Mixed harmonic flow correlations

Recent results on symmetric cumulants

Matthew Luzum

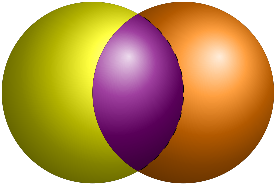
F. Gardim, F. Grassi, ML, J. Noronha-Hostler;
arXiv:1608.02982

Universidade de São Paulo

NBI Mini Workshop
19 September, 2016

Evolution of heavy-ion collision

- Hydro picture: system thermalizes and expands as fluid
- Particles emitted independently at end of evolution



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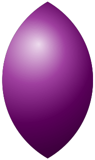

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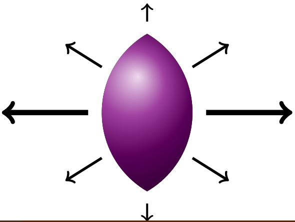

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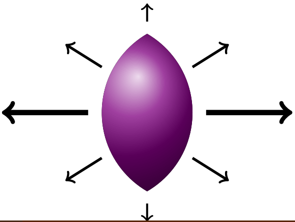

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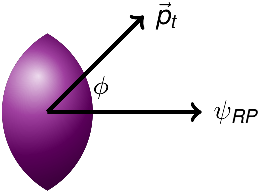

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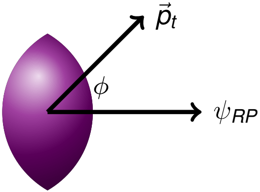

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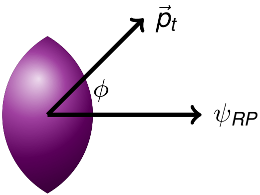

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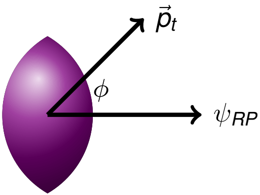
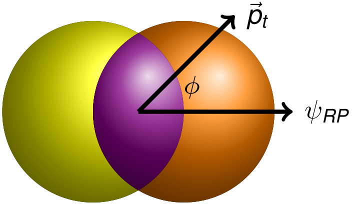


FLUCTUATIONS; A HISTORY

$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + 2v_2 \cos 2\phi + \ldots$$

- If no fluctuations: only v_2 at midrapidity (+ small v_4)
- Fluctuations are important!
- No symmetry $\implies$ more harmonics, flow coefficients are *vectors*
- Flow vectors fluctuate from event-to-event



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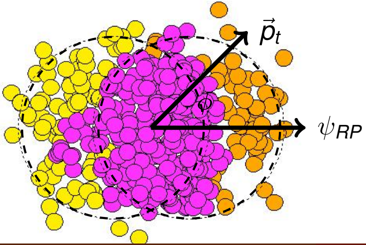

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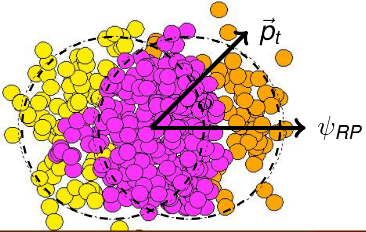

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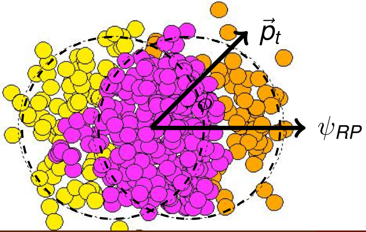

