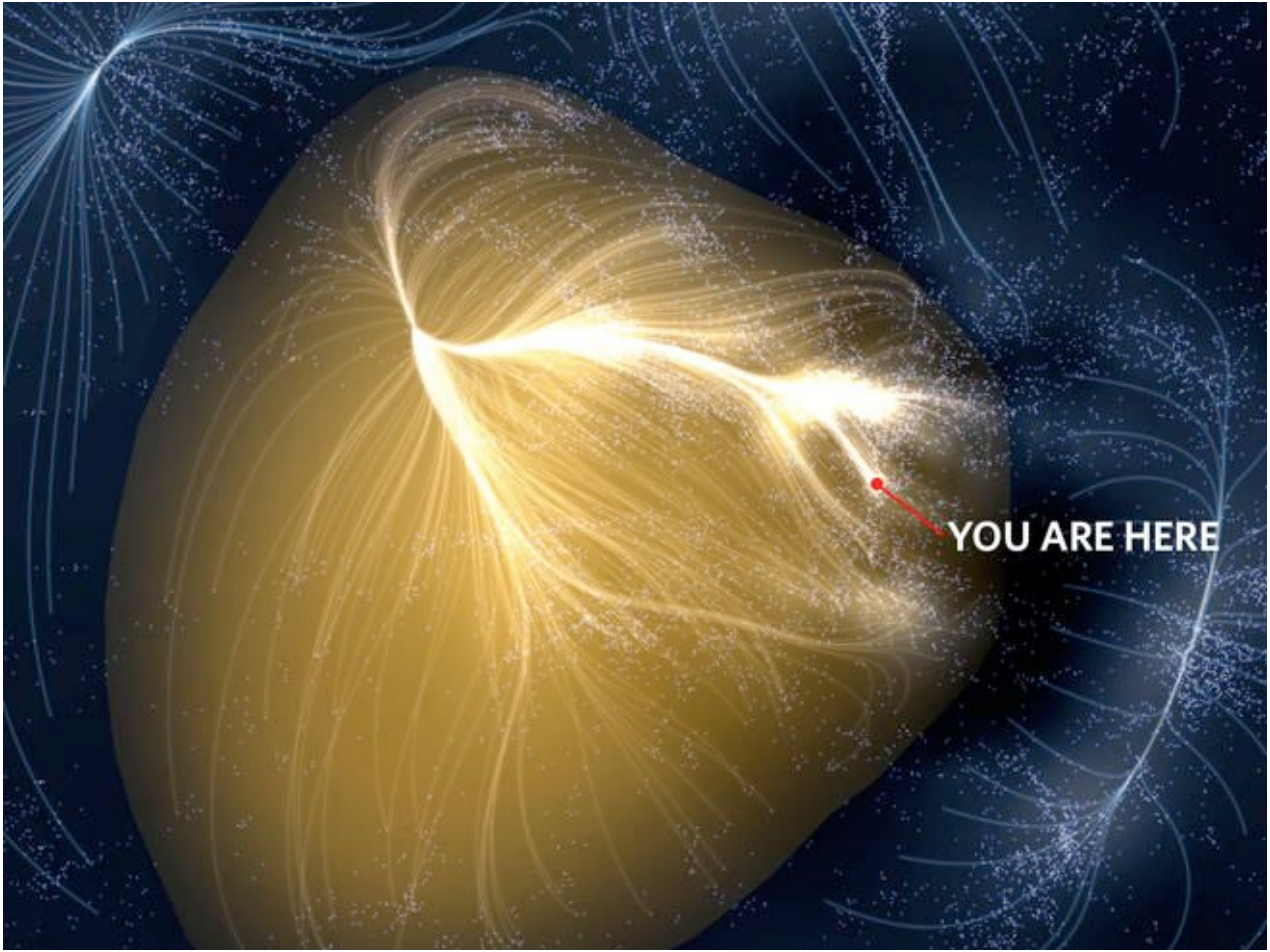
Collisionless, self-gravitating systems

Steen H. Hansen,
Dark Cosmology Centre,
Niels Bohr Institute,

NBIA summer school, Aug 2017

Dark Cosmology Centre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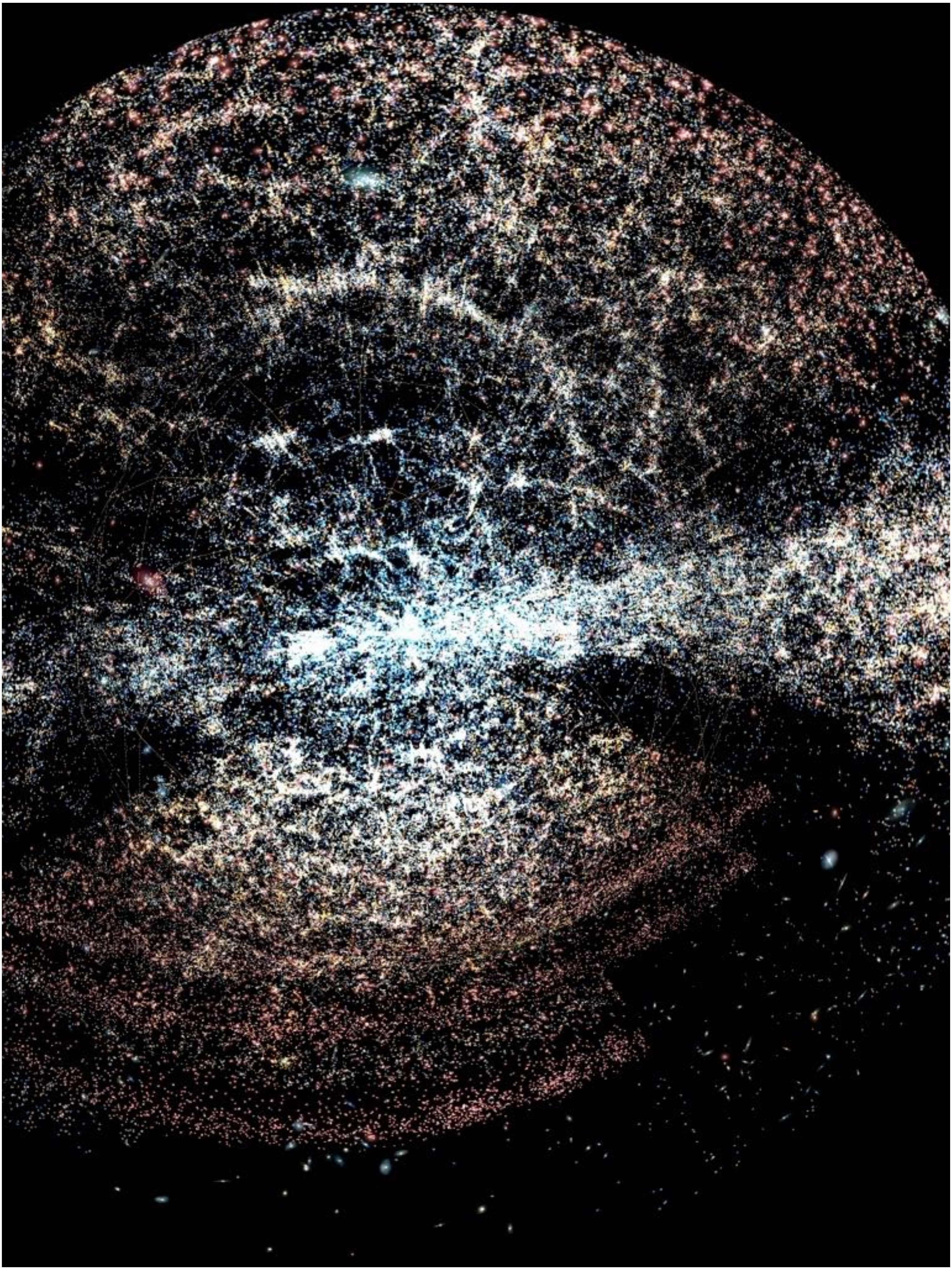
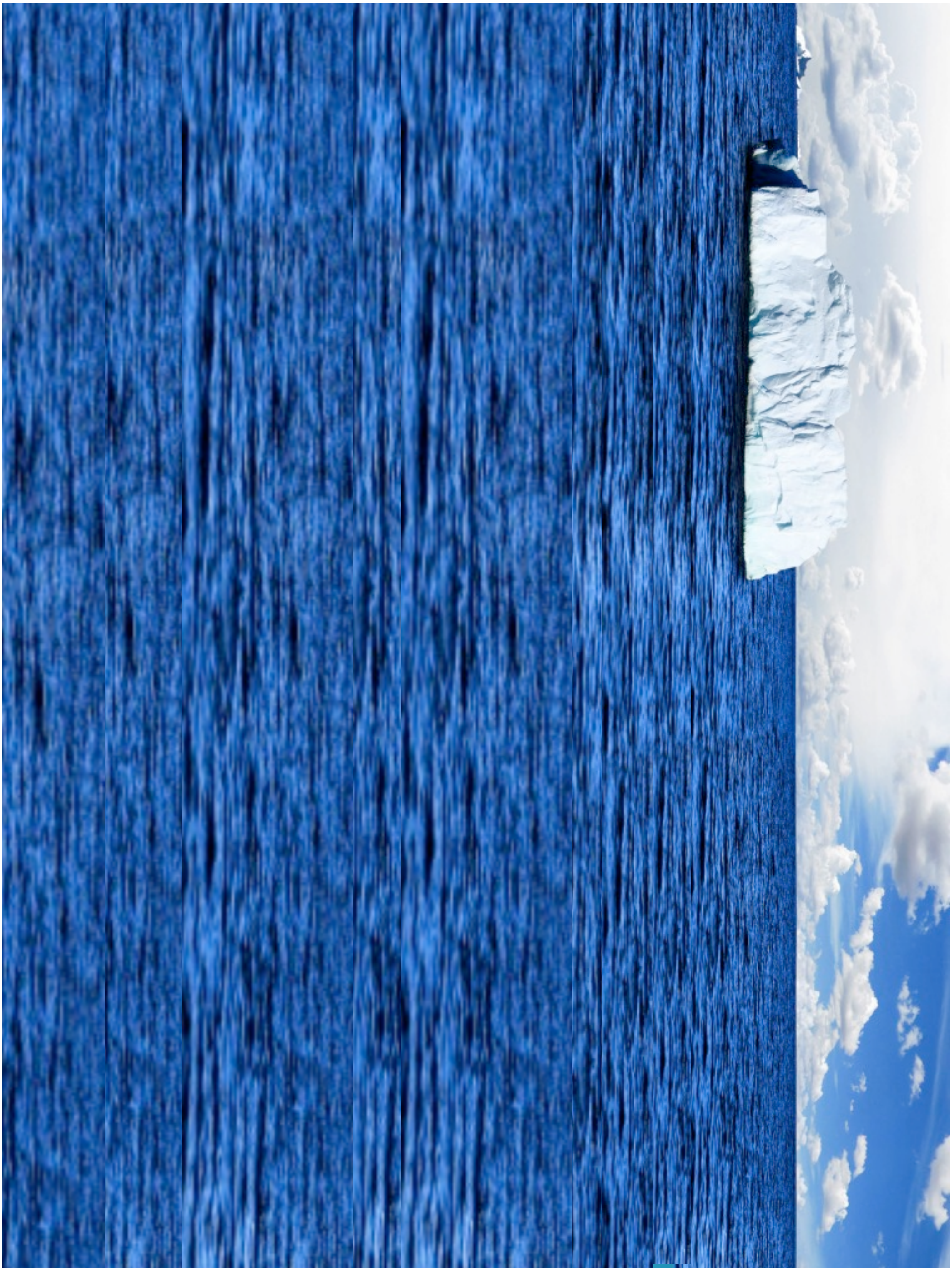
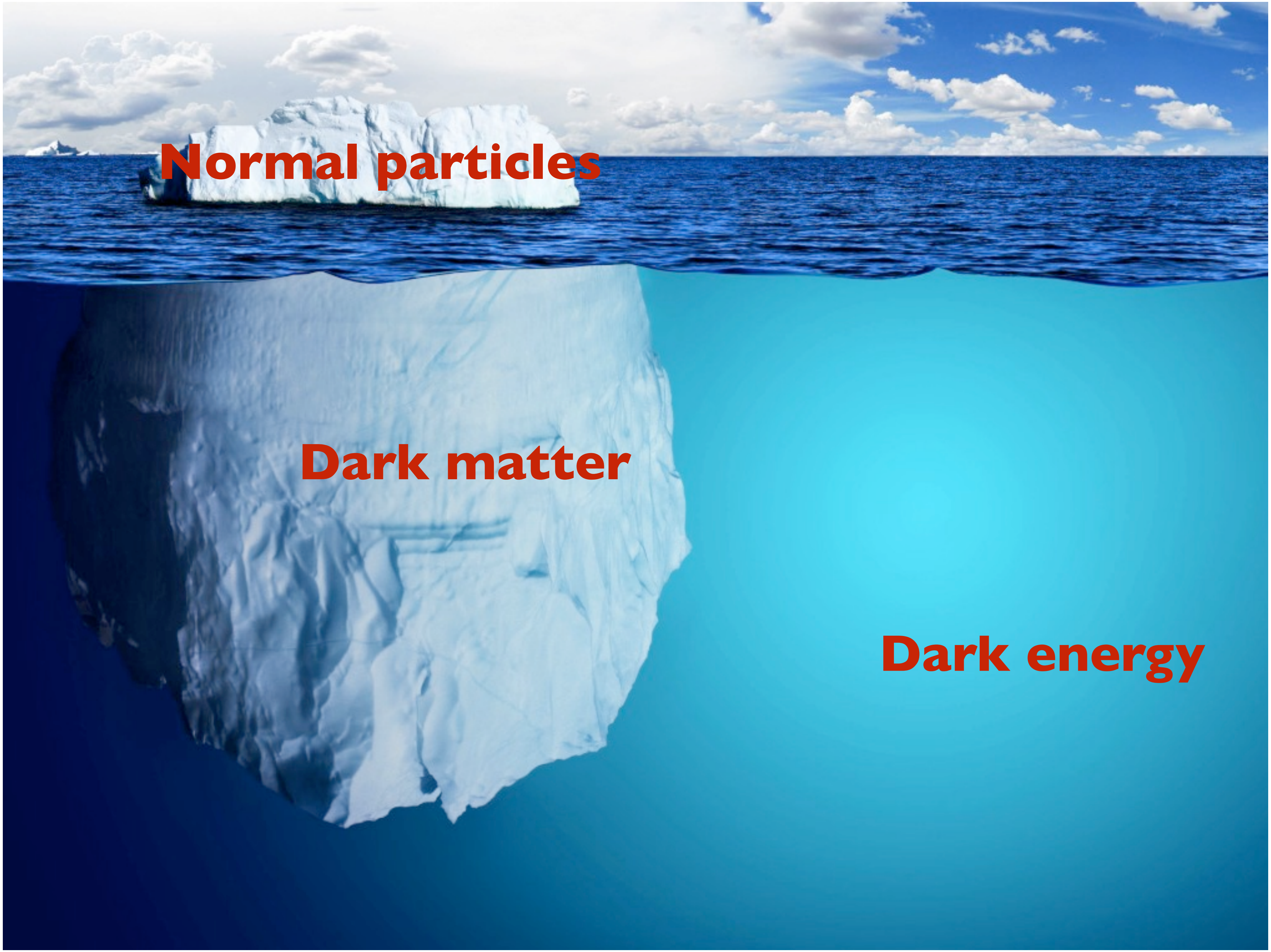





Normal particles
Dark matter
Dark energy

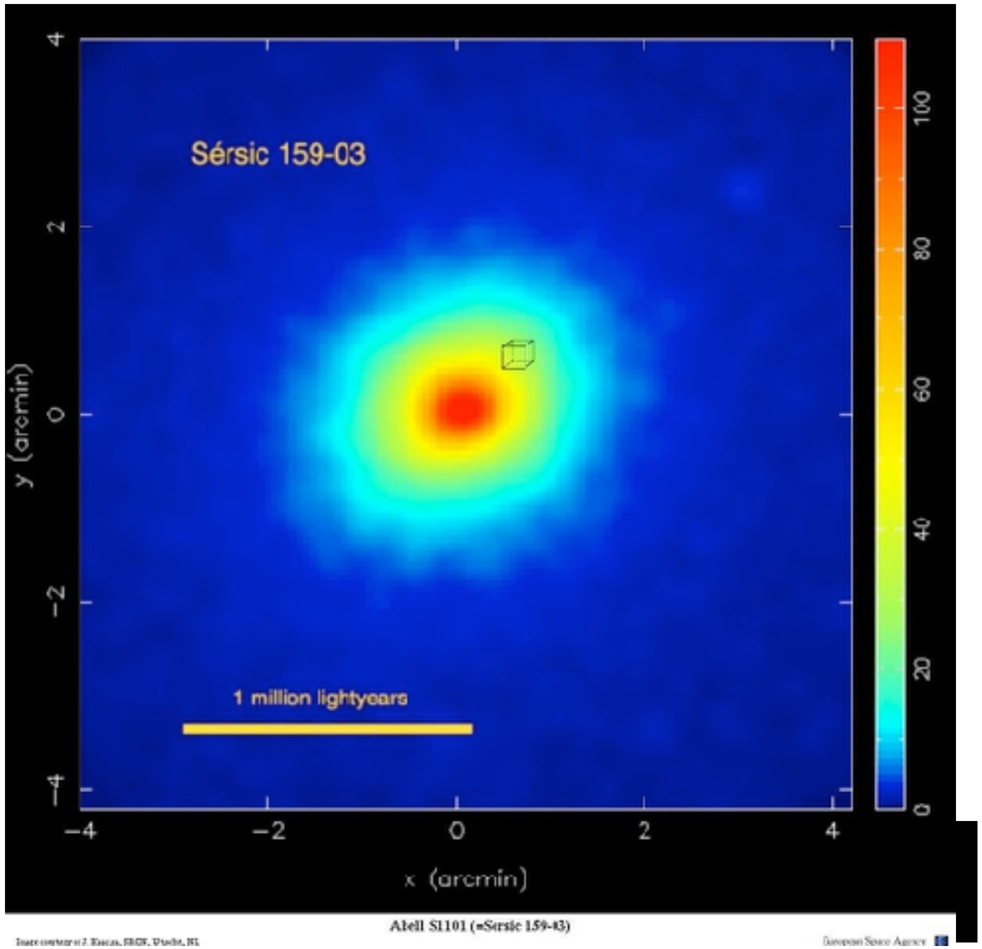
Dark matter

The magic of momentum conservation

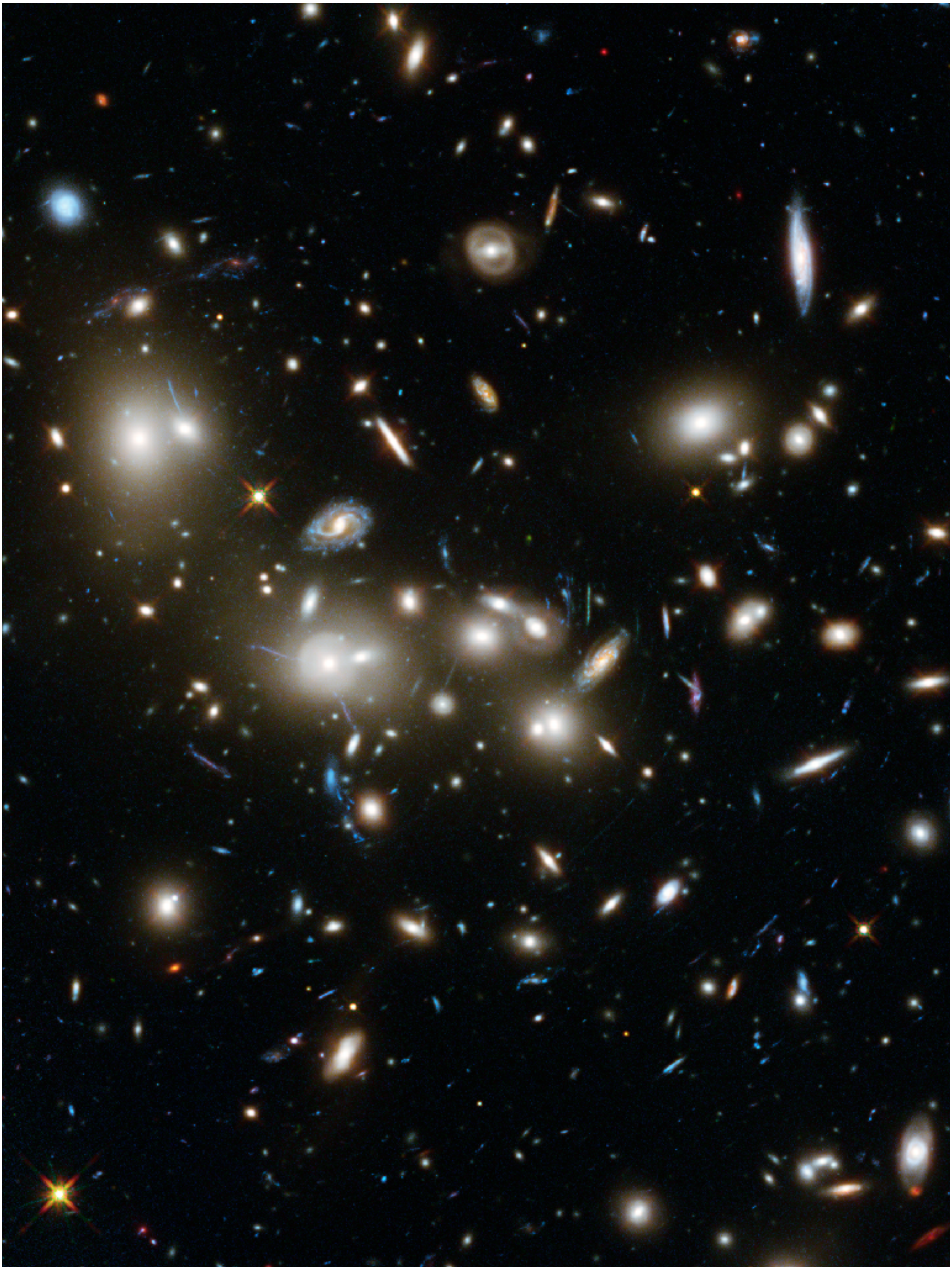
Steen H. Hansen
Dark Cosmology Centre

By measuring temperature and density of the X-ray emitting gas, we find that there is 5-10 times more dark matter than visible matter in a galaxy cluster.

