High-energy scattering on *Dp*-branes meets unitarity and causality

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Current Themes in High-Energy Physics and Cosmology

Foreword

- This talk is based on the work done together with Giuseppe D'Appollonio, Rodolfo Russo and Gabriele Veneziano and published in the following papers

  - "High-energy string-brane scattering: leading eikonal and beyond", JHEP 1011 (2010) 100, arXiv:1008:4773 [hep-th].
  - "Microscopic unitary description of tidal excitations in high-energy string-brane collisions", arXiv:1310.1254 [hep-th], JHEP **1311** (2013) 126.
  - "Regge behavior saves string theory from causality violations", arXiv:1502.01254 [hep-th], JHEP **1505** (2015) 144.
  - "A microscopic description of absorption in high-energy string-brane collisions" arXiv:1510.03837 [hep-th], JHEP **1603** (2016) 030.
  - Work in progress with R. Russo and A. Koemans Collado.

Plan of the talk

Introduction

- High-energy scattering can be studied for two kinds of processes.
- One involves two light objects.
- The other involves a light object and a heavy classical one at rest of mass M.
- They can be embedded in string theory.
- The light objects are perturbative string states (graviton, dilaton..).
- The heavy objects are a system of $N >> 1$ Dp-branes.
- In both kinds of processes the scattering amplitude diverges at high energy.
- The reason is that the process is dominated by graviton-exchange and the graviton couples to energy.
- An amplitude that diverges at high energy violates unitarity.

- This is the same problem that occurs for the elastic scattering of two longitudinally polarized massive gauge bosons, but the resolution of the problem is different.
- In the SM unitarity is recovered by adding the Higgs field.
- In the case of gravitational interactions unitarity is recovered by summing higher orders obtaining the so-called eikonal.
- In this talk we consider the scattering of a massless closed string against a maximally supersymmetric stack of N Dp-branes.
- It is a cleaner process because the external metric is not produced by the other particle, but it is given by the presence of the Dp-branes.
- This allows to go to a smaller impact parameter that is more complicated to do in string-string scattering.
- In particular, in field theory, the re-summation gives a large c-number phase 2δ and unitarity is recovered in a trivial way without needing additional degrees of freedom.
- The case of string is more interesting.

- Elastic unitarity is violated already for large impact parameter and the full unitarity is restored by including inelastic processes.
- The re-summation gives the eikonal operator that acts in the space of the closed string states.
- This allows us to directly identify the microstates that are necessary to restore unitarity.
- But this is not the full story.
- If we go to an impact parameter of the order of the string length ($b \sim \sqrt{\alpha'}$) we get again violations of unitarity and possibly problems with causality.
- In this talk I will show how the problems with causality, present in field theory, are instead avoided in string theory [Camanho, Edelstein, Maldacena and Zhiboedov, JHEP **1602** (2016) 020].
- I will also discuss how the problems with unitarity present at low impact parameter $b \sim \sqrt{\alpha'}$ can in principle be solved.
- They follow from the fact that the closed string can be absorbed by the brane, decaying in open strings.

- Then, the inclusion of open string degrees of freedom should eliminate the remaining problems with unitarity.
- But, unlike the case of the closed string, where, with a large number N of parallel Dp-branes, we can limit ourselves to just one closed string, in the absorption of a closed string by a Dp-brane an arbitrary number of open strings can be created.
- This problem is very close, in spirit, to the famous information puzzle arising from the formation and evaporation of a black hole from a pure initial state.
- Therefore, studying it, we hope to learn something about black hole physics.
- We will see that, in string theory, curved space-time is not put by hand, as in GR, but emerges from string scattering amplitudes computed in flat Minkowski space-time with suitable boundary conditions.
- A massless spin 2 requires general covariance to be consistent.

High-energy scattering for light states

- In an elastic two-particle scattering there are two high-energy regimes: small angle and large angle scattering.
- In terms of the two Mandelstam variables and the scattering angle:

$$s = -(p_1 + p_2)^2 \quad ; \quad t = -(p_2 + p_3)^2 = -s \sin^2 \frac{\theta}{2} \quad \textit{in the c.m.f.}$$

  with all incoming massless particles, one can identify the two regimes.
- The first one is the high-energy Regge limit:

$$t << s \Longrightarrow \infty \quad ; \quad \theta << 1$$

  that will be considered in the following.
- The second one is the large angle scattering:

$$s \sim |t| \Longrightarrow \infty \quad ; \quad \theta \sim 1$$

  that will not be considered in this talk.

- ACV studied the high energy scattering of two closed strings [Amati, Ciafaloni and Veneziano, 1987....].
- At leading order in the impact parameter b, the effective geometry turns out to be the Aichelburg-Sexl shock-wave metric produced by an energetic massless particle + string corrections.
- In fact, in the field theory limit ($\alpha' \to 0$), the produced phase shift agrees with the one computed in field theory for the scattering of a particle on the Aichelburg-Sexl metric (D non-compact dims):

$$2\delta_D(s,b) = (D-2)\frac{\sqrt{s\pi}R_s}{8}\left(\frac{R_s}{b}\right)^{D-4}\frac{\Gamma(\frac{D-4}{2})}{\Gamma(\frac{D-1}{2})}$$

$$\theta_D = \frac{\sqrt{\pi}\Gamma(\frac{D}{2})}{\Gamma(\frac{D-1}{2})}\left(\frac{R_s}{b}\right)^{D-3} \quad ; \quad R_S^{D-3} = \frac{16\pi G_N\sqrt{s}}{(D-2)\Omega_{D-2}}$$

They reduce for $D=4$ to ['t Hooft, 1987]:

$$2\delta = -\sqrt{s}R_s \log\frac{b}{R_s} \quad ; \quad \theta = \frac{2R_s}{b} \quad ; \quad R_s = 2G_N\sqrt{s}$$

R_s is the Schwarzschild radius.

Classical description of a *Dp*-brane

- A maximally supersymmetric *Dp*-brane is described by the 10-dim classical solution (α, β along the brane; i, j outside the brane):

$$ds^2 = [H(r)]^{2A} \left(\eta_{\alpha\beta} dx^\alpha dx^\beta \right) + [H(r)]^{2B} \left(\delta_{ij} dx^i dx^j \right)$$

$$A = -\frac{7-p}{16} \quad , \quad B = \frac{p+1}{16} \quad ; \quad r^2 \equiv \delta_{ij} x^i x^j$$

$$\mathrm{e}^{-\sqrt{2}\kappa_{10}\phi(x)} = [H(r)]^{\frac{p-3}{4}} \quad , \quad \sqrt{2}\kappa_{10}\mathcal{C}_{01\ldots p}(x) = \left([H(r)]^{-1} - 1 \right)$$

- $H(r)$ is an harmonic function given by

$$H(r) = 1 + \left(\frac{R_p}{r} \right)^{7-p} \; ; \; R_p^{7-p} = \frac{2\kappa_{10} T_p N}{(7-p)\Omega_{8-p}} \; ; \; \Omega_q = \frac{2\pi^{\frac{q+1}{2}}}{\Gamma(\frac{q+1}{2})}$$

R_p corresponds to R_s in particle-particle scattering.
Unlike R_s, R_p is independent of the energy.
N is the number of coincident parallel *Dp*-branes.

- It is a classical solution of the bulk action ($a(p) = \frac{3-p}{2}$):

$$S = \int d^{10}x\sqrt{-g}\left[\frac{1}{2\kappa_{10}^2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2(p+2)!}\mathrm{e}^{a(p)\sqrt{2}\kappa_{10}\phi}\left(F_{p+2}\right)^2\right]$$

and of the DBI brane action that will be called boundary action:

$$S_{DBI} = -NT_p\left[\frac{1}{\kappa_{10}}\int d^{p+1}x\ \mathrm{e}^{-\frac{1}{\sqrt{2}}a(p)\kappa_{10}\phi}\sqrt{-\det g_{\alpha\beta}} + \sqrt{2}\int C_{p+1}\right]$$

$$= \int d^{p+1}x\ NT_p\left[-\eta_{\alpha\beta}h^{\alpha\beta} + \frac{3-p}{2\sqrt{2}}\phi + \sqrt{2}C_{01\ldots p} + \ldots\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa_{10}h_{\mu\nu}$$

They depend on the two quantities: κ_{10} and T_p.

- The information of what kind of D branes we are considering is encoded in the boundary action.
- The previous boundary action corresponds to that of the maximally supersymmetric Dp-branes of type II string theory.

