Supergravity at Five Loops

Current Themes in High Energy Physics and Cosmology
August 24, 2017

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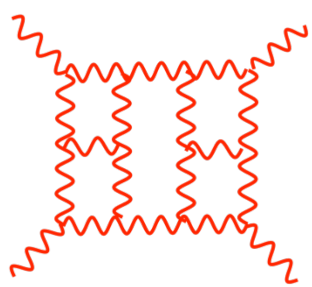
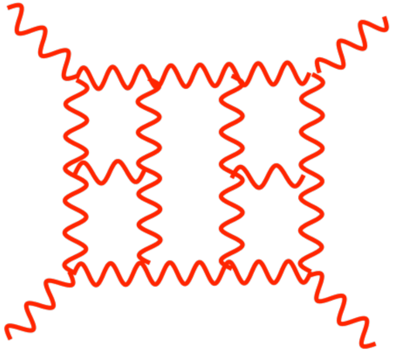
ZB, John Joseph Carrasco, Wei-Ming Chen, Henrik Johansson, Radu Roiban arXiv:1701.02519

ZB, John Joseph Carrasco, Wei-Ming Chen, Henrik Johansson, Radu Roiban, Mao Zeng, arXiv:1708.06807 (today)



Outline

1. Why we care about supergravity amplitudes at high-loop orders. Understand structure of scattering. UV.
2. Color-kinematics duality and double-copy construction.
3. Difficulties at 5 loops, causing multi-year delay.
4. Breaking the logjam: striking structure at high loops.
5. Exploiting the structure: Converting gauge theory to gravity.

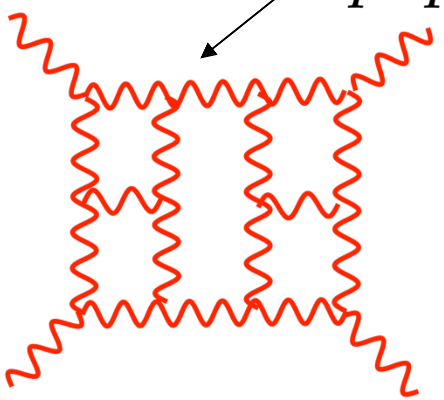


We finally have five-loop four-point integrand of $N = 8$ supergravity

UV Behavior of Gravity?

$\kappa = \sqrt{32\pi G_N}$ ← Dimensionful coupling

$\kappa\, p^\mu p^\nu$



Gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots \kappa p_j^\mu p_j^\nu \cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots g p_j^\nu \cdots}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must diverge at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

- **With more supersymmetry expect better UV properties.**
- **Need to worry about "hidden cancellations".**
- ***N* = 8 supergravity *best* theory to study.**

UV Behavior of Gravity

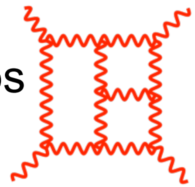
What is the actual UV behavior of *N* = 8 supergravity?

Not a philosophical question, but a technical question.

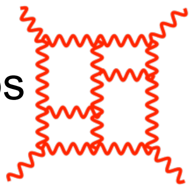
By trying to answer this question we learn a lot about gravity

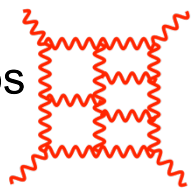
Feynman Diagrams for Gravity

Suppose we want to check UV properties of gravity theories:

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

Superspace helps, but not enough to make a difference. Standard techniques hopeless.

$N = 8$ supergravity: Where is First $D = 4$ UV Divergence?

3 loops $N = 8$	Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985)	✗
5 loops $N = 8$	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009)	✗
6 loops $N = 8$	Howe and Stelle (2003)	✗
7 loops $N = 8$	Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011)	?
3 loops $N = 4$	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops $N = 5$	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops $N = 4$	Vanhove and Tourkine (2012)	✓
9 loops $N = 8$	Berkovits, Green, Russo, Vanhove (2009)	✗

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

← **Don't bet on divergence**

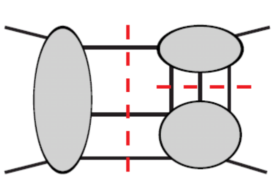
← **Weird structure. Anomaly-like behavior of divergence.**

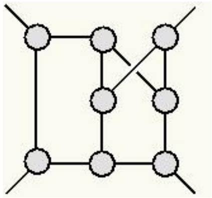
← **retracted**

- **Conventional wisdom holds that it will diverge sooner or later.**
- **But every detailed prediction either wrong or misleading.**

Our Basic Tools

We have powerful tools for computing scattering amplitudes and studying their UV properties:





- **Generalized unitarity method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson, Kosower

- **Duality between color and kinematics. Gravity scattering amplitudes directly from gauge-theory ones. Double copy.**

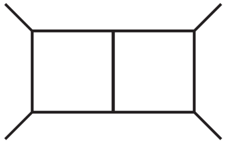
  ZB, Carrasco and Johansson (BCJ)

- **Advanced loop-integration technology.**

  Chetyrkin, Kataev and Tkachov; Laporta; A.V. Smirnov; V. A. Smirnov; Vladimirov; Marcus, Sagnotti; Czakon; Laporta; Kosower; Larsen and Zhang; Zeng, etc

- **I won't explain these tools in much detail but they underlie everything.**
- **Many other tools and advances that I won't discuss here.**

See Nima's and David's talks



Divergences in Pure Gravity

Goroff and Sagnotti, ZB, Cheung, Chi, Davies, Dixon and Nohle; ZB, Chi, Dixon, Edison

Even in pure gravity, leading 2-loop UV divergences don't work quite the way you were taught.

$$D = 4 - 2\epsilon$$

$$\mathcal{M}_4 = \left[\frac{1}{\epsilon}\left(\frac{209}{24} - \frac{15}{2} n_3\right) - \frac{1}{4} \ln \mu^2\right] \mathcal{K} + \text{finite} + \text{IR}$$

divergence

number of 3 forms

robust

Renormalization scale

$$\mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

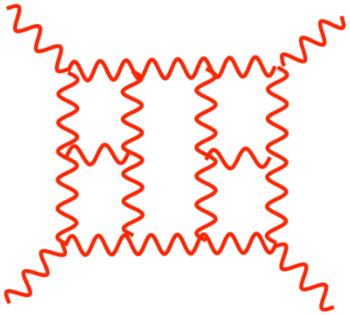
- **The UV divergence depends on the details of the regularization prescriptions. 3 forms not dynamical.**
- **Weird that renorm. scale and UV divergence not linked.**
- **The divergence itself comes from anomaly-like behavior:** $\epsilon/\epsilon \times 1/\epsilon$
- **Only other pure (super)gravity theory with known divergences is $N = 4$ supergravity: Similar anomaly-like behavior.**

See Kosower's talk

- **No such anomaly expected in $N \geq 5$ sugra.**

Carrasco, Kallosh, Tseytlin and Roiban; Kallosh; Freedman, Kallosh, Murli, Van Proeyen, Yamada

New Structures?



Might there be a new unaccounted structure in gravity theories that suggests the UV might be is tamer than conventional arguments suggest?

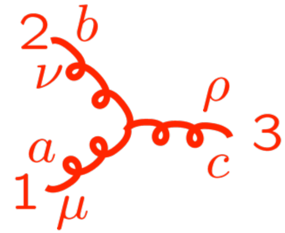
Yes!

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

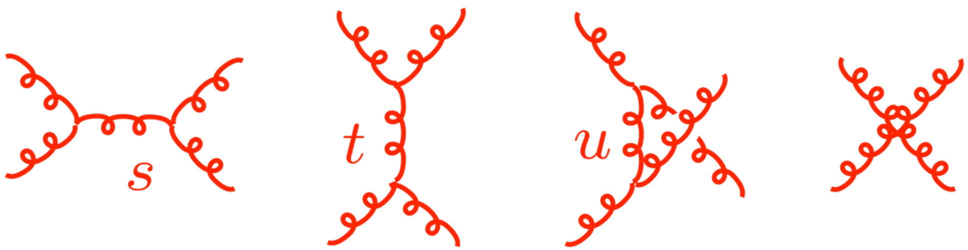
coupling constant — color factor — momentum dependent kinematic factor

$$-g f^{abc} (\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$



Color factors based on a Lie algebra: $[T^a, T^b] = i f^{abc} T^c$

Jacobi Identity $f^{a_1 a_2 b} f^{b a_4 a_3} + f^{a_4 a_2 b} f^{b a_3 a_1} + f^{a_4 a_1 b} f^{b a_2 a_3} = 0$



Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.

