From non-Abelian soliton vortices to Critical Strings: 10D strings bottom up

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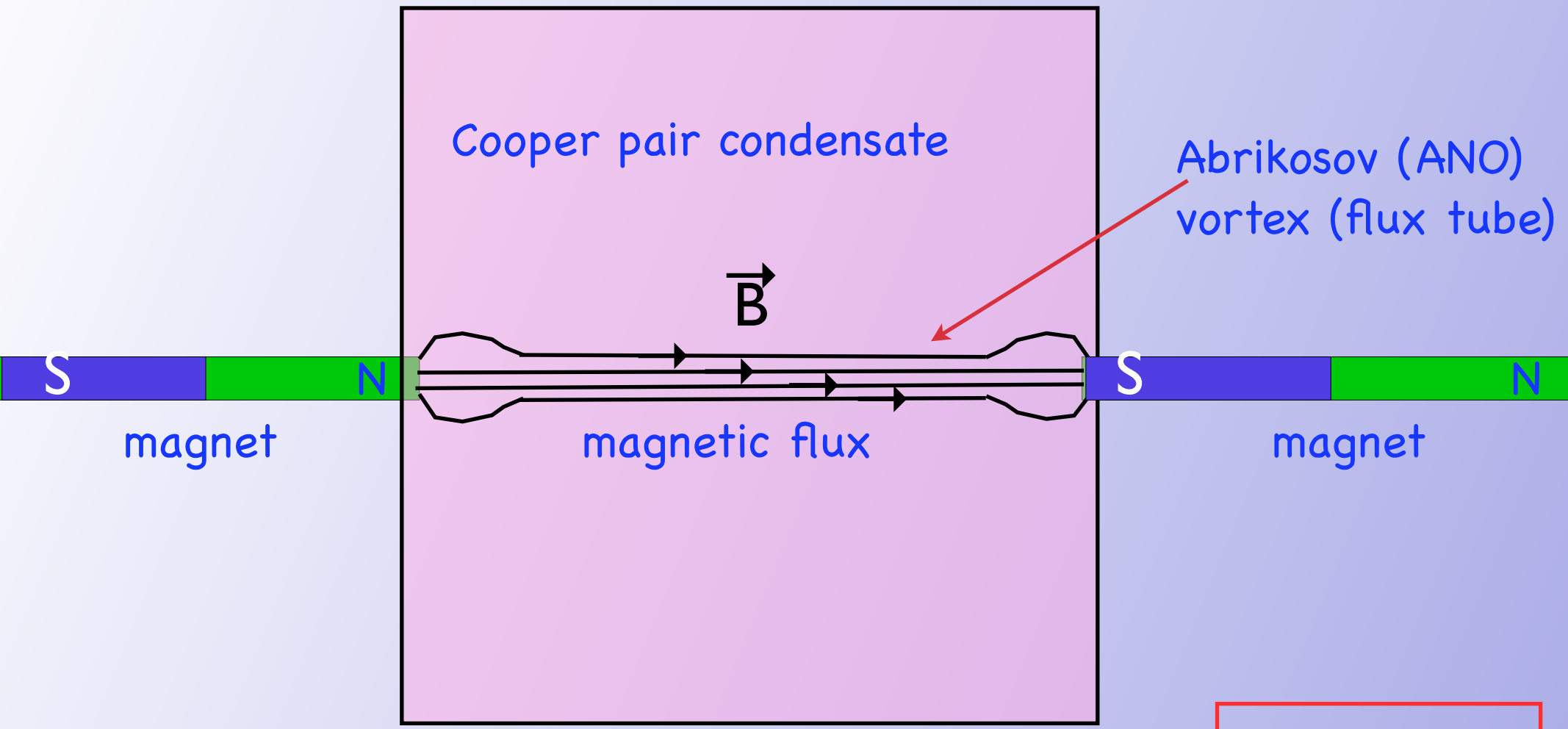
Giant Swirl Phenomenon, Crater Lake National Park, Oregon