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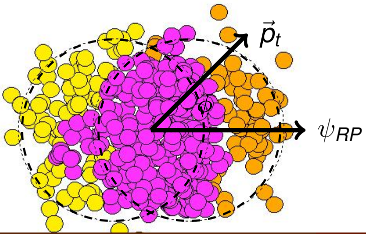
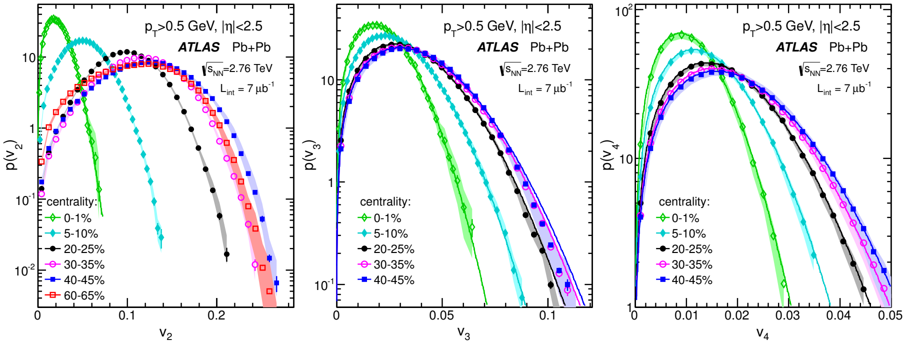
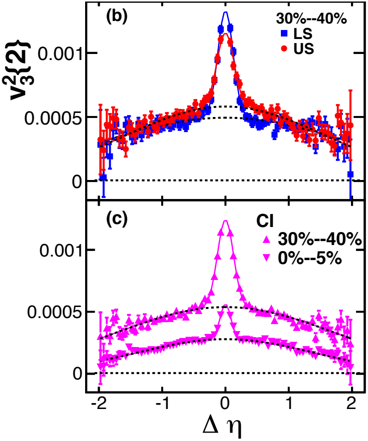




Fluctuations imply a set of statistical properties:

- Magnitude of flow coefficients have event-by-event distribution
- Flow vectors at different momenta can have non-trivial correlation
- Different harmonics can also have non-trivial correlations



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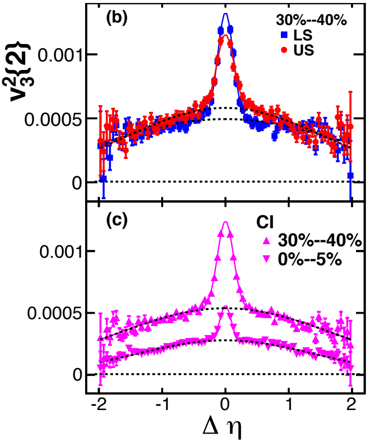

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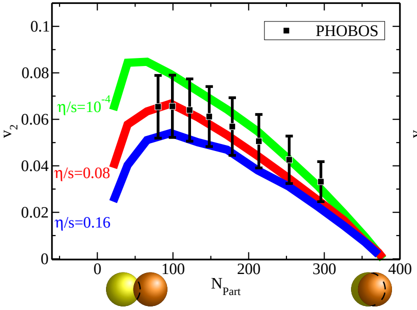
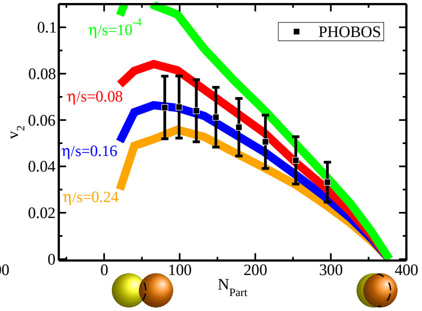
ELLIPTIC FLOW ALONE

“Glauber” initial conditions



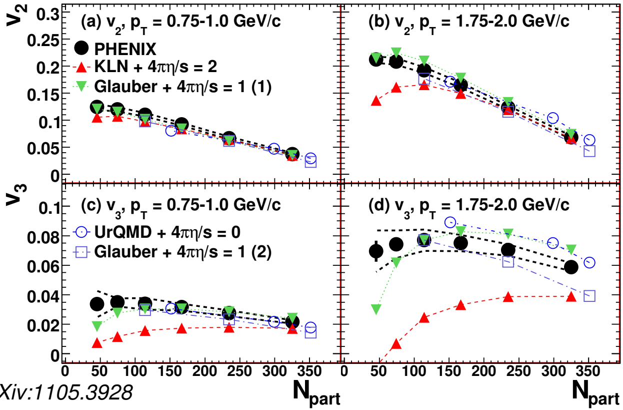
“CGC” initial conditions



(ML & Romatschke, Phys.Rev. C78 (2008) 034915)

- Can fit v_2 with different combinations of initial cond. + viscosity
- $\Longrightarrow$ Neither initial conditions nor viscosity individually constrained

ADD v_3:



PHENIX, arXiv:1105.3928

- Combining v_2 and v_3 can rule out models of initial conditions
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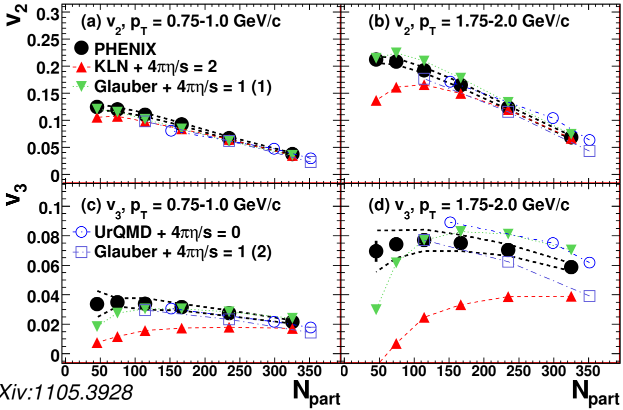

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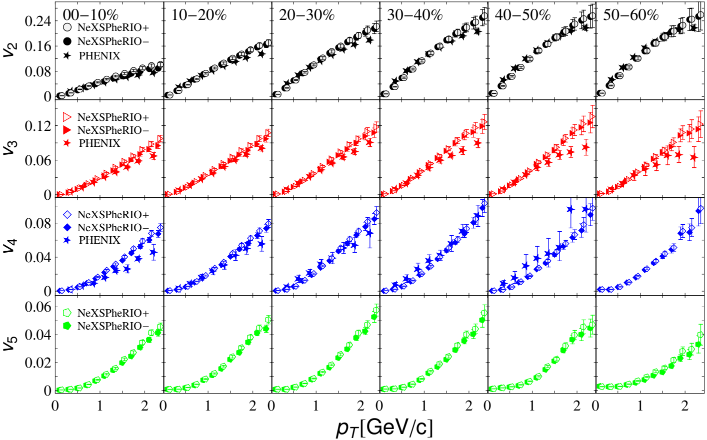
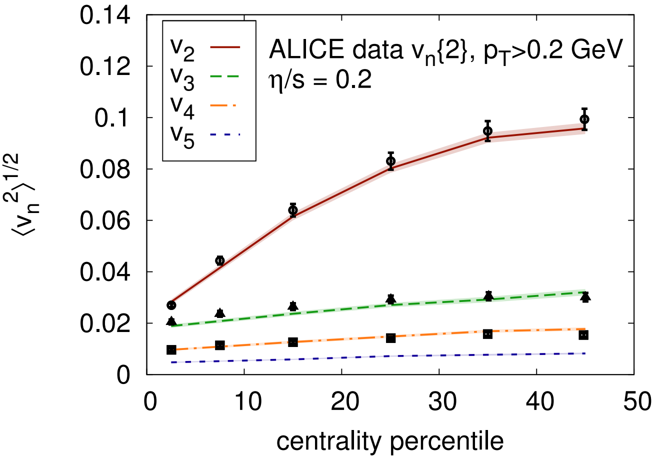
CALCULATIONS VS RHIC DATA



(F. Gardim, F. Grassi, ML, J.-Y. Ollitrault, Phys.Rev.Lett. 109 (2012) 202302)

- NeXus initial conditions + $\eta/s = 0$ at RHIC

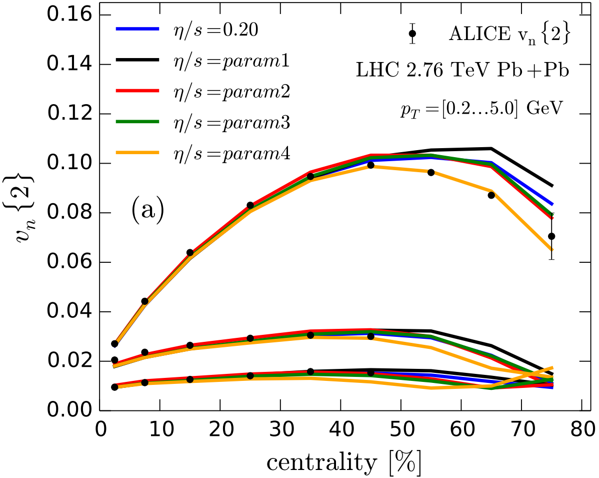
CALCULATIONS VS LHC1 DATA



(C. Gale, S. Jeon, B. Schenke, P. Tribedy, R. Venugopalan, Phys.Rev.Lett. 110 (2013) 012302)

- IP-Glasma initial conditions + $\eta/s = 0.2$ at LHC run 1

CALCULATIONS VS LHC1 DATA



(H. Niemi, K.J. Eskola, R. Paatelainen, Phys.Rev. C93 (2016) 024907)

