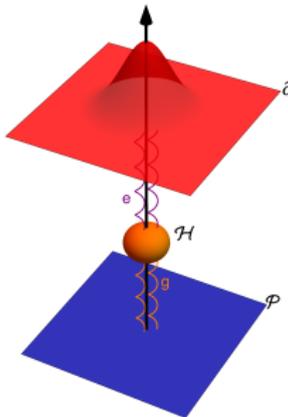


Violating weak cosmic censorship in AdS_4

Jorge E. Santos

NBI Current Themes in High Energy Physics and Cosmology



In collaboration with
T. Crisford, G. T. Horowitz, N. Iqbal, B. Way

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Weak cosmic censorship (Penrose 69 - page 1162 and 1164):

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Weak cosmic censorship (executive summary):

Is it possible to **form a region of arbitrarily large** curvature that is visible to **distant observers**?

Operationalising 2/2:

Weak cosmic censorship (Geroch and Horowitz 79):

Let (Σ, h_{ab}, K_{ab}) be a geodesically complete, **asymptotically flat**, initial data set. Let the matter fields obey second order **quasi-linear hyperbolic** equations and satisfy the **dominant energy condition**. Then, generically, the maximal development of this initial data is an asymptotically flat spacetime (in particular \mathcal{I}^+ is **complete**) that is strongly asymptotically predictable.

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Claims to fame - Gregory Laflamme type - $d \geq 5$:

Lehner, Pretorius '10 - [Black String](#)

Figueras, Kunesch, and Tunyasuvunakool '16 - [Black Rings](#)

Figueras, Kunesch, Lehner, and Tunyasuvunakool '17 - [Myers-Perry](#)

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Weak cosmic censorship meets AdS:

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Wish list:

Remain in **4D**, and start in the **vacuum of the theory**.

- 1 Setup
- 2 Adiabatic approximation: $\mathcal{A}(t) = \mathcal{A}$
- 3 The Conjecture
- 4 Violation of the Weak Cosmic Censorship Conjecture
- 5 Results
- 6 Conclusion & Outlook

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- At $T = 0$, moduli space of solutions is **1D**: $a_0(t) \equiv \mathcal{A}(t)\sigma$.

Adiabatic approximation: $\mathcal{A}(t) = \mathcal{A}$

G. T. Horowitz, N. Iqbal, JES, B. Way '14

M. Blake, A. Donos, D. Tong '14

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A simple criterion:

$\alpha < 1$: impurity destroys the IR, *i.e.* is **relevant**.

$\alpha = 1$: impurity **marginally** deforms the IR.

$\alpha > 1$: impurity should be **irrelevant** in the IR.

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$$ds^2 = \frac{L^2}{z^2} \left[-A dt^2 + S_1 (dr + K dz)^2 + S_2 r^2 d\varphi^2 + B dz^2 \right],$$

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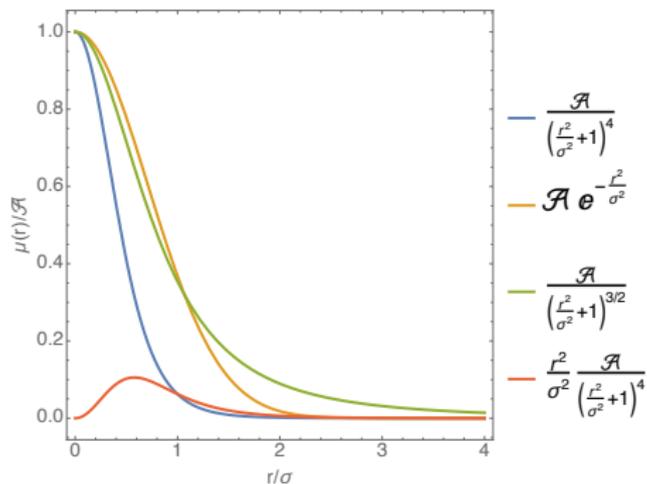
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 - Equations of motion **solve for gauge** defined by $\xi = 0$.