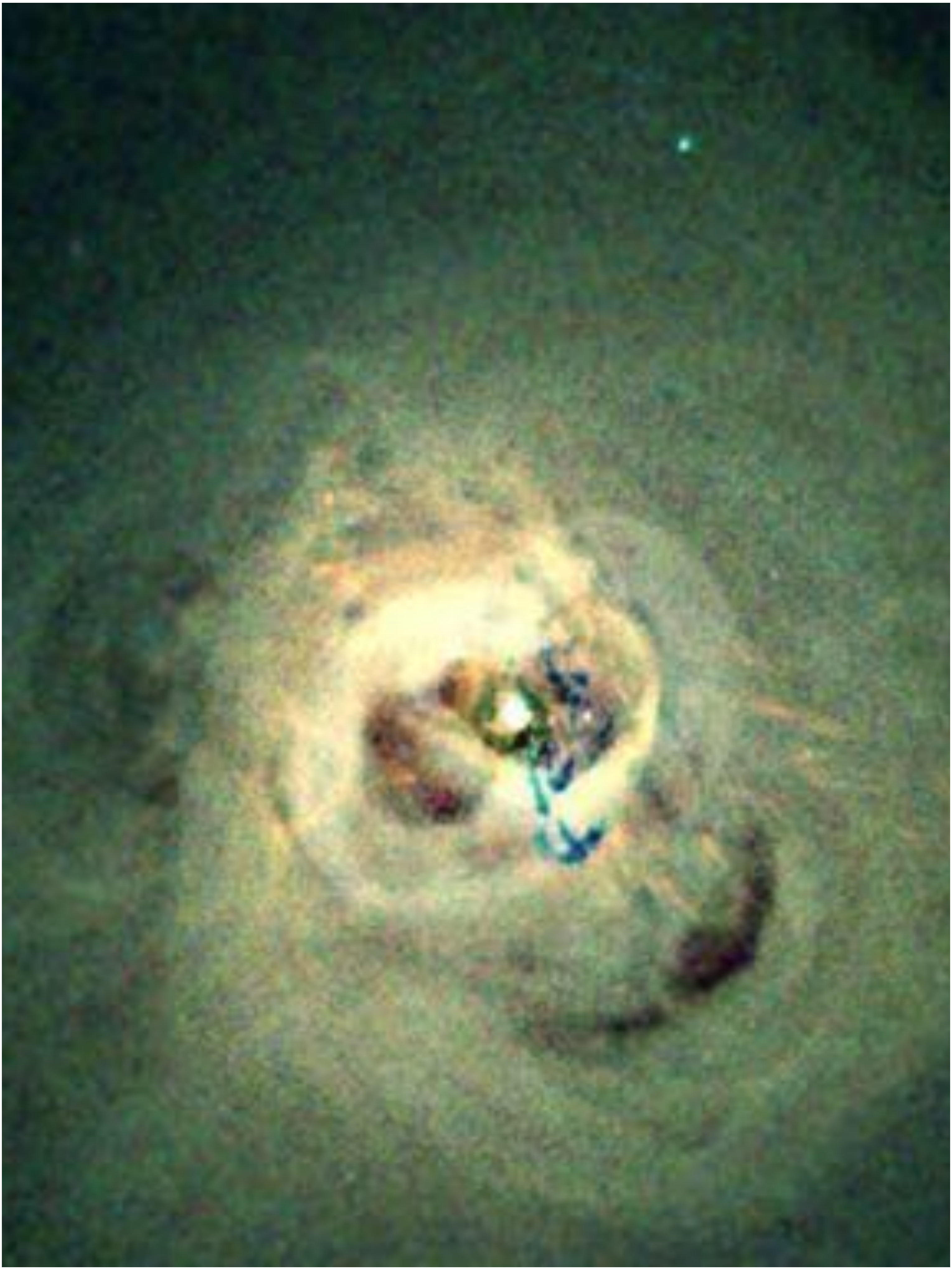
The devil is in the details…

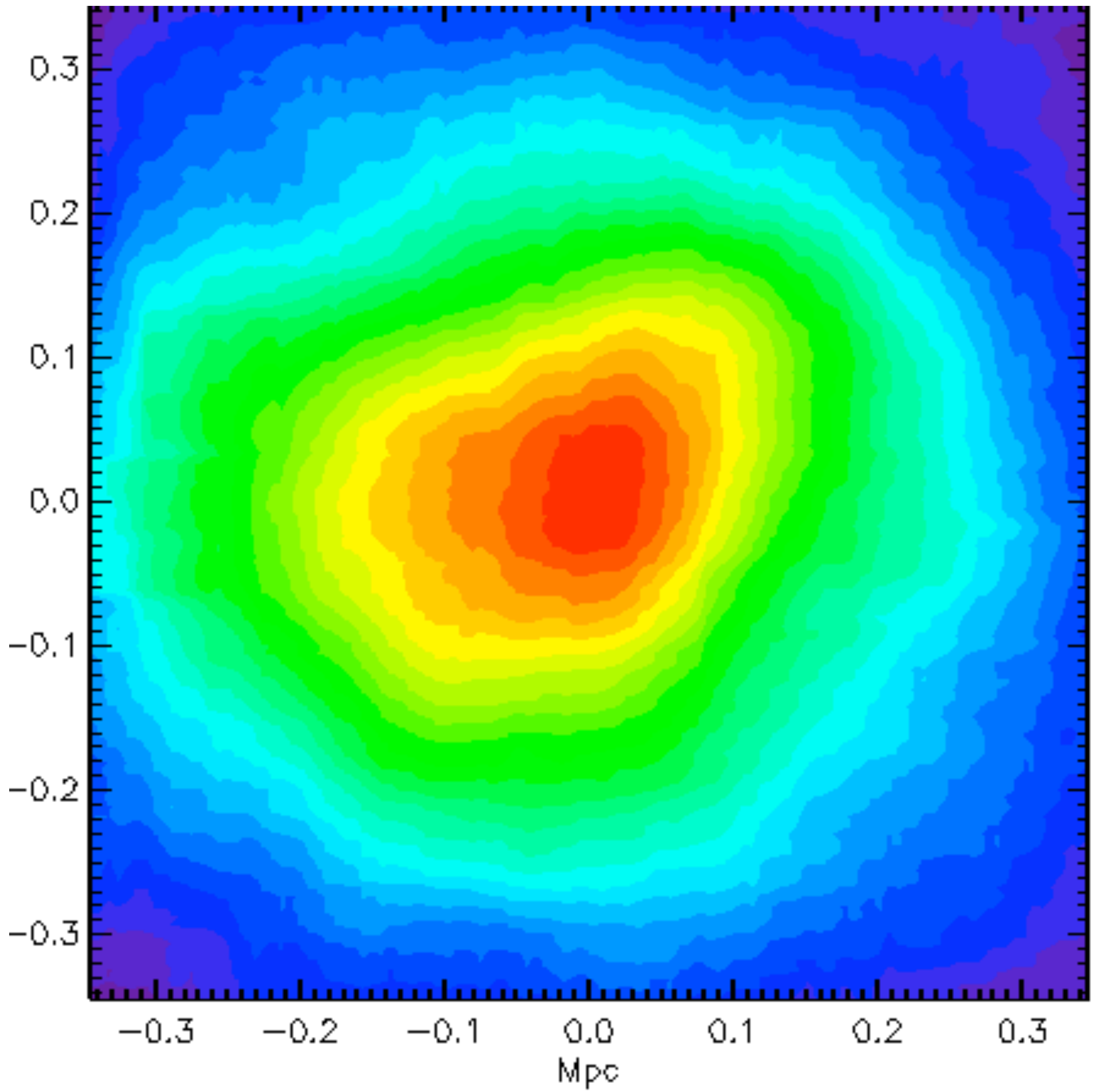


Observed galaxy cluster - visible light



Observed galaxy cluster - X-ray emitting gas

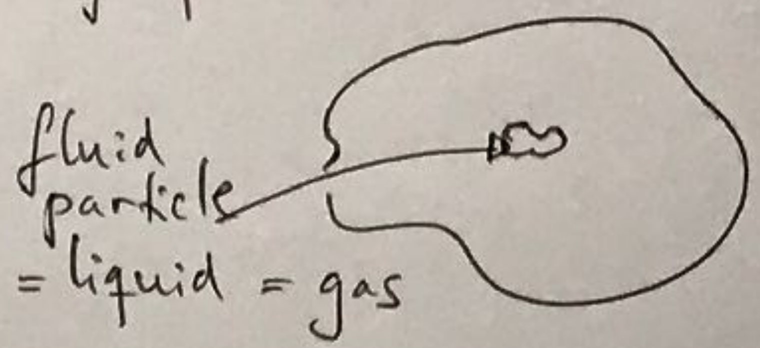



0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
Mpc

Momentum conservation in the gas gives you the equation of **Hydrostatic Equilibrium** (HE)

$$\frac{GM_{\rm tot}}{r} = -\frac{k_B T}{\mu m_p}\left(\frac{d{\rm ln}T}{d{\rm ln}r} + \frac{d{\rm ln}n_e}{d{\rm ln}r}\right)$$

A fluid particle has frequent collisions



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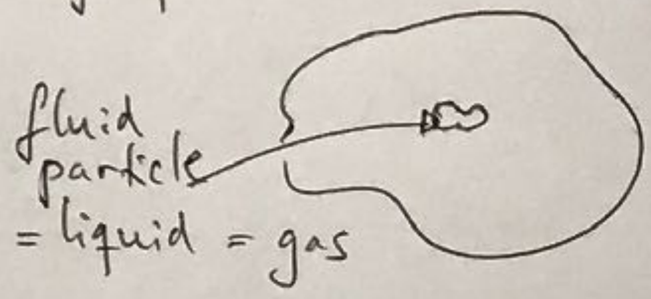


We can derive equations for :

m , mv , $\frac{1}{2} m v^2$, ...

0th order in vel $\quad \frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \vec{v}) = 0 \quad$ [Conservation of matter]

1st order in vel $\quad \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = - \frac{\nabla P}{\rho} + \vec{g} \quad$ [Conservation of momentum]

$$0 = -\frac{\nabla P}{\rho} + \vec{g}$$

For spherical systems we have

$$\vec{g}(r) = -\frac{G M_{tot}(r)}{r^2}\,\hat{e}_r$$

$$\Rightarrow \quad \frac{G M_{tot}}{r^2} = -\frac{\frac{\partial P}{\partial r}}{\rho}$$

If we have an ideal gas where $P = \frac{\rho k_B T}{\mu m_p}$

m_p = proton mass and $\mu = 0.59$ if we have 0.24 of mass in ^4He and the rest e and p.

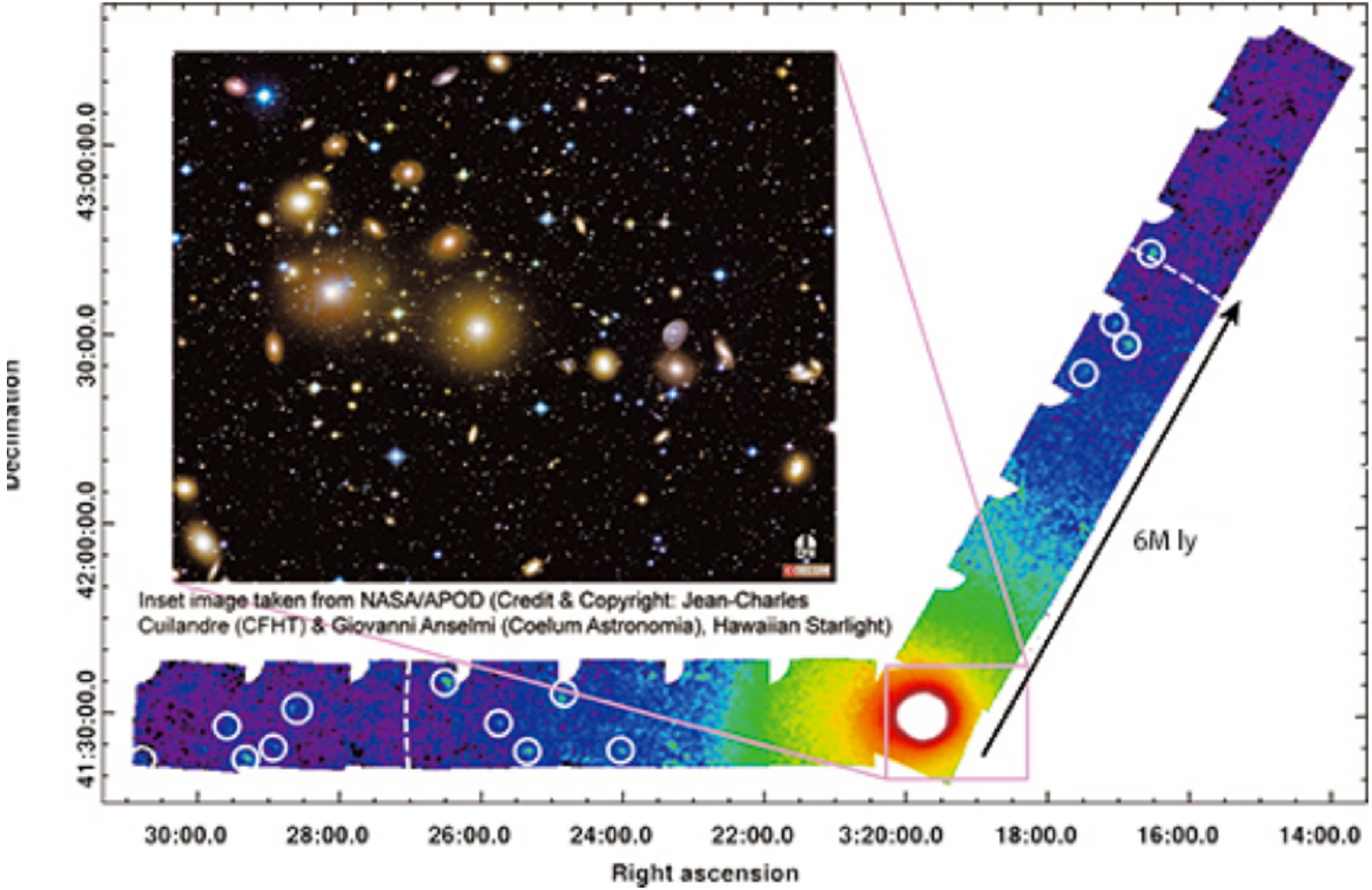
$$\Rightarrow \quad M_{tot}(r) = -\frac{k T r}{G \mu m_p}\left[\frac{d\log\rho}{d\log r} + \frac{d\log T}{d\log r}\right]$$

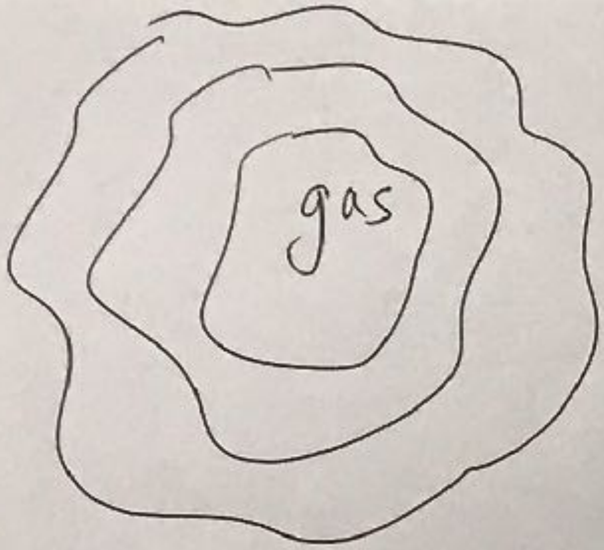
measure (T)

measure (ρ, T)

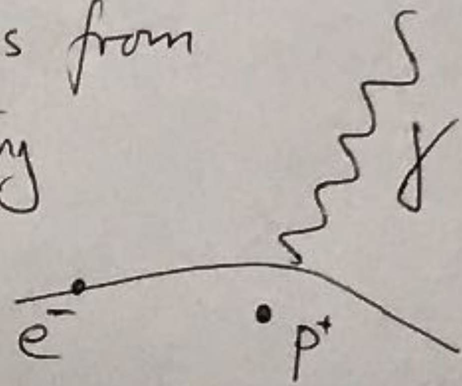
total (M_{tot}) = gas + dark matter + black hole + ...

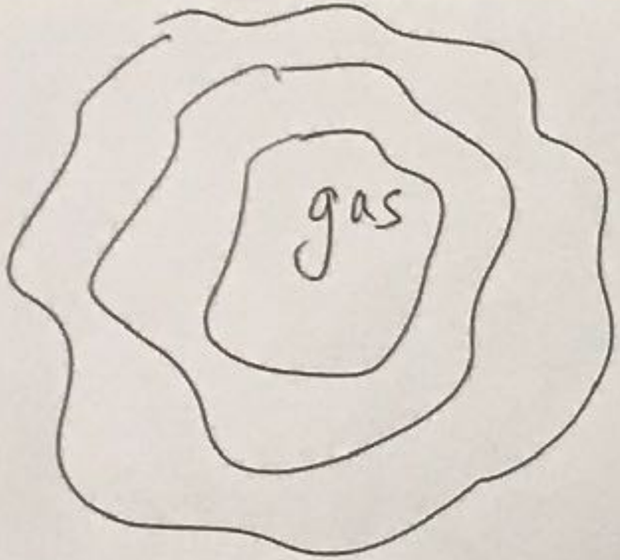
How do we measure the
temperature and density
of the hot gas?