- EKRT initial conditions + various $\eta/s(T)$ at LHC run 1

FLOW MEASUREMENTS

Basic building block is *n*-particle correlation:

$$\langle m\rangle_{n_1,n_2,\ldots,n_m} \equiv \left\langle \langle\cos(n_1\phi + n_2\phi \ldots + n_m\phi)\rangle_{m \text{ particles}} \right\rangle$$

$$\overset{\text{(flow)}}{=} \langle V_{n_1} V_{n_2} \ldots V_{n_m}\rangle$$

$$\sum n_i = 0$$

$$\langle\ldots\rangle_{2 \text{ particles}} \equiv \frac{1}{N_{\text{pairs}}}\sum_{i=1}^{M}\sum_{j\neq i}\ldots$$

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Typical weights: $W_{\langle 2\rangle} = M(M-1)$

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Flow measurements

Examples:

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Enforce gap in rapidity between pair $\Longrightarrow$ suppressed non-flow

$$-v_n\{4\}^4 \equiv \langle 4\rangle_{n,n,-n,-n} - 2\langle 2\rangle^2_{n,-n} \stackrel{\text{(flow)}}{=} \langle v_n^2\rangle - 2\langle v_n^2\rangle^2$$

Suppresses non-flow without rapidity gaps

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Symmetric Cumulants

Can also define a mixed-harmonic cumulant:

$$\mathrm{SC}(n,m) \equiv \langle 4\rangle_{n,m,-n,-m} - \langle 2\rangle_{n,-n}\langle 2\rangle_{m,-m}$$
$$\stackrel{\text{(flow)}}{=} \langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle.$$

Independent information better encoded in normalized quantity:

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(Bilandzic, Christensen, Gulbrandsen, Hansen, Zhou, Phys. Rev. C 89, no. 6, 064904 (2014))

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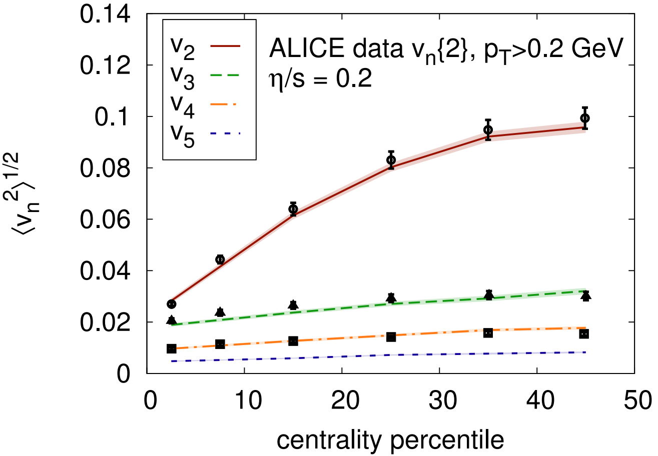
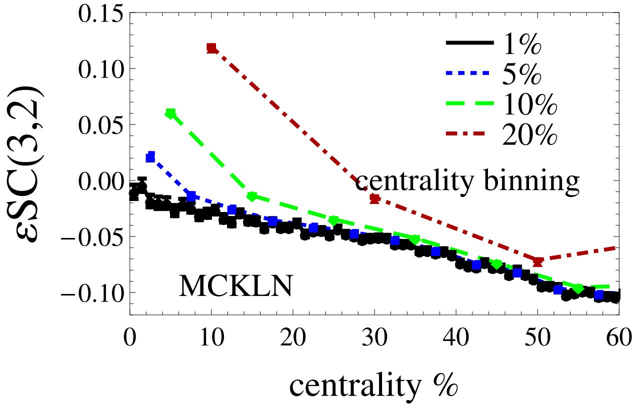
CENTRALITY BINNING



- The magnitude of all v_n increase from central $\to$ peripheral
- $\Longrightarrow$ impact parameter fluctuations increase NSC(n, m)

Centrality binning



- Large centrality bins produce a bias
- Biggest effect in central collisions, where v_n vs. centrality steepest

EVENT WEIGHTING

$$\mathrm{NSC}(n,m) \stackrel{\text{(flow)}}{=} \frac{\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle}{\langle v_n^2\rangle\langle v_m^2\rangle}$$