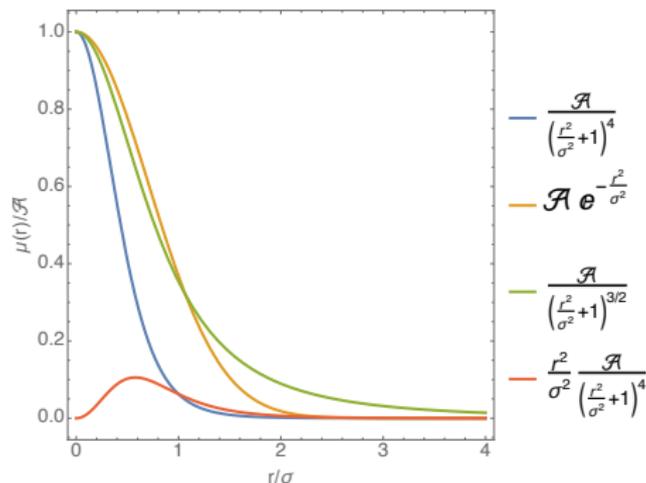
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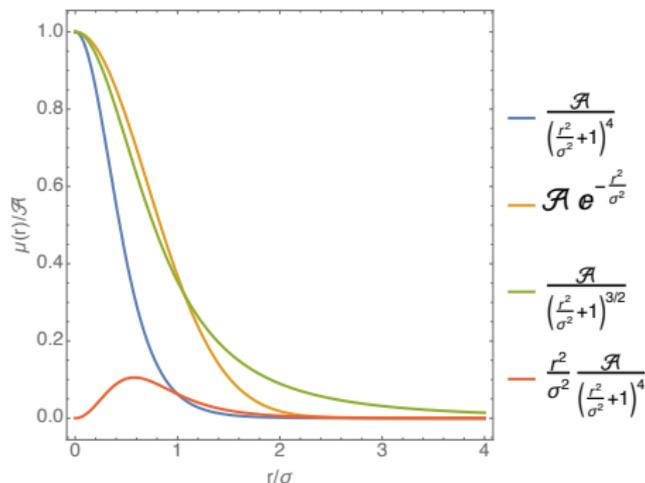
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- In all cases, the **IR geometry** is always **AdS₄** - irrelevant.

The irrelevant case 2/5

Static orbits:

We searched for locus in our manifold where

$$U^a \nabla_a U_b = \frac{q}{m} F_{ba} U^a \quad \text{with} \quad U_a U^a = -1,$$