Declination
43:00:00.0
30:00.0
42:00:00.0
41:30:00.0
Inset image taken from NASA/APOD (Credit & Copyright: Jean-Charles Cuilandre (CFHT) & Giovanni Anselmi (Coelum Astronomia), Hawaiian Starlight)
6M ly
30:00.0
28:00.0
26:00.0
24:00.0
22:00.0
3:20:00.0
18:00.0
16:00.0
14:00.0
Right ascension

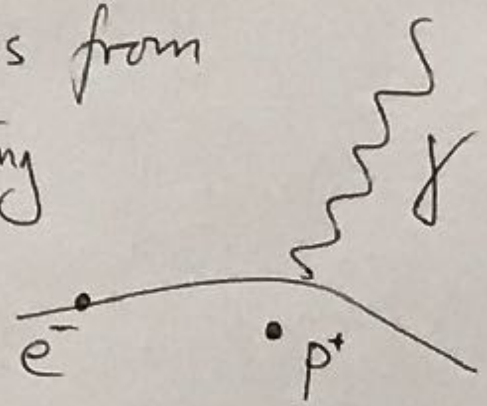


hot gas shines from
Bremsstrahlung

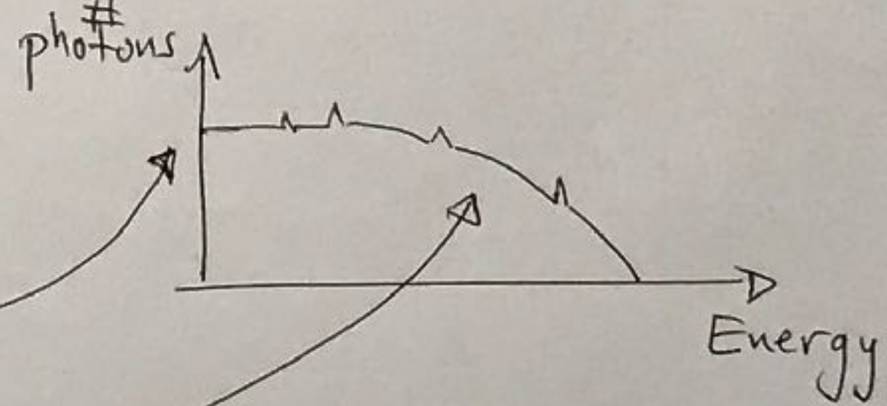




hot gas shines from Bremsstrahlung



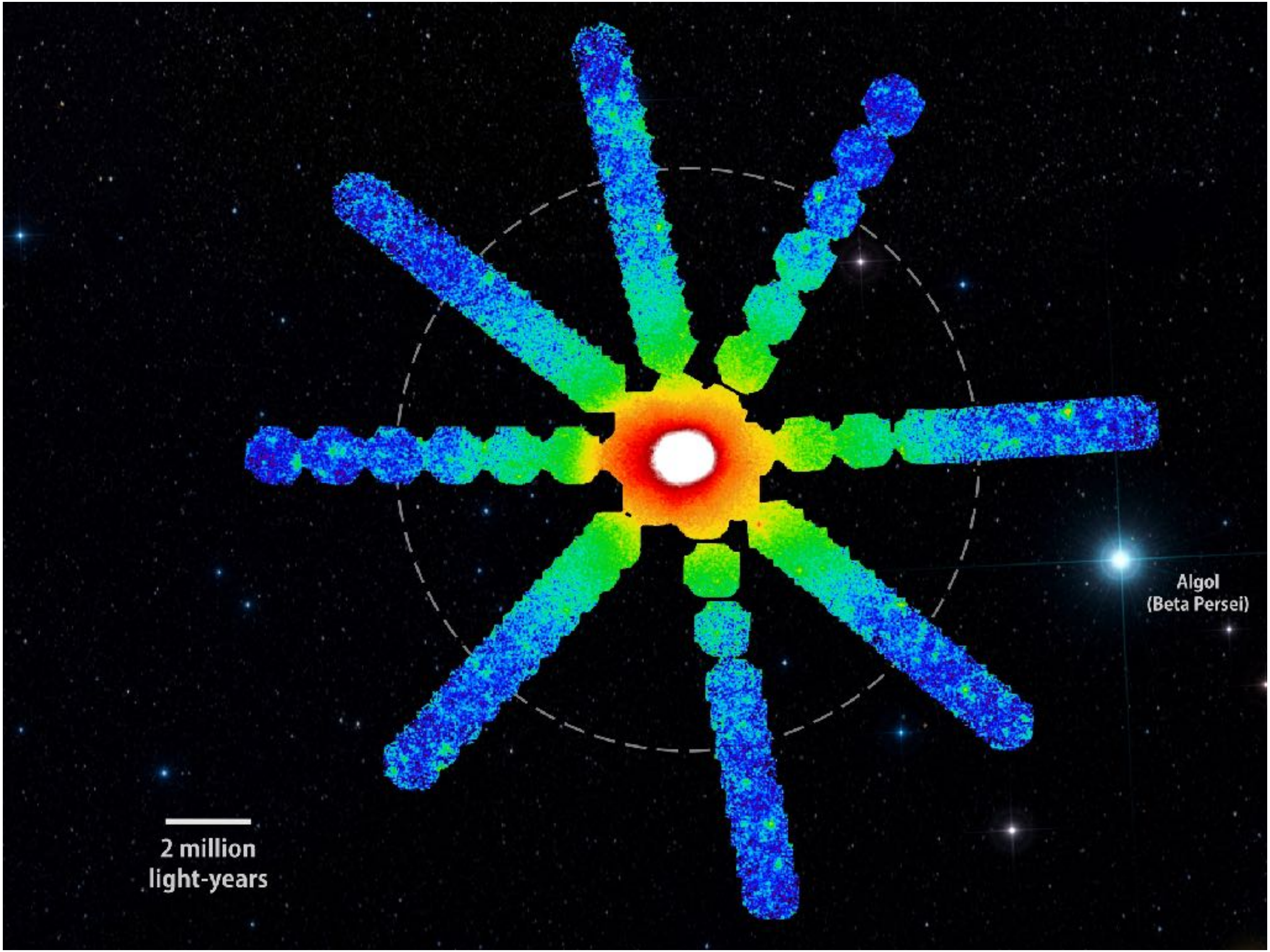
We measure a <u>spectrum</u> which looks like this

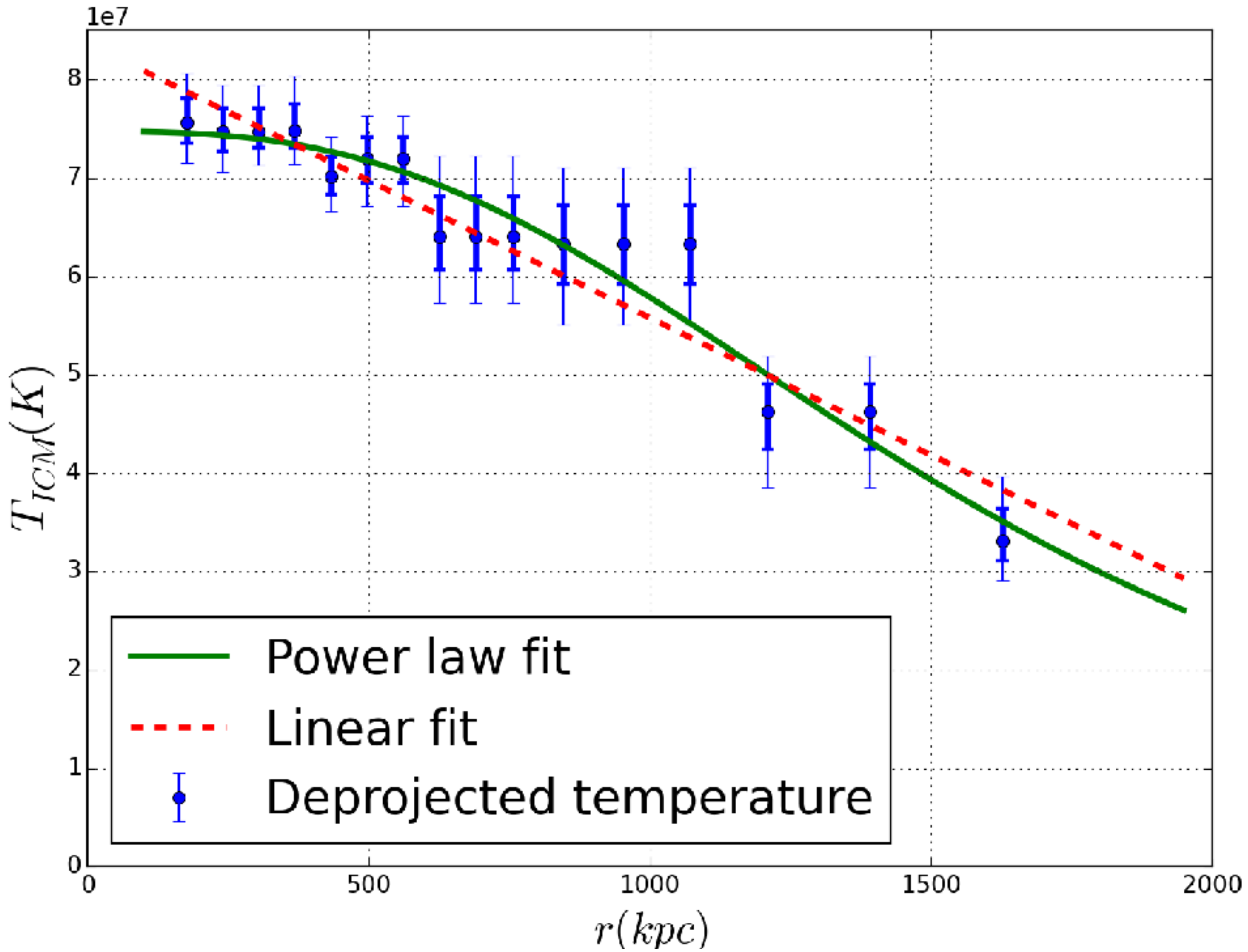


height of curve

The shape of the curve is $\sim \exp\left(-\frac{E}{k_B T}\right) \Rightarrow$ Measure T

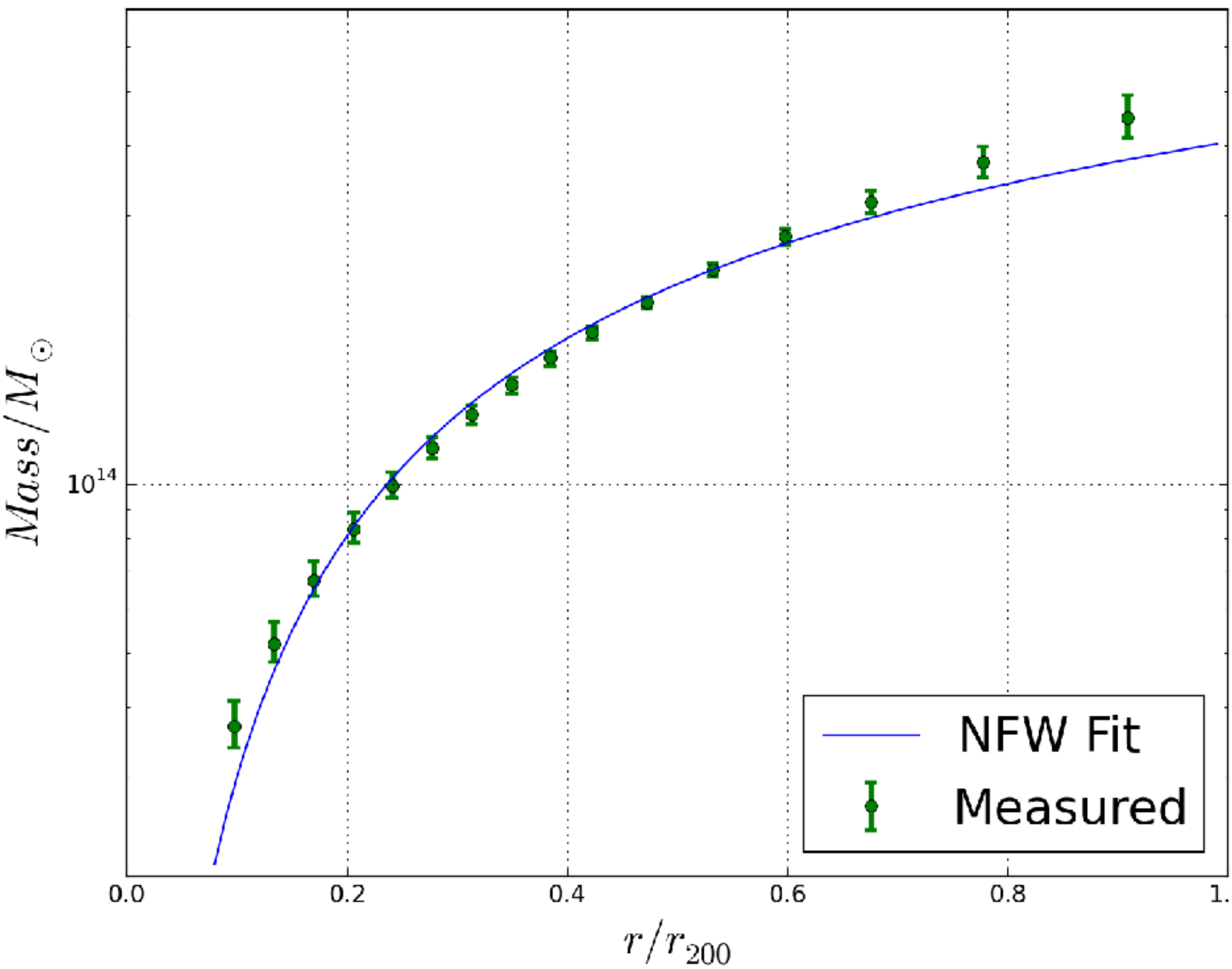
Total luminosity $\sim \rho^2 T^{1/2} \Rightarrow$ Measure ρ


2 million
light-years
Algol
(Beta Persei)



1e7
8
7
6
5
4
3
2
1
0
0
500
1000
1500
2000
$T_{ICM}(K)$
$r(kpc)$
Power law fit
Linear fit
Deprojected temperature

$$\frac{GM_{\rm tot}}{r} = -\frac{k_B T}{\mu m_p} \left(\frac{d{\rm ln}T}{d{\rm ln}r} + \frac{d{\rm ln}n_e}{d{\rm ln}r} \right)$$



Mass/M⊙
10^14
0.0
0.2
0.4
0.6
0.8
1.0
r/r200
NFW Fit
Measured

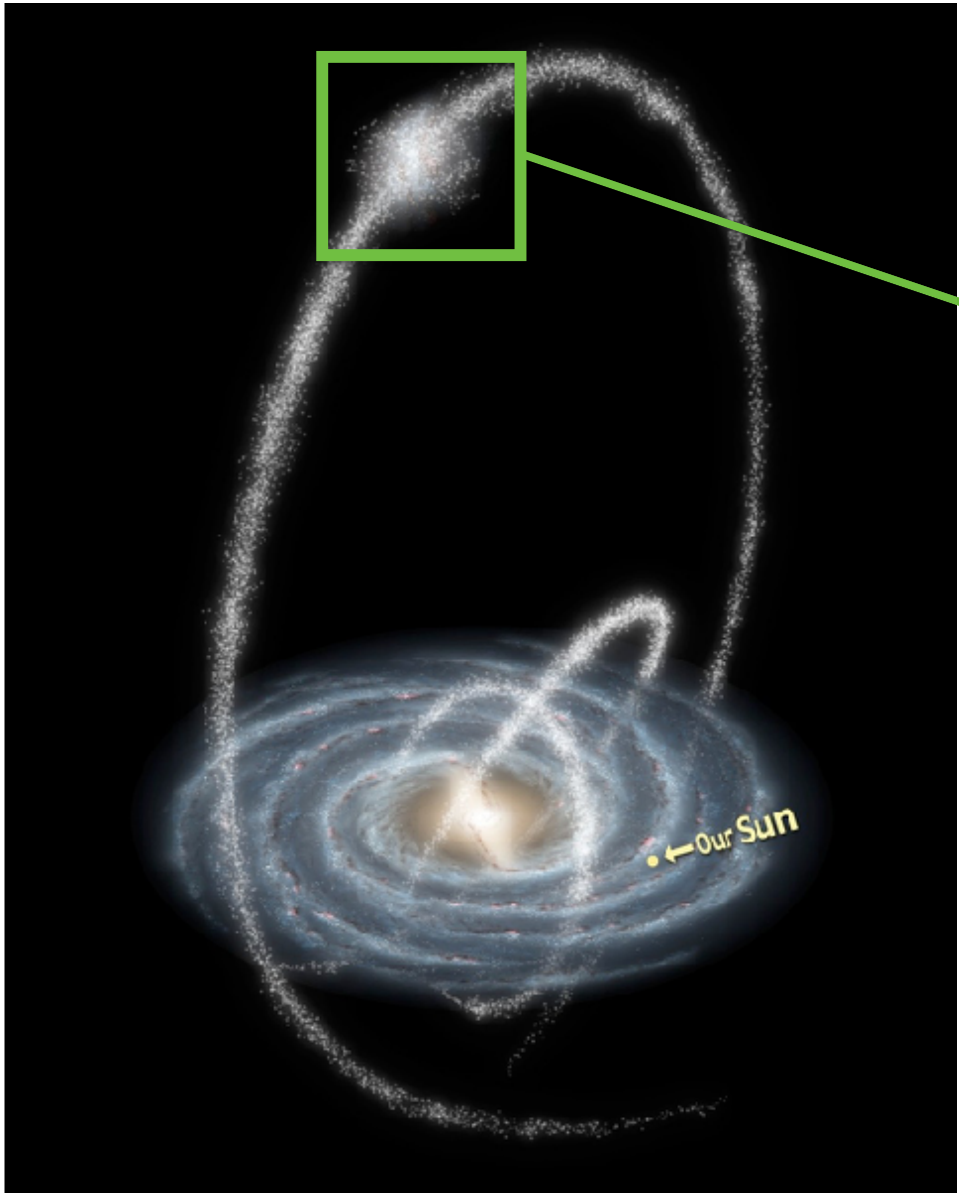
In conclusion
(concerning hydrostatic equilibrium),
by measuring the temperature and density
of a "**tracer**" (in this case the hot gas)
then we can derive the
total mass
as a function of radius.

That was for a collisional gas.

What about collisionless stars?

Steen H. Hansen
Dark Cosmology Centre

Dwarf galaxy



Does it weight more than just the mass of the stars?

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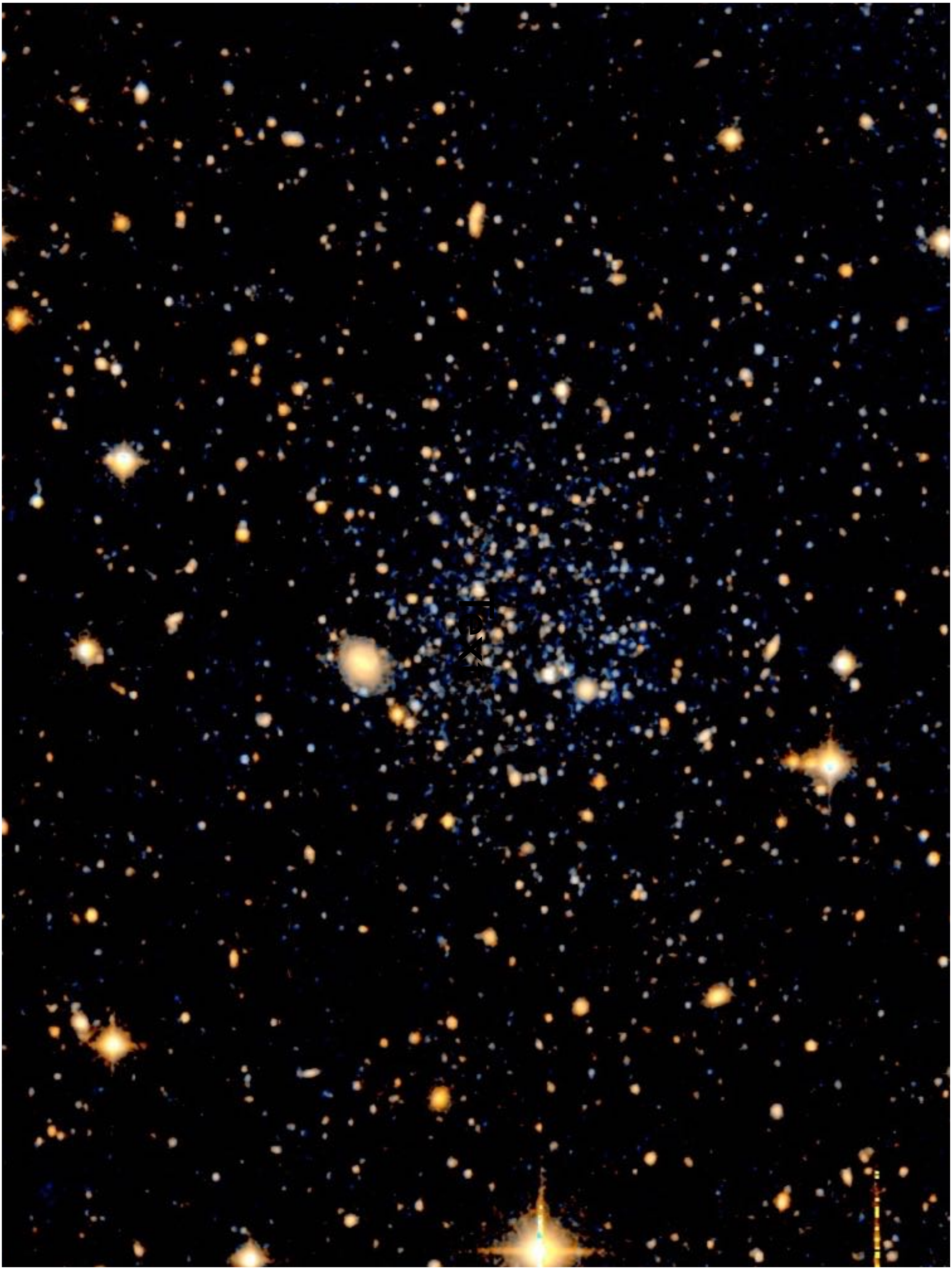
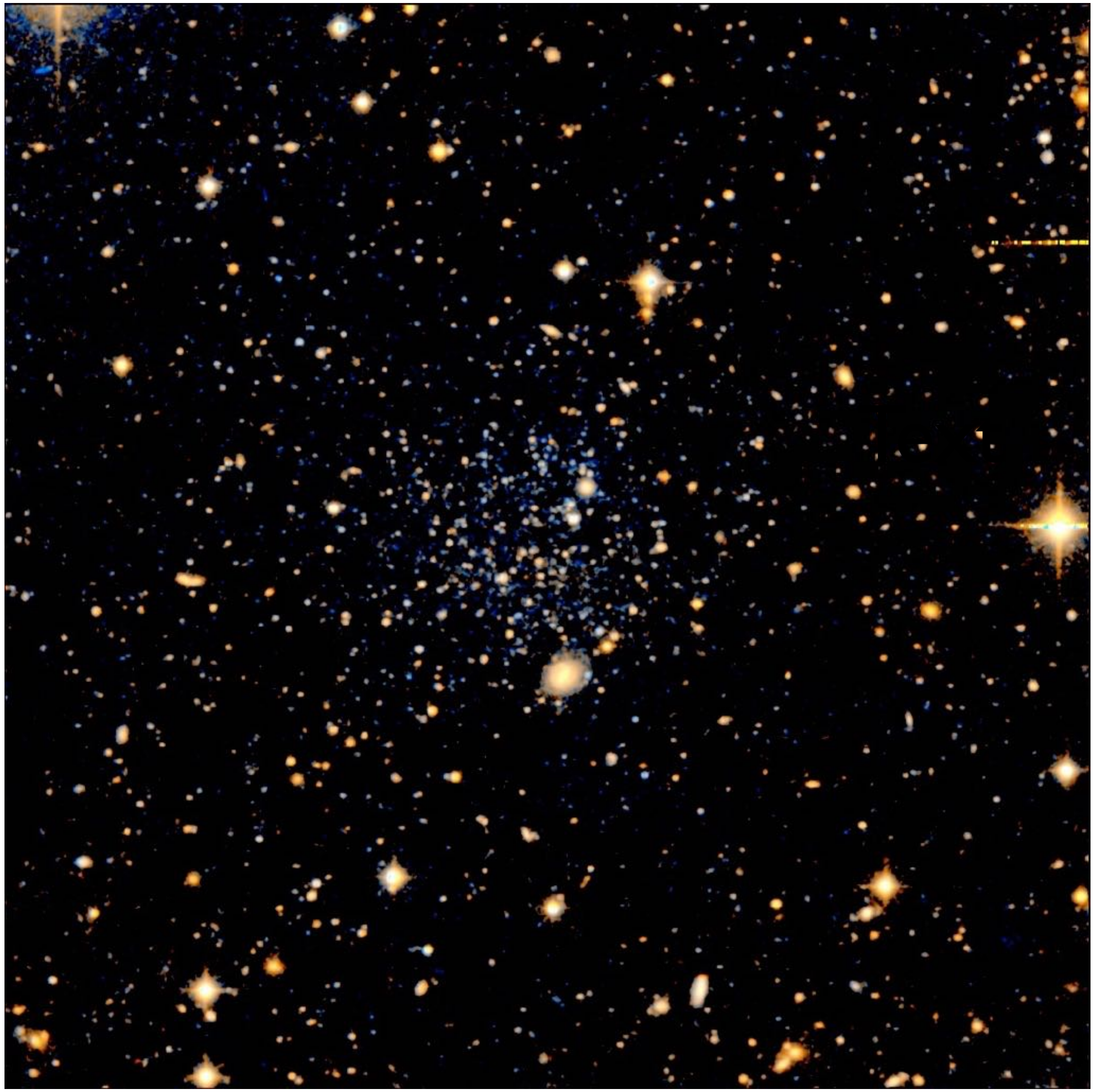