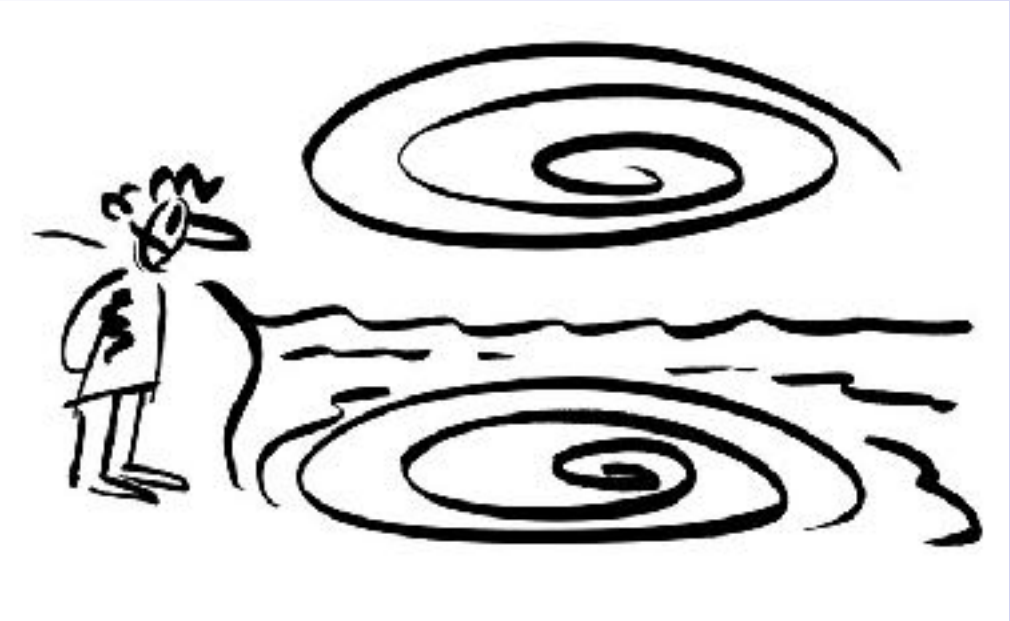
☞ The Meissner effect! 1930s



Superconductor of the 2nd kind

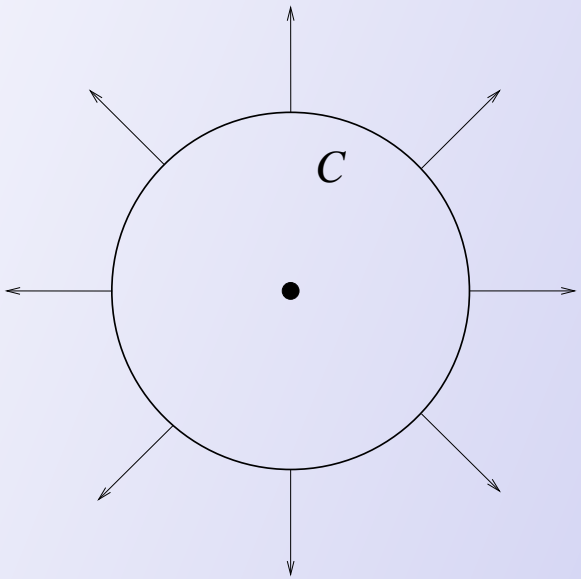
Abelian ☞

$$\mathcal{L} = -\frac{1}{4e^2}F_{\mu\nu}^2 + |\mathcal{D}^\mu\phi|^2 - U(\phi)$$



φ complex

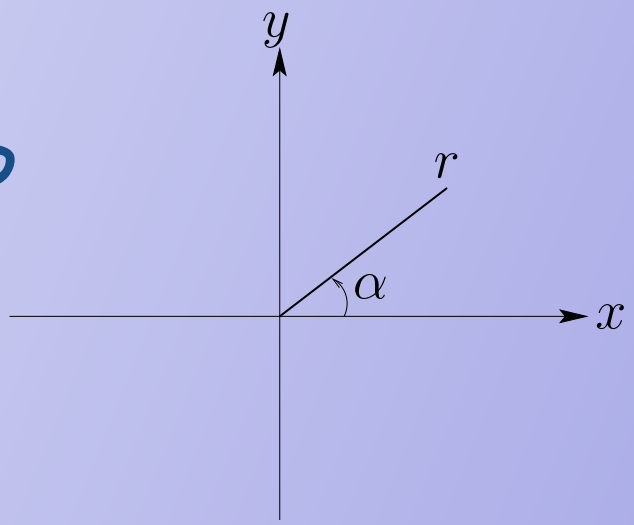
$$U(\phi) = \lambda\left(|\phi|^2 - v^2\right)^2$$



