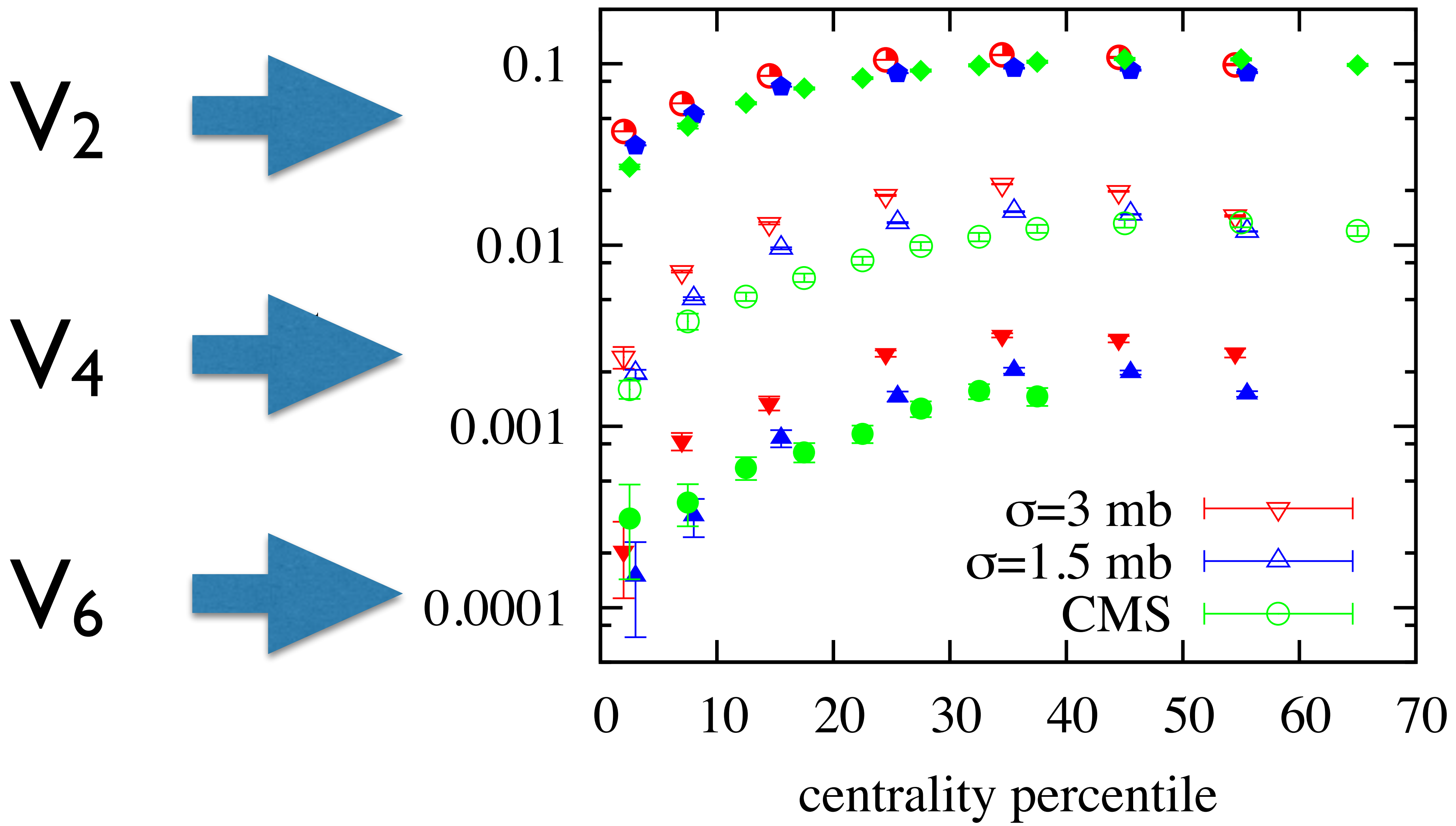


If the system expands for a longer time, the nonlinear couplings increase. Still a modest effect.