$s = (k_1 + k_2)^2 \quad t = (k_1 + k_4)^2$

$u = (k_1 + k_3)^2$

$$\mathcal{A}_4^{\text{tree}} = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right)$$

Color factors satisfy Jacobi identity:

Numerator factors satisfy similar identity:

$$\boxed{\begin{aligned} c_u &= c_s - c_t \\ n_u &= n_s - n_t \end{aligned}}$$

Proven at tree level and conjectured at loop level.

ZB, Carrasco, Johansson; Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove; Cachazo, etc

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

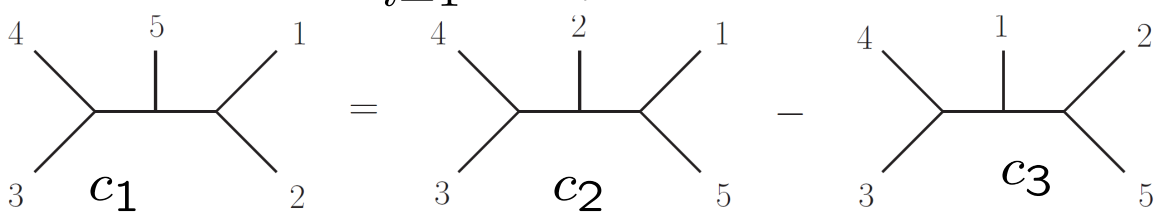
Consider five-point tree amplitude:

color factor

kinematic numerator factor

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i n_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

Feynman propagators



$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5 \, k_2 \cdot \varepsilon_1 \, \varepsilon_2 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 + c_2 + c_3 = 0 \;\; \Leftrightarrow \; n_1 + n_2 + n_3 = 0}$$

Claim: We can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.

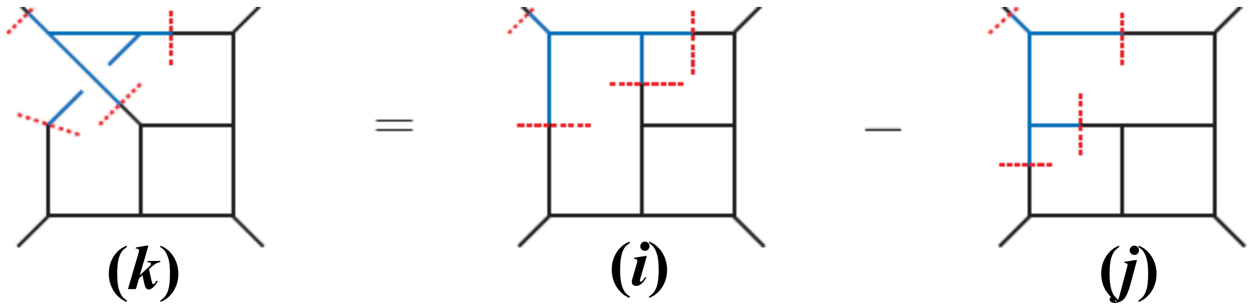
Progress on unraveling relations.

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer;
Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer
O'Connell and Montiero; Bjerrum-Bohr, Damgaard, O'Connell and Montiero; O'Connell, Montiero, White, etc.

Gravity Loop Integrands from YM

BCJ

Ideas conjectured to generalize to loops:



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

To get:

gauge theory $\longrightarrow$ gravity theory

simply take

color factor $\longrightarrow$ kinematic numerator

$$c_k \longrightarrow n_k$$

Gravity loop integrands follow from gauge theory!

Gravity From Gauge Theory

BCJ

$$-i\left(\frac{2}{\kappa}\right)^{(n-2)}\mathcal{M}_n^{\rm tree}(1,2,\ldots,n)=\sum_i\frac{n_i\,\tilde{n}_i}{\prod_{\alpha_i}p^2_{\alpha_i}}$$

Here we consider only simplest constructions:

$N=8$ sugra: $(N=4$ sYM$)\times(N=4$ sYM$)$
$N=5$ sugra: $(N=4$ sYM$)\times(N=1$ sYM$)$
$N=4$ sugra: $(N=4$ sYM$)\times(N=0$ sYM$)$

Spectrum controlled by simple tensor product of gauge theories.

More sophisticated lower-susy cases: QCD, magical supergravities, Einstein-YM with and without Higgsing, twin supergravities.

Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; ZB, Davies, Dennen, Huang and Nohle; Nohle; Chiodaroli, Günaydin, Johansson, Roiban. A. Anastasiou, L. Borsten, M.J. Duff, M.J. Hughes, Marrani, Nagy, Zoccali.

Many other theories in double-copy story, including open and closed string theory, NLSM, Dirac Born Infeld, Galileon and Z theory.

Cachazo, He, Yuan; Chen Du, Broedel, Schlotterer and Stieberger; Carrasco, Mafra, Schlotterer;

Recent Related Activities

- **Examples of exact classical solutions, including black holes.**
  Monteiro, O'Connell, White; Luna,Monteiro, O'Connell, White (2015)
- **Perturbative constructions of general classical solutions, including gravitational radiation problems (LIGO)**
  Goldberger, Ridgway (2016); Luna, Monterio, Nicholson, O'Connell, Ochirov, Westerberg, White (2016)
- **Loop level KLT and BCJ, using CHY, ambitwistor string, Q-cuts** Song He, Oliver Schlotterer (2016), Tourkine, Vanhove (2016,2017); Hohenegger, S. Stieberger (2017).
- **Analytic properties of gravity integrands.** Herrmann and Trnka (2016)
- **Kinematic algebra behind BCJ duality.**
  Monteiro and O'Connell; Cheung and Shen (2016).
- **Simplified gravity Lagrangian from left-right factorization.**
  Cheung and Remmen (2016,2017)
- **Double copy as consequence of gauge invariance.**
  Chiodaroli; Boels, Medina (2016), Arkani-Hamed, Rodina, Trnka (2016), Feng et al (2016)
- **Applications in string theory.** Steiberger; Vahhove
  Carrasco, Mafra, Schlotterer, (2016); Mafra and Schlotterer (2015,2016)

Supergravity and Ultraviolet Divergences

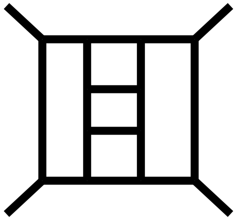
Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

- **First quantized formulation of Berkovits' pure-spinor formalism.** Bjornsson and Green
- **Unitarity method.** ZB, Davies, Dennen

Key point: *all* supersymmetry cancellations are exposed.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense" Bjornsson and Green

- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.** ? ← **Want to check this.**
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.** ?
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$.** ✗
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$.** ✗



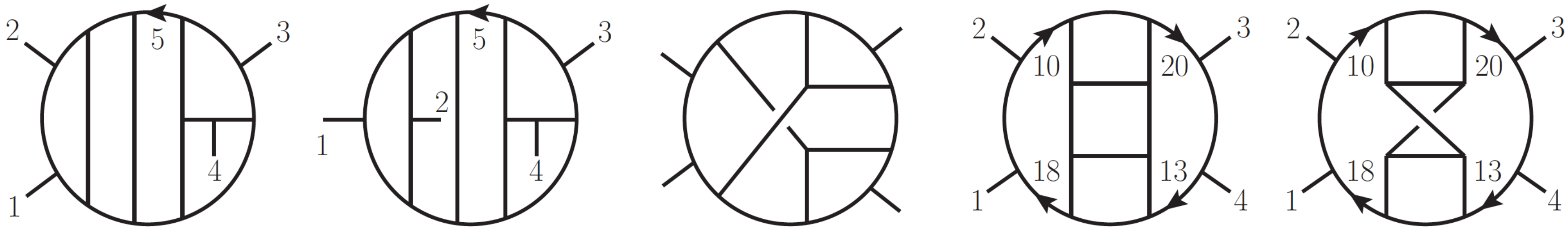
Consensus agreement from all methods

But new types of cancellations do exist: "enhanced cancellations".

ZB, Davies, Dennen

$N = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Chen Johannson, Roiban, Zeng



Place your bets:

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**

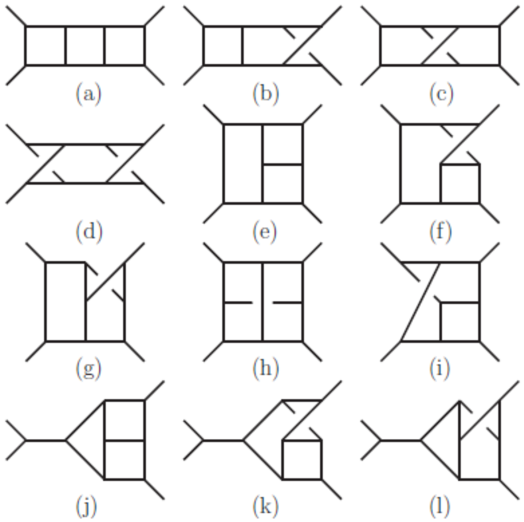
5 loops

Kelly Stelle:
English wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$\mathcal{N} = 4$ Supergravity UV Cancellation

BCJ duality works easily



$D = 4 - 2\epsilon$ ZB, Davies, Dennen, Huang

Graph	(divergence)$/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

($\mathcal{N} = 4$ sugra) = ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 0$ YM)

***All* three-loop divergences and subdivergences cancel completely!**

Still no standard-symmetry explanation, despite valiant attempt.