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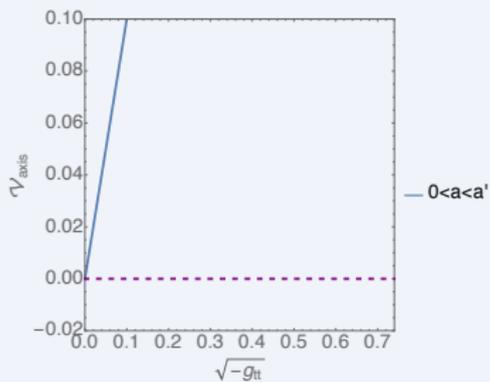
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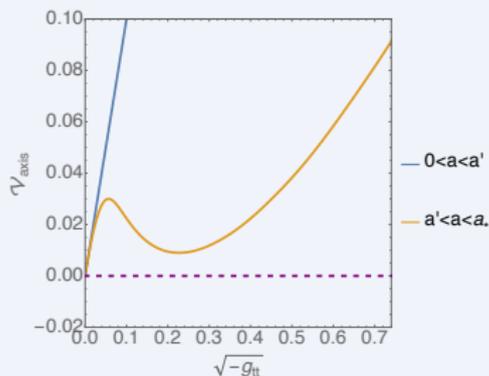
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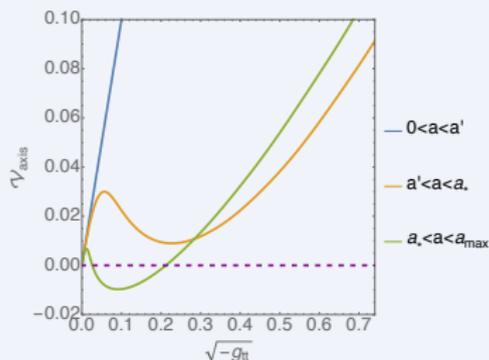
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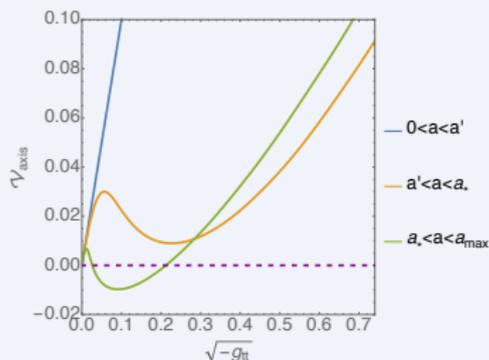
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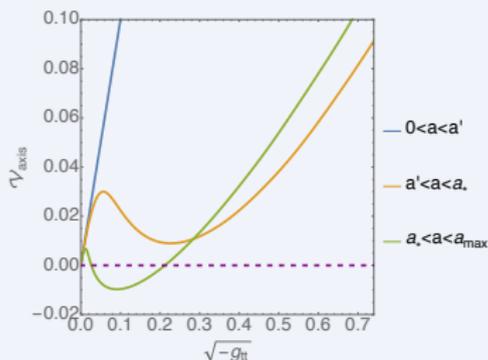
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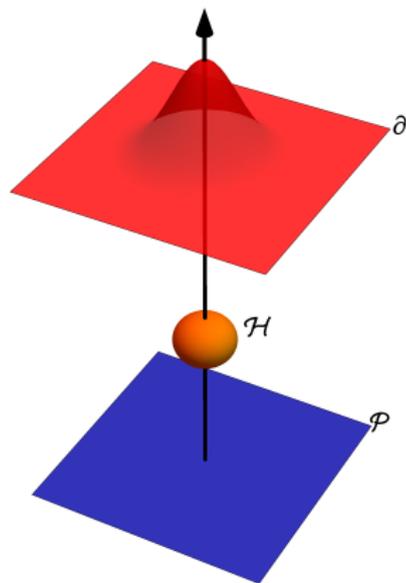
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Can we go **beyond the probe approximation** and construct the solutions where the **extremal hole hovers the Poincaré horizon**?