- In string theory everything is determined in terms of α' and g_s

$$\kappa_{10} = \frac{(2\pi)^{7/2}}{\sqrt{2}} g_s (\alpha')^2 \quad ; \quad T_p = \frac{\sqrt{\pi}}{(2\pi\sqrt{\alpha'})^{p-3}}$$

$$R_p^{7-p} = \frac{2\kappa_{10} T_p N}{(7-p)\Omega_{8-p}} = \frac{g_s N (2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}}$$

- *Dp*-branes are non-perturbative BPS states of string theory preserving 16 supersymmetries.
- Their mass per unit of p-volume and their RR charge are given by

$$M_p = \frac{T_p}{\kappa_{10}} N = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} N = \frac{\mu_p}{\sqrt{2}\kappa_{10}} \quad ; \quad \mu_p = \sqrt{2} T_p N$$

- For future use, notice that

$$R_p \sim (g_s N)^{\frac{1}{7-p}} \ell_s \quad ; \quad \ell_s \equiv \sqrt{\alpha' \hbar}$$

The classical deflection angle in brane background

- We want to compute the deflection angle of a massless probe moving in the metric created by a stack of N Dp-branes.
- Consider a general metric of the kind:

$$ds^2 \equiv g_{\mu\nu}(x)dx^\mu dx^\nu = -\alpha(r)dt^2 + \beta(r)(dr^2 + r^2 d\theta^2)$$

  where we have neglected coordinates that are not involved in the geodesic where only t, r and θ vary.
- The deflection angle can be best derived from the action of a massless point-particle in this metric:

$$S = \frac{1}{2}\int \frac{d\tau}{e}\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}(x) = \frac{1}{2}\int \frac{d\tau}{e}\left(-\dot{t}^2\alpha(r) + \beta(r)\left(\dot{r}^2 + r^2\dot{\theta}^2\right)\right)$$

  e is the einbein and S is invariant under arbitrary reparametrizations of the world line coordinate τ.
- The conjugate momenta are given by:

$$p_t \equiv \frac{\partial L}{\partial \dot{t}} = -\frac{\dot{t}\alpha}{e} \;\; ; \;\; p_r \equiv \frac{\partial L}{\partial \dot{r}} = \frac{\beta(r)\dot{r}}{e} \;\; ; \;\; p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{\dot{\theta}r^2\beta(r)}{e}$$

- The Eq. of motion for e gives:

$$\beta(r)\dot{r}^2 + \beta(r)r^2\dot{\theta}^2 = \alpha(r)\dot{t}^2$$

- Since the Lagrangian does not depend explicitly on either t or θ there are two conserved quantities: the energy and the angular momentum

$$E = -\alpha(r)\dot{t} \quad ; \quad J = \beta(r)r^2\dot{\theta}$$

where a dot denotes derivative with respect to τ and we have taken $e = 1$.

- Combining the three previous equations we get

$$\frac{\dot{\theta}}{\dot{r}} \equiv \frac{d\theta}{dr} = \frac{J}{\beta r^2}\frac{1}{\sqrt{\frac{E^2}{\alpha\beta} - \frac{J^2}{\beta^2 r^2}}} = \frac{b}{r^2}\frac{1}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}}$$

where $b \equiv J/E$ is the impact parameter.

- Integrating it, one gets the deflection angle:

$$\Theta_p = 2\int_{r*}^{\infty} \frac{dr}{r^2}\frac{b}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}} - \pi = 2\int_{r*}^{\infty} \frac{dr}{r^2}\frac{b}{\sqrt{1 + \left(\frac{R_p}{r}\right)^{7-p} - \frac{b^2}{r^2}}} - \pi$$

$r*$ is the turning point i.e. the largest root of the equation $1 + \left(\frac{R_p}{r}\right)^{7-p} - \frac{b^2}{r^2} = 0$.

- The result depends only on α/β: invariance under a r-dependent rescaling of the whole metric.
- Therefore, we can work in either the string or the Einstein frame.
- The integral can be done exactly for the cases $p = 5, 6$:

$$\tan\frac{\Theta_6}{2} = \frac{R_6}{2b} \quad ; \quad \Theta_5 = \frac{\pi}{\sqrt{1 - \left(\frac{R_5}{b}\right)^2}} - \pi$$

- No singularity for $p = 6$: Smooth behaviour for $0 \leq b \leq \infty$.
- Θ_5 diverges when $b \sim R_5$: the probe particle is captured.

- Same singular behaviour for $p = 3$ when $b \sim R_3$:

$$\Theta_3 = 2\sqrt{1 + k_3^2}K(k_3) - \pi \;\; ; \;\; k_3 = -1 + \frac{b}{2R_3}\sqrt{\left(\frac{b}{R_3}\right)^2 - 1}$$

$$K(k) = \int_0^1 \frac{dv}{\sqrt{(1 - v^2)(1 - k^2 v^2)}} = \frac{\pi}{2}\sum_{n=0}^{\infty}\left(\frac{(2n)!}{(n!)^2 2^{2n}}\right)^2 k^{2n}$$

  K is the complete elliptic integral of first kind.
- For general p we have not been able to write the deflection angle in a closed form: Still singular behaviour for $b \sim R_p$.
- We have computed the leading and the next to the leading behaviour for large impact parameter:

$$\Theta_p = \sqrt{\pi}\left[\frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})}\left(\frac{R_p}{b}\right)^{7-p} + \frac{1}{2}\frac{\Gamma(\frac{15-2p}{2})}{\Gamma(6-p)}\left(\frac{R_p}{b}\right)^{2(7-p)} + \dots\right]$$

The Shapiro time delay

- Compute the Shapiro time-delay for a probe particle moving in the metric created by a stack of N Dp-branes.
- In order to include also the case of the bosonic string we write the metric keeping an arbitrary space-time dimension D

$$ds^2 = (H(r))^{-\frac{D-p-3}{D-2}} \left(-dt^2 + dx_p^2\right) + (H(r))^{\frac{p+1}{D-2}} dx_{D-1-p}^2$$

- where

$$H(r) = 1 + \left(\frac{R_p}{r}\right)^{D-p-3} \quad , \qquad R_p^{D-p-3} = \frac{2\kappa_D T_p N}{(D-3-p)\Omega_{D-p-2}}$$

- Assume that the probe particle moves along one of the transverse directions $x^{D-1} \equiv z$.
- In general, moving in the metric of the branes, its trajectory will be deflected.
- However, if the impact parameter is much larger than R_p the deflection angle will be very small.

- and, in the first approximation, we can neglect it and assume that the probe moves along z.
- In this case we can also expand the metric for large r and keep only the dominant terms.
- For a massless probe particle moving in the metric of the Dp-branes we get

$$ds^2 = -dt^2 \left(1 + \frac{D-p-3}{D-2}\left(\frac{R_p}{r}\right)^{D-p-3} + \dots\right)$$
$$+dz^2 \left(1 + \frac{p+1}{D-2}\left(\frac{R_p}{r}\right)^{D-p-3} + \dots\right) = 0$$

- It implies

$$\frac{dt}{dz} = 1 + \frac{1}{2}\left(\frac{R_p}{r}\right)^{D-p-3} + \dots$$

where $r^2 = b^2 + z^2$.

- From this expression we can immediately compute the Shapiro time delay

$$\Delta t = \frac{R_p^{D-p-3}}{2} \int_{-\infty}^{+\infty} \frac{dz}{(b^2 + z^2)^{\frac{D-p-3}{2}}} = \frac{R_p^{D-p-3} \sqrt{\pi}\Gamma(\frac{D-p-4}{2})}{2b^{D-p-4}\Gamma(\frac{D-p-3}{2})}$$

that is a positive quantity.

The field theory eikonal

- We present an alternative way of computing the deflection angle and the Shapiro time delay that will be used later in string theory.
- The elastic scattering of a dilaton ϕ on a *Dp*-brane is obtained from:

$$\langle \phi(x)\phi(y)\mathrm{e}^{i(S_{bulk}+S_{boundary})}\rangle$$

- Bringing down one boundary action gives the amplitude with one boundary.
- In general, bringing down n boundary actions one gets the scattering amplitude (with *n* boundaries).
- The two dilatons have respectively momentum p_1 and p_2.
- Along the directions of the world-volume of a *Dp*-brane there is conservation of energy and momentum:

$$(p_1 + p_2)_{||} = 0 \quad ; \qquad p_1^2 = p_2^2 = 0$$

- The scattering is described by two Mandelstam-like variables:

$$t = -(p_{1\perp} + p_{2\perp})^2 = -4E^2 \sin^2 \frac{\Theta}{2} \quad ; \quad s = E^2 = |p_{1\perp}|^2 = |p_{2\perp}|^2$$

$\Theta =$ the angle between the $(9-p)$-dim vectors $p_{1\perp}$ and $-p_{2\perp}$.

- At high energy and low transfer momentum we have the following kinematical configuration:

$$p_1 = (E, \underbrace{0 \ldots 0}; \mathbf{p}_1, (E - \frac{\mathbf{p}_1^2}{2E} + \ldots)) \; ; \; p_1^2 = 0$$

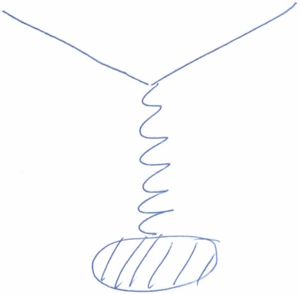
$$p_2 = (-E, \underbrace{0 \ldots 0}; \mathbf{p}_2, -(E - \frac{\mathbf{p}_2^2}{2E} + \ldots)) \; ; \; p_2^2 = 0$$

$$q \equiv p_1 + p_2 = (0, \underbrace{0 \ldots 0}; \mathbf{p}_1 + \mathbf{p}_2, \frac{\mathbf{p}_1^2 - \mathbf{p}_2^2}{2E} + O(\frac{1}{E^2}))$$

$\mathbf{p}_1, \mathbf{p}_2$ and the momentum transfer q are $(8-p)$-dim vectors orthogonal to the 9*th* (very energetic) space direction.