Text

Dwarf galaxy



The stellar velocities are much “too big”…

There must be about 100 times more **dark matter** than there is visible matter in some dwarf galaxies

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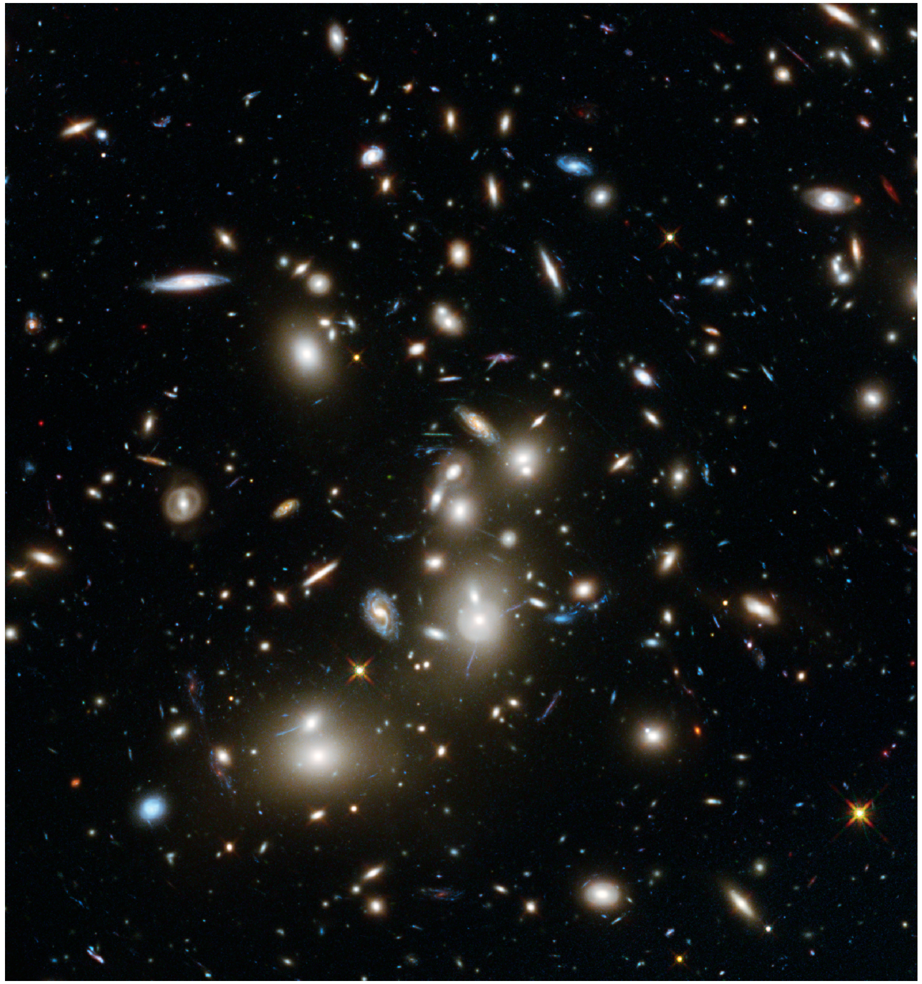
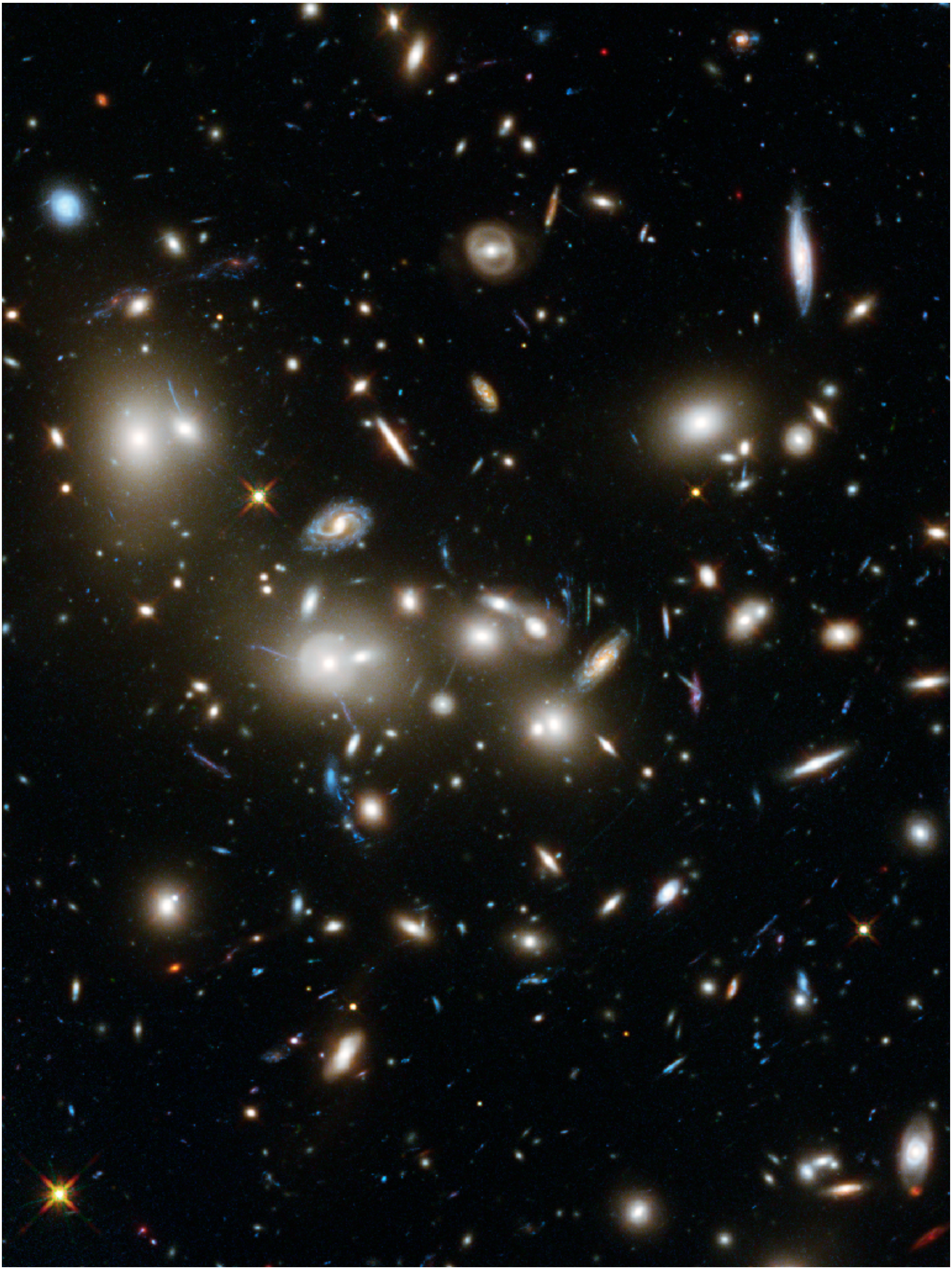
Dark Cosmology Centre

By measuring the velocity dispersion and density of the stars (galaxie), we can find the amount of dark matter in a dwarf galaxy (galaxy cluster).