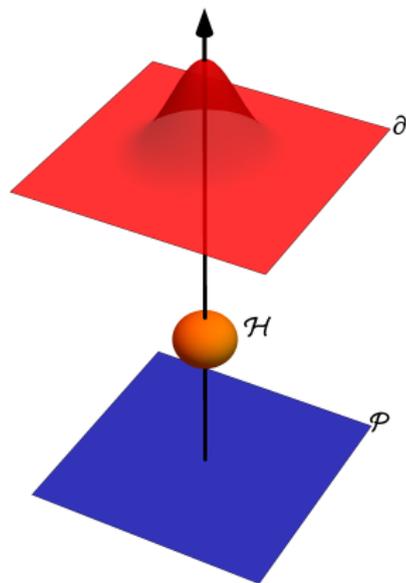
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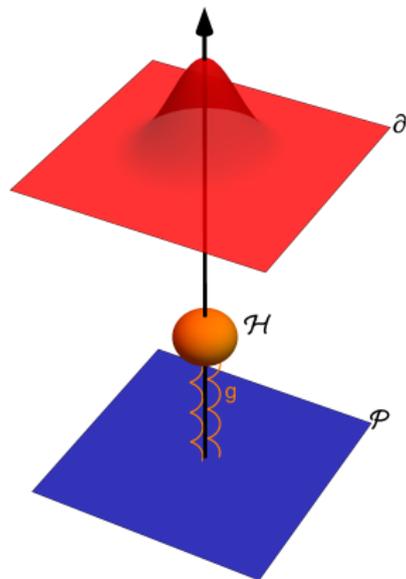
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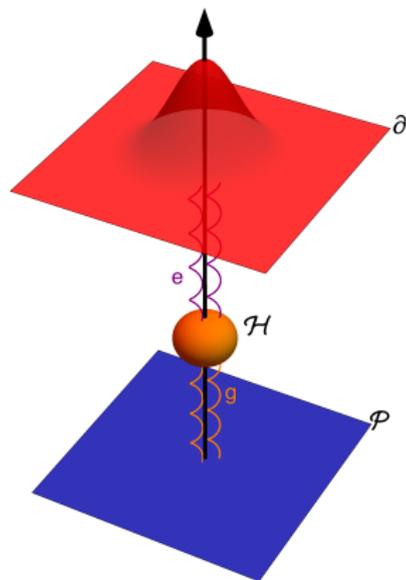
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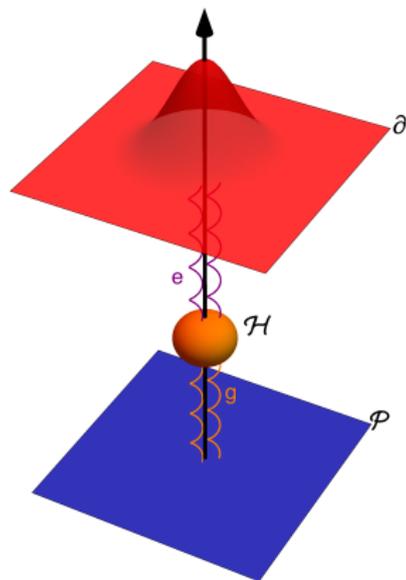
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The irrelevant case 4/5

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- This **cannot be** the **all** story because it would suggest an **unstable solution**.



The irrelevant case 5/5

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- 6 Phase transition is **second order**.
- 7 Perturbative **evidence in favour of stability**.

The marginal case

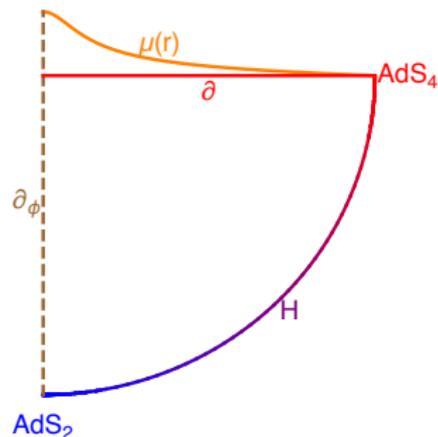
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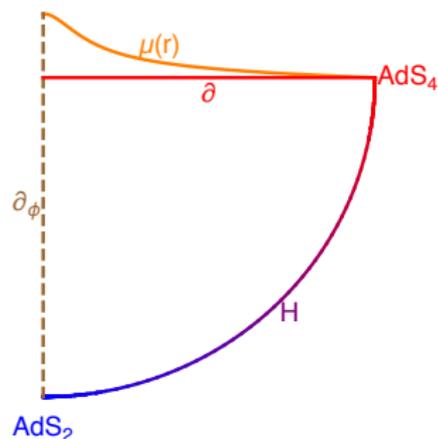
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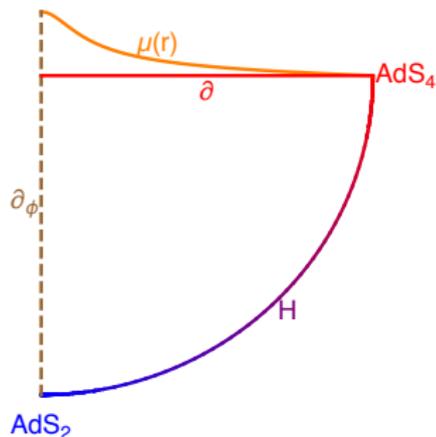
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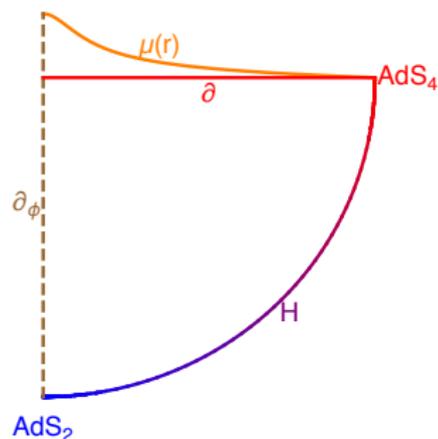
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- In this case, small hovering black holes also form!
- Also have a maximum amplitude a_{\max} which can be computed analytically.