- The T-matrix ($S = 1 + iT = 1 + i\frac{\mathcal{A}}{2E}$) with one boundary is equal to

$$T_1(E, t) \equiv \frac{\mathcal{A}_1(E, t)}{2E} = \frac{\pi^{\frac{9-p}{2}} R_p^{7-p}}{\Gamma(\frac{7-p}{2})} \cdot \frac{2E}{(-t)}$$



- A T-matrix that diverges with energy violates unitarity.
- In our case unitarity is restored by summing over the number of boundaries.
- Going to impact parameter space

$$T(E,b) \equiv \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}}\ \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}}\ T(E,t=-q^2)$$

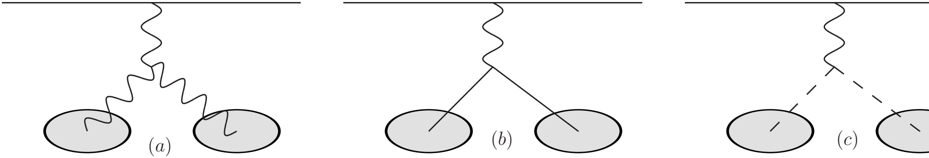
- one gets:

$$iT_1(E,b) = i\left(\frac{R_p^{7-p}\sqrt{\pi}E}{2b^{6-p}} \cdot \frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})}\right)$$

- The T-matrix with two boundaries is equal to

$$iT_2(E,b) \;=\; -\frac{1}{2}(T_1)^2 + i\sqrt{\pi}E\ b\ \left(\frac{R_p}{b}\right)^{2(7-p)} \cdot \frac{\Gamma(\frac{13-2p}{2})}{4\Gamma(6-p)}$$


(a)
(b)


(a)
(b)
(c)

- Up to two boundaries we get the following S matrix:

$$S = 1 + iT_1 - \frac{1}{2}(T_1)^2 + iT_2^{(1)} \sim \mathrm{e}^{iT_1(E,b) + iT_2^{(1)} + \dots}$$

- The terms that diverge with the energy exponentiate in a phase and in this way unitarity is recovered.
- In general

$$T_n(E, b) \sim A_n^{(n)} E^n + A_n^{(n-1)} E^{n-1} + \dots + A_n^{(1)} E + 0(1) + \dots$$

- $A_n^{(n)}(E, b)$ contributes to the leading eikonal, while $A_n^{(n-1)}(E, b)$ to the next to the leading eikonal and so on.
- To restore unitarity all terms divergent with energy must exponentiate.

- Assuming that all terms (up to two boundaries) divergent with the energy exponentiate, we get:

$$S(E,b) \equiv \mathrm{e}^{2i\delta(E,b)} = \mathrm{e}^{i\sqrt{\pi}\,\frac{Eb}{2}\left[\left(\frac{R_p}{b}\right)^{7-p}\cdot\frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})}+\left(\frac{R_p}{b}\right)^{2(7-p)}\cdot\frac{\Gamma(\frac{13-2p}{2})}{2\Gamma(6-p)}+\dots\right]}$$

- The first term is called leading eikonal and the second one next to the leading eikonal.
- In conclusion, for particles the S-matrix is just an infinite phase and it is unitary.
- From the eikonal one can compute the deflection angle.
- Going back to momentum space, we get:

$$\int d^{8-p}b\ \mathrm{e}^{i(-\mathbf{b}\cdot\mathbf{q}+2\delta(E,b))}$$

- For large impact parameter we have the saddle point equation:

$$\mathbf{q} = \frac{\mathbf{b}}{b}\ \frac{\partial(2\delta(E,b))}{\partial b}$$

- From which we compute the deflection angle:

$$\Theta_p = -\frac{\widehat{\mathbf{b}} \cdot \mathbf{q}}{E} = -\frac{1}{E} \; \frac{\partial (2\delta(E,b))}{\partial b}$$
$$= \sqrt{\pi} \left[\left(\frac{R_p}{b} \right)^{7-p} \cdot \frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})} + \frac{1}{2} \left(\frac{R_p}{b} \right)^{2(7-p)} \frac{\Gamma(\frac{13-2p}{2})}{\Gamma(6-p)} + \dots \right]$$

- and the Shapiro time-delay

$$\Delta t = \frac{\partial}{\partial E} 2\delta(E,b) = \frac{R_p^{7-p} \sqrt{\pi}\, \Gamma(\frac{6-p}{2})}{2b^{6-p}\Gamma(\frac{7-p}{2})}$$

- They are both obtained from the phase shift $2\delta(E,b)$.
- They agree with the classical calculation for large impact parameter!!

Comparison with Scharzschild

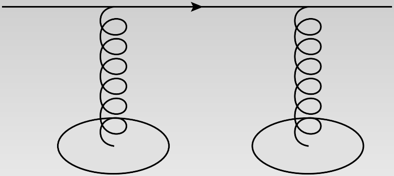
- Recently the leading and subleading eikonals for the scattering of a massless particle in the Schwarzschild metric have been computed: Akhouri et al, arXiv:1308.5204
  Bjerrum-Bohr et al, arXiv:1609.07477
  Luna et al, arXiv:1611.02172
- They find (*M* is the mass of the heavy object)

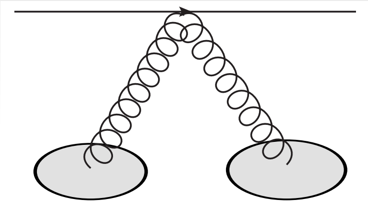
$$\chi(b) = -4G_N^{(4)} ME \log b + (G_N^{(4)} M)^2 E \frac{15\pi}{4b}$$

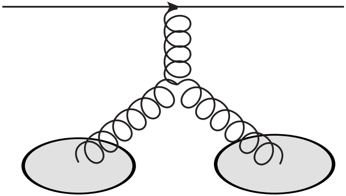
- Then the deflection angle in Einstein gravity is equal to:

$$\Theta_0 = \frac{4G_N^{(4)} M}{b} + \frac{15\pi (G_N^{(4)} M)^2}{4b^2}$$

- In order to compare with them we should eliminate the contribution of the dilaton and RR-field and compute with arbitrary *D*.
- In addition to the tree diagram, we get only the contribution of three of the five previous diagrams.







$$
\begin{aligned}
\chi_{TOT}(b,E) &= \frac{\kappa_D^2\tau_p}{4} E \frac{\Gamma(\frac{D-p-4}{2})}{\pi^{\frac{D-p-2}{2}} b^{D-p-4}} \\
&+ \frac{(\kappa_D^2\tau_p)^2\, E\, \Gamma^2(\frac{D-p-3}{2})\Gamma(D-p-\frac{7}{2})}{16\,\pi^{D-p-\frac{3}{2}}\Gamma(D-p-4)\ b^{2D-2p-7}} \\
&+ \frac{(\kappa_D^2\tau_p)^2\, E\, \Gamma(D-p-\frac{7}{2})\Gamma^2(\frac{D-p-1}{2})}{16(3+p-D)\Gamma(D-p-1)\ \pi^{D-p-\frac{3}{2}}\ b^{2D-2p-7}} \\
&\times \Big[-\frac{8(D-p-2)(D-p-3)}{D-2} \\
&+ \frac{(p+1)(D-p-3)}{D-2} + \frac{4(D-p-2)(D-2p-4)}{D-2} \\
&+ [4(D-p-2)-2] + [2 - \frac{(p+1)(D-p-3)}{D-2}]\Big]
\end{aligned}
$$

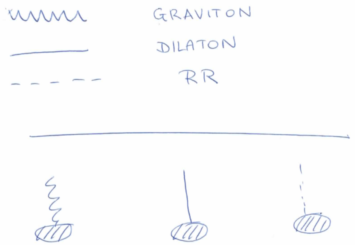
- For $D=4$ and $p=0$ the second line is vanishing and the last line is not present in EH. The rest reduces to the previous expression.
- For arbitrary D and p only the 1st and 2nd lines contribute.

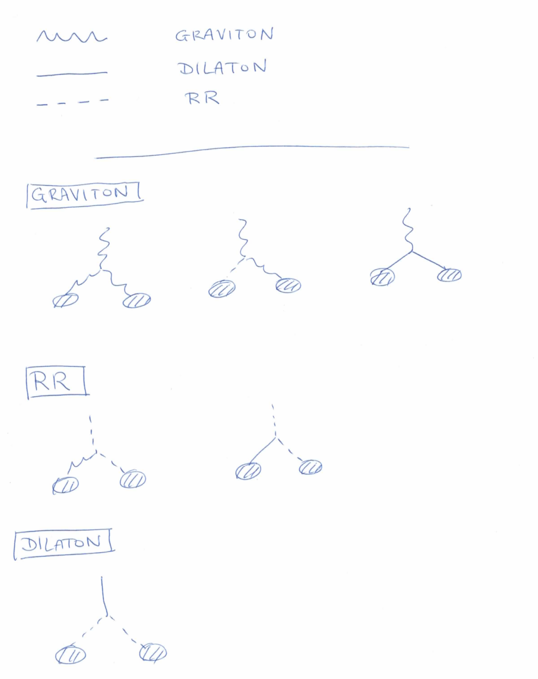
Classical solution from Feynman diagrams

- There is also an alternative (diagrammatical) way of also obtaining the previous classical solution.
- It consists in computing the one-point function for the metric, the dilaton and the RR field respectively, in an action that is the sum of the previous bulk and boundary actions:

$$\langle \Phi(x) \ e^{i(S_{bulk}+S_{boundary})} \rangle$$

- The term with one boundary action gives the leading term for $r \to \infty$ of the classical solution.
- The term with two boundaries gives the next to the leading term [M. Bertolini et al (2000)] and so on.
  For Schwarzschild see [M. Duff (1973)].
- The sum over the boundaries gives the complete classical solution.