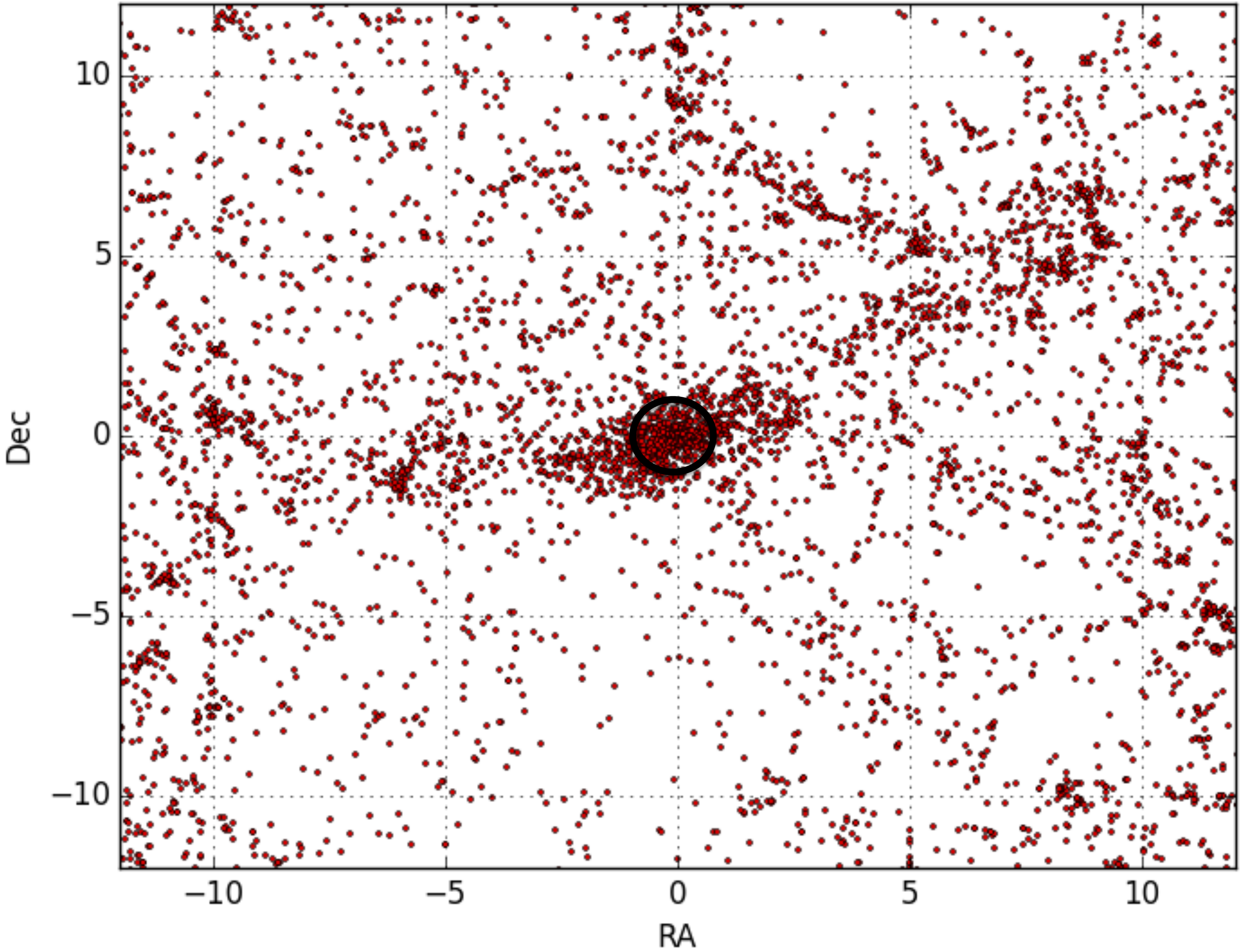
The devil is in the details…

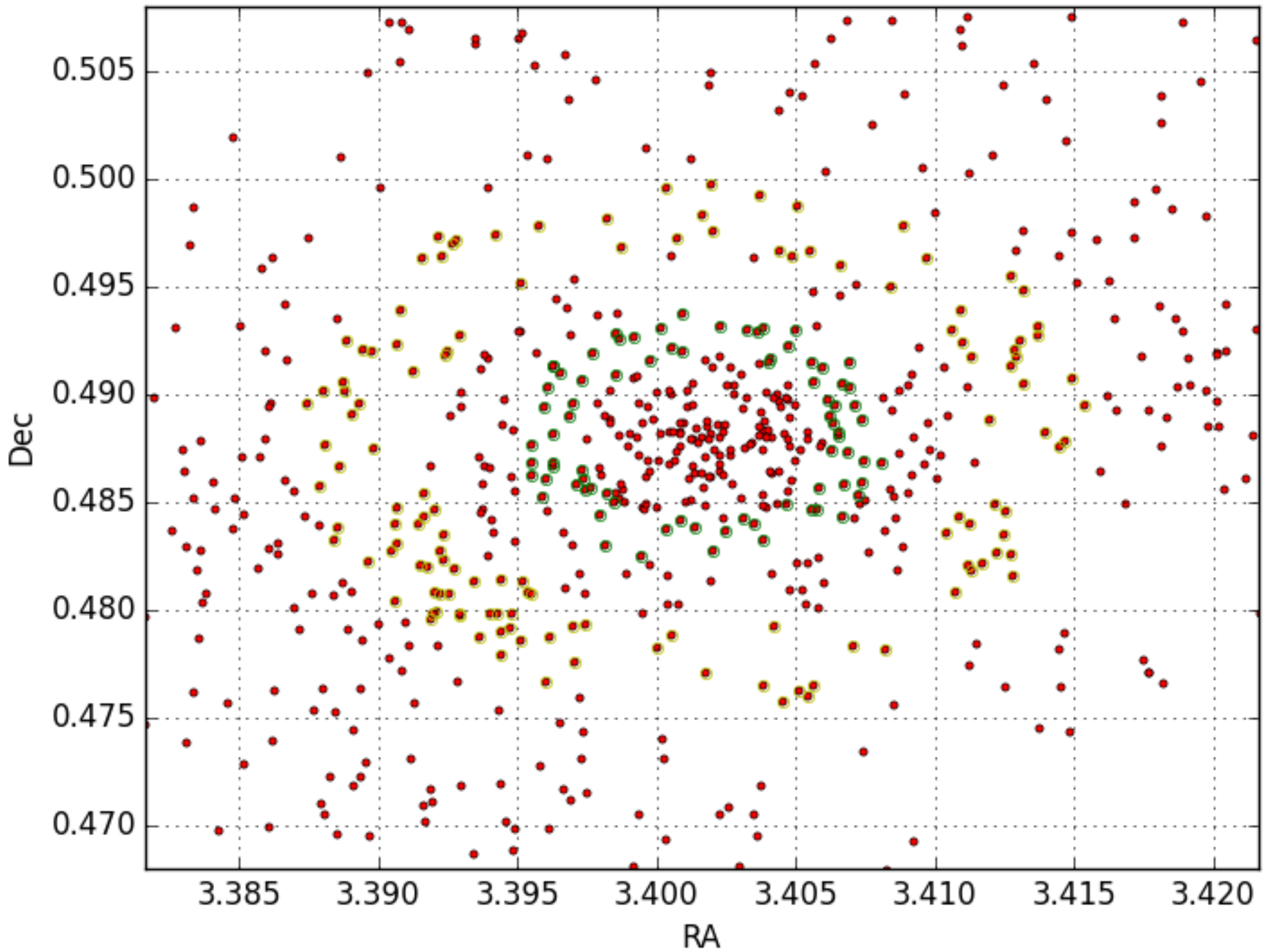


Observed galaxy cluster - visible light



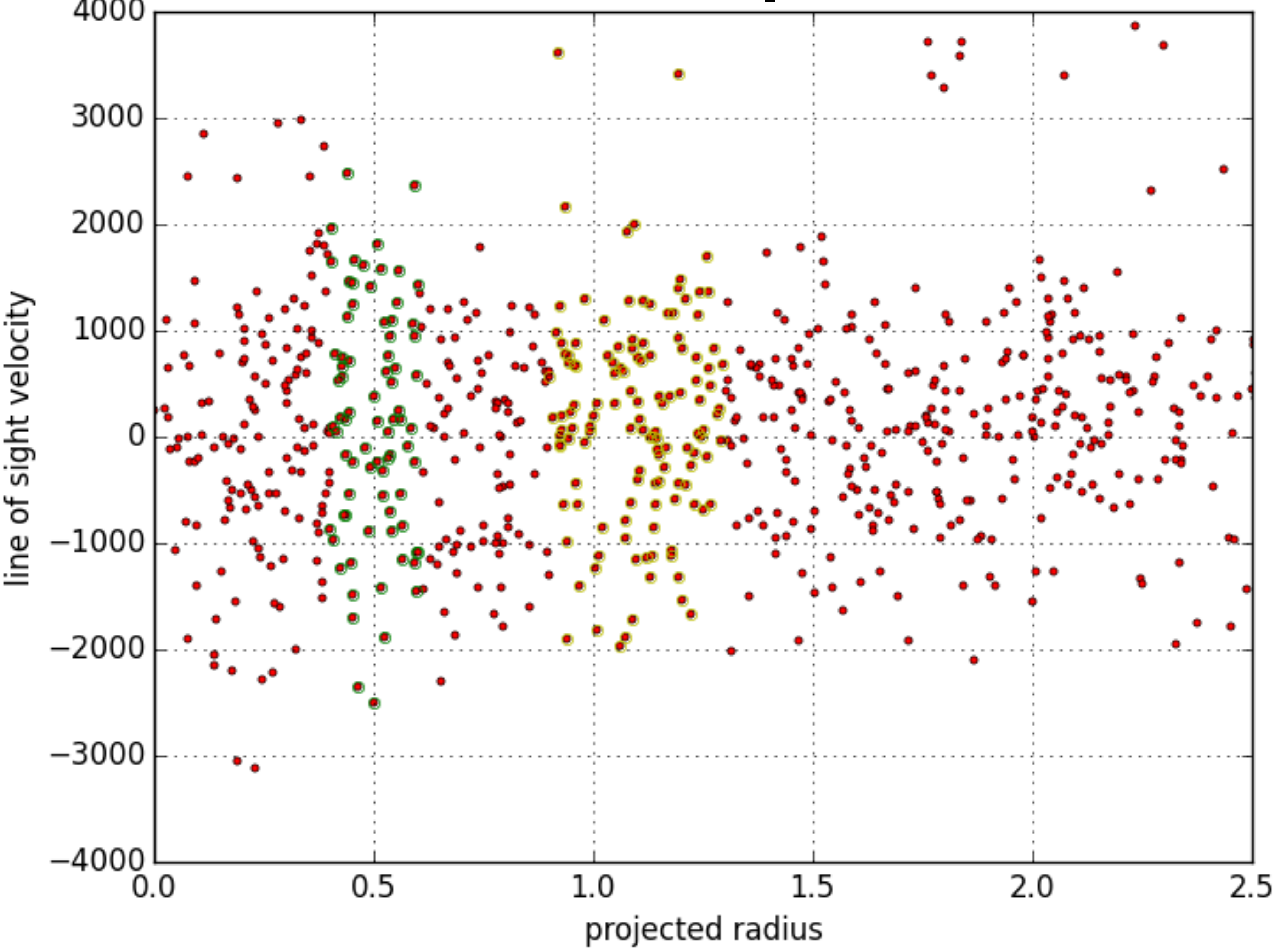
Coma galaxy cluster



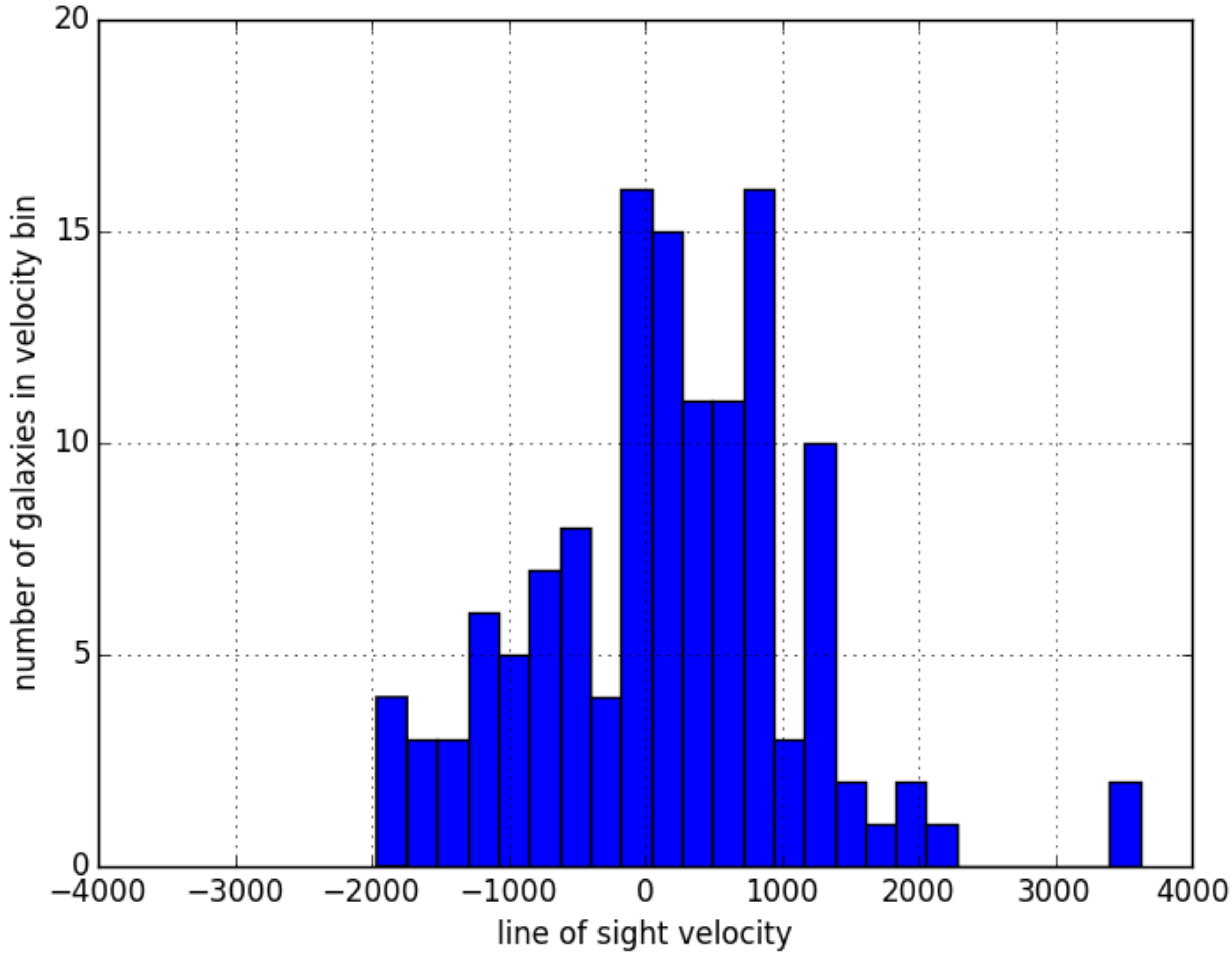


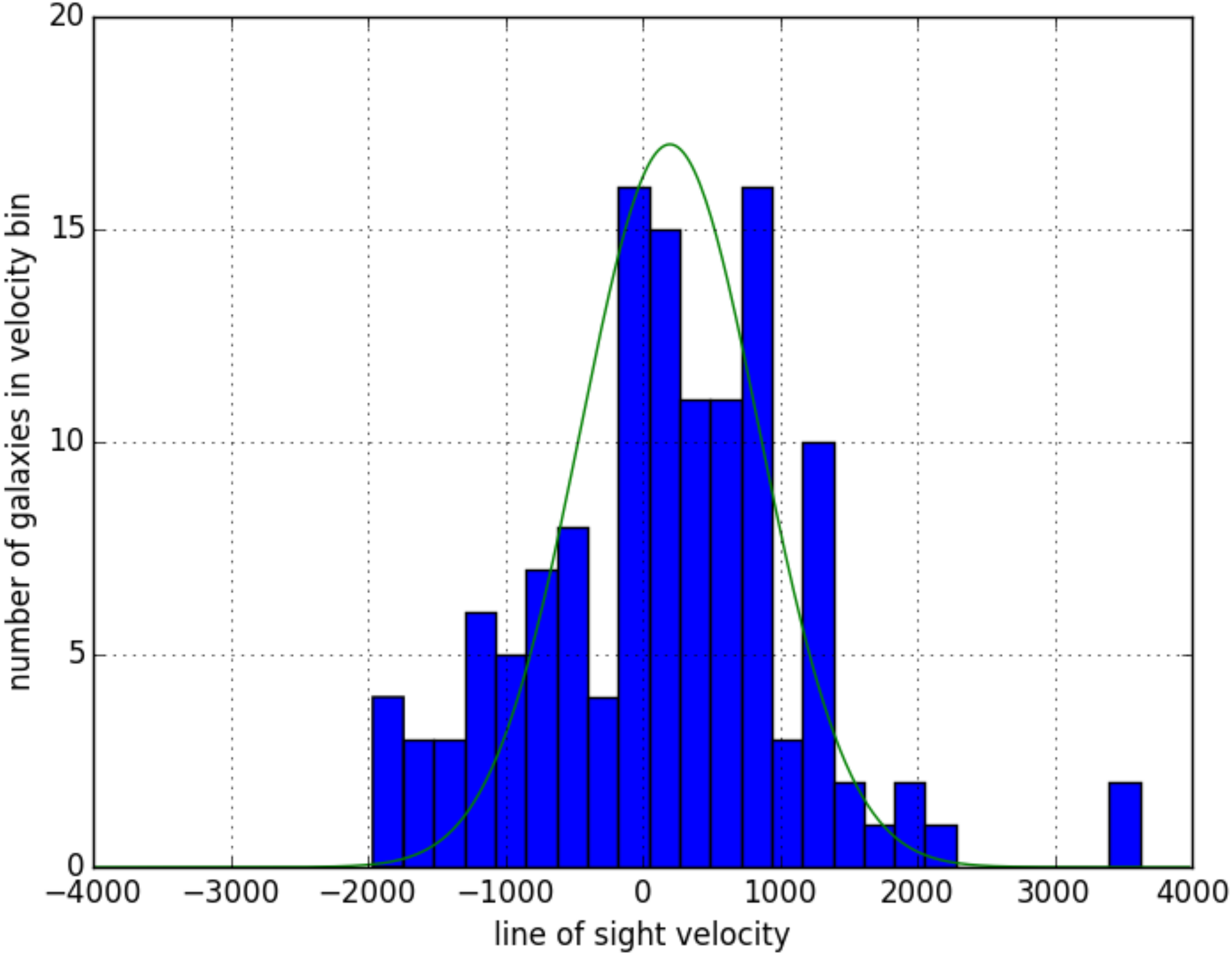
Dec
0.505
0.500
0.495
0.490
0.485
0.480
0.475
0.470
3.385
3.390
3.395
3.400
3.405
3.410
3.415
3.420
RA

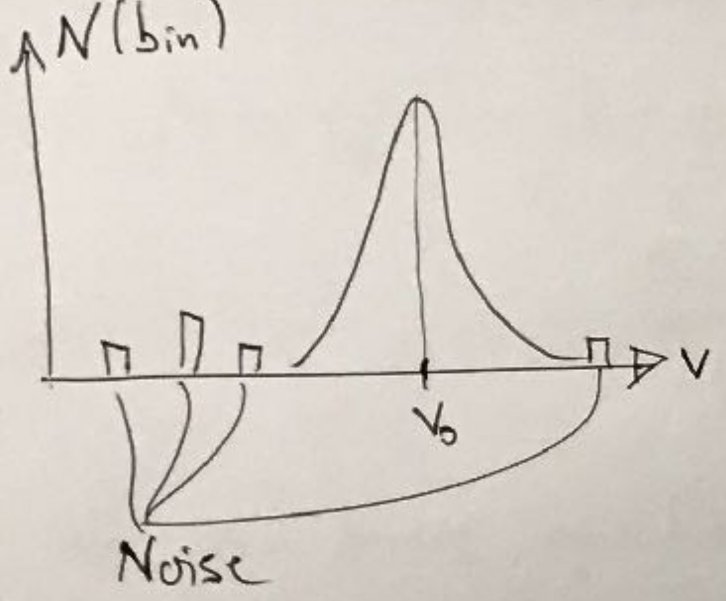
Phase space



line of sight velocity
4000
3000
2000
1000
0
−1000
−2000
−3000
−4000
0.0
0.5
1.0
1.5
2.0
2.5
projected radius

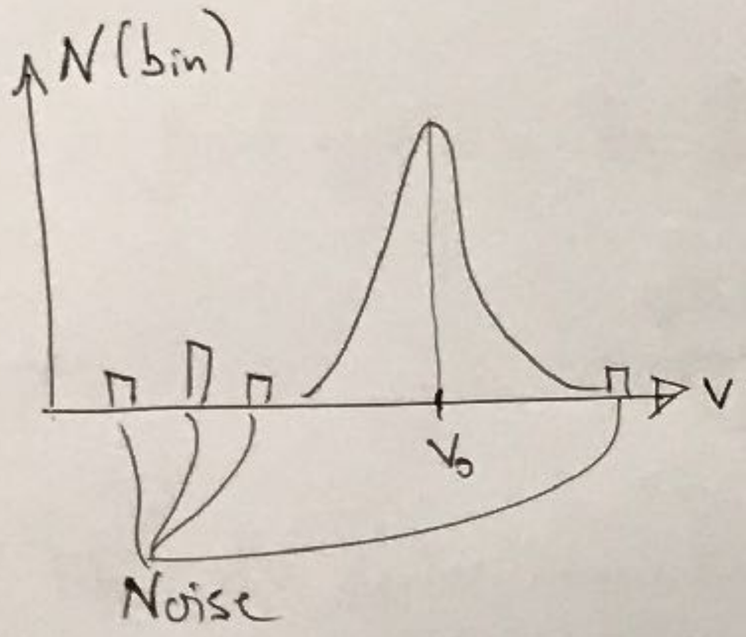

number of galaxies in velocity bin
20
15
10
5
0
−4000
−3000
−2000
−1000
0
1000
2000
3000
4000
line of sight velocity


number of galaxies in velocity bin
0
5
10
15
20
−4000
−3000
−2000
−1000
0
1000
2000
3000
4000
line of sight velocity



For the galaxies we find for each radial bin

$$N(\text{bin}) = N_0 \cdot \exp\left(-\frac{(v-v_0)^2}{2\sigma^2}\right) + \text{Noise}$$



For the galaxies we find for each radial bin

$$N(\text{bin}) = N_0 \cdot \exp\left(-\frac{(v-v_0)^2}{2\sigma^2}\right) + \text{Noise}$$

Remember, for a classical gas we have the velocity distribution

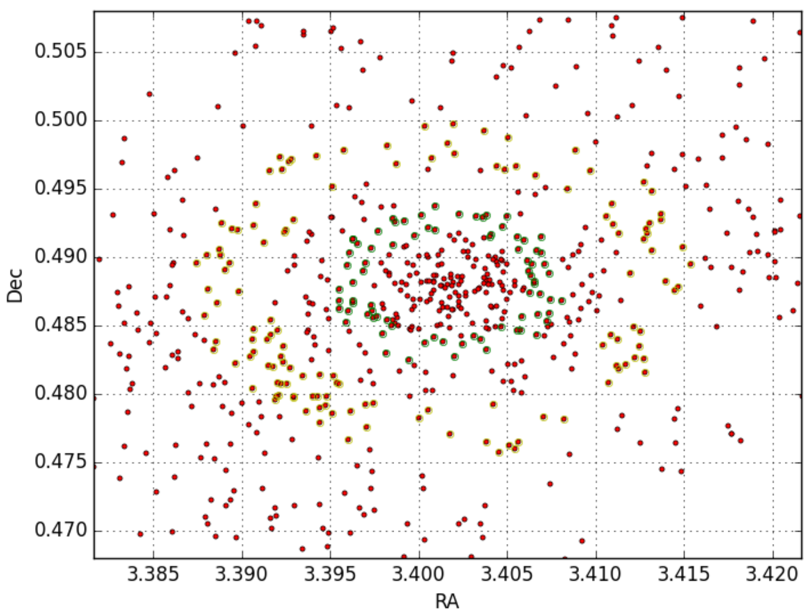
$$f_{gas}(v) \sim \exp\left(-\frac{\frac{1}{2} m v^2}{k_B T}\right)$$

Compare: the gas container is at rest $\Rightarrow v_0 = 0$

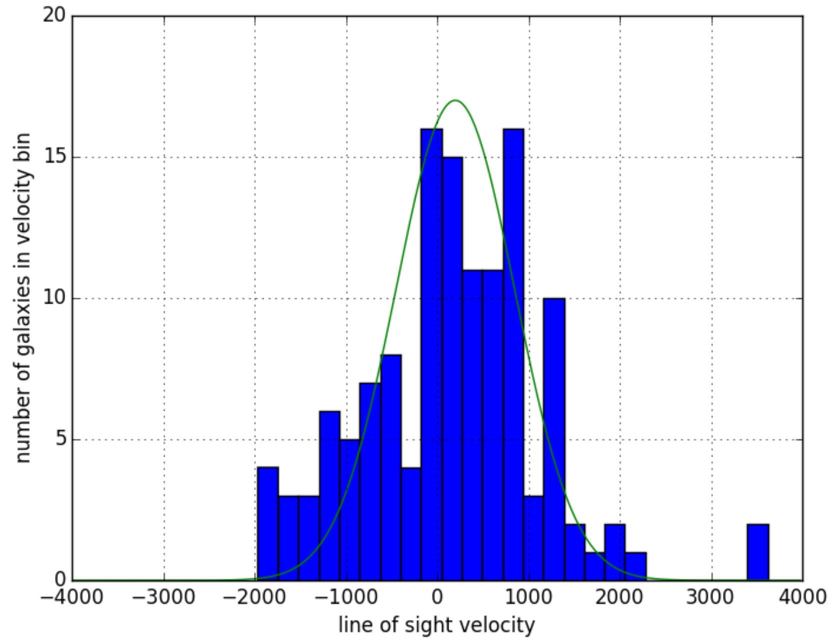
and $\sigma^2 \sim T$ looks the same

(σ^2: galaxy motion; T: gas)

density of galaxies



velocity dispersion



However, galaxies only contribute 5% of the total mass of the galaxy cluster.

The galaxies are tracers, but we want to know the total mass of the cluster.

Galaxies do virtually never collide 

⇒ We cannot use equations from fluid dynamics.

Instead, we use the collisionless Boltzmann equation

$$\frac{df}{dt} = 0$$

[galaxies do not jump in phase-space]

Galaxies do virtually never collide

⇒ We cannot use equations from fluid dynamics.

Instead, we use the collisionless Boltzmann equation

$$\frac{df}{dt} = 0$$

[galaxies do not jump in phase-space]

This equation has two defects:

1) it is VERY difficult to solve

2) the relevant quantities are virtually impossible to observe

However, we can get easier equations by integrating away all velocities:

$$\frac{df}{dt} = 0$$

A) $\int \frac{df}{dt} \, d^3v = 0$

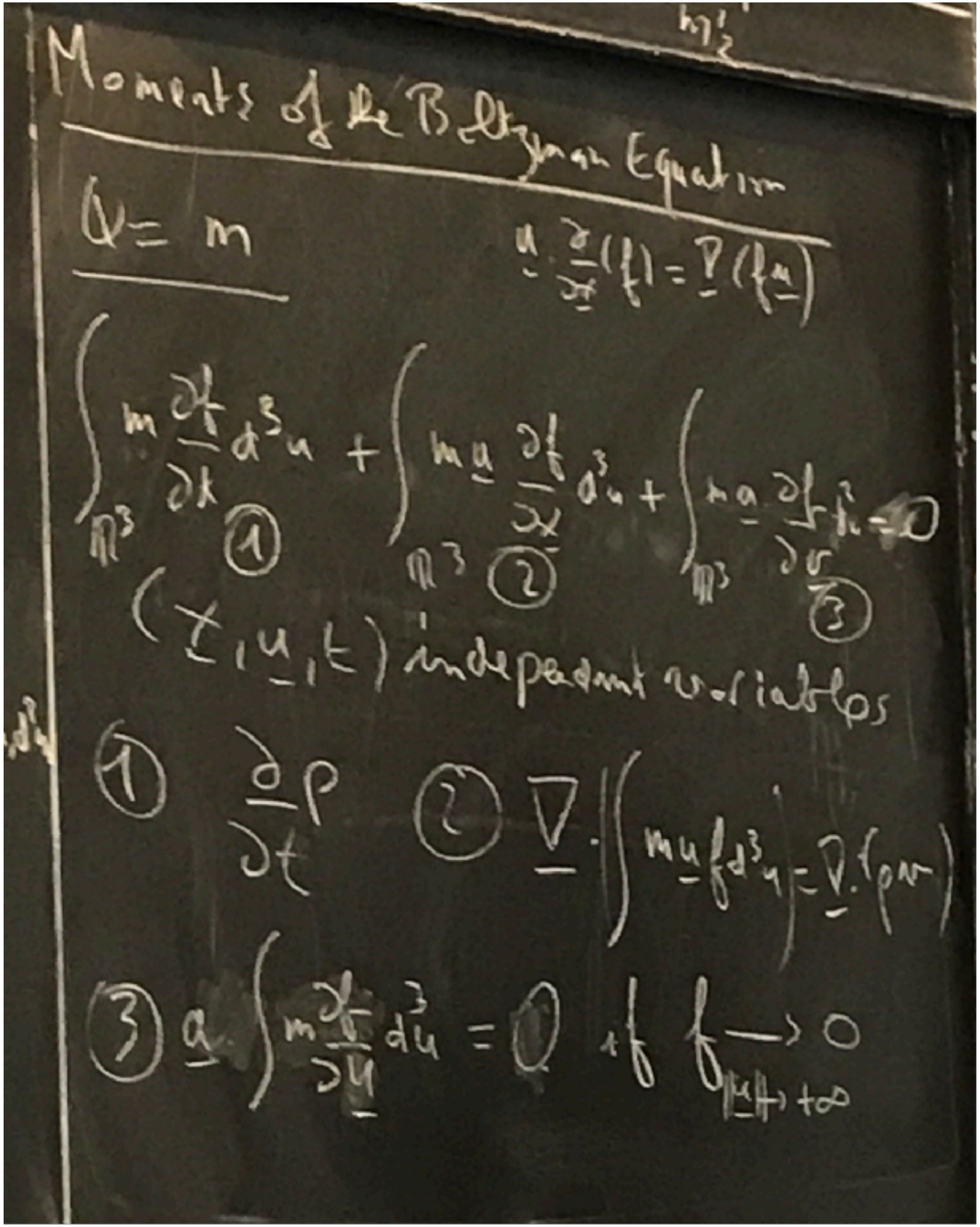
An infinite set of Jeans equations

B) $\int v \frac{df}{dt} \, d^3v = 0$

C) $\int v^2 \frac{df}{dt} \, d^3v = 0$

D) $\int v^3 \frac{df}{dt} \, d^3v = 0$

etc, etc, ...