The conjecture

G. T. Horowitz, JES, B. Way '16

The conjecture 1/2

- Impose **boundary electric profile** (with $\alpha \geq 1$):

$$f = \frac{a(t) r \alpha}{\sigma^2 \left(1 + \frac{r^2}{\sigma^2}\right)^{\frac{\alpha}{2} + 1}} dt \wedge dr .$$

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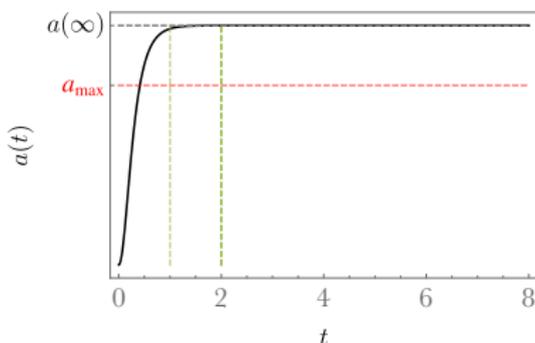
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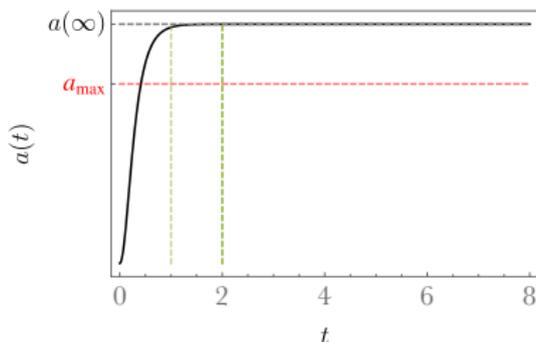


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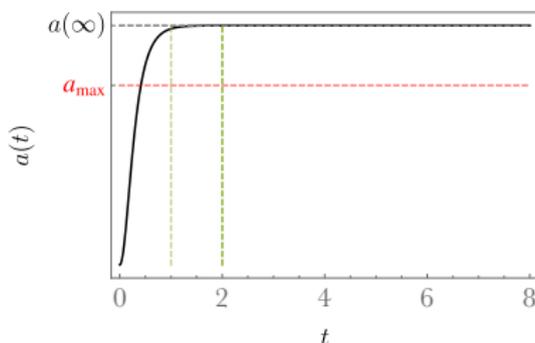
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- Take $a(t)$ of the form:



- Smallest $a_{\max} \approx 0.678$ occurs for $\alpha = 1$.

The conjecture 2/2

Possible outcomes:

- If $a(\infty) < a_{\max}$, we expect the solution to **settle down** to the simply connected solution found in the **static setup**.

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Conjecture - G. T. Horowitz, JES, B. Way '16:

For $a(\infty) > a_{\max}$, the resulting time evolution leads to **arbitrarily large curvatures** at late times, which are visible to boundary observers: **weak cosmic censorship is violated**.

Violation of the Weak Cosmic Censorship Conjecture

T. Crisford, JES '17

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Develop a characteristics code that can handle $T = 0$.

