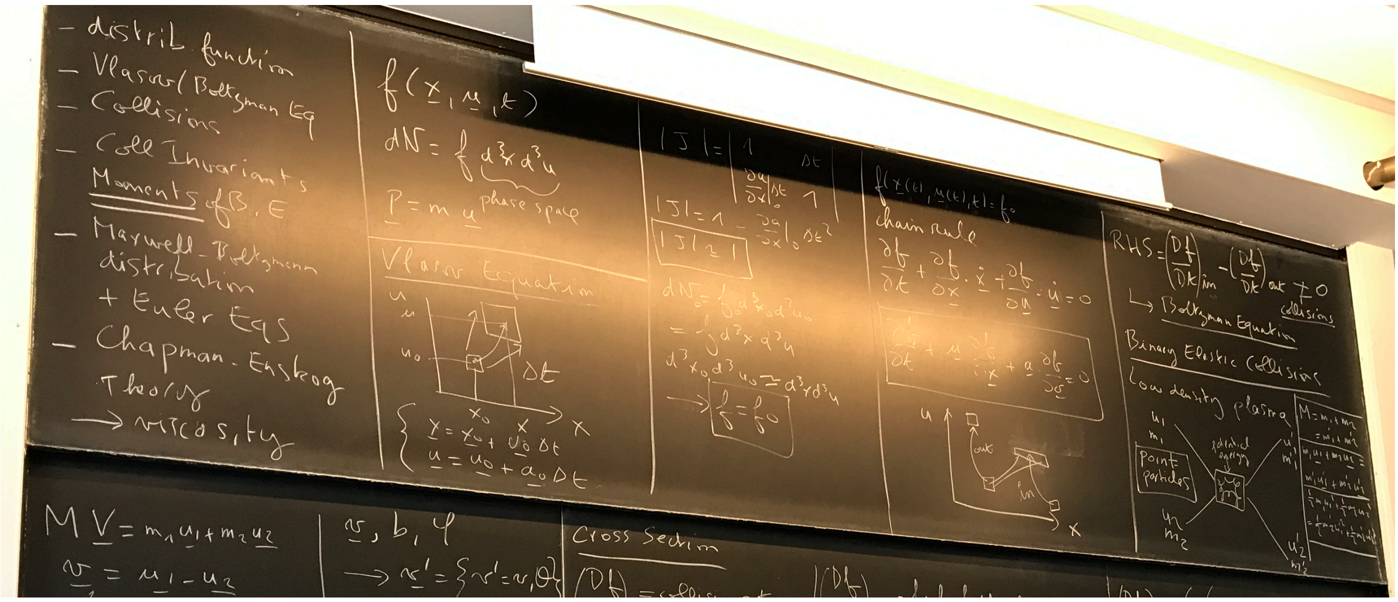
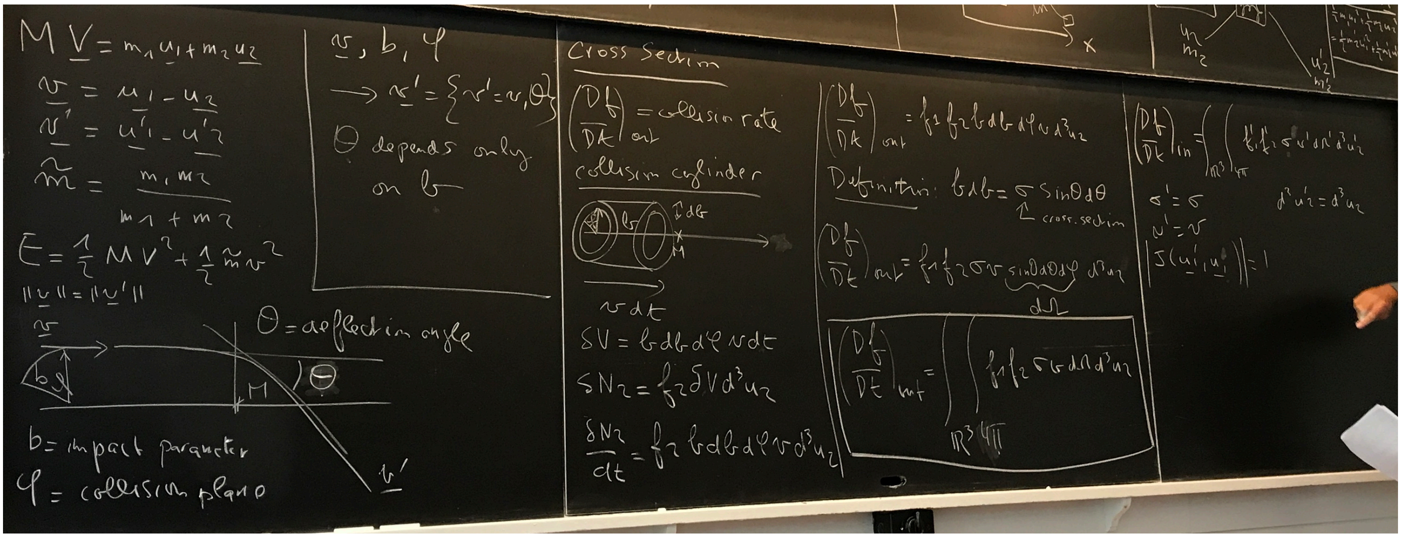
The Vlasov Equation and the Distribution Function