Fortunately,
Romain
did all the
detailed math
this monday
:-)

$$\int \frac{df}{dt}\, d^3v = 0$$ (is mass conservation)

$$\hookrightarrow \quad \frac{\partial \rho}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v_r\right)$$ [as expected]

$$\int v_r \frac{df}{dt}\, d^3v = 0$$ (is momentum conservation)

$$\hookrightarrow \quad \frac{\partial (\rho \sigma_r^2)}{\partial r} + \frac{\rho}{r}\left(2\sigma_r^2 - \sigma_\theta^2 - \sigma_\phi^2\right) = -\rho \frac{\partial \Phi}{\partial r}$$

(let us rewrite this a bit)

For spherical systems we have $\frac{\partial \Phi}{\partial r} = \frac{G M_{tot}(r)}{r^2}$

and thereby we get

$$M_{tot}(r) = - \frac{\sigma_r^2 \cdot r}{G} \left[\frac{d \log \rho}{d \log r} + \frac{d \log \sigma_r^2}{d \log r} + 2\beta \right]$$

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Remember, for a gas we have the hydrostatic equilibrium

$$M_{tot}(r) = -\frac{T_{gas} \cdot r}{G} \frac{k_B}{\mu m_p} \left[\frac{d \log \rho_{gas}}{d \log r} + \frac{d \log T_{gas}}{d \log r} \right]$$

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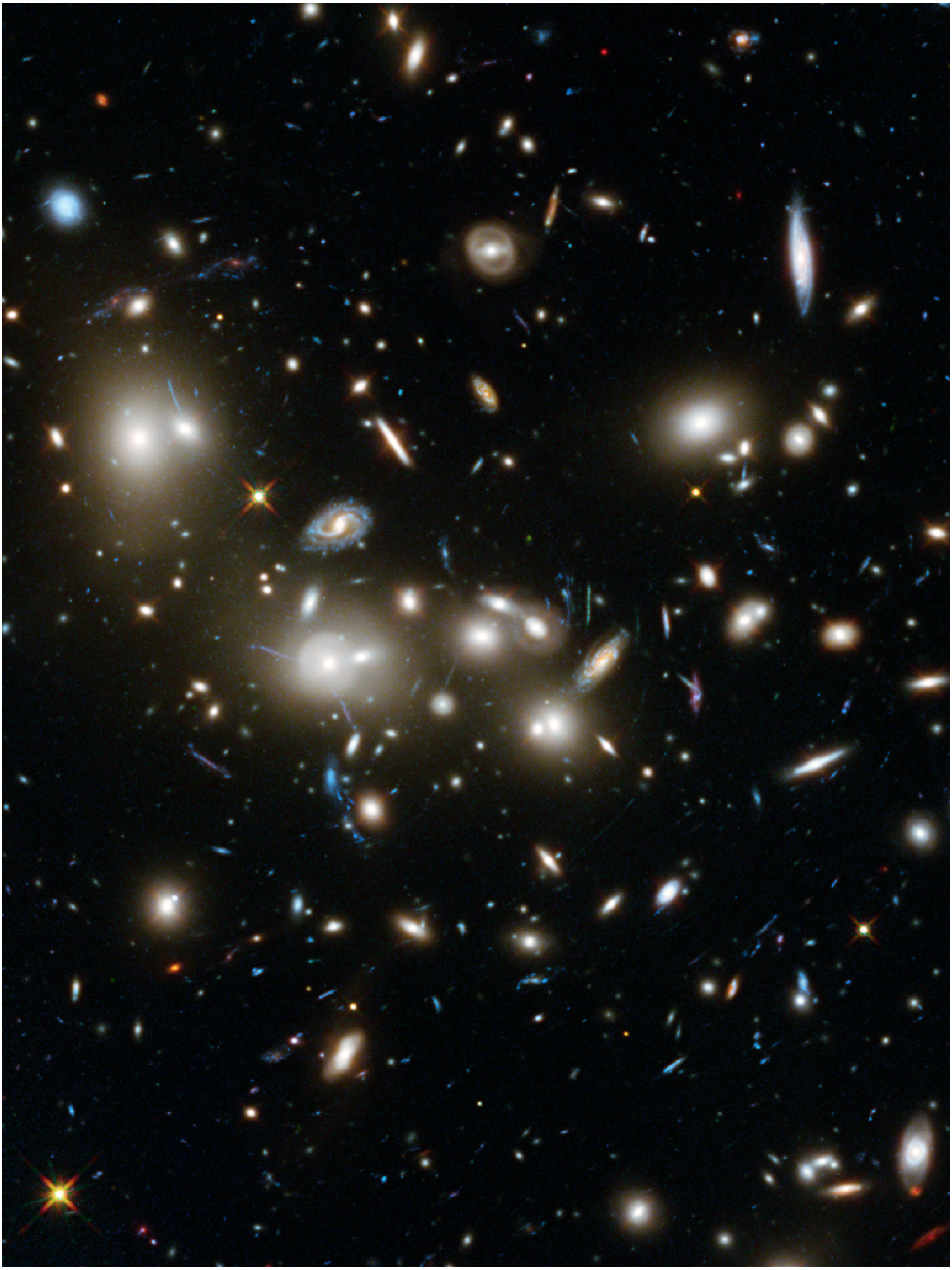
The magic of momentum conservation



The Jeans equation

$$\frac{GM(r)}{r} = -\sigma_r^2 \left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln \sigma_r^2}{\partial \ln r} + 2\beta \right)$$

The Jeans equation

$$\frac{GM(r)}{r} = -\sigma_r^2 \left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln \sigma_r^2}{\partial \ln r} + 2\beta \right)$$

The Jeans equation

$$\frac{GM(r)}{r} = -\sigma_r^2\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln \sigma_r^2}{\partial \ln r} + 2\beta\right)$$

Total mass includes everything, such as galaxies, gas, dark matter, black hole…

In conclusion
(concerning the Jeans equation)
by measuring the velocity dispersion and density
of a “**tracer**” (in this case the galaxies)
then we can derive the
total mass
as a function of radius.

The DM attractor

Steen H. Hansen
Dark Cosmology Centre

Difference between gas and DM

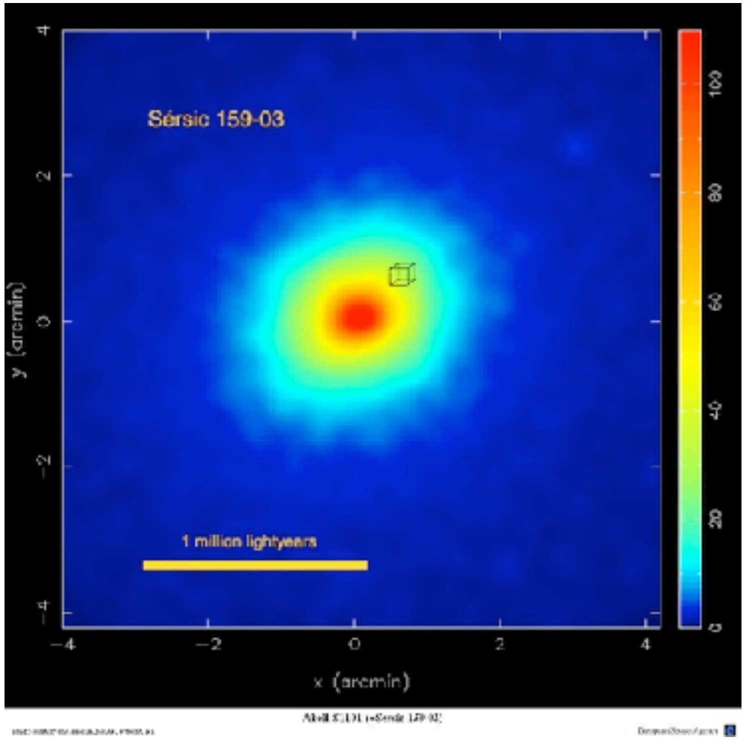
Hydrostatic equilibrium (gas)

$$\frac{GM_{\rm tot}}{r} = -\frac{k_B T}{\mu m_p}\left(\frac{d{\rm ln}T}{d{\rm ln}r} + \frac{d{\rm ln}n_e}{d{\rm ln}r}\right)$$

Jeans equation (dark matter)

$$\frac{GM_{\rm tot}}{r} = -\sigma_r^2\left(\frac{d{\rm ln}\sigma_r^2}{d{\rm ln}r} + \frac{d{\rm ln}\rho}{d{\rm ln}r} + 2\beta\right)$$

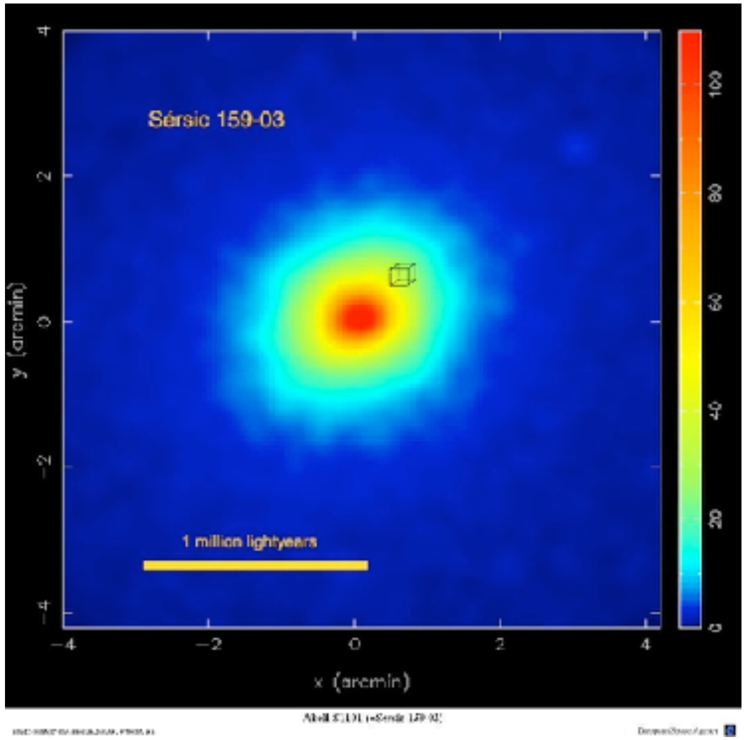
Gas has an equation of state



P ~ n T

$$\frac{GM_{\rm tot}}{r} = -\frac{k_B T}{\mu m_p}\left(\frac{d\ln T}{d\ln r} + \frac{d\ln n_e}{d\ln r}\right)$$

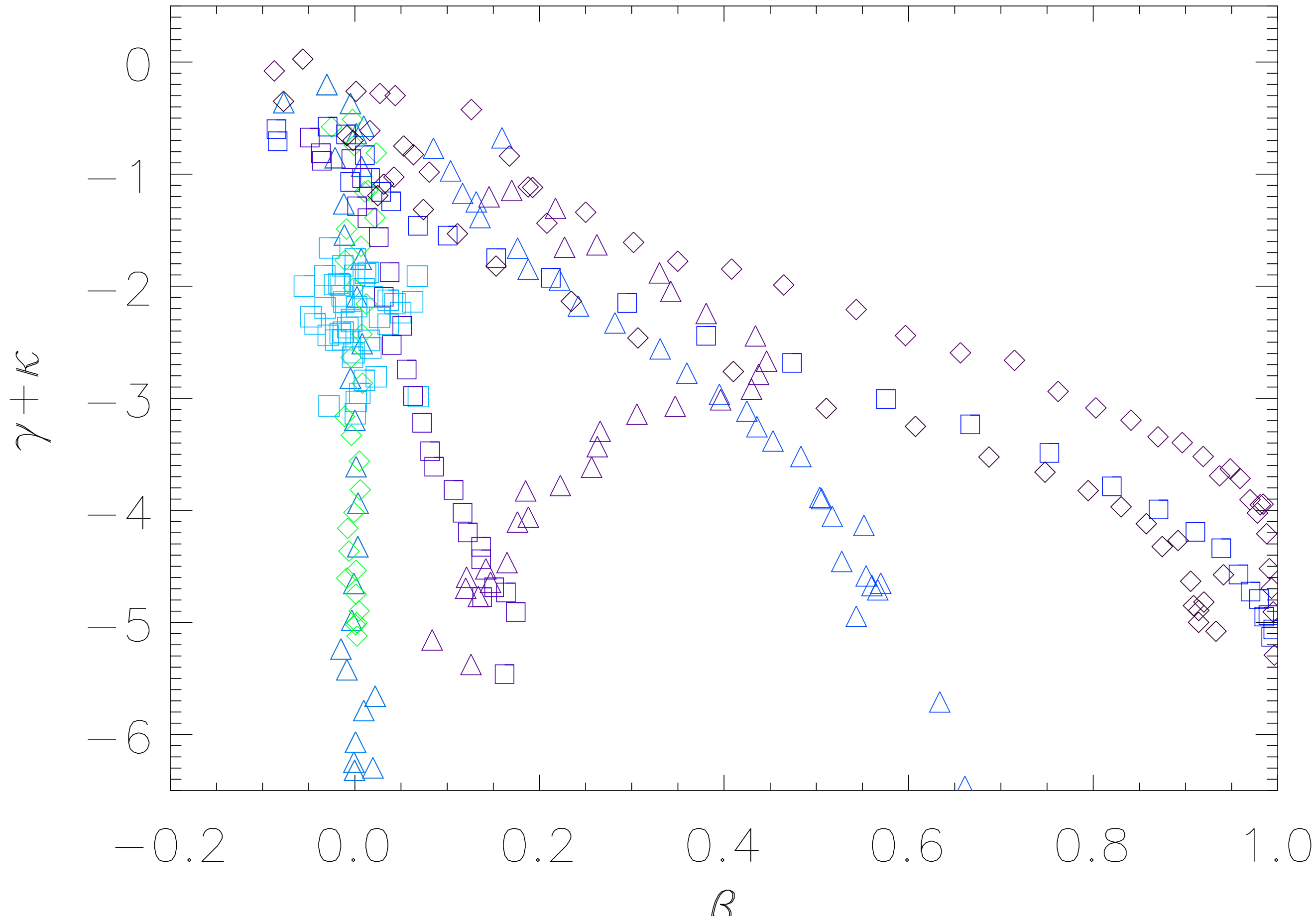
Rather few solutions to hydrostatic equilibrium



P ~ n T

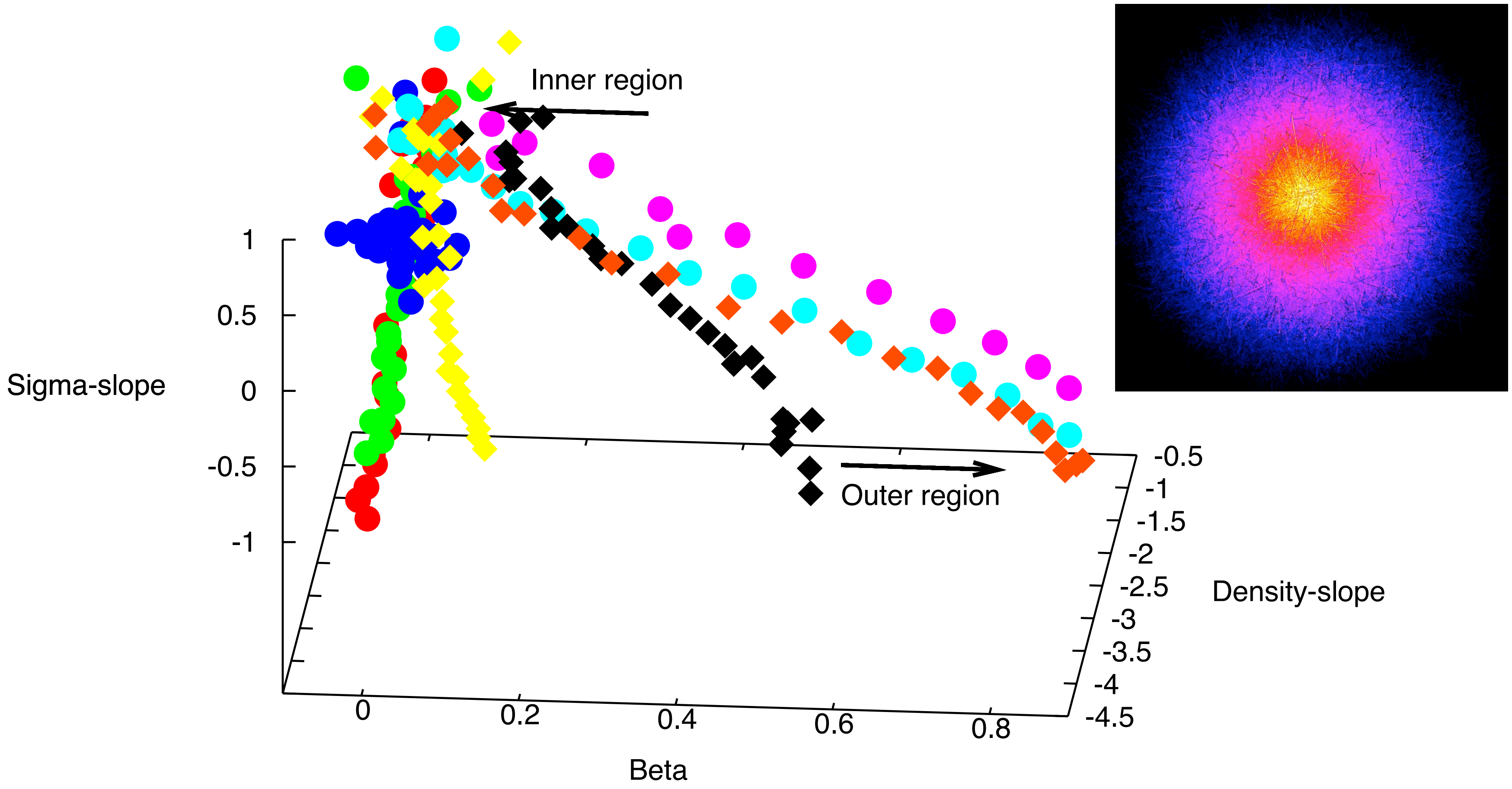
$$\frac{GM_{\mathrm{tot}}}{r} = -\frac{k_B T}{\mu m_p}\left(\frac{d\ln T}{d\ln r} + \frac{d\ln n_e}{d\ln r}\right)$$

Many many solutions to Jeans



$$\frac{GM_{\rm tot}}{r} = -\sigma_r^2 \left(\frac{d\ln\sigma_r^2}{d\ln r} + \frac{d\ln\rho}{d\ln r} + 2\beta \right)$$

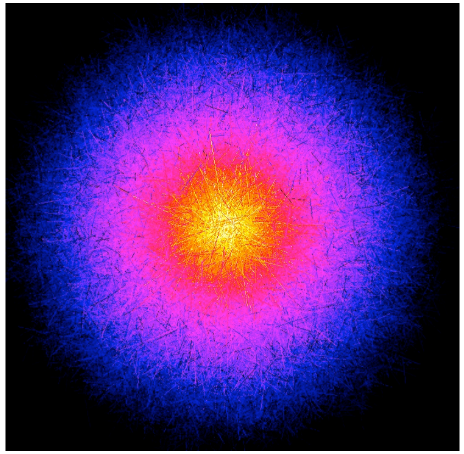
Many many solutions to Jeans



$$\frac{GM_{\rm tot}}{r} = -\sigma_r^2 \left(\frac{d\ln\sigma_r^2}{d\ln r} + \frac{d\ln\rho}{d\ln r} + 2\beta \right)$$

Are all these structures stable
to small perturbations?