Easier said, than done! Thankfully (or foolishly),
Toby Crisford was ready to embrace this!

Back to basics - 1/2:

$$ds^2 = \frac{L^2}{z^2} \left[- \left(1 - \frac{z^3}{z_+^3} \right) dt^2 + \frac{dz^2}{1 - \frac{z^3}{z_+^3}} + dr^2 + r^2 d\varphi^2 \right].$$

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- 5 r is a **bad** coordinate on the **intersection of the horizon** with a partial Cauchy surface with $v = \text{const.}$, since $g_{rr} \rightarrow 0$ as $z \rightarrow +\infty$.

The epiphany - 1/2:

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$$r = \frac{\sin \theta}{\rho} \quad \text{and} \quad z = \frac{\cos \theta}{\rho} .$$

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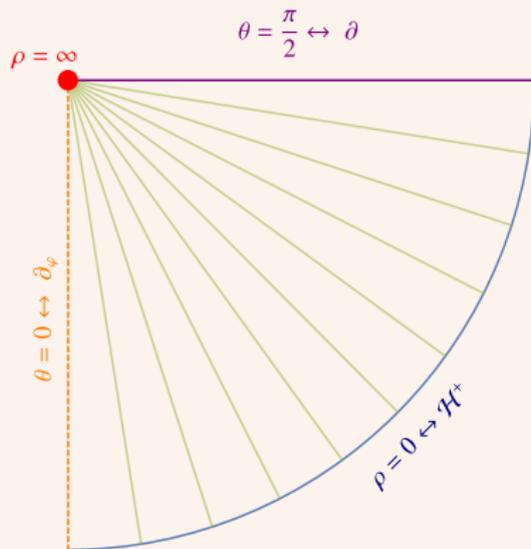
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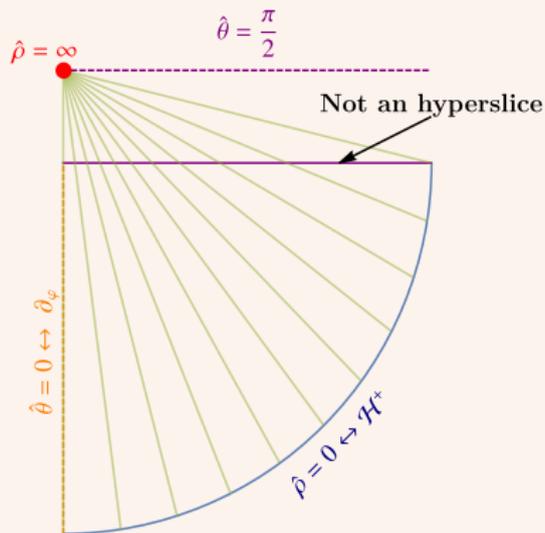
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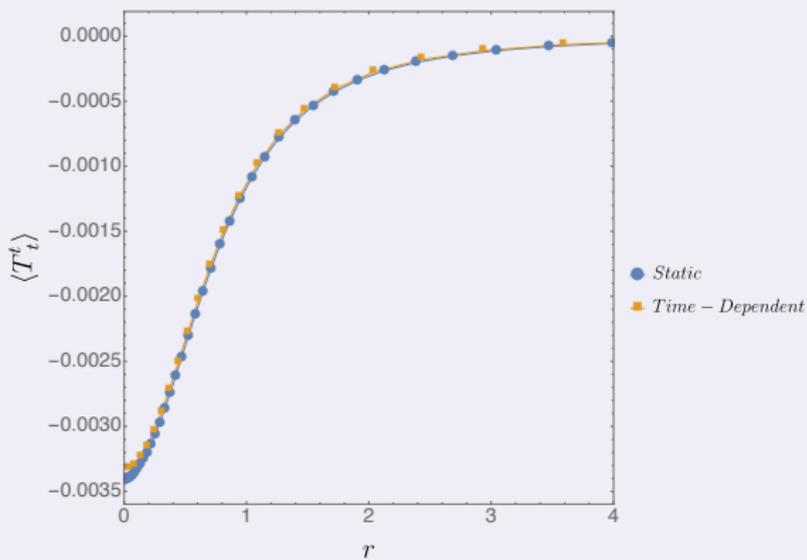