GRAVITON
DILATON
RR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GRAVITON
RR
DILATON

- For pure Einstein gravity in D dims the 2nd order comes entirely from the diagram with the 3-graviton vertex and one gets:

$$
\begin{aligned}
\langle g_{\mu\nu}\rangle &\equiv \langle \eta_{\mu\nu} + 2\kappa_D h_{\mu\nu}\rangle \\
&= \left(1 - A\frac{D-p-3}{D-2} + \frac{1}{2}A^2\left(\frac{D-p-3}{D-2}\right)^2 + \dots\right)\eta_{\parallel\mu\nu} \\
&+ \left[1 + A\frac{p+1}{D-2}\right. \\
&\left. + A^2\left(\frac{1}{2}\left(\frac{p+1}{(D-2)}\right)^2 - \frac{(D-p-3)(p+1)}{8(D-p-2)(D-2)}\right) + \dots\right]\eta_{\perp\mu\nu} \\
&+ A^2\left(\frac{(p+1)(D-p-3)}{2(D-2)} - \frac{(D-p-3)^2(p+1)}{8(D-2)(D-p-2)} + \dots\right) \\
&\times \frac{1}{(5+p-D)}\left(\eta_{\perp\lambda\tau} - 2(D-p-3)\frac{r_{\perp\lambda}r_{\perp\tau}}{r^2}\right)
\end{aligned}
$$

where $A \equiv 2\kappa_D^2\tau_p[(D-p-3)\Omega_{D-p-2}r^{D-p-3}]^{-1}$

- No red term in the harmonic gauge

- In the case of Einstein gravity for $D = 4$ and $p = 0$ one gets

$$\langle g_{\mu\nu} \rangle = \left(1 - \frac{2G_N M}{r} + 2\frac{G_N^2 M^2}{r^2} + \dots \right) \eta_{00} + \left(1 + \frac{2MG_N}{r} + \frac{M^2 G_N^2}{r^2} + \dots \right) \eta_{ij} + \left(\frac{M^2 G_N^2}{r^2} + \dots \right) \frac{r_i r_j}{r^2}$$

  in agreement with [Bjerrum-Bohr et al hep-th/0211071].
- The previous solution is generalized to a complete solution as follows:

$$\langle g_{\mu\nu} \rangle = \frac{r - MG_N}{r + MG_N} \eta_{00} + \left(1 + \frac{MG_N}{r} \right)^2 \eta_{ij} + \frac{G_N^2 M^2}{r^2} \left(\frac{r + G_N M}{r - G_N M} \right) \frac{r_i r_j}{r^2}$$

  that is the Schwarzschild metric in the harmonic gauge.

String theory

- In string theory the boundary action becomes the boundary state.
- It is a closed string state that describes a *Dp*-brane and that creates a boundary.
- It is given by:

$$|B\rangle \equiv \frac{T_p}{2} |B_X\rangle |B_\psi\rangle$$

- The bosonic part of the boundary state is equal to

$$|B_X\rangle = \delta^{D-p-1}(\hat{q}^i - y^i) \left(\prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n} S \cdot \widetilde{\alpha}_{-n}} \right) |0\rangle_\alpha |0\rangle_{\widetilde{\alpha}} |p = 0\rangle$$

$S \equiv (\eta_{\alpha\beta}; -\delta_{ij})$

- Using the boundary state and the vertex operators for open and closed strings one can compute any amplitude involving scattering of strings on *Dp*-branes.

- The disk (one boundary) amplitude for the elastic scattering of a massless state on the *Dp*-brane is given by:

$$\mathcal{A}_1(E,t) \sim \langle 0| \int \frac{d^2 z_1 d^2 z_2}{dV_{abc}} W_1(z_1,\bar{z}_1) W_2(z_2,\bar{z}_2)|B\rangle$$

$$= -\frac{\pi^{\frac{9-p}{2}} R_p^{7-p}}{\Gamma(\frac{7-p}{2})} \mathcal{K}(p_1,\epsilon_1;p_2,\epsilon_2) \frac{\Gamma(-\alpha' E^2)\Gamma(-\frac{\alpha'}{4}t)}{\Gamma(1-\alpha' E^2 - \frac{\alpha'}{4}t)}$$

[Ademollo et al, 1974, Klebanov and Thorlacius, 1995; Klebanov and Hashimoto, 1996, Garousi and Myers, 1996]

- At high energy

$$\mathcal{K}(p_1,\epsilon_1;p_2,\epsilon_2) = (\alpha' E^2)^2 Tr(\epsilon_1 \epsilon_2^t)$$

- The poles in the *t*-channel correspond to exchanges of closed strings, while those in the *s*-channel correspond to exchanges of open strings:

$$2 + \frac{\alpha'}{2} t = 2m \;\; ; \;\; m = 1,2,\ldots \;\; ; \;\; 1 + \alpha' E^2 = n \;\; ; \;\; n = 1,2\ldots$$

- Regge behaviour at high energy $(\alpha' s >> \alpha' t \sim 0)$ $(s \equiv E^2)$:

$$T_1(E,t) \equiv \frac{\mathcal{A}_1(E,t)}{2E} = \frac{R_p^{7-p}\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} \frac{\pi e^{-i\pi\frac{\alpha'}{4}t}(\alpha' s)^{1+\frac{\alpha'}{4}t}}{2E \ \sin(\pi\frac{\alpha'}{4}(-t)) \ \Gamma(1+\frac{\alpha' t}{4})}$$

- Unlike field theory, there is also an imaginary part.
- The real part describes the scattering of the closed string on the *Dp*-brane, while the imaginary part describes the absorption of the closed string by the *Dp*-brane (creating an open string).
- When $\alpha' \to 0$ the real part reduces to the field theoretical result (graviton exchange).
- For $\alpha' \neq 0$ we have the graviton exchange dressed with string corrections.
- Assuming that also the imaginary part exponentiates, we get the absorption amplitude:

$$S^{abs}(E,b) = e^{-\frac{\pi}{2\Gamma\left(\frac{7-p}{2}\right)}\sqrt{\frac{\pi\alpha' s}{\ln\alpha' s}}\left(\frac{R_p}{\ell_s(s)}\right)^{7-p} e^{-\frac{b^2}{\ell_s^2(s)}}} \ ; \ \ell_s(s) = \ell_s\sqrt{\ln\alpha' s}$$

that is a purely stringy effect, negligible for $b >> \ell_s(s)$.

- Also in this case the amplitude diverges at high energy.
- The high energy behaviour ($E \to \infty$) of the annulus (two-boundary) diagram has also been studied, by the saddle point technique.
- One gets the leading term for $E \to \infty$:

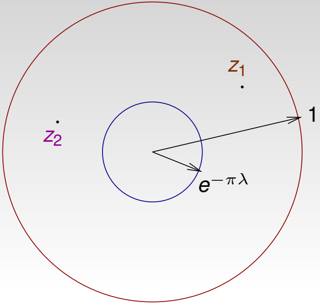
$$\frac{\mathcal{A}_2^{(3)}(E,t)}{2E} \to \frac{i}{2} \prod_{i=1}^{2} \left[\int \frac{d^{8-p}\mathbf{k_i}}{(2\pi)^{8-p}} \frac{\mathcal{A}_1(E,t_i)}{2E} \right]$$

$$\times \delta^{(8-p)}(\sum_{i=1}^{2} k_i - q) \;\; V_2(t_1,t_2,t) \quad ; \quad t_i \equiv -\mathbf{k}_i^2 \quad ; \quad t = -\mathbf{q}^2$$

where

$$V_2(t_1,t_2,t) = \frac{\Gamma(1+\frac{\alpha'}{2}(t_1+t_2-t))}{\Gamma^2(1+\frac{\alpha'}{4}(t_1+t_2-t))}$$

- Also the next to the leading term can be extracted from the annulus.
- Both the leading and the next to the leading terms reduce to the ones already computed In the field theory when $\alpha' \to 0$.



$$z_1 = e^{2\pi(-\lambda\rho_1 + i\omega_1)}$$

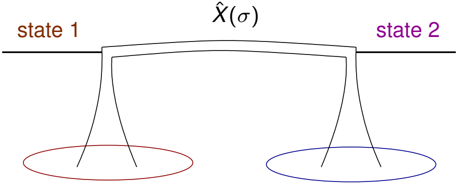
$$z_2 = e^{2\pi(-\lambda\rho_2 + i\omega_2)}$$

with

$$0 \leq \omega_1, \omega_2 \leq 1$$

$$0 \leq \rho_1, \rho_2 \leq \frac{1}{2}$$

World-sheet picture



Space-time picture

The leading eikonal operator

- In order to find the complete leading eikonal operator we go back to V_2 and write it in a more suggestive way, in terms of a set of $(8-p)$-dim bosonic transverse oscillators:

$$V_2(t_1, t_2, t) = \langle 0| \prod_{i=1}^{2} \left[\int_0^{2\pi} \frac{d\sigma_i}{2\pi} : \mathrm{e}^{i\mathbf{k}_i \cdot X(\sigma_i)} : \right] |0\rangle$$

  where

$$\hat{X}(\sigma) = i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{\alpha_n}{n} e^{in\sigma} + \frac{\tilde{\alpha}_n}{n} e^{-in\sigma} \right)$$

  without the zero mode part.

- The two vacuum states correspond to the two external massless states (states with no bosonic excitations: $(\epsilon_{\mu\nu} \psi^{\mu}_{-\frac{1}{2}} \tilde{\psi}^{\nu}_{-\frac{1}{2}} |0\rangle)$.