$\varphi = ve^{i\alpha}$ at $r \to \infty$

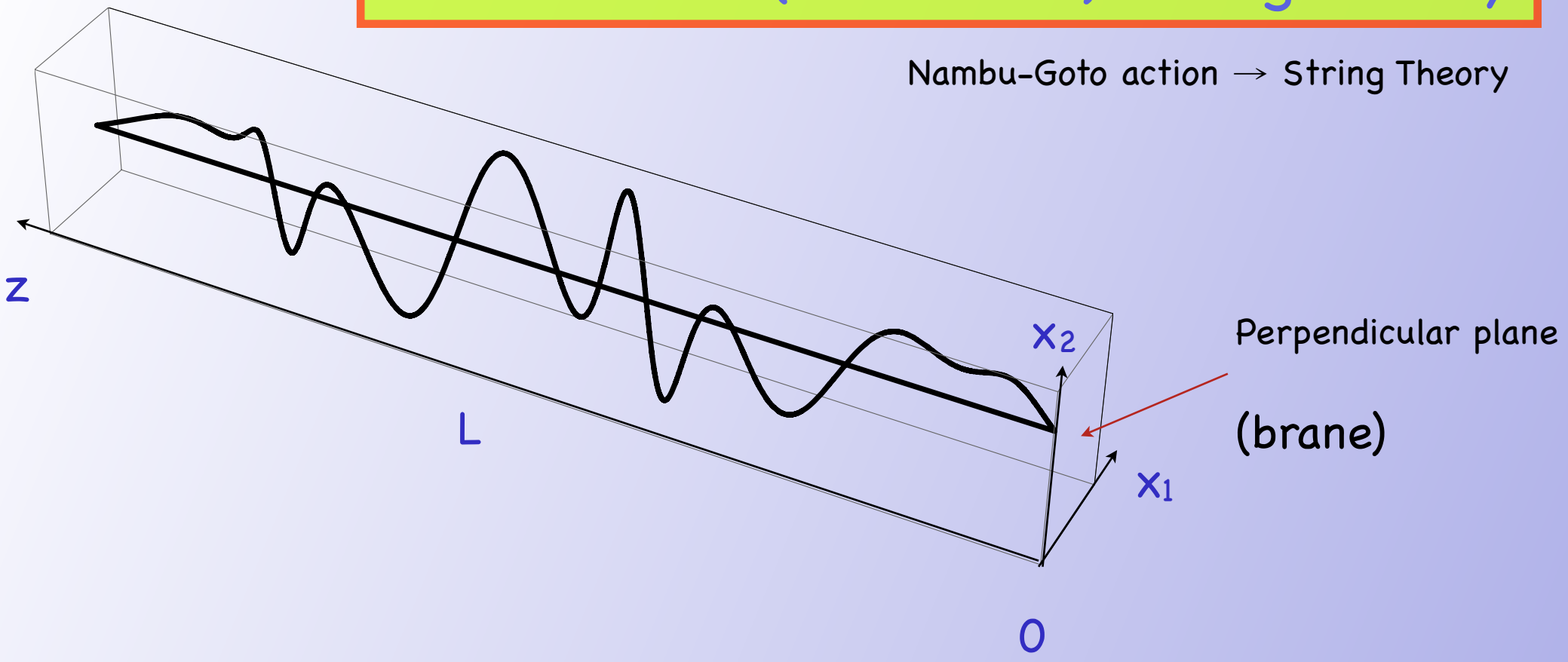
or

$\varphi = ve^{i(n)\alpha}$



Fundamental ("classical") string theory

Nambu-Goto action → String Theory



Infinitely thin string, fully defined by its coordinates

Quantization possible in 10 dimensions

2003: Hanany, Tong

Auzzi, Yung, et al.

Yung + M.S.