Bossard, Howe, Stelle; ZB, Davies, Dennen

A nontrivial example of "enhanced cancellations"

$\mathcal{N} = 5$ Supergravity Four Loop Cancellations

ZB, Davies and Dennen

We calculated four-loop divergence in $\mathcal{N} = 5$ supergravity.

Industrial strength software needed: FIRE5 and special purpose C++

$\mathcal{N} = 5$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 1$ sYM)

Crucial help from (Smirnov)^2

$\mathcal{N} = 4$ sYM

$\mathcal{N} = 1$ sYM

Diagrams necessarily UV divergent.

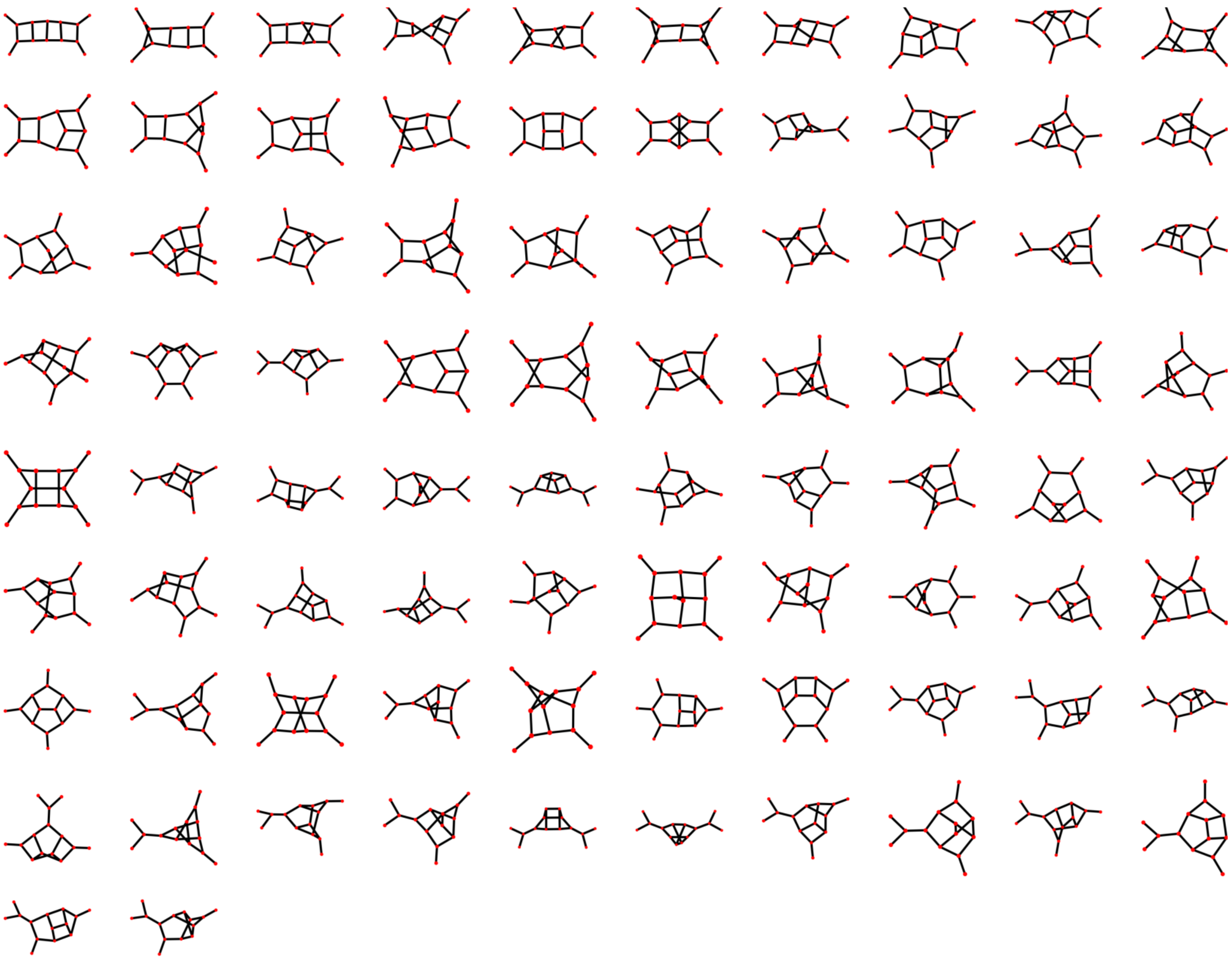
$\mathcal{N} = 5$ supergravity has no divergence at four loops.

Nontrivial example of an "enhanced cancellation".

No standard-symmetry explanation known.

82 nonvanishing numerators in BCJ representation

ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N} = 4$ sYM)



$\mathcal{N} = 5$ supergravity at Four Loops

Special purpose C++ and FIRE5

ZB, Davies and Dennen

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}[\frac{7358585}{7962624}s^2+\frac{2561447}{2654208}st-\frac{872683}{1990656}t^2]+\frac{1}{\epsilon^3}[\frac{75972559}{35389440}s^2+\frac{240984061}{26542080}st+\frac{1302037}{1310720}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{369234283}{11059200}s^2-\frac{257792411}{4915200}st-\frac{101847769}{14745600}t^2)+\zeta_2(\frac{7358585}{3981312}s^2+\frac{2561447}{1327104}st-\frac{872683}{995328}t^2) - \mathrm{S2}(\frac{1223621}{49152}s^2+\frac{46816475}{442368}st+\frac{2639903}{221184}t^2)+\frac{206093335871}{11466178560}s^2+\frac{320983191023}{3822059520}st+\frac{53309416589}{2866544640}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{84777347}{368640}s^2+\frac{382194721}{1474560}st+\frac{417476581}{1474560}t^2)-\zeta_4(\frac{3062401}{2457600}s^2+\frac{3881051}{3276800}st-\frac{112081813}{29491200}t^2) + \zeta_3(\frac{28162691399797}{53747712000}s^2+\frac{19354492750651}{35831808000}st-\frac{22092683352811}{107495424000}t^2)-\zeta_2(\frac{70861961}{17694720}s^2+\frac{227180689}{13271040}st + \frac{105727243}{53084160}t^2)+\mathrm{T1ep}(-\frac{1223621}{663552}s^2-\frac{46816475}{5971968}st-\frac{2639903}{2985984}t^2)-\mathrm{S2}(\frac{11916028151}{5898240}s^2 + \frac{72637733971}{13271040}st+\frac{17223563447}{53084160}t^2)+\mathrm{D6}(-\frac{9001177}{552960}s^2-\frac{264491}{10240}st-\frac{2610157}{552960}t^2) + \frac{110945914744727}{1146617856000}s^2+\frac{16989492195991}{127401984000}st-\frac{21362122998269}{573308928000}t^2]$
31–60	$\frac{1}{\epsilon^4}[-\frac{5502451}{2654208}s^2-\frac{3675877}{884736}st+\frac{11269}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{38102993}{26542080}s^2-\frac{291607201}{106168320}st-\frac{565798829}{318504960}t^2] + \frac{1}{\epsilon^2}[\zeta_3(\frac{108955183}{2211840}s^2+\frac{653019571}{8847360}st+\frac{9453043}{1769472}t^2)+\zeta_2(-\frac{5502451}{1327104}s^2-\frac{3675877}{442368}st+\frac{11269}{248832}t^2) + \mathrm{S2}(\frac{16797481}{1327104}s^2+\frac{1172969}{16384}st+\frac{978427}{82944}t^2)-\frac{304243754383}{19110297600}s^2-\frac{2032063711381}{19110297600}st-\frac{257798086613}{7166361600}t^2] + \frac{1}{\epsilon}[\zeta_5(\frac{33327659}{122880}s^2+\frac{13276219}{24576}st+\frac{22251887}{184320}t^2)+\zeta_4(\frac{12299887}{1474560}s^2+\frac{258056147}{5898240}st+\frac{46913759}{5898240}t^2) + \zeta_3(-\frac{26846001990157}{42998169600}s^2-\frac{337106527201}{265420800}st-\frac{5298324906787}{42998169600}t^2)+\zeta_2(\frac{282283789}{39813120}s^2+\frac{975199319}{53084160}st + \frac{60394451}{159252480}t^2)+\mathrm{T1ep}(\frac{16797481}{17915904}s^2+\frac{1172969}{221184}st+\frac{978427}{1119744}t^2)+\mathrm{S2}(\frac{10516980893}{4976640}s^2 + \frac{389045625329}{53084160}st+\frac{216032337589}{159252480}t^2)+\mathrm{D6}(\frac{503413}{23040}s^2+\frac{12342607}{552960}st+\frac{3661}{184320}t^2) - \frac{166777358259461}{1146617856000}s^2-\frac{565137511429117}{1146617856000}st-\frac{21629055712141}{191102976000}t^2]$
61–82	$\frac{1}{\epsilon^4}[\frac{285899}{248832}s^2+\frac{1058273}{331776}st+\frac{275869}{663552}t^2]+\frac{1}{\epsilon^3}[-\frac{380329649}{106168320}s^2-\frac{74703227}{11796480}st+\frac{124701919}{159252480}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{1371419}{86400}s^2-\frac{236241539}{11059200}st+\frac{4326077}{2764800}t^2)+\zeta_2(\frac{285899}{124416}s^2+\frac{1058273}{165888}st+\frac{275869}{331776}t^2) + \mathrm{S2}(\frac{8120143}{663552}s^2+\frac{1893289}{55296}st+\frac{92293}{663552}t^2)-\frac{58867708103}{28665446400}s^2+\frac{71191292711}{3185049600}st+\frac{83016363427}{4777574400}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{1520563}{36864}s^2-\frac{1178767861}{1474560}st-\frac{595491677}{1474560}t^2)-\zeta_4(\frac{6539029}{921600}s^2+\frac{313837819}{7372800}st+\frac{21665663}{1843200}t^2) + \zeta_3(\frac{20790944575597}{214990848000}s^2+\frac{6505876281371}{8957952000}st+\frac{70676991239557}{214990848000}t^2)+\zeta_2(-\frac{491377507}{159252480}s^2-\frac{66476563}{53084160}st + \frac{128393639}{79626240}t^2)+\mathrm{T1ep}(\frac{8120143}{8957952}s^2+\frac{1893289}{746496}st+\frac{92293}{8957952}t^2)+\mathrm{S2}(-\frac{14810628499}{159252480}s^2 - \frac{19698937889}{10616832}st-\frac{10272602953}{9953280}t^2)+\mathrm{D6}(-\frac{616147}{110592}s^2+\frac{1939907}{552960}st+\frac{1299587}{276480}t^2) + \frac{9307894793789}{191102976000}s^2+\frac{206124003456599}{573308928000}st+\frac{21562322533673}{143327232000}t^2]$