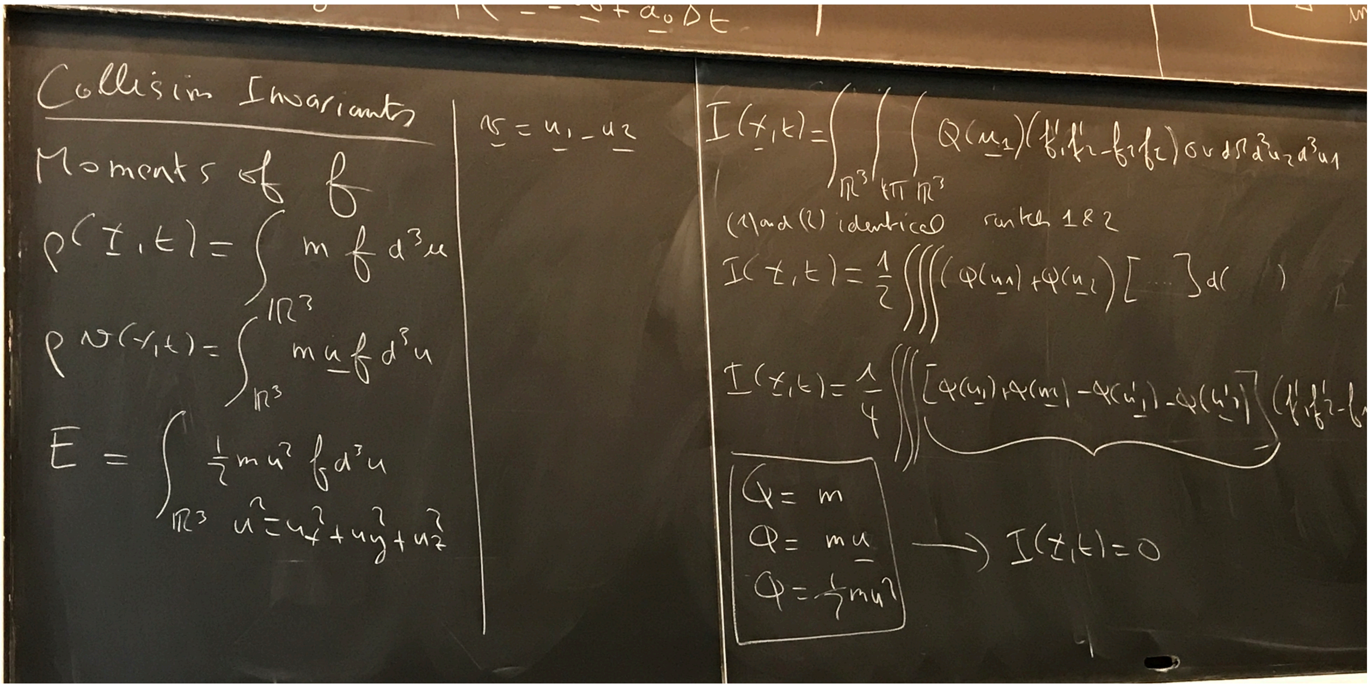


Collisions

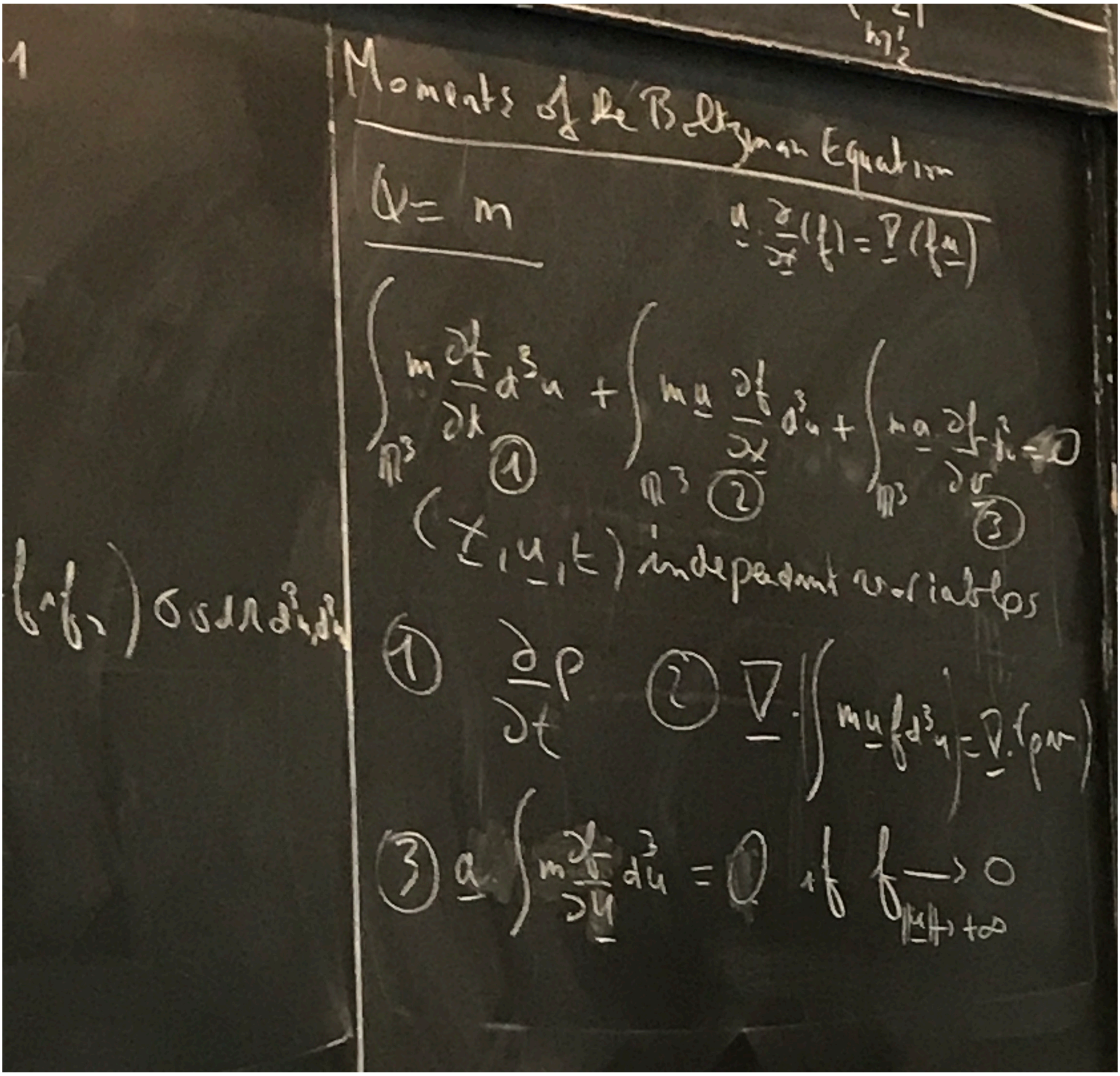


Cross Section
collision rate
collision cylinder
θ = deflection angle
θ depends only on b
b = impact parameter
φ = collision plane
Definition:
cross section

Collision Invariants



Taking Moments to get the Fluid Equations



Moments through Partial Integrals

$$Q(u_x) = m u_x$$

$$\int m u_x \frac{\partial f}{\partial t}\, d^3u + \int m u_x\, \underline{u}\cdot\frac{\partial f}{\partial \underline{x}}\, d^3u + \int m u_x\, \underline{a}\cdot\frac{\partial f}{\partial \underline{u}}\, d^3u = 0$$

(1) (2) (3)

(1) $\frac{\partial}{\partial t}(\rho v_x)$ $\underline{w}$ = thermal velocity

(2) $u_x = v_x + w_x$ $\int m \underline{w} f\, d^3u = 0$