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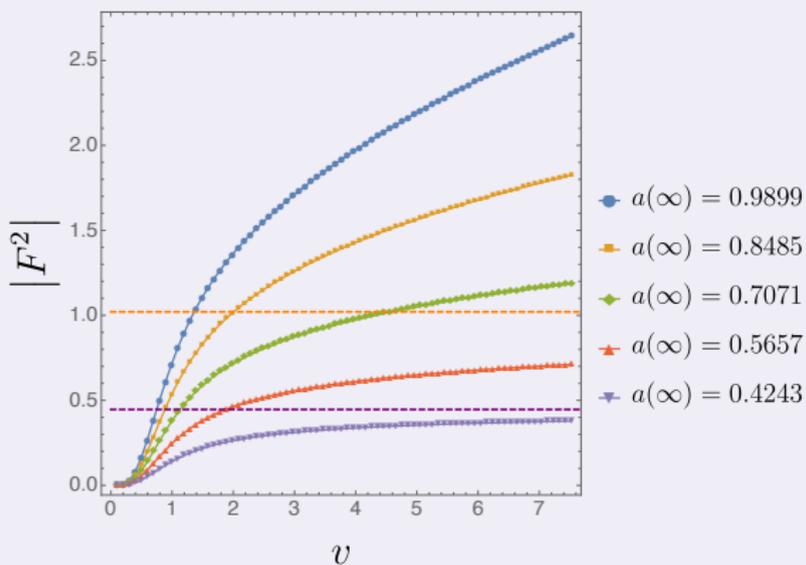
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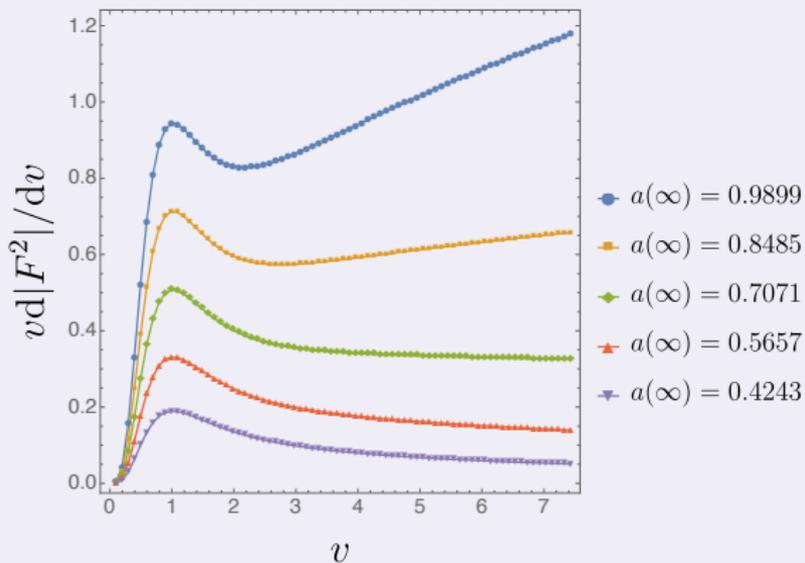
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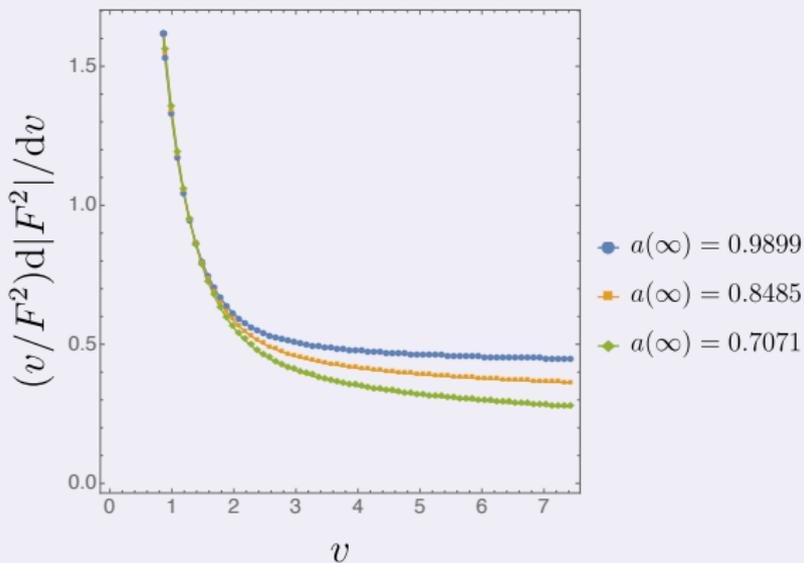
Matching old results with a time-dependent code!