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}[\frac{1052159}{995328}s^2+\frac{509789}{331776}st-\frac{121001}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{9042569}{1474560}s^2+\frac{34360945}{1327104}st+\frac{73518401}{13271040}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{11443919}{2764800}s^2+\frac{32520079}{552960}st+\frac{5836531}{230400}t^2)+\zeta_2(\frac{1052159}{497664}s^2+\frac{509789}{165888}st-\frac{121001}{248832}t^2) - \mathrm{S2}(\frac{637991}{6144}s^2+\frac{10978729}{27648}st+\frac{5080825}{55296}t^2)+(\frac{270806866183}{7166361600}s^2+\frac{89848068067}{597196800}st+\frac{218093645149}{7166361600}t^2)] + \frac{1}{\epsilon}[\zeta_5(\frac{100843}{360}s^2+\frac{17118043}{30720}st-\frac{30266471}{92160}t^2)+\zeta_4(\frac{11435323}{614400}s^2+\frac{232002227}{1843200}st+\frac{22211783}{460800}t^2) + \zeta_3(\frac{223300432349}{3359232000}s^2-\frac{178732984847}{716636160}st+\frac{951659436383}{53747712000}t^2) - \zeta_2(\frac{5492357}{245760}s^2+\frac{53468887}{663552}st+\frac{129714599}{6635520}t^2)+\mathrm{T1ep}(-\frac{637991}{82944}s^2-\frac{10978729}{373248}st-\frac{5080825}{746496}t^2) + \mathrm{S2}(-\frac{5700088747}{3686400}s^2-\frac{69470348491}{16588800}st-\frac{713512871}{6635520}t^2)+\mathrm{D6}(-\frac{357421}{43200}s^2-\frac{2891743}{230400}st-\frac{470219}{138240}t^2) - \frac{3571506237341}{28665446400}s^2-\frac{1611591325291}{5971968000}st+\frac{2301084608777}{143327232000}t^2]$
31–60	$\frac{1}{\epsilon^4}[-\frac{150715}{82944}s^2-\frac{668333}{221184}st-\frac{7213}{1990656}t^2]+\frac{1}{\epsilon^3}[-\frac{68021833}{13271040}s^2-\frac{36852103}{1327104}st-\frac{298377299}{39813120}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{36448033}{2764800}s^2-\frac{455889533}{2764800}st-\frac{82059281}{1382400}t^2)+\zeta_2(-\frac{150715}{41472}s^2-\frac{668333}{110592}st-\frac{7213}{995328}t^2) + \mathrm{S2}(\frac{13910839}{165888}s^2+\frac{1340033}{4096}st+\frac{26303855}{331776}t^2)-\frac{68286245653}{2388787200}s^2-\frac{20649690431}{119439360}st-\frac{351701043553}{7166361600}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{2362679}{9216}s^2-\frac{178668311}{92160}st-\frac{1268313}{10240}t^2)+\zeta_4(-\frac{124344121}{1843200}s^2-\frac{491722333}{1843200}st-\frac{68141309}{921600}t^2) - \zeta_3(\frac{630084012997}{53747712000}s^2-\frac{1250670277213}{663552000}st-\frac{6913218302303}{13436928000}t^2) + \zeta_2(\frac{352368061}{19906560}s^2+\frac{35509679}{663552}st+\frac{227699801}{19906560}t^2)+\mathrm{T1ep}(\frac{13910839}{2239488}s^2+\frac{1340033}{55296}st+\frac{26303855}{4478976}t^2) + \mathrm{S2}(\frac{188312318729}{99532800}s^2+\frac{110749829741}{16588800}st+\frac{5056299197}{3981312}t^2)+\mathrm{D6}(\frac{1220779}{76800}s^2+\frac{44791}{6912}st-\frac{1159831}{230400}t^2) + \frac{2755666297013}{28665446400}s^2+\frac{5622513975899}{35831808000}st-\frac{196197363193}{1769472000}t^2]$
61–82	$\frac{1}{\epsilon^4}[\frac{756421}{995328}s^2+\frac{985421}{663552}st+\frac{163739}{663552}t^2]+\frac{1}{\epsilon^3}[-\frac{1670161}{1658880}s^2+\frac{415193}{221184}st+\frac{4863881}{2488320}t^2] + \frac{1}{\epsilon^2}[\zeta_3(\frac{110861}{6400}s^2+\frac{16293841}{153600}st+\frac{9408019}{276480}t^2)+\zeta_2(\frac{756421}{497664}s^2+\frac{985421}{331776}st+\frac{163739}{331776}t^2) + \mathrm{S2}(\frac{1657459}{82944}s^2+\frac{7734025}{110592}st+\frac{4181095}{331776}t^2)-\frac{8243516153}{895795200}s^2+\frac{558349337}{24883200}st+\frac{11133949867}{597196800}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{1094509}{46080}s^2+\frac{63657091}{46080}st+\frac{5210161}{11520}t^2)+\zeta_4(\frac{11254769}{230400}s^2+\frac{129860053}{921600}st+\frac{23717743}{921600}t^2) - \zeta_3(\frac{2745647960587}{53747712000}s^2+\frac{3654260151947}{2239488000}st+\frac{5720906529119}{10749542400}t^2) + \zeta_2(\frac{11564107}{2488320}s^2+\frac{2244901}{82944}st+\frac{40360999}{4976640}t^2)+\mathrm{T1ep}(\frac{1657459}{1119744}s^2+\frac{7734025}{1492992}st+\frac{4181095}{4478976}t^2) + \mathrm{S2}(-\frac{420043}{1215}s^2-\frac{825589625}{331776}st-\frac{5785239343}{4976640}t^2)+\mathrm{D6}(-\frac{210731}{27648}s^2+\frac{4196129}{691200}st+\frac{1457647}{172800}t^2) + \frac{33976742047}{1194393600}s^2+\frac{4046536311847}{35831808000}st+\frac{212357840779}{2239488000}t^2]$

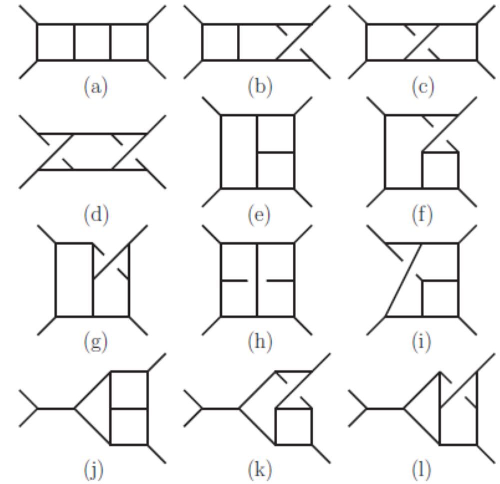
Adds up to zero: no divergence. Enhanced cancellations!
No standard (super)symmetry explanation exists.