$$u_x\, \underline{u}\cdot\frac{\partial f}{\partial \underline{x}} = \underline{\nabla}\cdot(u_x f \underline{u})$$

$$\underline{\nabla}\cdot\left(\int m u_x f \underline{u}\, d^3u\right)$$

Moments through Partial Integrals

$$M_\times \underline{u} = v_\times \underline{v} + w_\times \underline{v} + v_\times \underline{w} + w_\times \underline{w}$$

$= 0$

$$\underline{\nabla}\cdot\left(\rho v_\times \underline{v} + 0 + 0 + \underline{\underline{P}}\right) \quad (3)$$

integration by parts

$$\boxed{P_{ij} = \int m w_i w_j f \, d^3 u}$$

Pressure tensor

$$\boxed{\frac{\partial}{\partial t}(\rho \underline{v}) + \underline{\nabla}\cdot\left(\rho \underline{v} \otimes \underline{v} + \underline{\underline{P}}\right) = \rho \underline{a}}$$

Moments through Partial Integrals

$$E = \int \frac{1}{2} m u^2 f \, d^3u = \frac{1}{2}\rho v^2 + e$$

$$\boxed{e = \int \frac{1}{2} m w^2 f \, d^3u \qquad w^2 = w_x^2 + w_y^2 + w_z^2}$$

internal energy

$$\boxed{e = \frac{3}{2} \mathrm{Tr}\, \underline{\underline{P}}}$$

$$\boxed{\underline{Q} = \int m \underline{w} \frac{w^2}{2} f \, d^3u}$$

$$\boxed{\frac{\partial E}{\partial t} + \underline{\nabla} \cdot \left(E \underline{v} + \underline{\underline{P}} \cdot \underline{v} + \underline{Q} \right) = \rho \underline{v} \cdot \underline{a}}$$

The Maxwell-Boltzmann Distribution

Parts

$$\boxed{f_0 = \frac{\rho/m}{(\,\pi\,)^{3/2}}\, e^{-\frac{1}{2}\frac{(u-v)^2}{\sigma^2}}}$$

$$\underline{\underline{P}} = P\,\underline{\underline{\mathbb{1}}}$$

$$\underline{Q} = 0$$

↳ Euler Equ.

$$\underline{\underline{P}}) = \rho\, \underline{a}$$