- Then the leading order from the annulus can be written as follows:

$$\frac{\mathcal{A}_2^{(3)}(E,t)}{2E} \to \frac{i}{2}\prod_{i=1}^{2}\left[\int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(E,-\mathbf{k}_i^2)}{2E}\right]\delta^{(8-p)}(\sum_{i=1}^{2}\mathbf{k}_i - \mathbf{q})$$
$$\times\langle 0|\prod_{i=1}^{2}\left[\int_0^{2\pi}\frac{d\sigma_i}{2\pi} : \mathrm{e}^{i\mathbf{k}_i\cdot X(\sigma_i)} :\right]|0\rangle$$

  where the two vertex operators correspond to the two leading Reggeons exchanged in the two t-channels: t_1 and t_2.
- It can naturally be generalized to the leading term coming from a surface with h boundaries:

$$\frac{\mathcal{A}_h^{(h+1)}(s,t)}{2E} \sim \frac{i^{h-1}}{h!}\prod_{i=1}^{h}\left[\int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(s,-\mathbf{k}_i^2)}{2E}\right]$$
$$\times\delta^{(8-p)}(\sum_{i=1}^{h}\mathbf{k}_i - \mathbf{q})\ \langle 0|\prod_{i=1}^{h}\left[\int_0^{2\pi}\frac{d\sigma_i}{2\pi} : \mathrm{e}^{i\mathbf{k}_i\cdot X(\sigma_i)} :\right]|0\rangle$$

- Going to impact parameter space

$$i\frac{\mathcal{A}_h^{(h+1)}(s,\mathbf{b})}{2E} = \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}}\, e^{i\mathbf{bq}}\, i\frac{\mathcal{A}_h^{(h+1)}(s,t)}{2E}$$

$$= \frac{i^h}{h!}\prod_{i=1}^{h}\left[\int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(s,-\mathbf{k}_i^2)}{2E}\right]$$

$$\langle 0|\prod_{i=1}^{h}\left[\int_0^{2\pi}\frac{d\sigma_i}{2\pi} : \mathrm{e}^{i\mathbf{k}_i(\mathbf{b}+\hat{X}(\sigma_i))} :\right]|0\rangle$$

- Summing all contributions:

$$\sum_{h=1}^{\infty}\frac{\mathcal{A}_h^{(h+1)}(s,\mathbf{b})}{2E} \sim \langle 0|\frac{1}{i}\left[e^{2i\hat{\delta}(s,b)} - 1\right]|0\rangle$$

we get the leading eikonal operator

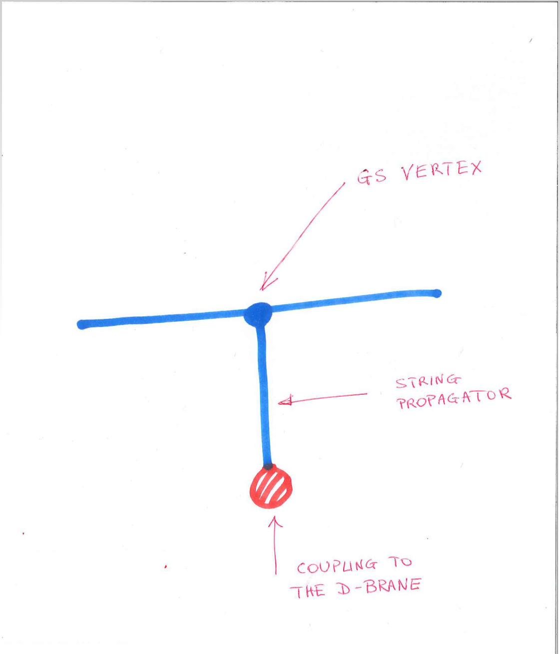
$$2\hat{\delta}(s,b) = \int_0^{2\pi}\frac{d\sigma}{2\pi}\int\frac{d^{8-p}\mathbf{k}}{(2\pi)^{8-p}}\frac{\mathcal{A}_1(s,-\mathbf{k}^2)}{2E} : e^{i\mathbf{k}(\mathbf{b}+\hat{\mathbf{X}}(\sigma))} :$$

- The final result (that includes all string corrections) is obtained from the field theoretical one with the substitution:

$$\mathbf{b} \Longrightarrow \mathbf{b} + \hat{\mathbf{X}} \;\; ; \;\; \hat{X}(\sigma) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \left(\frac{\alpha_n}{n} e^{in\sigma} + \frac{\tilde{\alpha}_n}{n} e^{-in\sigma} \right)$$

  and normal ordering.
- We have constructed the leading eikonal operator that, when saturated with the vacuum of bosonic oscillators, reproduces the leading term of the elastic scattering amplitude of massless states at high energy.
- Are the bosonic oscillators (here introduced to rewrite the elastic scattering amplitude at high energy) related to the string bosonic oscillators?
- If yes, why don't we have also the string fermionic oscillators?
- Why do we have only $8 - p$ bosonic oscillators?

- The eikonal operator has now been derived from string theory starting from the light-cone Green-Schwarz three-string vertex.
- It is an object that contains three infinite sets of harmonic oscillators, each describing one of the 3 states.
- It has the important property that, when it is saturated with three arbitrary physical states in the light-cone gauge, it provides the 3-point coupling among them.
- At high energy its structure simplifies.
- Inserting in one of the three legs a string propagator and closing it with the boundary state, one gets an object that contains only two infinite sets of harmonic oscillators.
- This object turns out to be the eikonal operator!

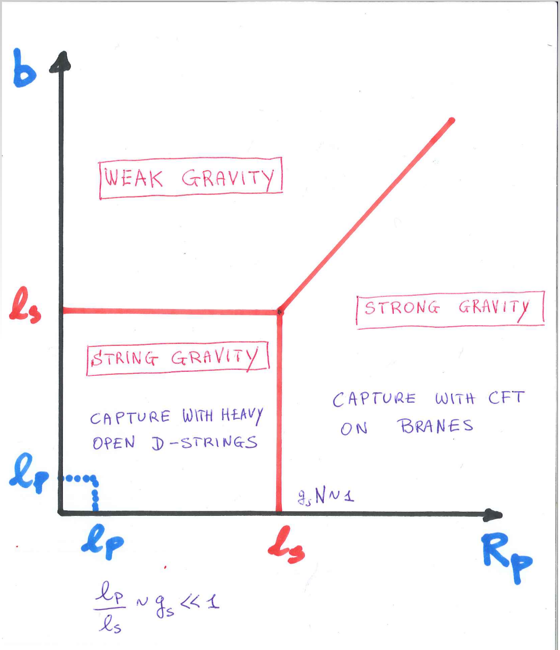

GS VERTEX
STRING PROPAGATOR
COUPLING TO THE D-BRANE

Parameter space at high energy

- We need to use string perturbation theory: $g_s << 1$.
- This means that $\frac{\ell_P}{\ell_s} \sim g_s << 1$.
- Closed string loops and production of many closed strings are negligible if the number of Dp-branes $N >> 1$.
- Three relevant length scales (neglecting ℓ_P):

$$b \quad ; \quad R_p \sim (g_s N)^{\frac{1}{7-p}} \ell_s \quad ; \quad \ell_s = \sqrt{\alpha' \hbar}$$

- Playing with N and g_s, we can make $\frac{R_p}{\ell_s}$ arbitrary.


b
WEAK GRAVITY
l_s
STRONG GRAVITY
STRING GRAVITY
CAPTURE WITH CFT
ON BRANES
CAPTURE WITH HEAVY
OPEN D-STRINGS
l_P
$g_s N \sim 1$
l_P
l_s
R_P
$\frac{l_P}{l_s} \sim g_s \ll 1$

Physical consequences of the eikonal operator

- The leading eikonal operator can be expanded in string corrections:

$$2\hat{\delta} = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n T_1(E,b)}{\partial b_{i_1} \partial b_{i_2} \ldots \partial b_{i_n}} : \overline{X^{i_1} X^{i_2} \ldots X^{i_n}} :$$

where

$$T_1 = \frac{\sqrt{\pi} E}{2} \frac{\Gamma(\frac{6-p}{2})}{\Gamma(\frac{7-p}{2})} R_p^{7-p} (b^2)^{\frac{p-6}{2}}$$

and

$$: \overline{X^{i_1} X^{i_2} \ldots X^{i_n}} :\equiv \int_0^{2\pi} \frac{d\sigma}{2\pi} : X^{i_1}(\sigma) X^{i_2}(\sigma) \ldots X^{i_n}(\sigma) :$$

$$\hat{X}(\sigma) = i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \left(\frac{\alpha_n}{n} e^{in\sigma} + \frac{\tilde{\alpha}_n}{n} e^{-in\sigma} \right)$$

- The first string correction is given by:

$$2\,\hat{\delta}(s,\mathbf{b}+\hat{\mathbf{X}}) \sim \frac{1}{2E}\left[\mathcal{A}_1(s,b) + \frac{1}{2}\frac{\partial^2\mathcal{A}_1(s,b)}{\partial b^i \partial b^j}\,\overline{\hat{X}^i\hat{X}^j} + \dots\right]$$

- with

$$\frac{1}{4\sqrt{s}}\,\frac{\partial^2\mathcal{A}_1(s,b)}{\partial b_i \partial b_j} = Q_\perp(s,b)\,\left[\delta_{ij} - \frac{b_i b_j}{b^2}\right] + \,Q_\parallel(s,b)\,\frac{b_i b_j}{b^2}$$

where

$$Q_\perp(s,b) = \frac{1}{4\sqrt{s}}\,\frac{1}{b}\frac{d\mathcal{A}_1(s,b)}{db} \;\; ; \;\; Q_\parallel(s,b) = \frac{1}{4\sqrt{s}}\,\frac{d^2\mathcal{A}_1(s,b)}{db^2}$$

- They are given by:

$$Q_\perp(s,b) \equiv -\frac{E}{2b}\Theta_p = -\frac{\sqrt{\pi}}{2}\sqrt{s}\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\frac{R^{7-p}}{b^{8-p}}$$

$$Q_\parallel(s,b) = -(7-p)\;Q_\perp(s,b)$$

- The phase shift $\delta(s,b)$ gets an imaginary part

$$\left|\langle 0|e^{2i\hat{\delta}(s,\mathbf{b})}|0\rangle\right| \sim e^{-\frac{1}{2\sqrt{s}}\mathrm{Im}\mathcal{A}_1(s,b)}\left(2\pi\alpha'|Q_\perp(s,b)|\right)^{\frac{8-p}{2}}\sqrt{7-p}$$
$$\times e^{-\pi\alpha'(7-p)|Q_\perp(s,b)|}$$

where $Q_\perp(s,b) = -\frac{\sqrt{\pi}}{2}\sqrt{s}\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\frac{R^{7-p}}{b^{8-p}}$ and

$$\mathcal{A}_1(s,b) \sim s\sqrt{\pi}\,\frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\frac{R_p^{7-p}}{b^{6-p}}$$
$$+\frac{i\pi\sqrt{s}}{\Gamma\left(\frac{7-p}{2}\right)}\sqrt{\frac{\pi\alpha' s}{\ln(\alpha' s)}}\left(\frac{R_p}{\ell_s(s)}\right)^{7-p} e^{-\frac{b^2}{\ell_s^2(s)}}$$

$\ell_s(s) \equiv \sqrt{\alpha'}\sqrt{\ln(\alpha' s)}$ is the effective string length.