Enhanced UV Cancellations

ZB, Davies, Dennen

Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but the amplitude is well behaved.

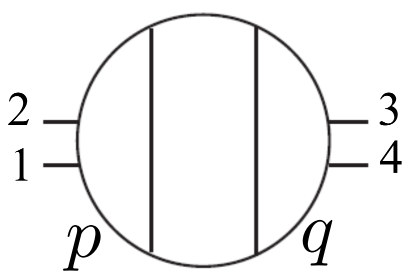
- **By definition this is an enhanced cancellation.**
- **Not the way nonabelian gauge theory works.**



already log divergent

***N* = 4 sugra: pure YM x *N* = 4 sYM**

***N* = 4 sugra**



$$n_i \sim s^3 t A_4^{\text{tree}} (p \cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

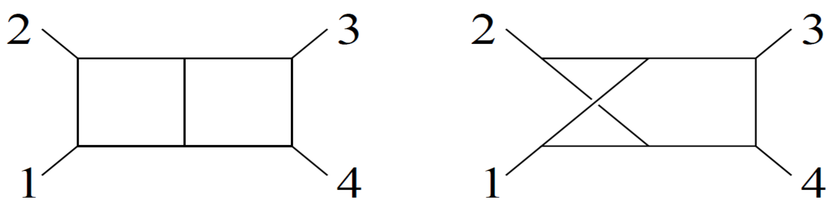
- **3 loop UV finiteness of *N* = 4 supergravity proves existence of "enhanced cancellation" in supergravity theories.**
- **No known standard symmetry explanation.**

Where does new magic come from?

ZB, Davies, Dennen, Huang; Bossard, Howe, Stelle

To analyze we need a simpler example: Half-maximal supergravity in $D = 5$ at 2 loops. No known symmetry explanation in this case.

Similar to $N = 4$, $D = 4$ sugra at 3 loops, except much simpler.



$D = 5$ half max sugra
$N = 4$ sYM x $N = 0$ YM

Quick summary:

— Finiteness in $D = 5$ tied to double-copy structure.

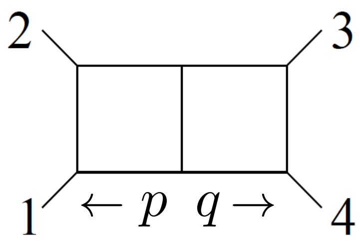
— Cancellations in certain forbidden gauge-theory color structures imply hidden UV cancellations in supergravity.

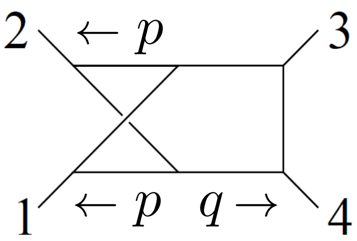
Double-copy structure implies extra cancellations!

Unfortunately, argument relies on special two-loop property: integrals of $N = 4$ sugra are identical to those of QCD.

Need a more general approach

Multiloop Cancellations





ZB, Enciso, Parra-Martinez, Zeng

route momenta in different ways

- **Demonstrated enhanced cancellations require integration properties. Not like non-abelian gauge theory.**
- **Standard proofs of UV properties are ruled out.**

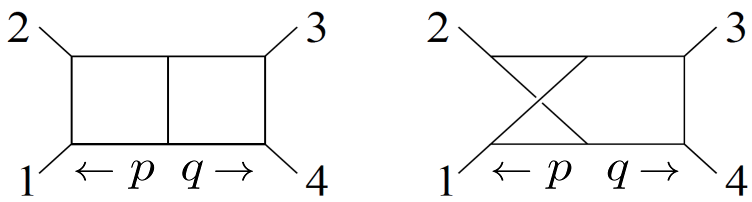
We make use of enormous advances in understanding relations between integrals based on IBP technology.

Gluza, Kajda, Kosower ; Johansson, Kosower and Larsen; Ita; Larsen and Zhang

Allows us to write integrands manifestly UV finite up to terms that integrate to zero.

Is there a generic structure for the enhanced cancellations?

Multiloop Enhanced Cancellations



ZB, Enciso, Parra-Martinez, Zeng (2017)

Analysis of cancellations in half-maximal supergravity in $D = 5$ reveals following interesting pattern.

Conjecture: At large loop momentum enhanced cancellations follow from Lorentz symmetry and SL(L) relabeling symmetry.

Lorentz symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D\ell_a\Big) \sum_{a=1}^{L} \Big(\ell_{a\mu}\frac{\partial}{\partial \ell_a^\nu} - \ell_{a\nu}\frac{\partial}{\partial \ell_a^\mu}\Big) \frac{\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

SL(L) relabeling symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D\ell_a\Big) \sum_{a=1}^{L} \frac{\partial}{\partial \ell_a^\nu} \frac{\omega_{ab}\ell_{b\mu}\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

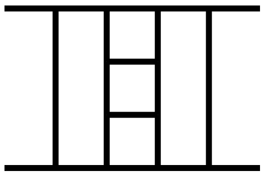
L loops

Symmetries generate a generic set of identities between integrals

Finding BCJ Forms Nontrivial

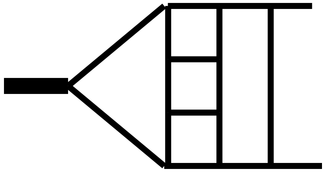
Gravity integrands might be free, but gauge-theory ones are not. Trouble beyond four loops.

5-loop 4-pt $\mathcal{N} = 4$ sYM amplitude:



Despite considerable effort no one has succeeded in finding a BCJ form.

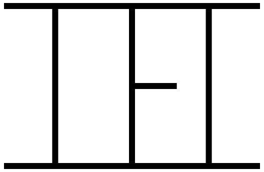
$\mathcal{N} = 4$ sYM 5 loop form factor:



On other hand, no trouble with form factor.

Gang Yang (2016)

Two-loop five-point QCD identical helicity:



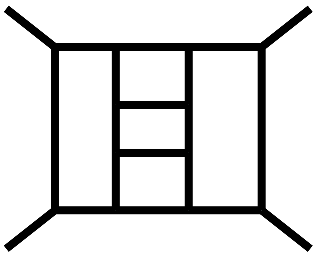
This required an ansatz with curiously high power counting.

O'Connell and Mogull (2015)

It can be difficult to find BCJ representations.

Five-Loop $N = 8$ Supergravity

$N = 4, 5$ supergravity complicated. $N = 8$ supergravity simpler.



Are the expected enhanced cancellations actually present?

Turns out to be quite nontrivial to find BCJ representations:

- **Need ansatz (guess) for numerators.**
- **Large linear systems (up to 10^6 parameters).**
- **No one has succeeded as yet, despite considerable effort.**