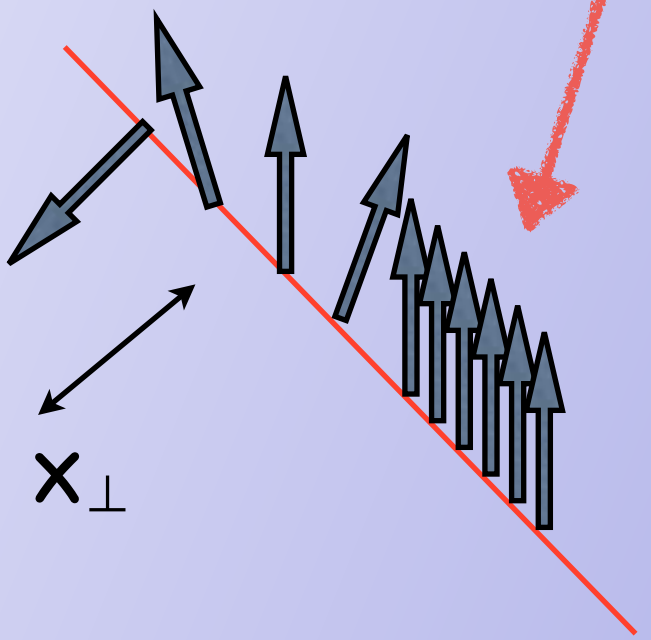
CP(N-1) $\mathcal{N}$=2 sigma model

Non-Abelian internal d.o.f

4D bulk, $\mathcal{N}$=2 Yang-Mills

with N flavors & N colors

+ FI



classically gapless excitations

Non-Abelian vortex strings are supported in 4D Yang-Mills theories with matter with N=2,1, and 0 supersymmetry.

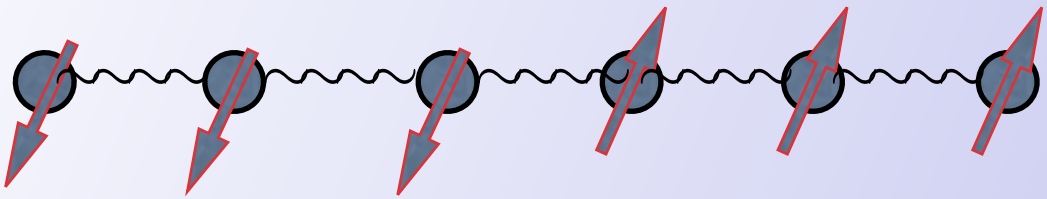
In QCD they are supported at hight density (chemical potential);

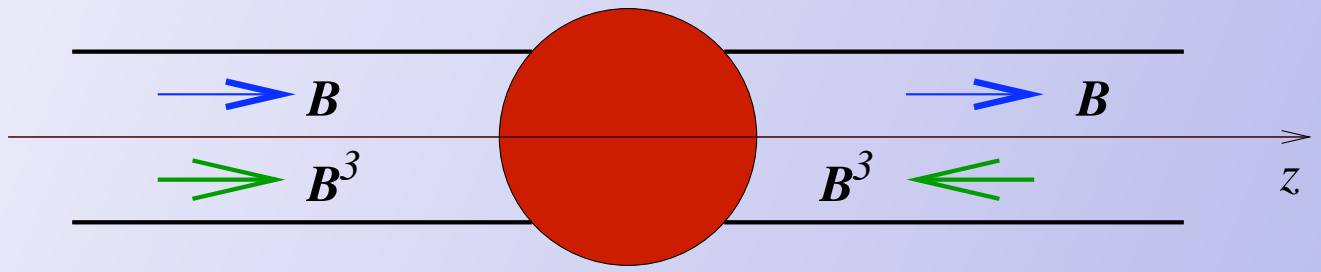
In condensed matter physics

$$\text{B phase } H_B^3 = SO(3)_{S+L}.$$

Low energy dynamics of $U(1)$ vortices in systems with cholesteric vacuum structure

$$\Delta\mathcal{L}_\varepsilon = -\eta\varepsilon_{ijk}\chi_i\partial_j\chi_k$$





Yung + M.S.
Hanany, Tong

Evolution in dimensionless parameter m^2/ξ

- Kinks are confined in 4D (attached to strings).
- Kinks are confined in 2D:

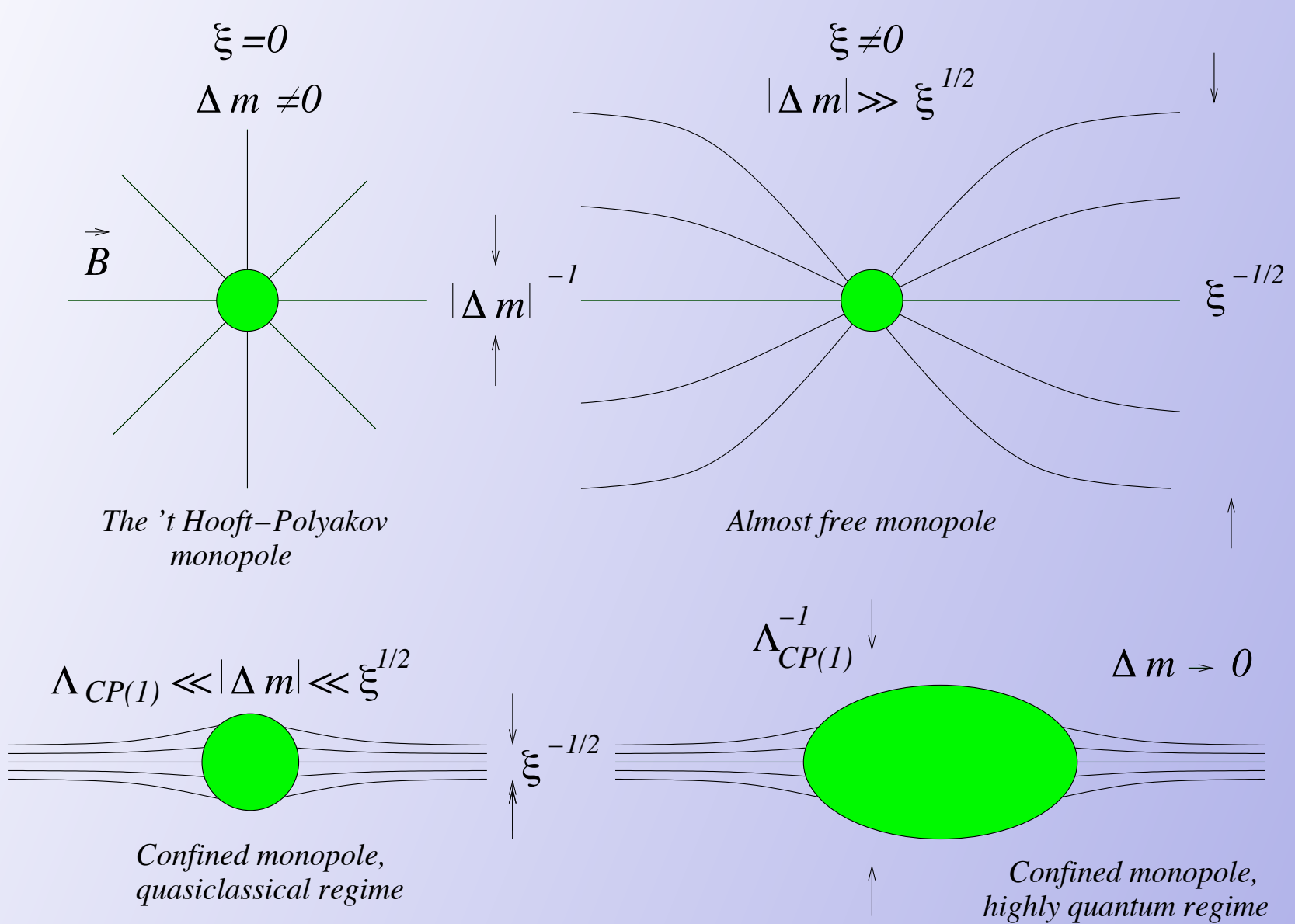
  ★only kink-antikink in the spectrum★

  if SUSY is unbroken (explained by Witten)

4D ↔ 2D correspondence

Kink = Confined Monopole

Why?