- Typically, 4-particle correlations have $\sim M^4$ event weights, while 2-particle correlations have $\sim M^2$ weighting
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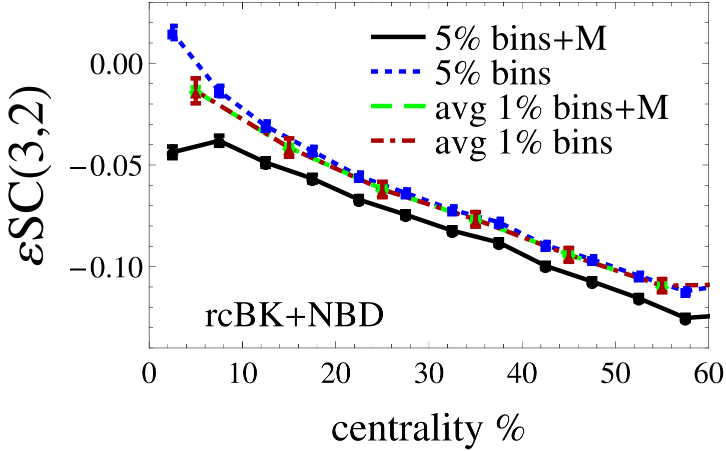
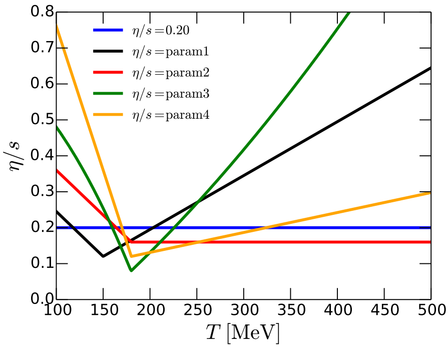
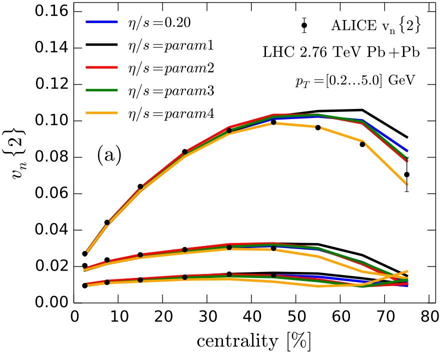


- $\Longrightarrow$ *negative* bias with large centrality bins
- No bias with small centrality bins

FIRST CALCULATION





(H. Niemi, K.J. Eskola, R. Paatelainen, Phys.Rev. C93 (2016) 024907)

- Various $\eta/s(T)$ fit single harmonic data at RHIC + LHC.
- Other mixed harmonic measurements ruled out some

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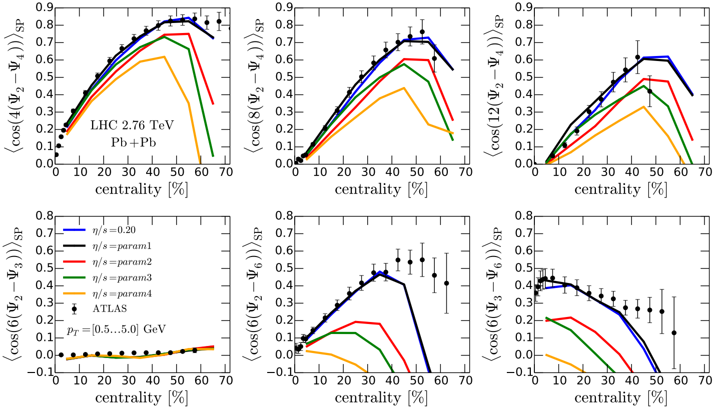

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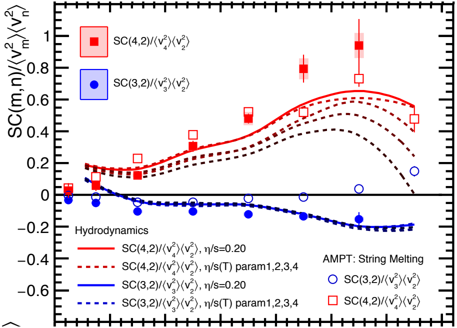

ALICE, arXiv:1604.07663; Niemi, Eskola, Paatelainen, 1505.02677

- Doesn't seem to fit NSC(3,2) and NSC(4,2)
- (may be affected by centrality binning effects)
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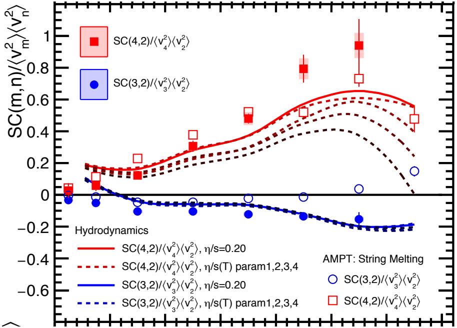