1. Create structure in equilibrium

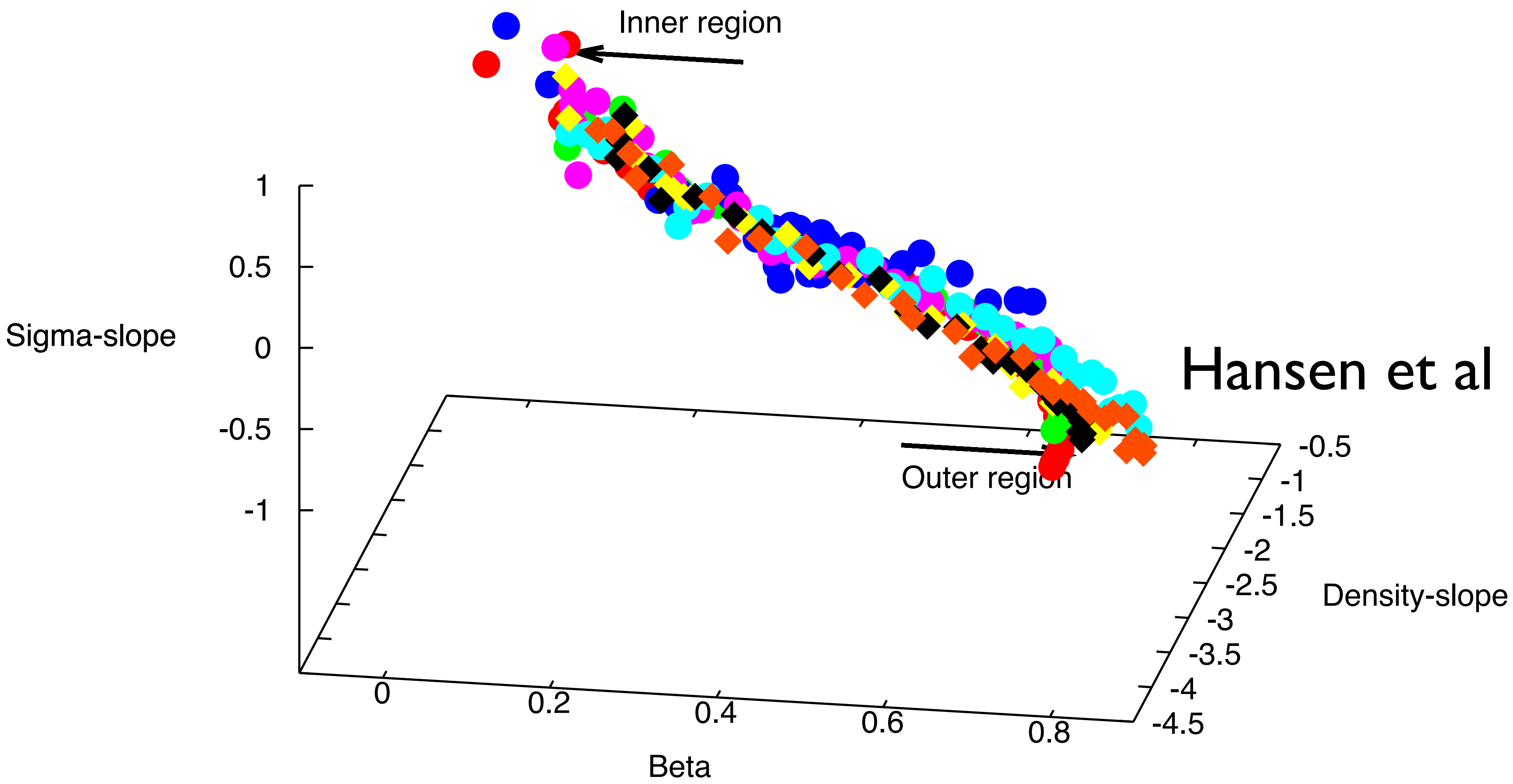


2a. Perturb structure

2b. Let structure relax

Repeat till happy

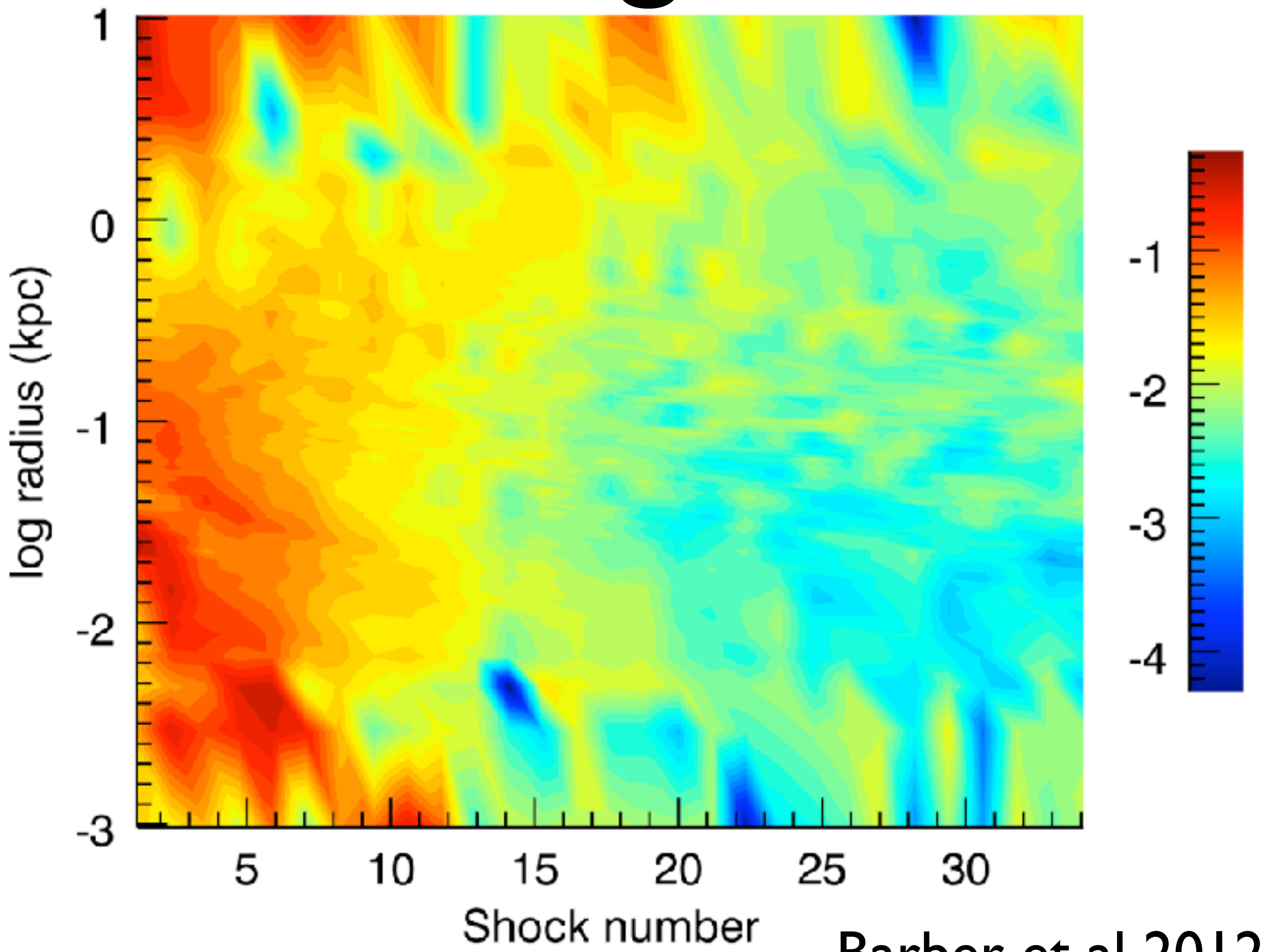
One attractor



Hansen et al 2010

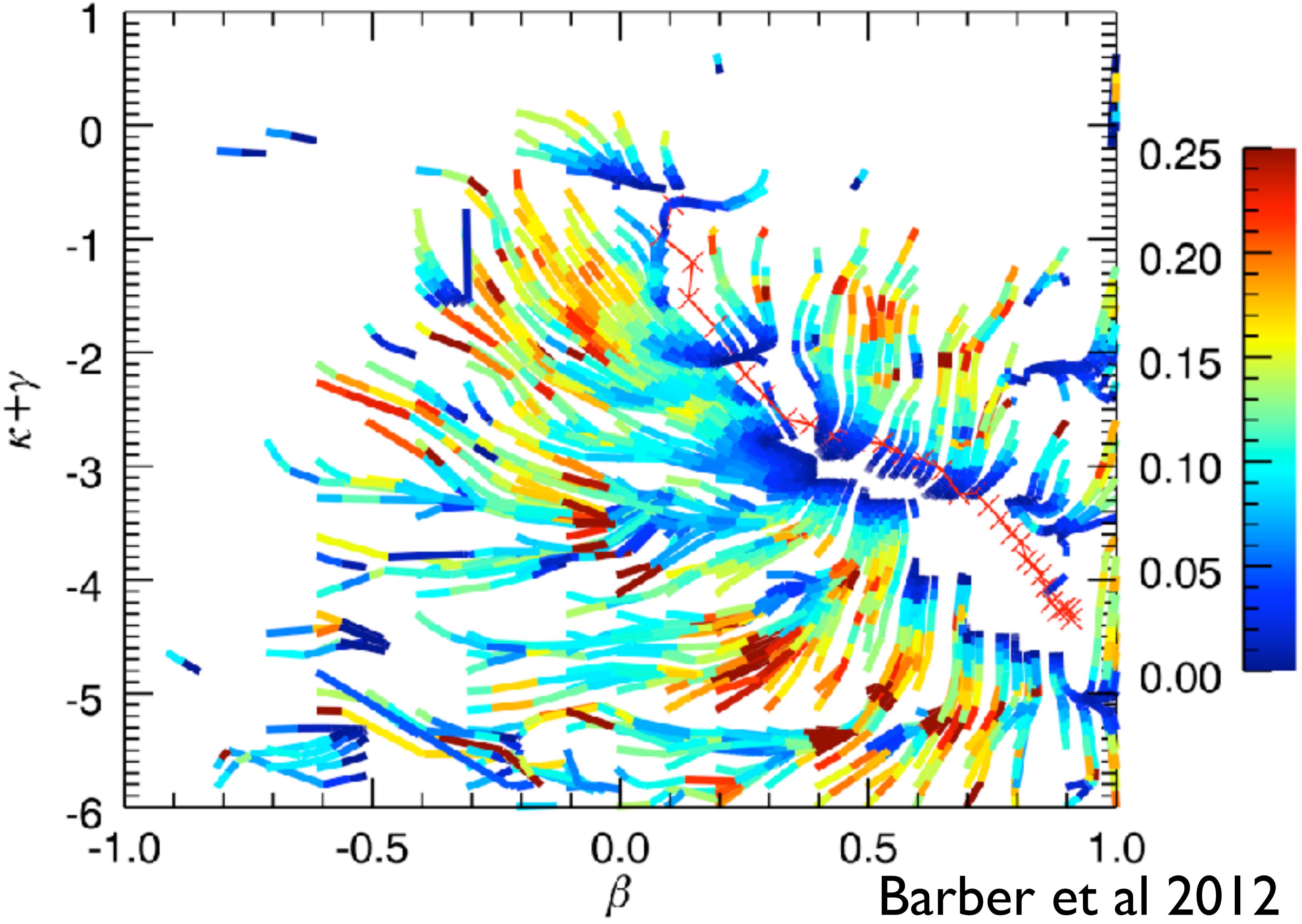
$$\frac{GM_{\rm tot}}{r} = -\sigma_r^2 \left(\frac{d\ln\sigma_r^2}{d\ln r} + \frac{d\ln\rho}{d\ln r} + 2\beta \right)$$

Convergence?

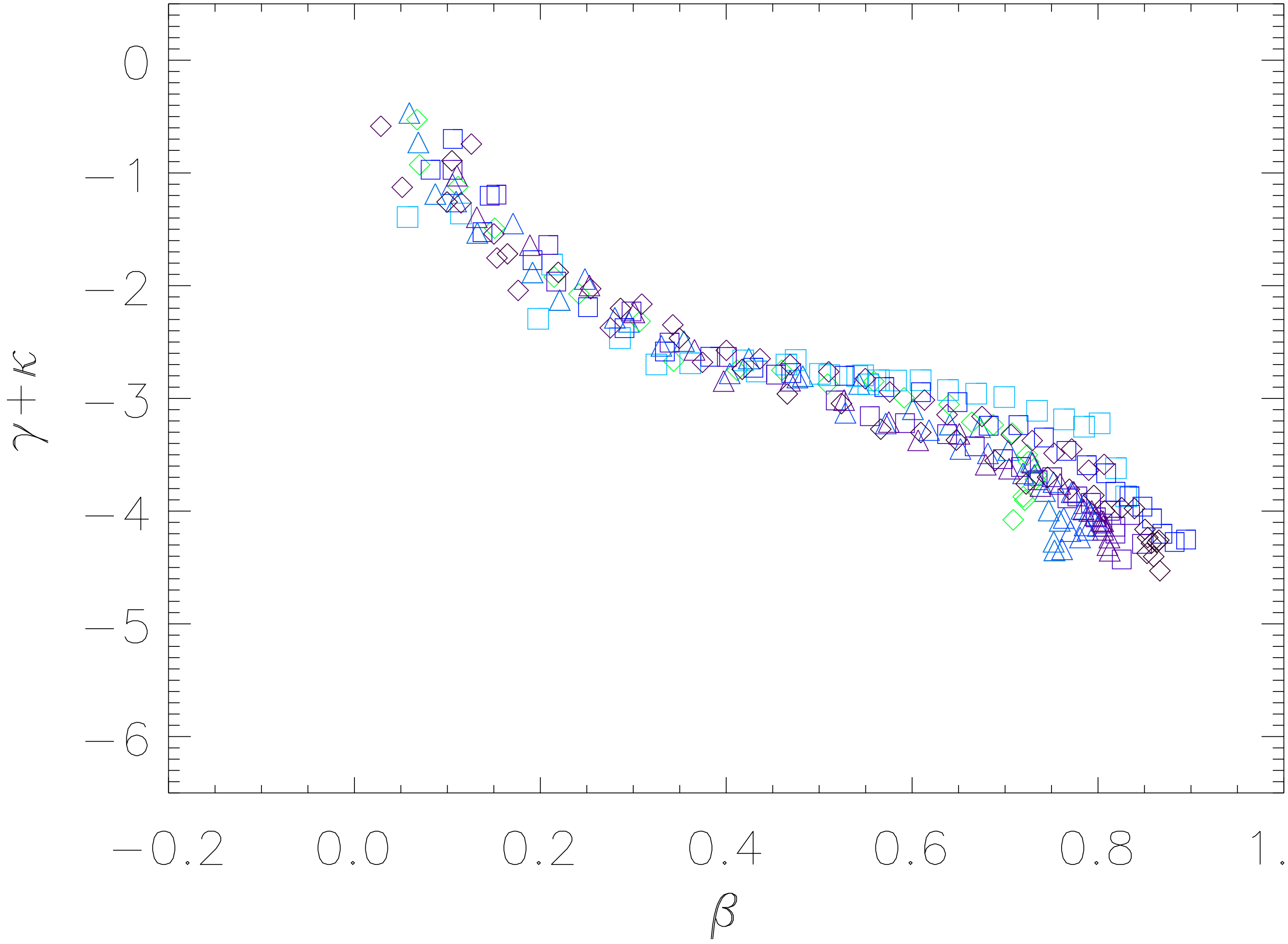


Barber et al 2012

An attractor!



Barber et al 2012

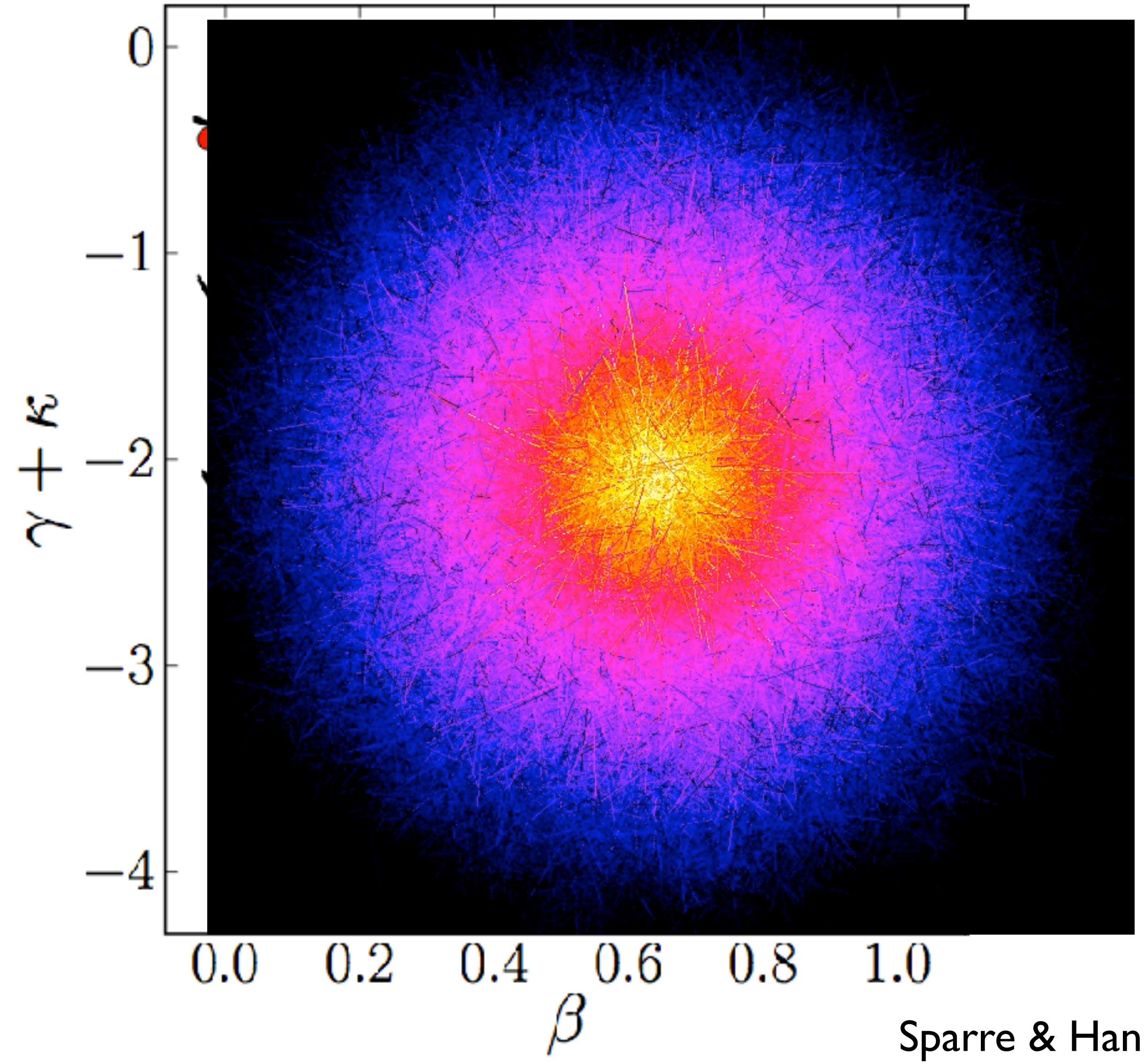


$$\frac{GM_{\rm tot}}{r} = -\sigma_r^2 \left(\frac{d\ln\sigma_r^2}{d\ln r} + \frac{d\ln\rho}{d\ln r} + 2\beta \right)$$

What does this mean?