$\xi = 0$
$\Delta m \neq 0$
$\vec{B}$
$|\Delta m|^{-1}$
$\xi \neq 0$
$|\Delta m| \gg \xi^{1/2}$
$\xi^{-1/2}$
The 't Hooft–Polyakov monopole
Almost free monopole
$\Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2}$
$\xi^{-1/2}$
$\Lambda_{CP(1)}^{-1}$
$\Delta m \to 0$
Confined monopole, quasiclassical regime
Confined monopole, highly quantum regime

Ten-dimensional critical string as a soliton in four-dimensional super-Yang-Mills theory

With A. YUNG

4D bulk, N=2 Yang-Mills

with N_f flavors and N colors;

U(N) gauge group

8 supercharges;

supports 1/2 BPS

"non-Abelian" vortices

For long time

$N_f=N$

$\Rightarrow$ AF CP(N-1)

UV incomplete

H.D.

If $N_f = 2N$, then

bulk theory CONFORMAL (mod ξ)

$$S_1 = \int d^2\sigma\sqrt{h}\left\{h^{\alpha\beta}\left(\tilde{\nabla}_\alpha \bar{n}_P \nabla_\beta n^P + \nabla_\alpha \bar{\rho}_K \tilde{\nabla}_\beta \rho^K\right)\right.$$

$$\left. + \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - \beta\right)^2\right\} + \text{fermions}\,,$$

$$\nabla_\alpha = \partial_\alpha - iA_\alpha\,, \qquad \tilde{\nabla}_\alpha = \partial_\alpha + iA_\alpha$$

Non-compact Calabi-Yau, Ricci-flat!!!!!

World-sheet theory → WCP(2,2): Verification:

Virasoro central charge (including ghosts) = 0

ghost

$$c_{\rm Vir} = \frac{3}{2}\left(D + \frac{2}{3}c_{\rm WCP} - 10\right),$$

$$c(\mathrm{WCP}(N,N)) = 3(N+N-1)$$

N=2

$$\ell^{-2} \to \xi \times \begin{cases} g^2, & g^2 \ll 1 \\ \infty, & g^2 \to 4\pi \\ 16\pi^2/g^2, & g^2 \gg 1 \end{cases}$$

β = 0 in the self dual point;

Target space develops conical singularity!

Massless 4D baryon as a deformation of the conifold complex structure (Supergravity approx.)

Strong coupling: $\beta \to 0$ (selfdual point, $g^2 = 4\pi$)

$$\delta G_{ij} = \phi_4(x)\, \delta g_{ij}(y)\,,$$

4D 6D

Must be normalizable

deformations of complex structure lead to one 4D massless scalar hyper

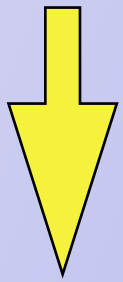
There are two 4D modular parameters in the case at hand:

(a) complex structure (determined by H{2,1});

(b) Kähler form (determined by H{1,1}).

(a) deformed conifold , H{2,1} (Hodge numbers) implies 1 normalizable mode

(b) Resolved manifold, H{1,1}, (Hodge numbers) non-normalizable modes in 6D



One massless scalar hypermultiplet of string states in 4D at strong coupling ($g^2 \longrightarrow 4\pi$)

- $U(1)_B$ in the bulk is unconventional

Take $U(1)$ from $U(2)_{gauge}$;

- Define $U(1)_{flavor}$ as a $U(1)$ rotation of f1 and f2 in one direction & f3 and f4 in opposite;
- $U(1)_{gauge} \times U(1)_{flavor} \rightarrow U(1)_{diag} = U(1)_B$

Unbroken by <f1>, <f2>

On the world sheet

	n	ρ
$U(1)_g$	-1/2	1/2
$U(1)_f$	1/2	1/2
$U(1)_{diag}$	0	1

Deformed conifold par.

$w \sim n \times \rho$; $w^2 \sim b$; $Q_B(b) = 2$

Massive excitations

- Unlike M=0 case, supergravity approximation cannot be used

- **Non-critical $c = 1$ string** (with Liouville, e.g. Kutasov et al.)