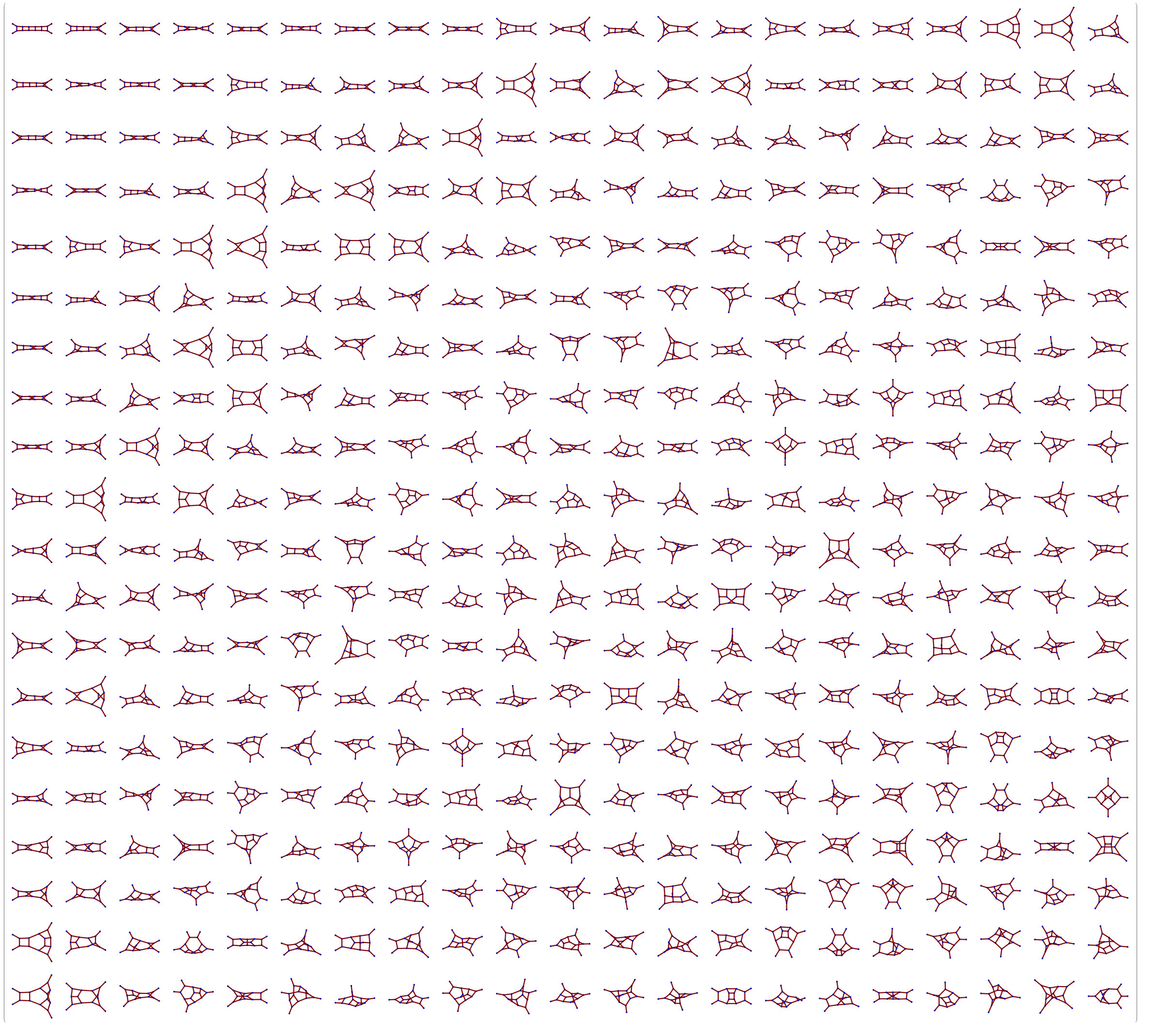
Need a better way:

- **Banish large Ansatze or guessing.**
- **No large linear systems. Get amplitude one piece at a time.**
- **Stick to sYM as long as possible and convert to sugra at end.**
- **Keep double-copy idea. Essential!**

The $N = 4$ sYM 5 Loop Integrand

ZB, Carrasco Johansson, Roiban, arXiv:1207.6666

410 diagrams. answer can be downloaded from arXiv



Want to convert this to $N = 8$ supergravity.

Contact Term Method

ZB, Carrasco, Chen, Johansson, Roiban

Task is to convert $\mathcal{N} = 4$ sYM 5-loop integrand into $\mathcal{N} = 8$ sugra.

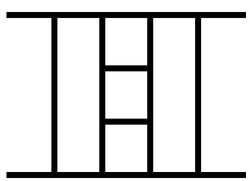
Start with "naïve double copy" of *any* correct sYM integrand:

$$c_i \rightarrow n_i$$ **even though not BCJ representation**

Without BCJ duality, not the correct $\mathcal{N} = 8$ integrand

$\mathcal{N} = 8$ cuts:

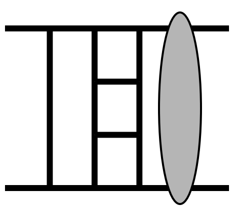
Max cuts:



Automatic

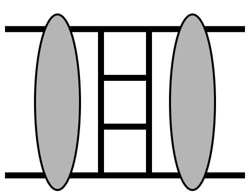
Generalized Unitarity
All exposed legs on shell

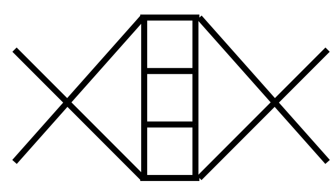
Nmax cuts:



Automatic via BCJ, 4pt trees always work

N^2max cuts:



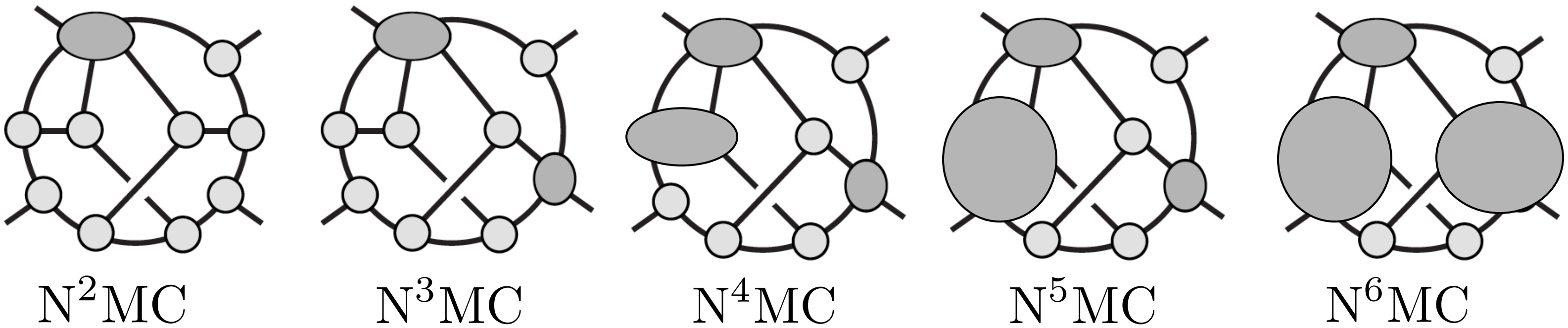


Add contact term to make it work

Blobs interfere: not automatic

Contact Term Method

contact = (gravity cut) – (cut of incomplete amplitude)



Cuts become complicated

analytical → **numerical**

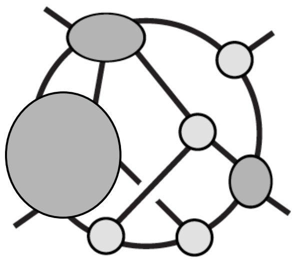
Missing contact terms become simple for complicated cuts

N^6 MC contact numerator: $a_1 s^2 + a_2 st + a_3 t^2$

- **Contact associated with cut directly giving missing piece of amplitude.**
- **75K cuts need to be evaluated.**
- **Sounds daunting. Not for faint of heart.**

Game for optimists: "Simplifying miracle is around the corner"

Generating SUGRA Cut



N^{5}MC

$$(\text{SUGRA cut}) = \sum_{\substack{N=8 \\ \text{states}}} M_1^{\text{tree}} M_2^{\text{tree}} \cdots M_m^{\text{tree}}$$

Gravity tree ampl

KLT kernel

Color order YM tree

KLT Relation:

$$M_n^{\text{tree}} = \sum_{i,j} K_{i,j} (A_n^{\text{tree}})_i (\tilde{A}_n^{\text{tree}})_j$$

Kawai, Lewellen and Tye

$$(N = 8 \text{ state sum}) = (N = 4 \text{ sYM state sum}) \times (N = 4 \text{ sYM state sum})$$

$$(\text{gravity cut}) = \sum_{\substack{i1 \cdots i_k \\ j1 \cdots j_k}} K_{\substack{i1 \cdots i_k \\ j1 \cdots j_k}} \sum_{N=4 \text{ states}} A_1^{\text{tree}} \cdots A_k^{\text{tree}} \sum_{N=4 \text{ states}} \tilde{A}_1^{\text{tree}} \cdots \tilde{A}_k^{\text{tree}}$$

$= (\text{gauge theory cut})$ x $(\text{gauge theory cut})$

Gravity cut generated directly from known 5 loop sYM result.

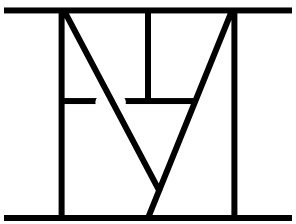
Apply KLT to cuts of known *N* = 4 sYM loop amplitudes.
Fast, but complicated analytic structure.