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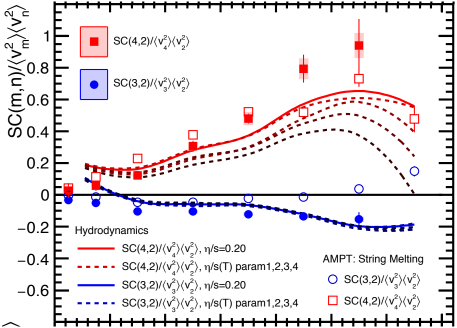

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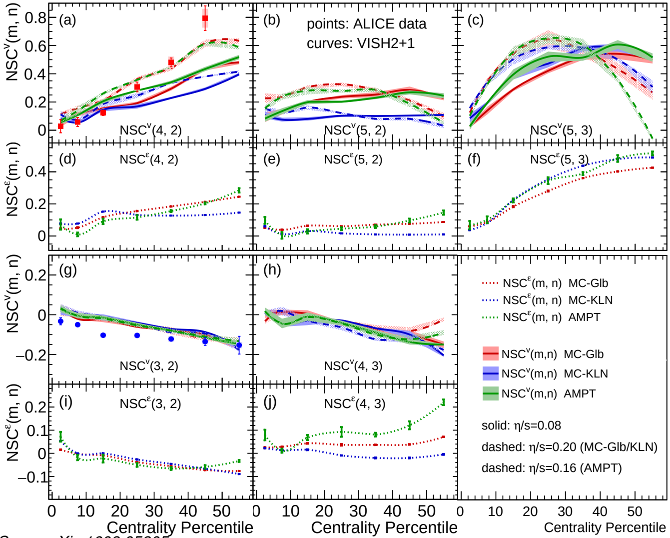
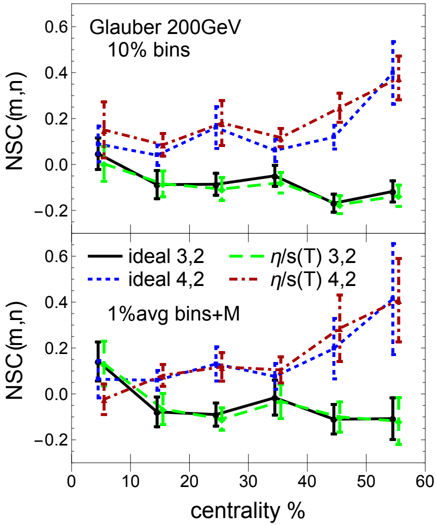


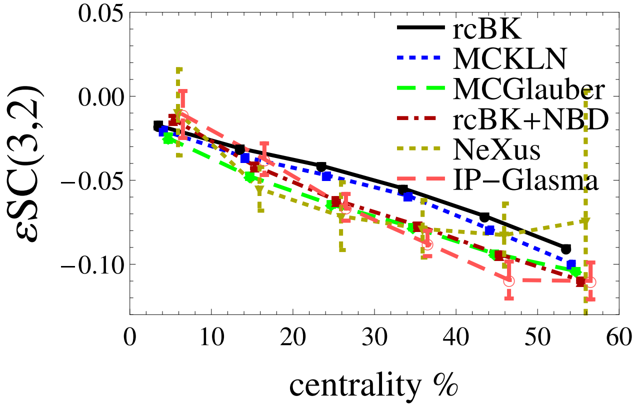
Zhu, Zhou, Xu, Song; arXiv:1608.05305

NSC(3,2) also insensitive to magnitude of η/s (NSC(4,2) sensitive)



Even that observation can depend on centrality binning!