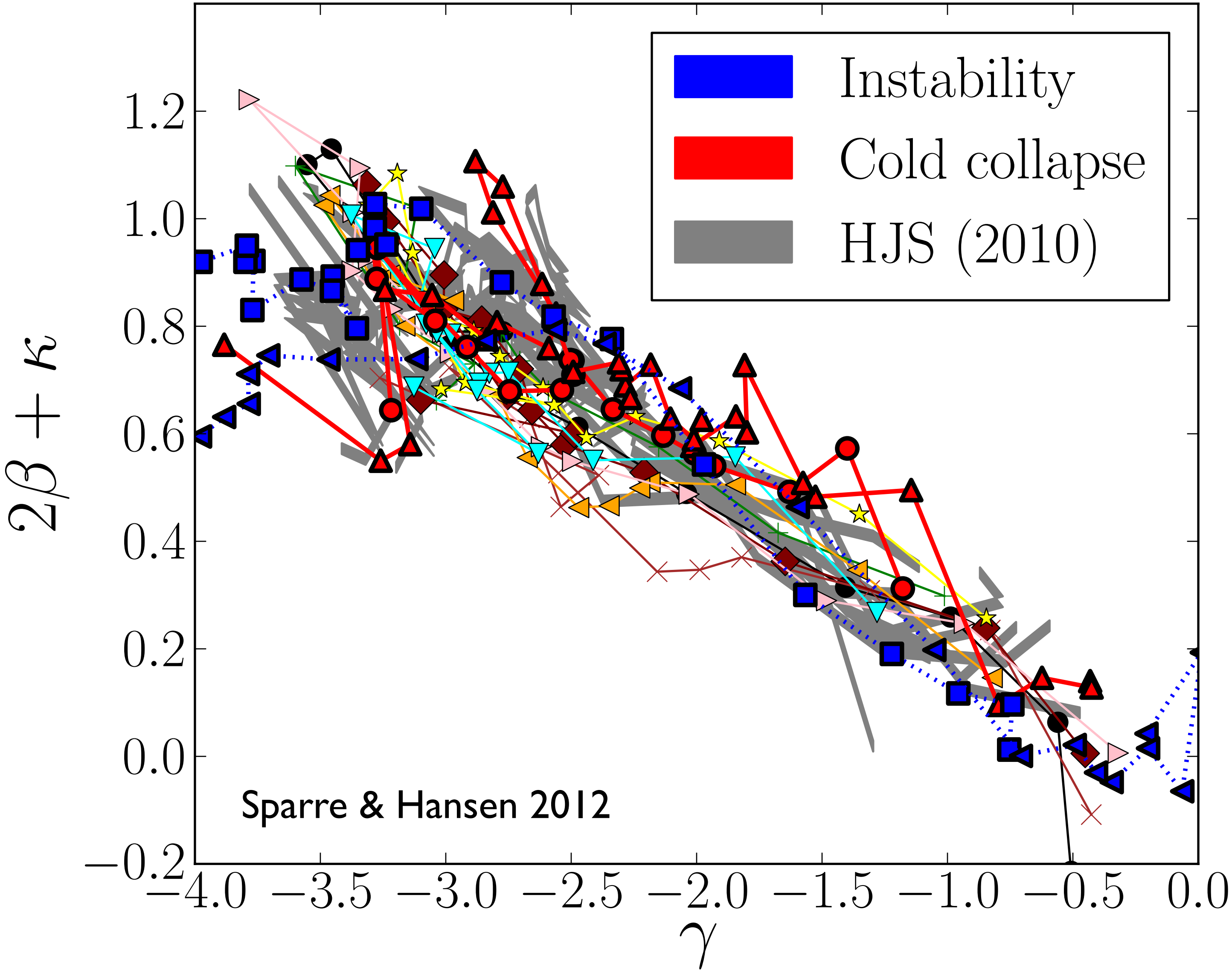
- All structures “want” to follow a connection between sigma, rho and beta (dispersion, density and anisotropy)
- This is almost an **equation of state** for dark matter
- ...if the attractor is real

G-perturbations

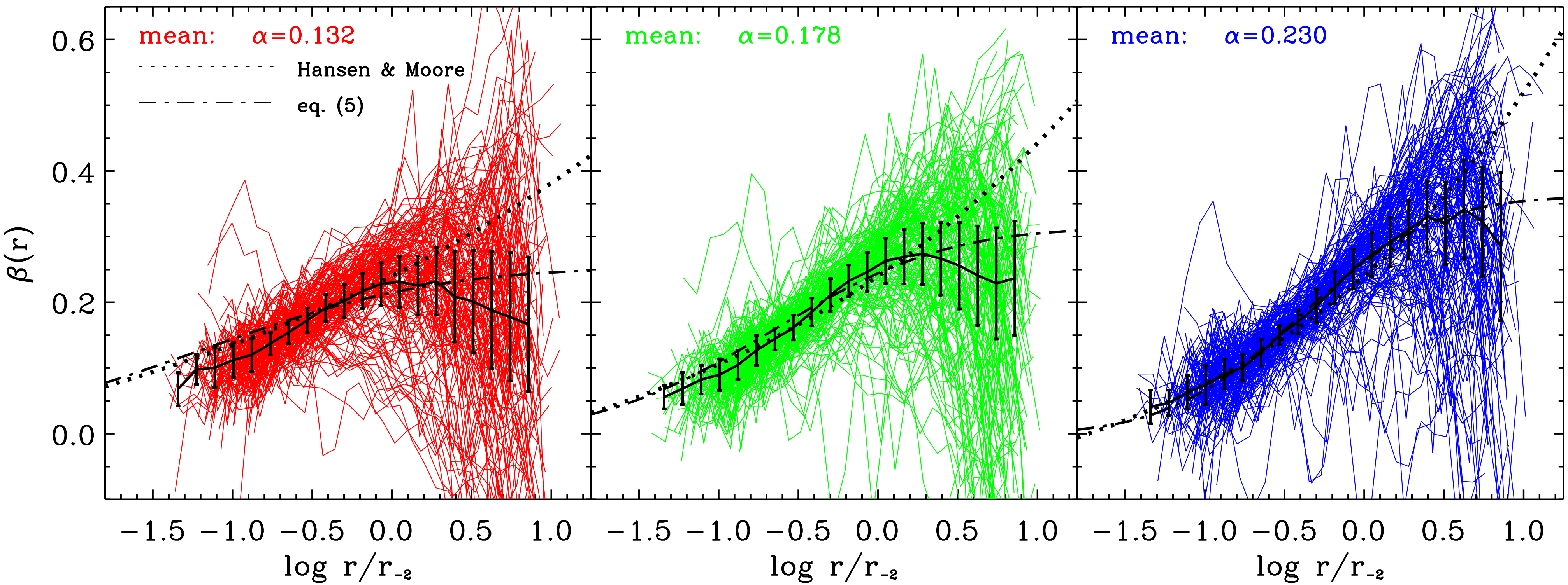


Sparre & Hansen 2012

Cold collapse

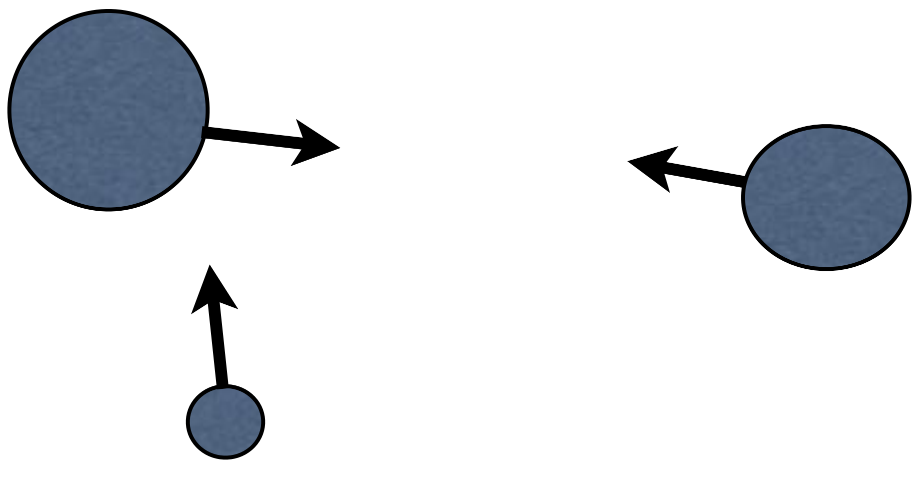


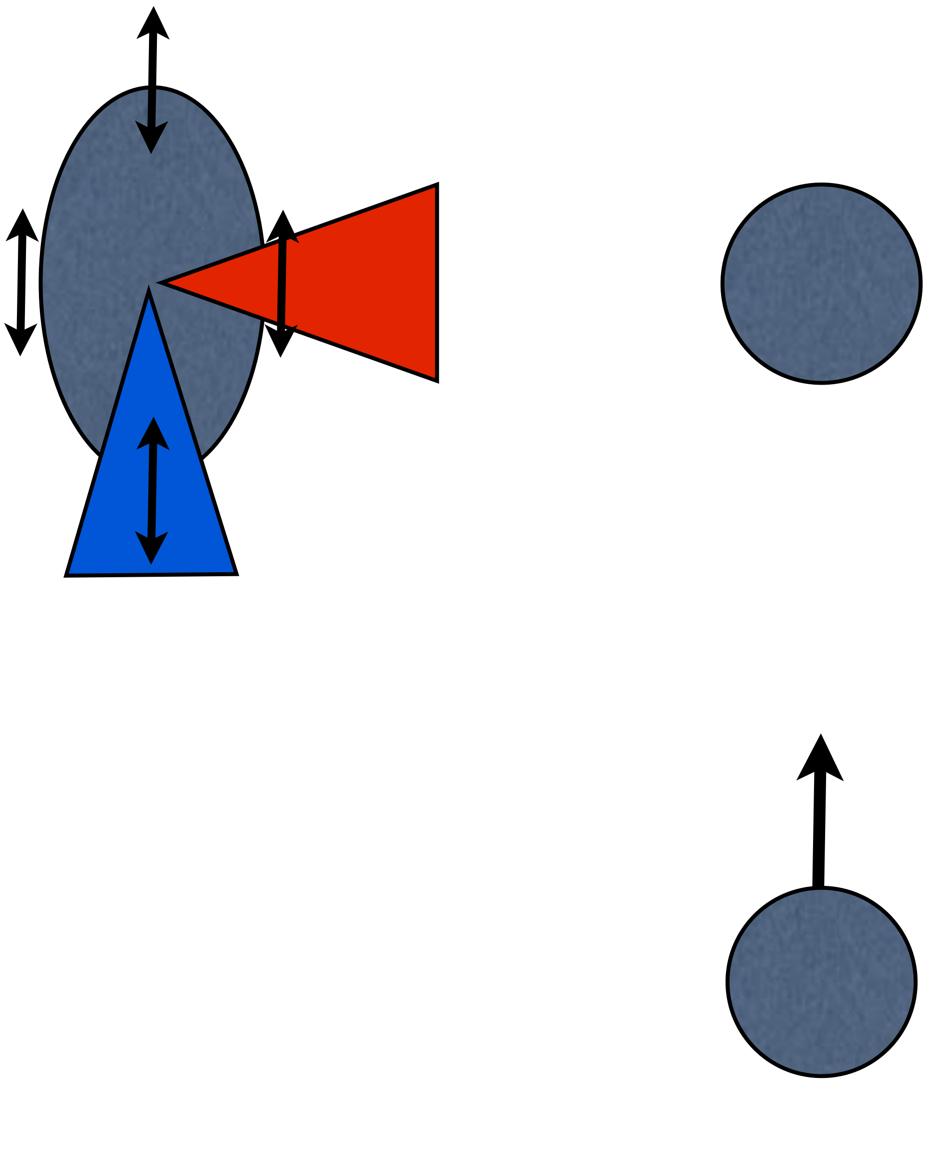
Cosmological simulations

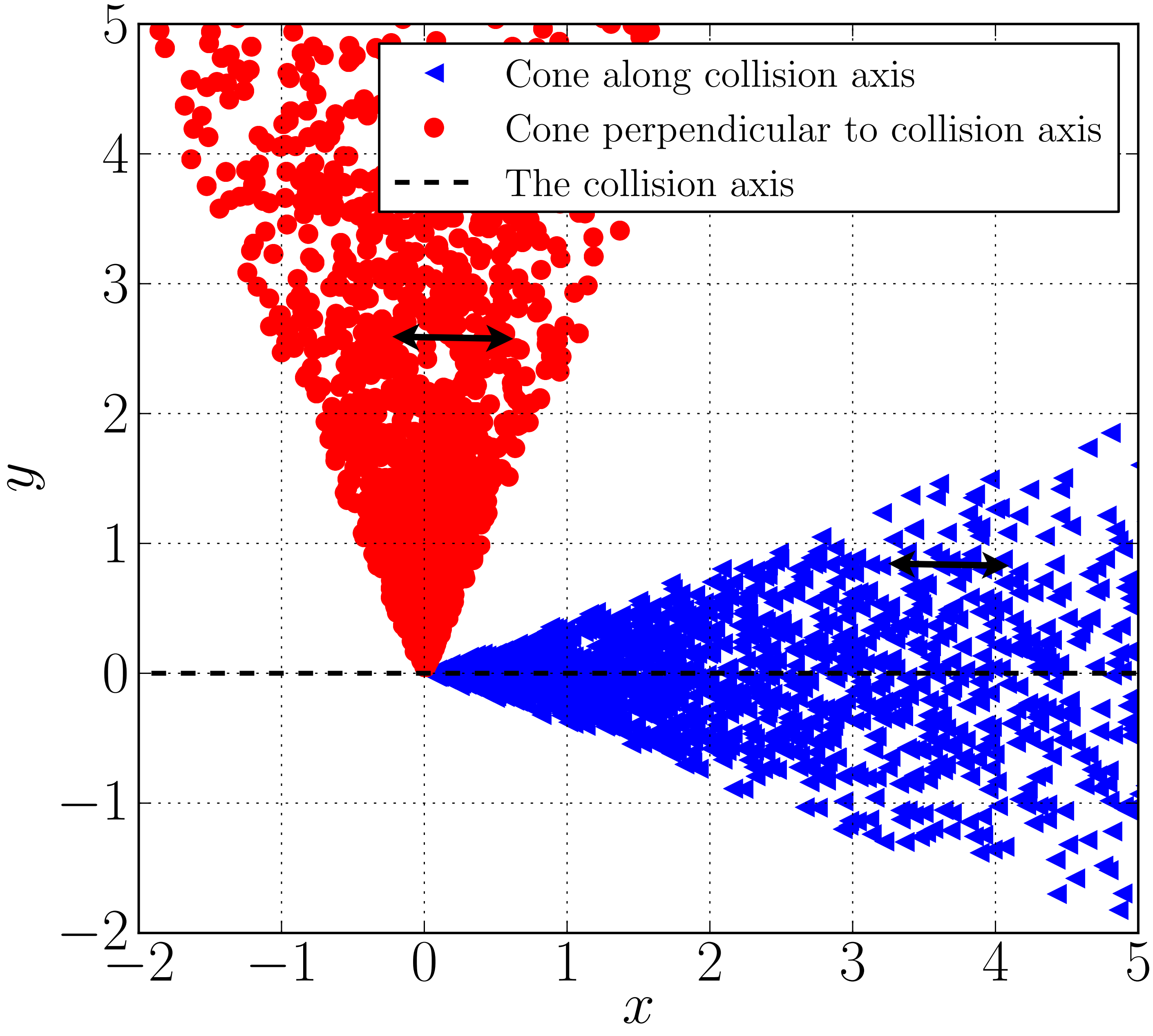


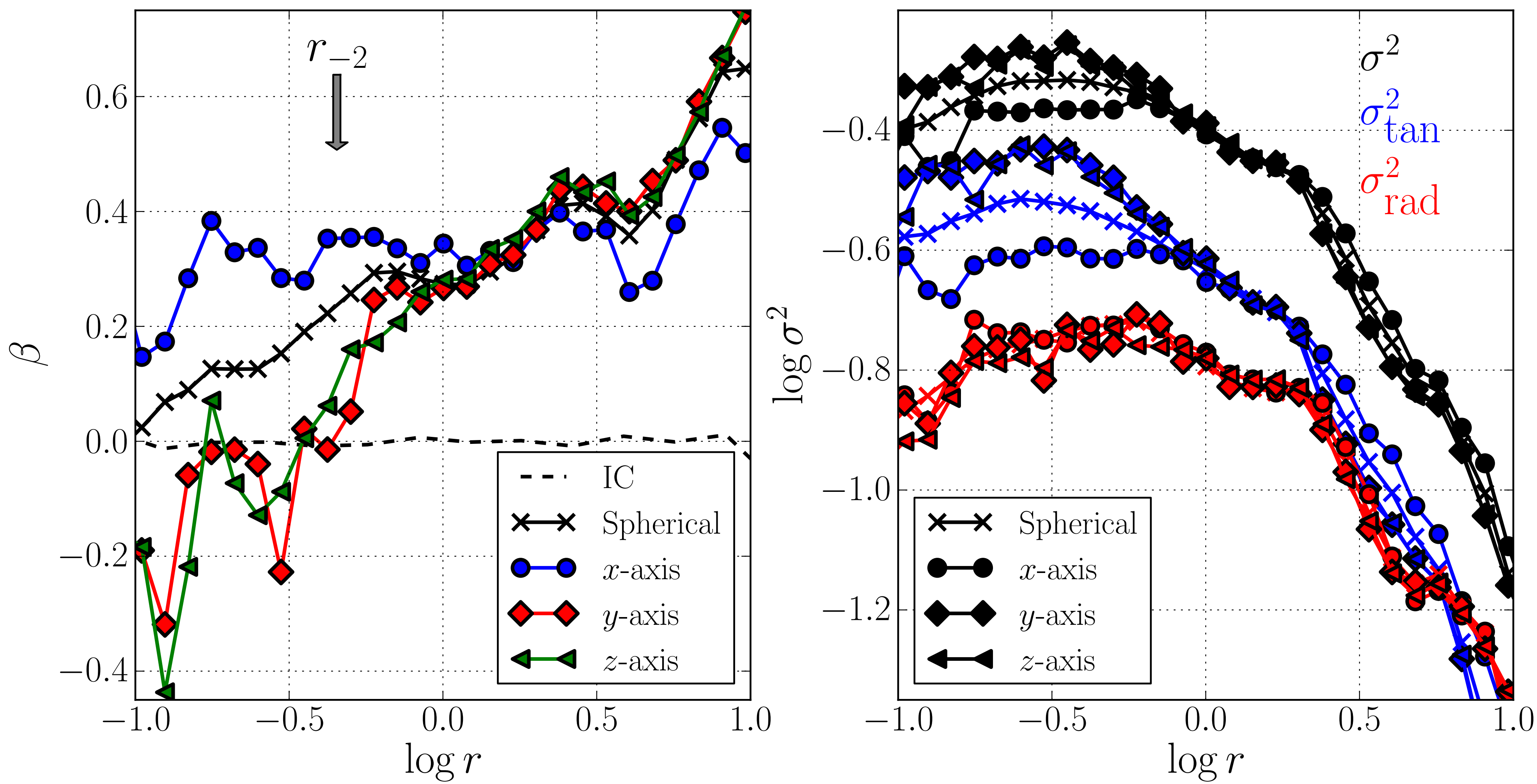
Ludlow et al 2010

the “problem” is mergers...



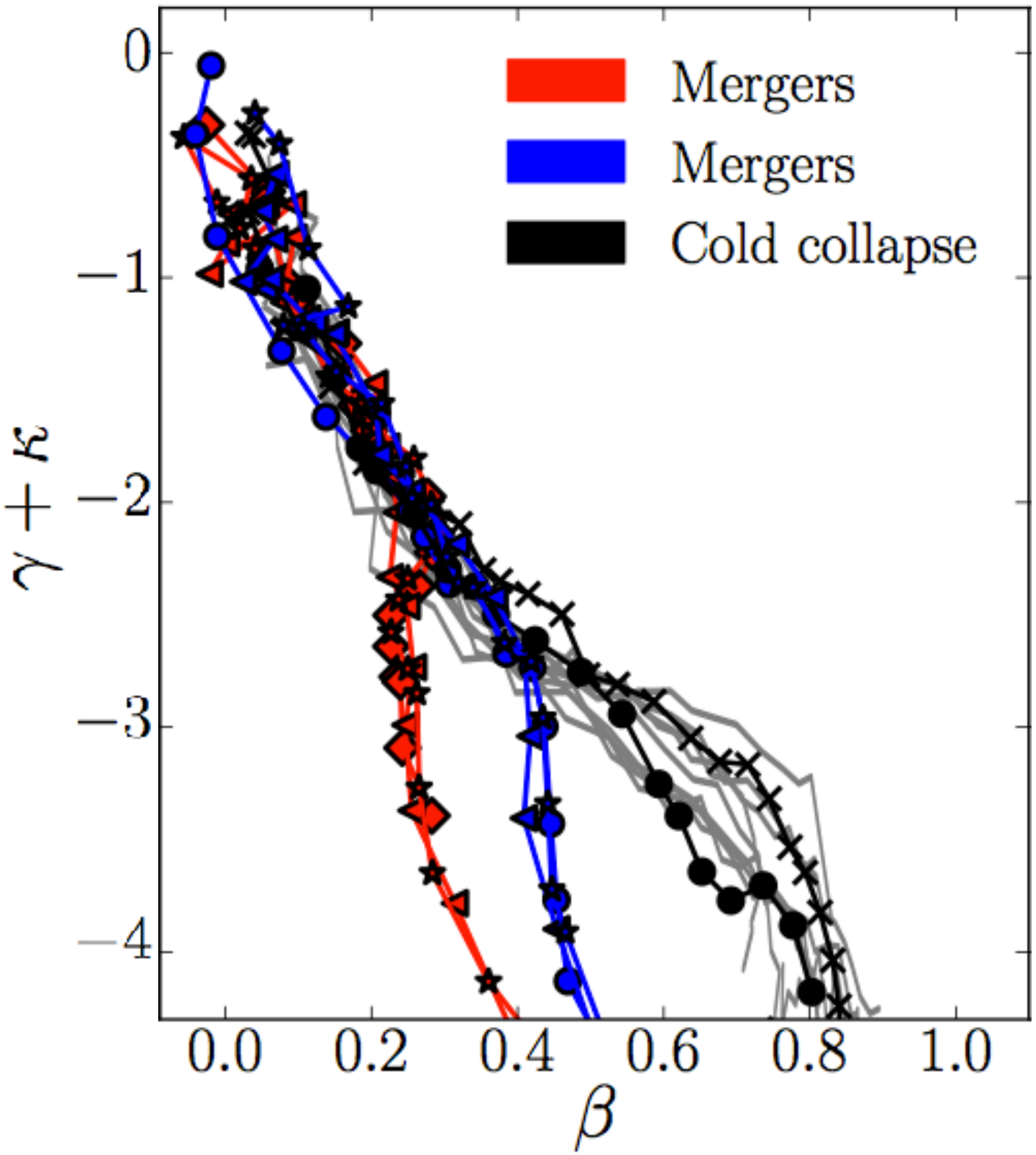



Cone along collision axis
Cone perpendicular to collision axis
The collision axis
x
y



β
log σ²
log r
r₋₂
IC
Spherical
x-axis
y-axis
z-axis
σ²
σ²tan
σ²rad

Mergers


Mergers
Mergers
Cold collapse
$\gamma + \kappa$
β
0
−1
−2
−3
−4
0.0
0.2
0.4
0.6
0.8
1.0

Where we stand today

- The attractor is (almost) an equation of state for dark matter

- We still don’t have a deep understanding of the origin of this “equation of state”

- Cosmological haloes have a strong merger history dependent beta in the outer regions (and be very careful if using spherical averages)

Take home message

- Gas collides, DM and stars do not. Therefore, the Hydrostatic Equilibrium equation is different from the Jeans equation - fundamentally different.
- The collisionless particles almost have an equation of state - but we still don't really understand why.

The end!