A Simplifying Miracle

ZB, Carrasco, Chen, Johansson, Roiban

contact = (gravity cut) – (cut of incomplete amplitude)

1. **Most contact terms vanish! Why?**
2. **In general gravity contacts far simpler than expected.**
3. **Four-point double-contacts factorize. Extremely striking.**



double 4pt contact

$$\left[2s^3 - s^2 u + 4s^2(2k_1 \cdot l_6) + \cdots\right]$$
$$\times\left[s^2 u + 2su^2 - s^2(2k_1 \cdot l_6) + \cdots\right]$$

Each factor looks like gauge theory

Reminds us of KLT factorization:

$$M^{\text{tree}}(1,2,3,4) = s_{12} A^{\text{tree}}(1,2,3,4) \times A^{\text{tree}}(1,2,4,3)$$

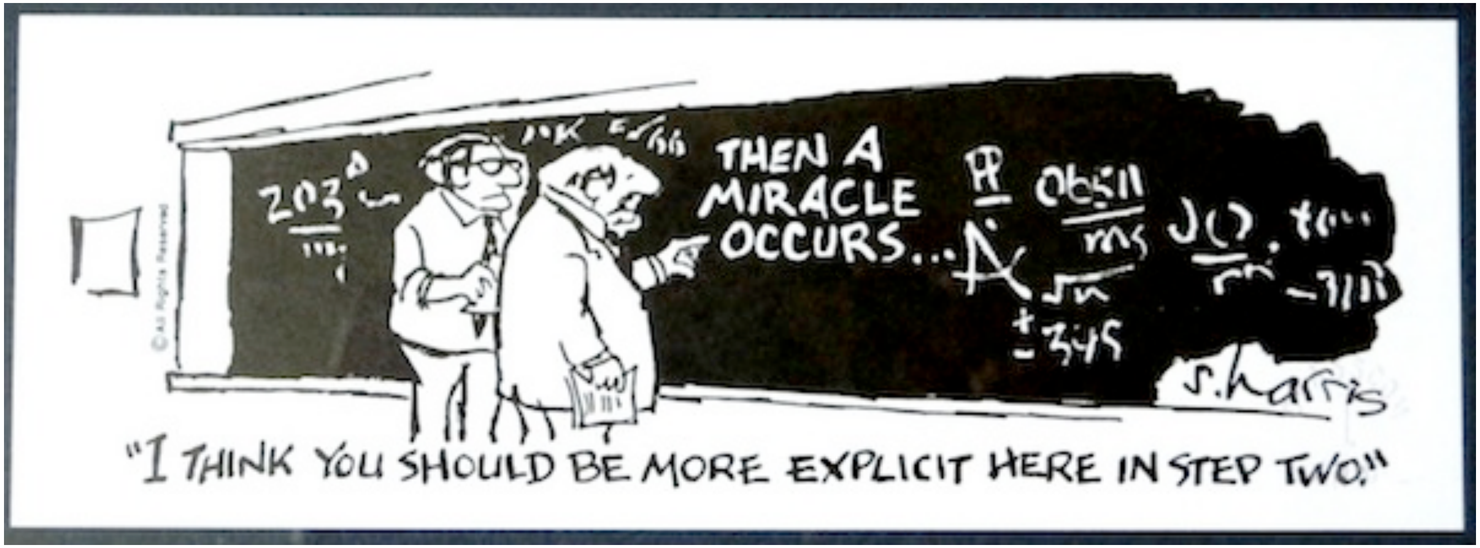
For 5 or higher-point contacts no overall factorization, as with KLT.

Can we write down formulas that give missing gravity pieces directly from gauge theory, bypassing gravity cuts?

A Miracle

1. Start from gauge-theory loop amplitude.
2. Construct naïve double copy.
3. Compute cut of naïve double copy.
4. Compute gravity cut from gauge-theory cuts via KLT.
5. Subtract and shake hard (nontrivial).
6. Extract surprisingly simple gravity contact.

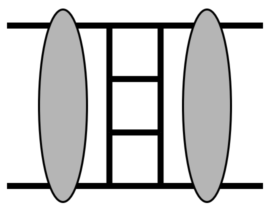
Miracle: The contact terms are so simple we should be able write down missing gravity contacts directly from gauge theory.

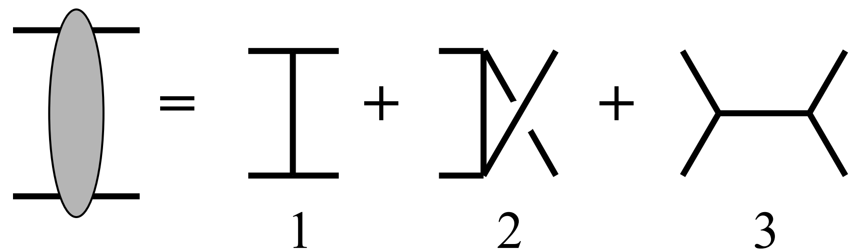


BCJ Discrepancy Functions

Need a function defined purely in gauge theory as building block for missing gravity pieces.

Inside multiloop diagram





kinematic numerators

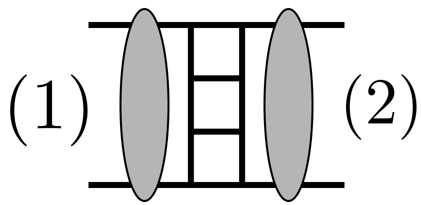
BCJ discrepancy function: $$J \equiv \sum_{i=1}^{3} n_i$$

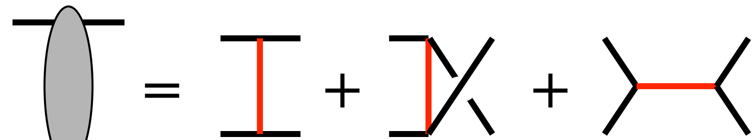
Vanishes if we have BCJ form of gauge theory.

Obvious guess is these are building blocks for missing gravity pieces.

Missing pieces: $$\sim \sum J \times J$$

Deriving Gravity Contact Formulas





color

numerator

propagator

$$\mathcal{C}^{4\times4}_{\mathrm{YM}} = \sum_{i_1,i_2} \frac{c_{i_1 i_2} n_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

Generalized gauge transformation

$$\delta_{i1,i2} \equiv n_{i1i2} - n^{\mathrm{BCJ}}_{i1,i2} = d^{(1)}_{i1} k^{(2)}(i_2) + d^{(2)}_{i_2} k^{(1)}(i1)$$

Generalized gauge invariance:

$$\sum_{i_1,i_2} \frac{c_{i_1 i_2} \delta_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} = 0 = \sum_{i_1,i_2} \frac{n^{\mathrm{BCJ}}_{i_1 i_2} \delta_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

BCJ discrepancy function:

$$J^{(1)}_{i_2} \equiv \sum_{i_1}^{3} n_{i_1 i_2} = d^{(1)}_{i_2} \sum_{i_1}^{3} k^{(1)}(i_1) \qquad J^{(2)}_{i_1} \equiv \sum_{i_2}^{3} n_{i_1 i_2} = d^{(2)}_{i_1} \sum_{i_2}^{3} k^{(2)}(i_2)$$

$$\mathcal{C}^{4\times4}_{\mathrm{SG}} = \sum_{i_1,i_2} \frac{n^{\mathrm{BCJ}}_{i_1 i_2} n^{\mathrm{BCJ}}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

cross term between numerators and discrepancy vanishes.

Formula for missing contact:

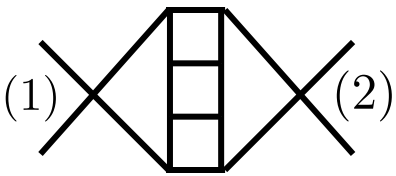
$$\mathcal{C}^{4\times4}_{\mathrm{SG}} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} n_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} - \frac{2}{d^{(1)}_{1} d^{(2)}_{1}} J^{(1)}_{1} J^{(2)}_{1}$$

Gravity from Gauge Theory

ZB, Carrasco, Chen, Johansson, Roiban

Missing gravity from any gauge theory representation

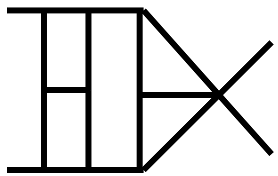
BCJ discrepancy functions



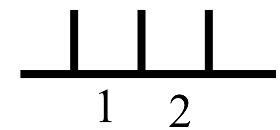
$$\mathcal{E}^{4\times 4}_{\rm GR} = -\frac{1}{d_1^{(1)} d_1^{(2)}}\Big(J_{\bullet,1}\tilde J_{1,\bullet} + J_{1,\bullet}\tilde J_{\bullet,1}\Big)$$