NSC(3,2) vs. initial conditions



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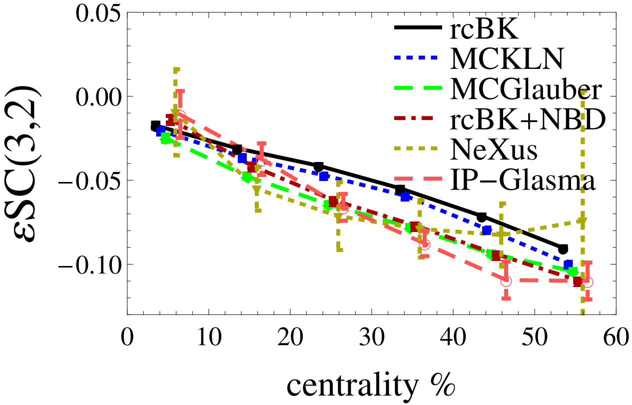

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Summary/Conclusions

- Mixed harmonic correlation measurements shed new light on heavy-ion collisions
- Symmetric cumulants measure correlation between magnitudes of flow vectors of different harmonics, with suppression of non-flow correlations
- Different sensitivity to initial conditions and viscosity than previous observables.

EXTRA SLIDES

Cumulant expansion

Idea: *(Teaney, Yan; Phys.Rev. C83 (2011) 064904)*

- Characterize density by moments (or cumulants) of 2-D Fourier transform

$$\rho(\mathbf{k}) = \int d^2x \rho(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

- (Small m = small power of k in Taylor series)
- Can write complete hydro response as set of functions

$$v_n e^{in\Psi_n} = f(W_{m,n})$$

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- Must make quantities with correct symmetries out of $W_{m,n} \sim \langle r^m e^{in\phi} \rangle$
- If anisotropies are small, can arrange in Taylor series. E.g., to first order:

$$V_n \equiv v_n e^{in\Psi_n} = \sum_{p=0}^{\infty} C_{n+2p,n} W_{n+2,p}$$

- If hydro is sensitive to large scale structure, first terms are most important (smaller powers of k in Fourier transform).
- If single term sufficient:

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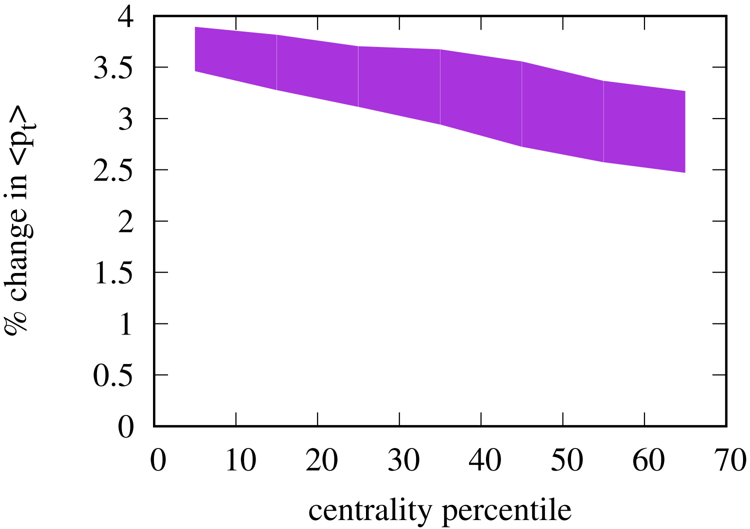
Final prediction



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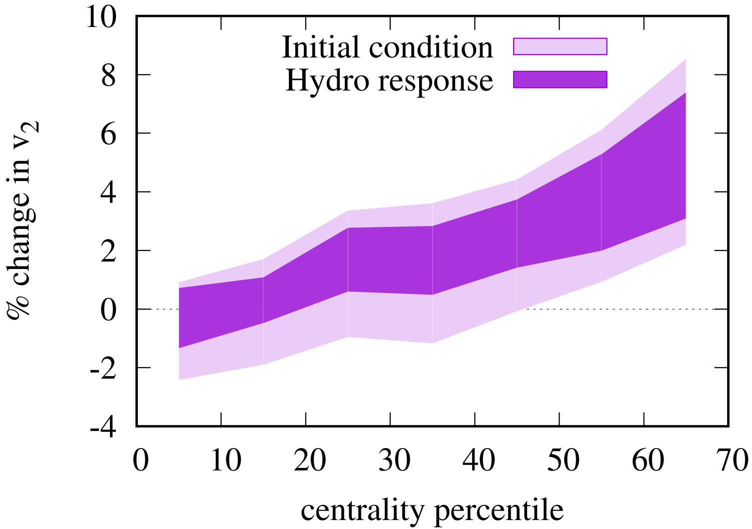