- We can write it as follows:

$$\left|\langle 0|e^{2i\hat{\delta}(s,\mathbf{b})}|0\rangle\right| \sim \mathrm{e}^{-\left(\frac{b}{b_D}\right)^{8-p}+\frac{8-p}{2}\log\left(\frac{2}{7-p}\left(\frac{b}{b_D}\right)^{8-p}\right)+c\sqrt{\frac{\pi\alpha' s}{\ln\alpha' s}}\left(\frac{R_p}{l_s(s)}\right)^{7-p}e^{-\frac{b^2}{l_s^2(s)}}}$$

where

$$b_D^{8-p} = \frac{\pi}{2}\alpha'\sqrt{\pi s}(7-p)\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}R_p^{7-p}$$

- We get three terms that violate elastic unitarity.
- The first two are due to the absorption of the elastic channel due to string excitations that become non negligible for $b \leq b_D$.
- The second one comes from the imaginary part of the disk amplitude: it is a pure string effect due to the creation of an open string.
- At large distances the first effect is more important than the inelastic absorption of the disc amplitude and becomes relevant already at impact parameters large compared with both l_s and R_p.

- For very large impact parameter $b >> \sqrt{\alpha'}, R_p, b_D$ elastic unitarity is satisfied and the scattering amplitude at high energy is as in the case of a point-particle with small string corrections.
- When we go to an impact parameter $b \sim b_D$, then elastic unitarity is not satisfied anymore.
- The string gets excited by the tidal forces and from the scattering of massless states on a Dp-brane one starts to produce massive closed string states.
- Remember that, if N is large with $g_s N$ fixed, the production of more than one closed string is depressed.

- The Newton potential generated by the Dp-branes is given by:

$$U_{p;D=10} = -\frac{7-p}{8}\left(\frac{R_p}{r}\right)^{7-p} \Longrightarrow U_{0;D=4} = -\frac{G_N M_0}{r}$$

- If we have a straight rigid string of length $\Delta r \sim \sqrt{\alpha'}$, then the potential acting on the two end-points is different:

$$U_p = -\frac{7-p}{8}\left(\frac{R_p}{r \pm \Delta r}\right)^{7-p}$$
$$\sim -\frac{7-p}{8}\left(\frac{R_p}{r}\right)^{7-p} \mp \frac{(7-p)^2}{8}\frac{R_p^{7-p}}{r^{8-p}}\Delta r + \dots$$

- The end-points of the string are pulled apart by the tidal forces.
- They start to be relevant when they become of the same order of the tension of a string of length $\sqrt{\alpha'}$:

$$\frac{(7-p)^2}{8}\frac{R_p^{7-p}}{r^{8-p}}\sqrt{\alpha'}E = \frac{\sqrt{\alpha'}}{2\pi\alpha'} \Longrightarrow r_D^{8-p} \sim \frac{\pi(7-p)^2}{4} R_p^{7-p}\alpha' E$$

that, apart from a numerical factor, reproduces b_D.

Regge behavior saves ST from causality violations

- CEMZ pointed out that the extensions of EH action with R^2 and/or R^3 terms suffer from causality violations (negative Shapiro time-delay).
- They conclude that this is avoided only in theories with an infinite number of such terms as in string theory.
- Let us consider the bosonic string where such terms appear and let us show how the Regge behavior, present in string theory, solves the problem of causality violations.
- The scattering of a tachyon on a Dp-brane is given by:

$$\mathcal{A}_1^{TT} = \frac{\kappa_{26} T_p N}{2} \frac{\Gamma(-1 - \alpha' s)\Gamma(-1 - \frac{\alpha'}{4} t)}{\Gamma(-2 - \alpha' s - \frac{\alpha'}{4} t)}$$

- In the Regge limit we get

$$\mathcal{A}_1^{TT} \sim \frac{\kappa_{26} T_p N}{2} \mathrm{e}^{-\mathrm{i}\pi \frac{\alpha' t}{4}} (\alpha' s)^{1 + \frac{\alpha' t}{4}} \frac{\Gamma\left(-\frac{\alpha' t}{4}\right)}{1 + \frac{\alpha' t}{4}} \equiv A_1(s, q)$$

- Similarly the tree-level scattering of a massless state, in the Regge limit, is given by

$$\mathcal{A}_1^{\mathcal{G}\mathcal{G}} \sim \frac{\kappa_{26} T_p N}{2} \mathrm{e}^{-\mathrm{i}\pi \frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}} \frac{\Gamma\left(-\frac{\alpha' t}{4}\right)}{1+\frac{\alpha' t}{4}} \to \color{red}{A_1(s,q)}$$
$$\times \left((\epsilon_1\epsilon_2) - \frac{\alpha'}{2}(\epsilon_1 q)(\epsilon_2 q)\right)\left((\bar{\epsilon}_1\bar{\epsilon}_2) - \frac{\alpha'}{2}(\bar{\epsilon}_1 q)(\bar{\epsilon}_2 q)\right)$$

- This amplitude can be compared with the one for the type II superstring theories:

$$\mathcal{A}_1^{\mathrm{II}} \sim (\epsilon_1\epsilon_2)(\bar{\epsilon}_1\bar{\epsilon}_2)\frac{\kappa_{10} T_p^{\mathrm{II}} N}{2}\Gamma\left(-\frac{\alpha' t}{4}\right) \mathrm{e}^{-\mathrm{i}\pi\frac{\alpha' t}{4}}(\alpha' s)^{1+\frac{\alpha' t}{4}}$$
$$\equiv (\epsilon_1\epsilon_2)(\bar{\epsilon}_1\bar{\epsilon}_2) A_1^{\mathrm{II}}(s,q)$$

- Two main differences between the two.
- The leading Regge trajectory in the bosonic string includes the tachyon.

- The bosonic string has a non-trivial dependence on the polarization tensors.
- This is a direct consequence of the modification of the three-graviton vertex in the bosonic string due a quadratic Gauss-Bonnet term and a cubic term (Riemann)^3.
- These corrections are absent in the maximally supersymmetric case.
- In superstring we have constructed the eikonal operator:

$$2\hat{\delta}^{\mathrm{II}}(s, \mathbf{b}) = \int_0^{2\pi} \frac{d\sigma}{2\pi} \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}} \frac{A_1^{\mathrm{II}}(s, \mathbf{q})}{2E} : \mathrm{e}^{\mathrm{i}\mathbf{q}(\mathbf{b}+\hat{X}(\sigma))} :$$

- A similar eikonal operator $2\hat{\delta}(s, \mathbf{b})$ can be introduced in the bosonic case, simply by changing the critical dimension to $d = 26$ and using $A_1(s, \mathbf{q})$:

$$2\hat{\delta}(s, \mathbf{b}) = \int_0^{2\pi} \frac{d\sigma}{2\pi} \int \frac{d^{24-p}\mathbf{q}}{(2\pi)^{24-p}} \frac{A_1(s, \mathbf{q})}{2E} : \mathrm{e}^{\mathrm{i}\mathbf{q}(\mathbf{b}+\hat{X}(\sigma))} :$$

- The eikonal operator reproduces the high-energy behavior of a tachyon:

$$\langle 0|2\hat{\delta}(s,\mathbf{b})|0\rangle = \int \frac{d^{24-p}\mathbf{q}}{(2\pi)^{24-p}} \frac{A_1(s,\mathbf{q})}{2E} \mathrm{e}^{i\mathbf{qb}}$$

- and also that of the massless state, represented by $|\epsilon,\bar{\epsilon}\rangle = \epsilon_I \bar{\epsilon}_J a_{-1}^I \tilde{a}_{-1}^J |0\rangle$:

$$\langle \epsilon_1,\bar{\epsilon}_1|2\hat{\delta}(s,\mathbf{b})|\epsilon_2,\bar{\epsilon}_2\rangle$$
$$= \int \frac{d^{24-p}\mathbf{q}}{(2\pi)^{24-p}} \frac{A_1(s,-\mathbf{q}^2)}{2E} \mathrm{e}^{i\mathbf{qb}} \int_0^{2\pi} \frac{d\sigma}{2\pi} \langle \epsilon_1,\bar{\epsilon}_1| : \mathrm{e}^{\mathrm{i}\mathbf{q}\hat{X}(\sigma)} : |\epsilon_2,\bar{\epsilon}_2\rangle$$