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- 4 At $t = 0$ **vacuum state of the theory**: inject finite energy.

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- Found a new phase, which is stable to finite N corrections, for which we don't have a field theory interpretation.
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What to ask me after the talk:

- Is there a relation between the violation weak cosmic censorship in this setup and the weak gravity conjecture?
- Have you tried different profiles?
- Is axisymmetry a restriction?
- Include charged scalar fields and test weak gravity conjecture.

Conclusions:

- Found a new phase, which is stable to finite N corrections, for which we don't have a field theory interpretation.
- We have found a four-dimensional counterexample to the weak cosmic censorship.

What to ask me after the talk:

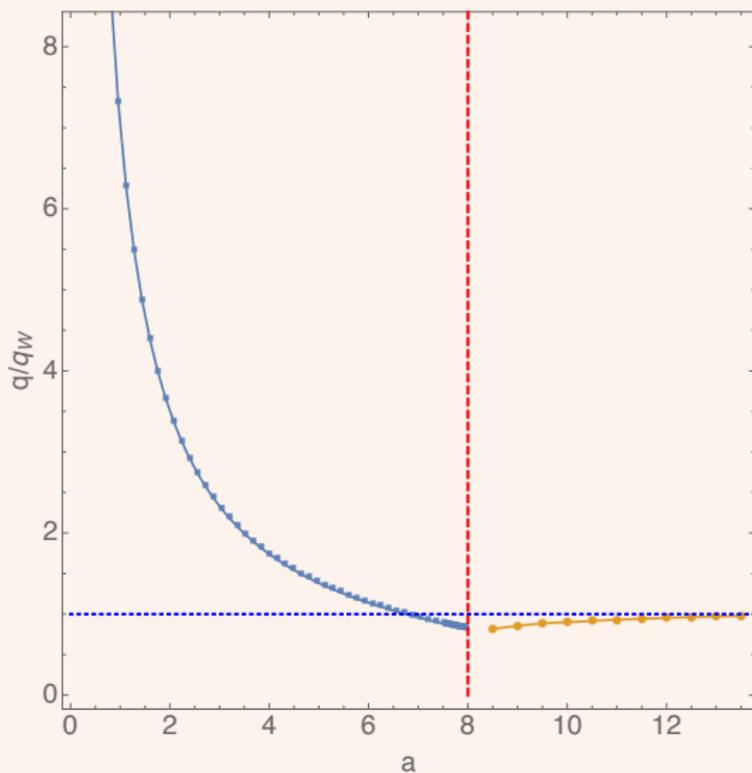
- Is there a relation between the violation weak cosmic censorship in this setup and the weak gravity conjecture?
- Have you tried different profiles?
- Is axisymmetry a restriction?
- Include charged scalar fields and test weak gravity conjecture.

Outlook:

- What is the field theory interpretation of this phenomenon?

Preliminary results - Crisford, Horowitz and Santos:

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Thank You!