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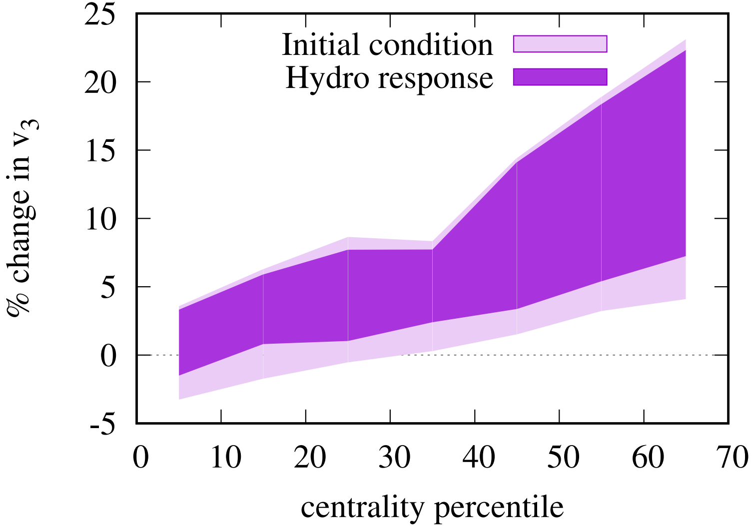
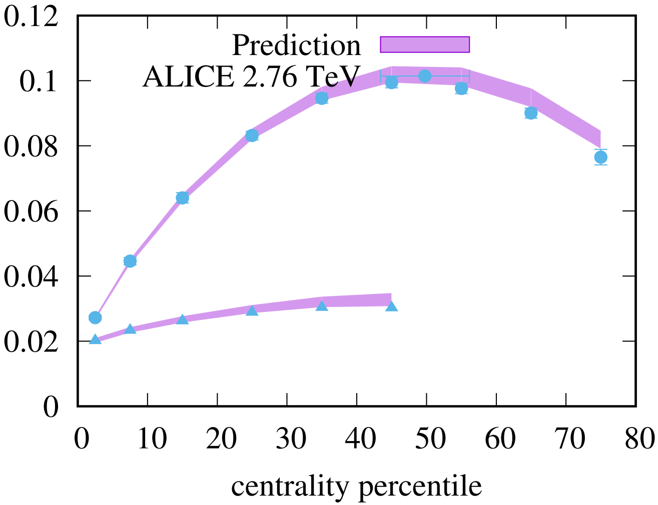
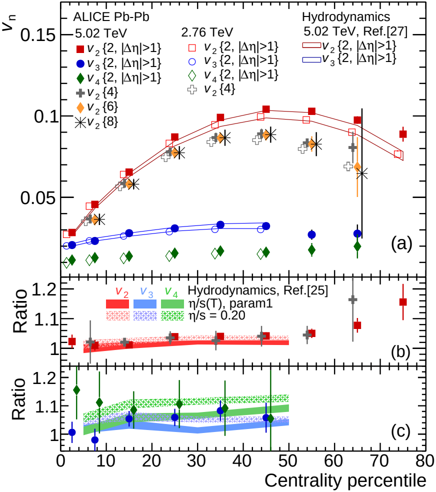


Prediction preview: absolute $v_2\{2\}$ and $v_3\{2\}$

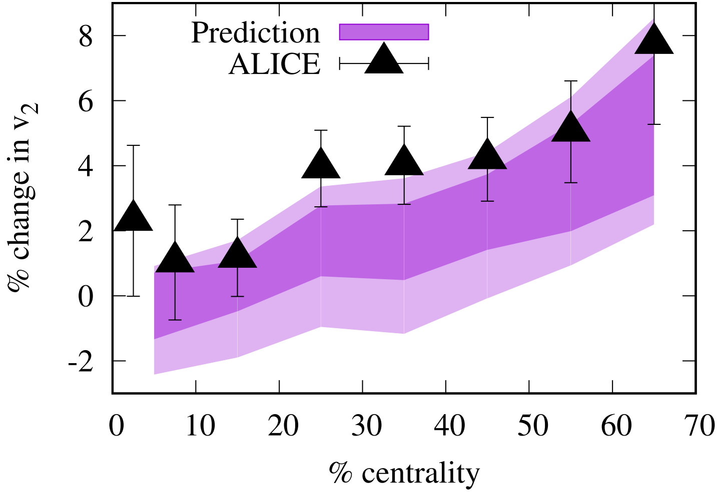


First data from run 2



(ALICE Collaboration, Phys.Rev.Lett. 116 (2016) 132302)

Comparison with data



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