$$\tilde{R}_4 \times Y_6 \rightarrow \mathbb{R}^4 \times \mathbb{R}_\phi \times S^1$$

$$g_s = e^{-\frac{Q}{2}\phi}.$$

Spin-2 states

$$(M^G)^2_{j,l} = 8\pi T \left(l^2 + \frac{1}{4} \right)$$

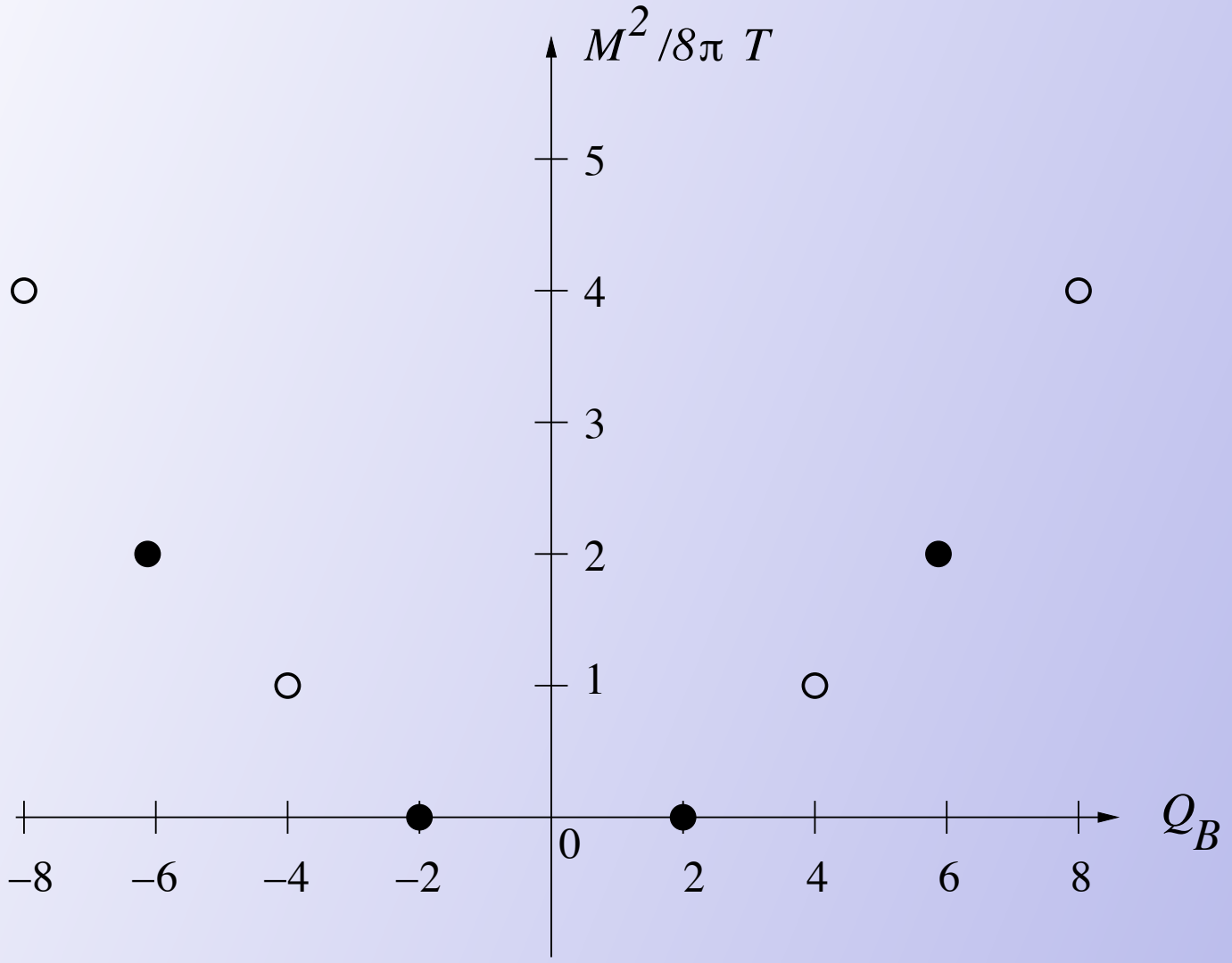


Figure 1: Spectrum of spin-0 and spin-2 states as a function of the baryonic charge. Closed and open circles denote spin-0 and spin-2 states, respectively.

Figure 2: Examples of the monopole "necklace" baryons: a) Massless b-baryon with $Q_B = 2$; b) Spin-2 baryon with $Q_B = 4$. Open circles denote monopoles.

Conclusion:

10D critical string IS reincarnation of a BPS soliton in 4D N=2 super-Yang-Mills with U(2) gauge and four quark flavors!

Reverse Holography!