- where the last matrix element is equal to

$$\int_0^{2\pi} \frac{d\sigma}{2\pi} \langle \epsilon_1,\bar{\epsilon}_1| \left(1 - \frac{1}{2} : (\mathbf{q}\hat{X}(\sigma))^2 : + \frac{1}{24} : (\mathbf{q}\hat{X}(\sigma))^4 :\right) |\epsilon_2,\bar{\epsilon}_2\rangle =$$
$$= \left((\epsilon_1\epsilon_2) - \frac{\alpha'}{2}(\mathbf{q}\epsilon_1)(\mathbf{q}\epsilon_2)\right)\left((\bar{\epsilon}_1\bar{\epsilon}_2) - \frac{\alpha'}{2}(\mathbf{q}\bar{\epsilon}_1)(\mathbf{q}\bar{\epsilon}_2)\right)$$

- As in the case of superstring, one can show that the eikonal operator exponentiates:

$$S = \mathrm{e}^{2\mathrm{i}\hat{\delta}(s,\mathbf{b})}$$

- The amplitude corresponding to massless states has the dependence on α' both in $A_1(s, -\mathbf{q}^2)$ (giving Regge behavior) and in the higher gravitational terms.
- Let us perform the field theory limit by taking $\alpha' \to 0$ in $A_1(s, -\mathbf{q}^2)$ keeping the dependence on α' in the higher gravitational terms.
- We get

$$2\delta^{\mathrm{ft}}_{IH,JK}(E, \mathbf{q}) = \kappa_{26} T_p N \frac{E}{\mathbf{q}^2} \left(\delta_{IJ} - \frac{\alpha'}{2} \mathbf{q}_I \mathbf{q}_J \right) \left(\delta_{HK} - \frac{\alpha'}{2} \mathbf{q}_H \mathbf{q}_K \right) ,$$

  that describes the transition from the state $a^I_{-1} \tilde{a}^H_{-1} |0\rangle$ to the state $a^J_{-1} \tilde{a}^K_{-1} |0\rangle$.
- For some of the polarizations we get negative Shapiro time-delay !

- Let us now show that, if we instead keep the dependence on α' in $A_1(s, -\mathbf{q}^2)$, we do not run into the field theory problems.
- Let us start with the superstring ($\bar{Y} = Y - i\pi \equiv \log(\alpha' s) - i\pi$)

$$\frac{\mathcal{A}_1(s, -\mathbf{q}^2)}{2E} = \frac{1}{2} N T_p \kappa_{10} \Gamma\left(-\frac{\alpha' t}{4}\right) \frac{(\alpha' s)}{2E} e^{\frac{\alpha' t}{4}\bar{Y}} ,$$

- In impact parameter space we find:

$$\frac{\mathcal{A}_1(s, \mathbf{b})}{2E} \equiv \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}} \frac{\mathcal{A}_1(s, t)}{2E} e^{i\mathbf{q}\mathbf{b}} = \frac{1}{2} N T_p \kappa_{10} \Gamma\left(1 + \frac{\alpha'}{4}\nabla^2\right)$$
$$\times \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}} \frac{4s}{2E\mathbf{q}^2} \exp\left(-\frac{\alpha'}{4}\mathbf{q}^2 \bar{Y} + i\mathbf{q}\mathbf{b}\right) , \quad t = -\mathbf{q}^2 = \frac{\partial^2}{\partial \mathbf{b}^i \partial \mathbf{b}^i}$$

- We can compute the integral in the second line as follows

$$\frac{4s}{2E}\int_0^\infty dT \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}} \mathrm{e}^{-\mathbf{q}^2\left(T+\frac{\alpha'}{4}\bar{Y}\right)+i\mathbf{q}\mathbf{b}}$$

$$= \frac{4s}{(4\pi)^{\frac{8-p}{2}}} \int_0^\infty dT \left(T+\frac{\alpha'}{4}\bar{Y}\right)^{\frac{p-8}{2}} \mathrm{e}^{-\frac{\mathbf{b}^2}{4\left(T+\frac{\alpha'}{4}\bar{Y}\right)}}$$

$$= \frac{2E}{(4\pi)^{\frac{8-p}{2}}} \int_{\frac{\alpha'}{4}\bar{Y}}^\infty d\hat{T}\, \hat{T}^{\frac{p-8}{2}} \mathrm{e}^{-\frac{\mathbf{b}^2}{4\hat{T}}}$$

$$= \frac{2E}{(4\pi)^{\frac{8-p}{2}}} \left(\frac{\mathbf{b}^2}{4}\right)^{\frac{p}{2}-3} \int_0^{\frac{\mathbf{b}^2}{\alpha'\bar{Y}}} dt\ t^{2-\frac{p}{2}} \mathrm{e}^{-t}$$

$$= \frac{2E}{(4\pi)^{\frac{8-p}{2}}} \left(\frac{\mathbf{b}^2}{4}\right)^{\frac{p}{2}-3} \gamma\left(3-\frac{p}{2}; \frac{b^2}{\alpha' \log(\alpha' s)}\right) \qquad (1)$$

- The last integral gives an incomplete gamma-function

$$\gamma(s;x) = \int_0^x dt t^{s-1} \mathrm{e}^{-t} = \sum_{k=0}^\infty \frac{x^{s+k}\mathrm{e}^{-x}}{s(s+1)\dots(s+k)} \Longrightarrow \frac{x^s}{s}\mathrm{e}^{-x} + \dots$$

- At high energy and small impact parameter $b \ll \sqrt{\alpha'} Y$

$$\frac{\mathcal{A}_1(s, \mathbf{b})}{2E} \sim \frac{1}{2} N T_p \kappa_{10} \, \Gamma \left(1 + \frac{\alpha'}{4}\nabla^2\right) \frac{2E}{(4\pi)^{\frac{8-p}{2}}}$$

$$\times \left(\frac{4}{\alpha' \log(\alpha' s)}\right)^{3-\frac{p}{2}} \frac{1}{3-\frac{p}{2}} \mathrm{e}^{-\frac{b^2}{\alpha' \log(\alpha' s)}} \left(1 + \mathcal{O}\left(\frac{\mathbf{b}^2}{\alpha' \log(\alpha' s)}\right)\right)$$

where we have neglected the imaginary part not relevant here (relevant only for absorption).

- In the bosonic string proceeding exactly as before and neglecting for the moment the polarization dependent prefactor, we obtain instead

$$\frac{\mathcal{A}_1(s, \mathbf{b})}{2E} = \frac{1}{2} N T_p \kappa_{26} \frac{\Gamma\left(1 + \frac{\alpha'}{4}\nabla^2\right)}{1 - \frac{\alpha'}{4}\nabla^2} \frac{2E}{(4\pi)^{\frac{24-p}{2}}} \left(\frac{\mathbf{b}^2}{4}\right)^{\frac{p}{2}-11}$$

$$\times \gamma \left(11 - \frac{p}{2}; \frac{\mathbf{b}^2}{\alpha' \log(\alpha' s)}\right) \quad ,$$

- which, for $\mathbf{b} \ll \sqrt{\alpha'} \log(\alpha' s)$ and at high energy, becomes

$$\frac{\mathcal{A}_1(s, \mathbf{b})}{2E} \sim \frac{1}{2} N T_p \kappa_{26} \frac{\Gamma\left(1 + \frac{\alpha'}{4}\nabla^2\right)}{1 - \frac{\alpha'}{4}\nabla^2}$$
$$\times \frac{2E}{(4\pi)^{\frac{24-p}{2}}} \left(\frac{4}{\alpha' \log(\alpha' s)}\right)^{11-\frac{p}{2}} \frac{1}{11 - \frac{p}{2}}$$
$$\times \mathrm{e}^{-\frac{b^2}{\alpha' \log(\alpha' s)}} \left(1 + \mathcal{O}\left(\frac{\mathbf{b}^2}{\alpha' \log(\alpha' s)}\right)\right) .$$

- If we further notice that the operator $\alpha'\nabla^2$ acts on a function of $x \equiv \frac{\mathbf{b}^2}{\alpha' \log(\alpha' s)}$, we see that, effectively, $\alpha'\nabla^2 \sim (\log(\alpha' s))^{-1} \partial_x^2$.
- At high-energy we can approximate the differential operators with the identity operator.
- In particular, for the bosonic string even the tachyonic pole at $\nabla^2 = \frac{4}{\alpha'}$ becomes harmless.
- This is essentially due to the fact that tachyon exchange is suppressed by two powers of the energy with respect to graviton exchange and therefore it is negligible in the high-energy limit.

- Then we simply obtain (elastic scattering for a tachyon)

$$\frac{Re\left(\mathcal{A}_1(s,\mathbf{b})\right)}{2E} \sim \frac{1}{2} N T_p \kappa_{26} \frac{2E}{(4\pi)^{\frac{24-p}{2}}} \left(\frac{4}{\alpha' \log(\alpha' s)}\right)^{11-\frac{p}{2}} \frac{1}{11-\frac{p}{2}}$$
$$\times \mathrm{e}^{-\frac{b^2}{\alpha' \log(\alpha' s)}} \left(1 + \mathcal{O}\left(\frac{\mathbf{b}^2}{\alpha' \log(\alpha' s)}\right)\right)$$

- The eikonal phase for the elastic scattering of a graviton can be obtained by acting on the previous expression with the Fourier transform of a polynomial in the momenta and the polarizations.
- For the bosonic string this polynomial is

$$\left((\epsilon_1 \epsilon_2) - \frac{\alpha'}{2}(\epsilon_1 \mathbf{q})(\epsilon_2 \mathbf{q})\right)\left((\bar{\epsilon}_1 \bar{\epsilon}_2) - \frac{\alpha'}{2}(\bar{\epsilon}_1 \mathbf{q})(\bar{\epsilon}_2 \mathbf{q})\right) .$$

- In sharp contrast with the QFT limit, the terms containing the momentum transfer **q** (which in the impact parameter space corresponds to $\frac{\partial}{\partial \mathbf{b}}$) are parametrically small at high energy.