Transport calculations

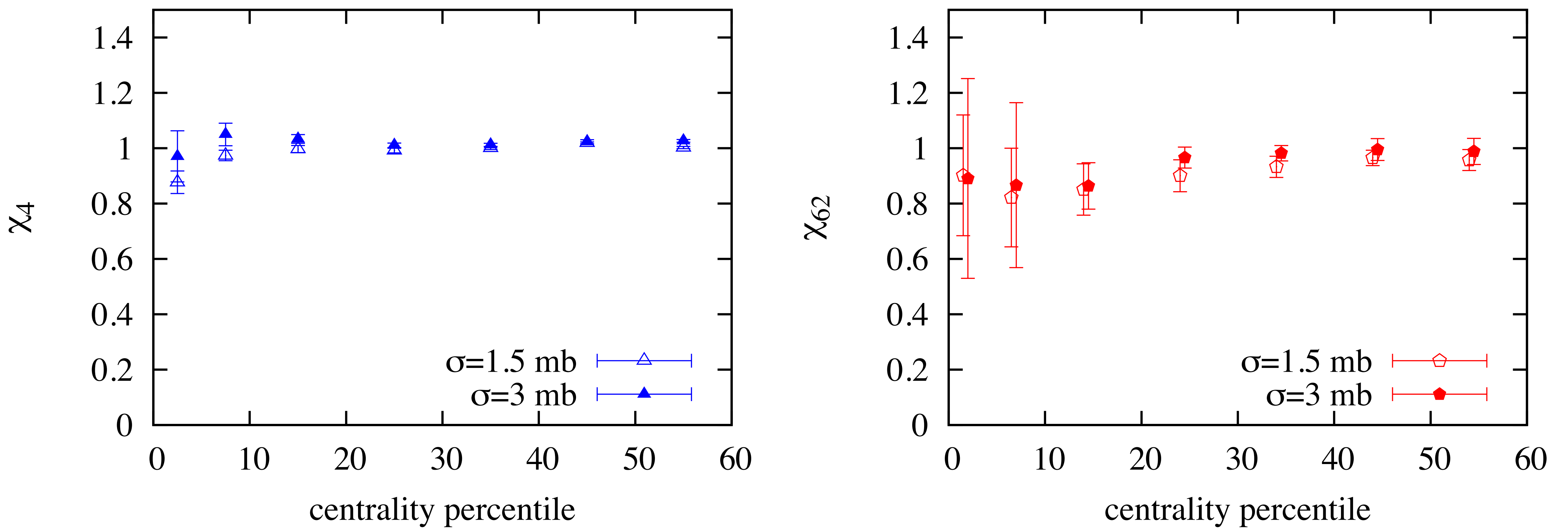
- Late stages where the system falls out of equilibrium seem to be the most important and are not correctly modeled in hydro.
- Therefore we carry out transport (AMPT) calculations where one follows trajectories and collisions until the last.
- Free parameter : parton-parton elastic cross section σ

Anisotropic flow in transport theory



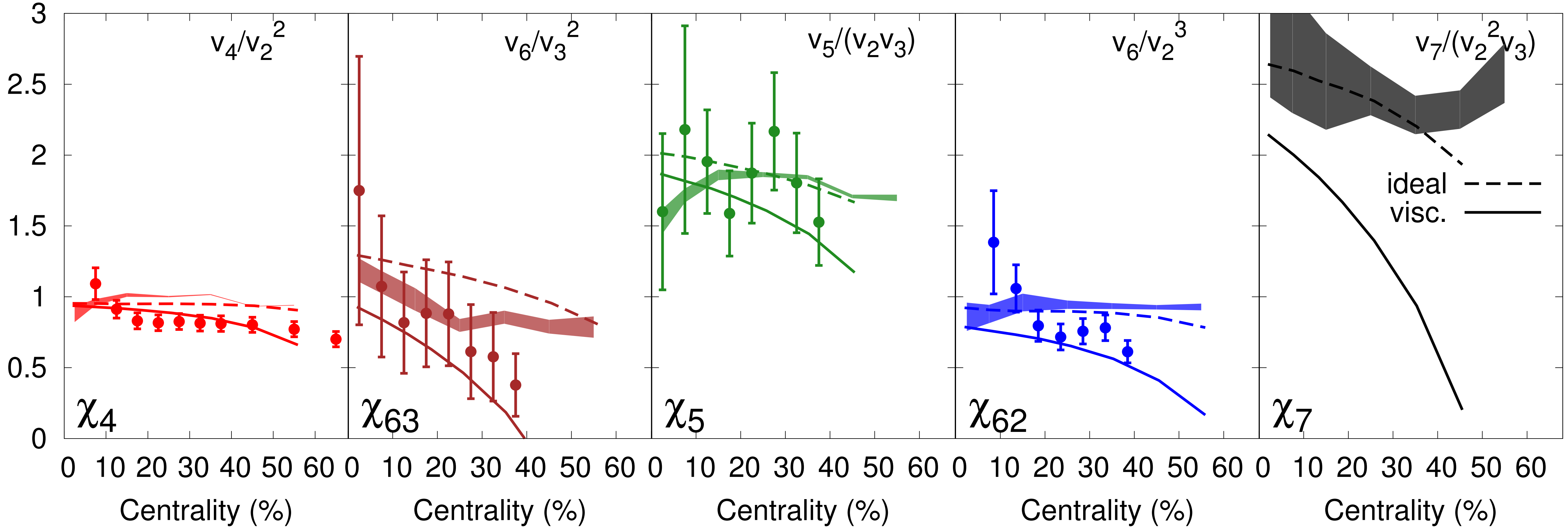
Anisotropic flow increases with interaction strength as expected

Nonlinear coupling in transport theory



Sensitivity to σ cancels out in the nonlinear couplings

Transport versus hydro versus data



Shaded bands: AMPT results with σ=1.5 mb

In fairly good agreement with ideal hydro results and data.