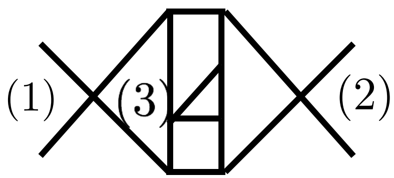
propagators cancel trivially

Expand into 15 diagrams



$$\mathcal{E}^{5}_{\rm GR} = -\frac{1}{6}\sum_{i=1}^{15} \frac{J_{\{i,1\}}\tilde J_{\{i,2\}} + J_{\{i,2\}}\tilde J_{\{i,1\}}}{d_i^{(1)} d_i^{(2)}}$$





$$\mathcal{E}^{4\times 4\times 4}_{\rm GR} = -\sum_{i_3=1}^{3}\frac{J_{\bullet,1,i_3}\tilde J_{1,\bullet,i_3}}{d_1^{(1)}d_1^{(2)}d_{i_3}^{(3)}} - \sum_{i_2=1}^{3}\frac{J_{\bullet,i_2,1}\tilde J_{1,i_2,\bullet}}{d_1^{(1)}d_{i_2}^{(2)}d_1^{(3)}} - \sum_{i_1=1}^{3}\frac{J_{i_1,\bullet,1}\tilde J_{i_1,1,\bullet}}{d_{i_1}^{(1)}d_1^{(2)}d_1^{(3)}} + \frac{J_{\bullet,1,1}\tilde J_{1,\bullet,\bullet}}{d_1^{(1)}d_1^{(2)}d_1^{(3)}} + \frac{J_{1,\bullet,1}\tilde J_{\bullet,1,\bullet}}{d_1^{(1)}d_1^{(2)}d_1^{(3)}} + \frac{J_{1,1,\bullet}\tilde J_{\bullet,\bullet,1}}{d_1^{(1)}d_1^{(2)}d_1^{(3)}} + \{J \leftrightarrow \tilde J\}$$

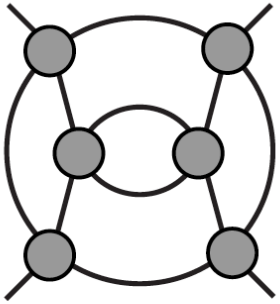
Etc.

- **Applies to *any* adjoint gauge theory, not just $N = 4$ sYM.**
- **Simple generalization for asymmetric double copies.**
- **Same constructions work at tree level! Five-point formula similar to known tree formula.** Bjerrum-Bohr, Damgaard, Søndergaad, Vanhove

Simple Recursive Pattern

Simple recursive pattern for adding four-point trees to cuts:

Starting point: $$-\sum_{i_1,i_p,\ldots,i_q=1}^{3} \frac{J_{i_1,\ldots,i_p,\ldots,i_q}\tilde{J}_{i_1,\ldots,i_p,\ldots,i_q}}{d^{(1)}_{i_1}\cdots d^{(p)}_{i_p}\cdots d^{(q)}_{i_q}}$$



Repeatedly substitute:

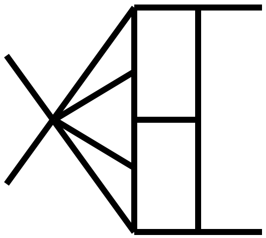
$$\sum_{i_p=1}^{3} \frac{J_{a_1,\ldots,i_p,\ldots,a_q}\tilde{J}_{b_1,\ldots,i_p,\ldots,b_q}}{d^{(p)}_{i_p}} \to -\frac{J_{a_1,\ldots\bullet,\ldots,a_q}\tilde{J}_{b_1,\ldots,1,\ldots,b_q}}{d^{(p)}_{1}} + \{J \leftrightarrow \tilde{J}\}$$

Correction to naïve double copy. Sum over unique terms generated, having at least one modification to starting *J* or $\tilde{J}$.

- **Have similar formula for single 5 point tree.**
- **Promising that there should be simple patterns for more general configurations.**

See arXiv:1708.06807 (today)

5 Loop $N = 8$ supergravity



The generalized double copy enormously simplifies the computation of missing gravity contact terms. The impossible becomes doable!

N^2 – N^3 maximal cuts: use formulas.
N^4 – N^6 maximal cuts: numerical analysis more efficient.

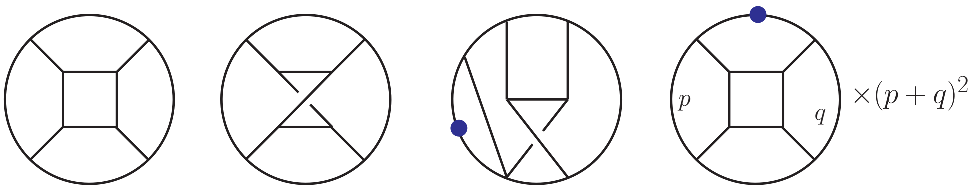
We have the five-loop integrand!

- **Of 76K potential diagrams with contact terms, 60K vanish.**
- **Details depends on arbitrary choices starting from naïve double copy.**
- **Contact-term representation is poor. Terms quartically divergent in $D = 24/5$ instead of log divergent.**
- **Have confirmed expected cancellations of divergences in $D = 22/5$. Tests integrand and integration methods.**

IBP for High Loop Supergravity

Next step is to integrate the expression for large loop momenta.

Constructed representation is poor: need to series expand in four powers of external momenta. Get vacuum diagrams.



+ many thousands

Lorentz symmetry

$$0=\int\Big(\prod_{a=1}^{L} d^D\ell_a\Big)\sum_{a=1}^{L}\Big(\ell_{a\mu}\frac{\partial}{\partial\ell_a^\nu}-\ell_{a\nu}\frac{\partial}{\partial\ell_a^\mu}\Big)\frac{\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

SL(5) relabeling symmetry

$$0=\int\Big(\prod_{a=1}^{L} d^D\ell_a\Big)\sum_{a=1}^{L}\frac{\partial}{\partial\ell_a^\nu}\frac{\omega_{ab}\ell_{b\mu}\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

Should be sufficient for finding enhanced cancellations.

ZB, Enciso, Parra-Martinez, Zeng

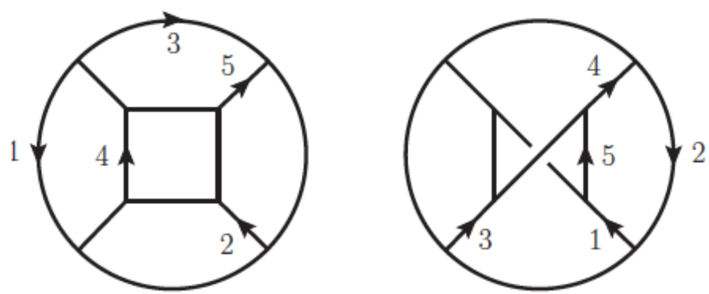
Integrating 5 Loop $N = 8$ supergravity

ZB, John Joseph Carrasco, Wei-Ming Chen, Henrik Johansson, Radu Roiban, Mao Zeng

As a warmup we calculated coefficient of two master integrals in $D = 22/5$.

Expand in large loop momentum.
Use modern unitarity compatible IBP methods.

Gluza, Kadja, Kosower; Kosower and Larsen; Schabinger; Søgaard, Zhang; Ita; Georgoudis and Zhang, etc



(P) (NP)

level	$V^{(P)}$	$V^{(NP)}$
0	$\frac{2439779211}{154000}$	$\frac{2911616507}{7392000}$
2	$\frac{374402283}{308000}$	$\frac{8846490651}{224000}$
3	$\frac{3535277}{800}$	$\frac{791440021}{35200}$
4	$-\frac{18900121}{880}$	$-\frac{1152620531}{18480}$
sum	0	0

$D = 22/5$

Cancellation in $D = 22/5$ checks integrand and our integration methods and efficiency. (Analogous to 5-loop QCD beta function.)

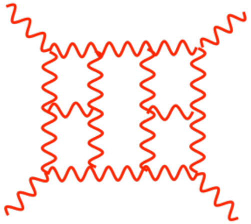
Currently working on much more challenging, but interesting $D = 24/5$ case. (The calculation for the bet.)

Some Remaining Challenges

- ***D*** **= 24/5 loop integrations need to be done before we have answer to bet.**
- **Want complete set of formulas for converting *any* gauge-theory loop amplitude to corresponding gravity ones.**
- **Want better 5-loop sYM starting point to make UV extraction easier.**
- **Systematic and complete understanding of "enhanced cancellations" still needed.**
- **Can we carry over ideas to general classical solutions, without starting from special gauges? Applications to gravitational radiation?**

Goldberger, Ridgway (2016); Luna, Monterio, Nicholson, O'Connell, Ochirov, Westerberg, White (2016)

Summary



1. Duality between color and kinematics.
2. Double copy idea offer nontrivial insight into gravity.
   — Gravity loops from gauge theory.
   — Classical solutions.
3. Even when duality not manifest, new ideas allow us to extract gravity generalized double copy from gauge theory.
4. Applied these ideas to solve 5-loop 4-point integrand.
5. Using modern integration ideas we have efficient loop integration. Stay tuned!

In the coming months we hope to finally determined UV behavior of $N = 8$ supergravity at 5 loops.

We have a powerful new method to construct multiloop gravity amplitudes from gauge theory. Expect to learn a lot more about gravity in the coming years.