- They are suppressed by inverse powers of $\log(\alpha' s)$ with respect to terms of the same order in $|\mathbf{b}|$.
- Since the leading (Einstein-Hilbert) term respects causality, the Shapiro time delay is positive for all possible choices of the polarizations of the graviton, the Kalb-Ramond form and the dilaton.
- The Regge behavior of string amplitudes seems to be the essential ingredient for the resolution of the causality problem.

Closed-string absorption: towards strong-gravity

- For impact parameter $b \leq \ell_s(s)$ there is a violation of unitarity due to the absorption of the closed string by the *Dp*-brane.
- If we go back to the elastic scattering of a tachyon on a *Dp*-brane:

$$A_{TT} = \frac{\kappa N T_p}{2} \frac{\Gamma(-\alpha' s - 1)\Gamma\left(-\frac{\alpha'}{4}t - 1\right)}{\Gamma\left(-\alpha' s - \frac{\alpha'}{4}t - 2\right)} \, , \qquad T_p = \frac{\sqrt{\pi}}{2^4}\left(2\pi\sqrt{\alpha'}\right)^{11-p}$$

- In the Regge limit the amplitude has a real and an imaginary part:

$$\mathrm{Re}\mathcal{A}_{TT} = \frac{\kappa N T_p}{2} \cos\pi\left(1 + \frac{\alpha'}{4}t\right)\Gamma\left(-1 - \frac{\alpha'}{4}t\right)(\alpha' s)^{1+\frac{\alpha'}{4}t} \, ,$$

$$\mathrm{Im}\mathcal{A}_{TT} = \pi\frac{\kappa N T_p}{2}\frac{(\alpha' s)^{1+\frac{\alpha'}{4}t}}{\Gamma\left(2 + \frac{\alpha'}{4}t\right)} \, .$$

- The imaginary part is due to the absorption of the closed string by the *Dp*-brane. It is a pure string effect.

- In impact parameter space one gets

$$\mathrm{Im}\mathcal{A}_{TT}(s,b) = \int \frac{d^{24-p}q}{(2\pi)^{24-p}}\, e^{-i\vec{b}\vec{q}}\, \mathrm{Im}\mathcal{A}_{TT}(s,t)$$
$$\sim \pi \frac{\kappa N T_p}{2} \frac{\alpha' s}{(\pi\alpha' \log \alpha' s)^{\frac{24-p}{2}}}\, e^{-\frac{b^2}{\alpha' \log \alpha' s}}$$

- At small impact parameter $b \leq \ell_s(s)$ the closed string can be absorbed by the *Dp*-brane becoming an open string (with its endpoints attached to the brane).
- In fact, the imaginary part is also given by:

$$\mathrm{Im}\mathcal{A}_{T,T}(p_1,p_2) = \pi\alpha' \sum_{\chi} B_{T,\chi}(p_1) B^*_{T,\chi}(p_2)$$

- This corresponds to the fact that the closed string becomes an open string living on the *Dp*-brane world volume and subsequently is emitted becoming again a closed string that can leave the brane.

- Classically this transition can only happen at $b = 0$ corresponding to the fact that the closed string touches the brane.
- Quantum fluctuations allow this process to happen even at $b \neq 0$ giving the exponential decay.
- The possibility of the closed string to become an open string was not included in the eikonal operator.
- As a consequence, one observed a violation of unitarity.
- It is believed that the extension of the eikonal operator to include also open strings will restore unitarity.
- Unlike the case of the closed strings, playing with g_s and N, one cannot limit the number of open strings.
- Therefore, one must include the possibility of any number of open strings and this is unfortunately not easy.
- As a first step in this direction, we have looked at the property of a single open string, created by a very energetic closed string at the level of the disk.

- In this case the open string belongs to a high level $n \sim \alpha' E^2$.
- In order to have arbitrary open and closed string we use the light-cone open-closed vertex.
- In this way we determine the microscopic description of the open string state created by the absorption of an arbitrary closed string.
- For a very energetic closed string tachyon, we get the following open string state:

$$\frac{1}{(\pi\alpha' \log(\alpha' s))^{\frac{24-p}{2}}} \mathcal{P}_n \left[e^{-\frac{1}{\alpha' \log \alpha' s}\left(b^i + i\sqrt{\frac{\alpha'}{2}} \sum_k \frac{1}{k} a^i_{-k}\right)^2} \mathcal{V}_{\vec{0}} \right] |0\rangle$$

  $\mathcal{P}_n$ is the projector into level n and $\mathcal{V}_{\vec{0}}$ is the state at $\vec{b} = 0$.
- It can also be written in a more suggestive form if we interpret the Gaussian factor as a squeezed state for the effective creation and destruction operators

$$B^i = \frac{1}{\sqrt{\log n}} \sum_{k=1}^{n} \frac{a^i_k}{k} \,, \qquad B^{i\dagger} = \frac{1}{\sqrt{\log n}} \sum_{k=1}^{n} \frac{a^i_{-k}}{k}$$

- In the high energy limit they satisfy

$$[B^i, B^{j\dagger}] = \frac{\delta_{ij}}{\log n} \sum_{k=1}^{n} \frac{1}{k} \sim \delta_{ij}$$

- Since the squeezed states represent the position eigenstates in the oscillator basis of the Hilbert space we can write

$$|b\rangle_X = \frac{1}{(\sqrt{\pi}\ell_n)^{\frac{24-p}{2}}} \; e^{-\frac{b^2}{2l_n^2} - i\frac{\sqrt{2}}{\ell_n} B^\dagger b + \frac{1}{2}\left(B^\dagger\right)^2} |0\rangle = e^{-ib^i P^i} |0\rangle_X$$

where $\ell_n^2 = \alpha' \log n$.

- $|b\rangle_X$ is an eigenstate of the position operators

$$X^i = i\frac{l_n}{\sqrt{2}}\left(B^i - B^{i\dagger}\right) = i\sqrt{\frac{\alpha'}{2}} \sum_{k=1}^{n} \left(\frac{a_k^i}{k} - \frac{a_{-k}^i}{k}\right) \equiv \sum_{k=1}^{n} x_k^i$$

with eigenvalue b^i.

- P^i are the corresponding momentum operators

$$P^i = \frac{1}{\sqrt{2}l_n}\left(B^i + B^{i\dagger}\right) = \frac{1}{\sqrt{2\alpha'}\log\alpha' s}\sum_{k=1}^{n}\left(\frac{a_k^i}{k} + \frac{a_{-k}^i}{k}\right) \equiv \frac{1}{\log\alpha' s}\sum_{k=1}^{n} p_k^i$$

Here x_k and p_k are the position and momentum modes of the open string,

- The state $|b\rangle_X$ is normalized in the standard way, $\langle b|b'\rangle = \delta(b-b')$.
- We can then write

$$\widetilde{\mathcal{V}}_{\vec{b}}|0\rangle = \frac{e^{-\frac{b^2}{2\alpha' \log\alpha' s}}}{(\pi\alpha'\log\alpha' s)^{\frac{24-p}{4}}}\mathcal{P}_n\,\mathcal{V}_{\vec{0}}\,|b\rangle_X$$

- To check the correctness of the open string state we compute:

$$\mathrm{Im}\mathcal{A}_{TT}(s,b) = \pi\alpha' \langle 0|\mathcal{V}^{\dagger}_{\vec{0}}\widetilde{\mathcal{V}}_{\vec{b}}|0\rangle$$

- We get

$$\mathrm{Im}\mathcal{A}_{TT}(s,b) = \frac{\kappa T_p N}{2}(\pi\alpha' s)\,\frac{e^{-\frac{b^2}{\alpha' \log \alpha' s}}}{(\pi\alpha' \log \alpha' s)^{\frac{24-p}{2}}}$$

in agreement with the computation of the imaginary part from the amplitude!

- This analysis can be repeated for the massless and the massive closed string states obtaining similar expressions in terms of the squeezed state.

Conclusion and outlook

- We have studied the high-energy scattering of a closed string on a system of N parallel Dp-branes for various values of the impact parameter b.
- In all cases the scattering amplitude is dominated by graviton exchange and diverges with the energy, violating unitarity.
- For very large values of $b >> b_D >> \sqrt{\alpha'}$, the scattering is as in the case of a point-like particle.
- Elastic unitarity is recovered by summing over higher orders obtaining a c-number eikonal that satisfies elastic unitarity.
- Below a certain value of the impact parameter b_D elastic unitarity is violated.
- For impact parameter $b \sim b_D$ string effects become important.
- Due to tidal forces the closed string gets excited and inelastic channels open and become more and more relevant with respect to the elastic one.

- Therefore, the eikonal becomes an operator and the S-matrix is a unitary operator that includes all inelastic processes.
- If we go to values of $b \sim \sqrt{\alpha'}$, then unitarity is again violated.
- In order to restore it, we should include also the possibility of the absorption of the closed string by the branes decaying in any number of open strings.
- We have done the first step in this direction by studying the disk amplitude where the closed string decays into a single open string.
- We have determined the open string state in the case of very energetic closed string.
- We have checked that this state reproduces the disk imaginary part of the elastic closed string amplitude.
- One should go on considering the annulus where two open strings can be created by the energetic closed string.
- It is expected that eventually full unitarity is recovered when we will be able to also include the contribution of open strings in the eikonal operator.

- This does not seem to be an easy task because, unlike the case of the closed string where only one closed string can be created for large N, in the case of the open string, any number of open strings can be created.
- Finally, for impact parameter $b \sim \sqrt{\alpha'}$, we have shown that the Regge behavior of the scattering amplitude saves the bosonic string from causality violations.