Exploring a QFT as a UV completion for gravity

We have a QFT for quantum general relativity --- EFT
- but here we mean a UV theory

Diagnosis and obstacles for the UV

Inducing the Einstein action using Yang-Mills theories

Unstable ghosts and unitarity

Work with Gabriel Menezes
arXiv:1712.04468 , arXiv:1804.04980 …

John Donoghue
Copenhagen
Aug 12, 2018

Motivation:

Most attempted UV completions of gravity are not renormalizeable QFTs
- very exotic by BSM standards

All other interactions in SM are renormalizeable QFTs at known energies

Can we have a renormalizeable QFT for gravity at all scales?

All in SM initially had obstacles to overcome
- SM has “variations” on the QFT theme

Known obstacles for gravity (plus sociology)
- will need another “variation”

Connections:

Early pioneers: Stelle, Fradkin-Tsetlyn, Adler, Zee, Smilga,
Tomboulis, Hasslacher-Mottola,
Lee-Wick, Coleman, Boulware-Gross….

Present activity: Einhorn-Jones, Salvio-Strumia, Holdom-Ren,
Donoghue-Menezes, Mannheim, Anselmi
Odintsov-Shapiro, Narain-Anishetty…

In the neighborhood: Lu-Perkins-Pope-Stelle, 't Hooft,
Grinstein-O'Connell-Wise ….

Diagnosis 1:

Matter loops renormalize R^2 terms

- at one loop – and at **all loop order**
- dimensionless coupling constant

R^2 terms must be in a fundamental QFT action

R^2 terms lead to quartic propagators

$$R^2 \sim \partial^2 g \partial^2 g \qquad\qquad D(q) = \frac{i}{q^4}$$

With quartic propagators, graviton loops also stop at R^2

- all orders
- dimensionless coupling constant – **scale invariant**
- R^2 theories renormalizeable (Stelle)

QFT "variation" needs quartic propagators

Diagnosis 2:

Need usual propagators at low energy
- well defined ground states
- experiment

Low energy action is Einstein-Hilbert

Observation: Strongly interacting theories generate changes in Cosm. Const and in G

Can we start with scale invariant R^2 action and induce Einstein action at low energy, hopefully in controlled fashion? Is this viable?

Overview of our path (simplified):

"Gauge assisted quadratic gravity"

Consider Yang-Mills plus Weyl gravity:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4g^2} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right]$$

$$\begin{aligned} C_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta} - \frac{1}{2}\left(R_{\mu\alpha} g_{\nu\beta} - R_{\nu\alpha} g_{\mu\beta} - R_{\mu\beta} g_{\mu\alpha} + R_{\nu\beta} g_{\mu\alpha}\right) \\ &+ \frac{R(g)}{6}\left(g_{\mu\alpha} g_{\nu\beta} - g_{\nu\alpha} g_{\mu\beta}\right) \quad . \end{aligned}$$

Both sectors are asymptocically free:

Take $\xi^2 \ll g^2$

$$\xi^2(q) = \frac{\xi^2(\mu)}{1 + \frac{\xi^2(\mu)(N_\infty + N_q)}{320\pi^2} \ln(q^2/\mu^2)} = \frac{320\pi^2}{(N_\infty + N_q) \ln(q^2/\Lambda_\xi^2)}$$

"Helper" YM becoming strong will define Planck scale

Gravity can stay weakly coupled at all scales

if $\xi = 0.1$ at the Planck scale, it would become strong at $\Lambda_\xi = 10^{-1006}$ eV.)

Yang-Mills will induce the Einstein action

$$\frac{1}{16\pi G_{\rm ind}} = \frac{i}{96}\int d^4x\; x^2\;\langle 0|T(T^{\mu}_{\ \mu}(x)T^{\nu}_{\ \nu}(0)|0\rangle$$

Adler-Zee

In QCD we find:

JFD, Menezes

$$\frac{1}{16\pi G_{\rm ind}} = 0.0095 \pm 0.0030\ {\rm GeV}^2$$

Scale up by 10^{19} to get correct Planck scale

Spin 2 propagator after induced G:

- one loop order in light fields

$$
\begin{aligned}
iD_{\mu\nu\alpha\beta} &= i\mathcal{P}^{(2)}_{\mu\nu\alpha\beta} D_2(q) \\
D_2^{-1}(q) &= \frac{q^2+i\epsilon}{\tilde{\kappa}^2(q)} - \frac{q^4}{2\xi^2(\mu)} - \frac{q^4 N_{\rm eff}}{640\pi^2}\ln\left(\frac{-q^2-i\epsilon}{\mu^2}\right) - \frac{q^4 N_q}{1280\pi^2}\ln\left[\frac{(q^2)^2}{\mu^4}\right]
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{1}{\tilde{\kappa}^2(q)} &\to \frac{1}{\kappa^2} \qquad q \ll M_P \\
&\to 0 \qquad q \gg M_P
\end{aligned}
\qquad\qquad \kappa^2 = 32\pi G
$$

and $N_{\rm eff}$ = active d.o.f. with quadratic propagators
N_q = active d.o.f. with quartic propagators

Here ignoring cosmological constant

<u>Three regions</u>:

$$
\begin{aligned}
iD_{\mu\nu\alpha\beta} &= i\mathcal{P}^{(2)}_{\mu\nu\alpha\beta} D_2(q) \\
D_2^{-1}(q) &= \frac{q^2 + i\epsilon}{\tilde{\kappa}^2(q)} - \frac{q^4}{2\xi^2(\mu)} - \frac{q^4 N_{\rm eff}}{640\pi^2} \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right) - \frac{q^4 N_q}{1280\pi^2} \ln\left[\frac{(q^2)^2}{\mu^4}\right]
\end{aligned}
$$

1) Beyond Planck mass:

YM plus quadratic gravity

$$
\begin{aligned}
&N_{\rm eff} = N_\infty = D + N_{SM}, \\
&N_q = 199/3
\end{aligned}
$$

2) Intermediate energy

- interplay of quadratic and quartic terms
- dominated by resonance

3) Low energy EFT region

quartic terms subdominant

$$
\begin{aligned}
N_{\rm eff} &= N_V + \frac{1}{4} N_{1/2} + \frac{1}{6} N_S + \frac{21}{6} \\
&= \frac{21}{6} + N_{SM} + N_{BSM}
\end{aligned}
$$

Resonance in spin-two propagator

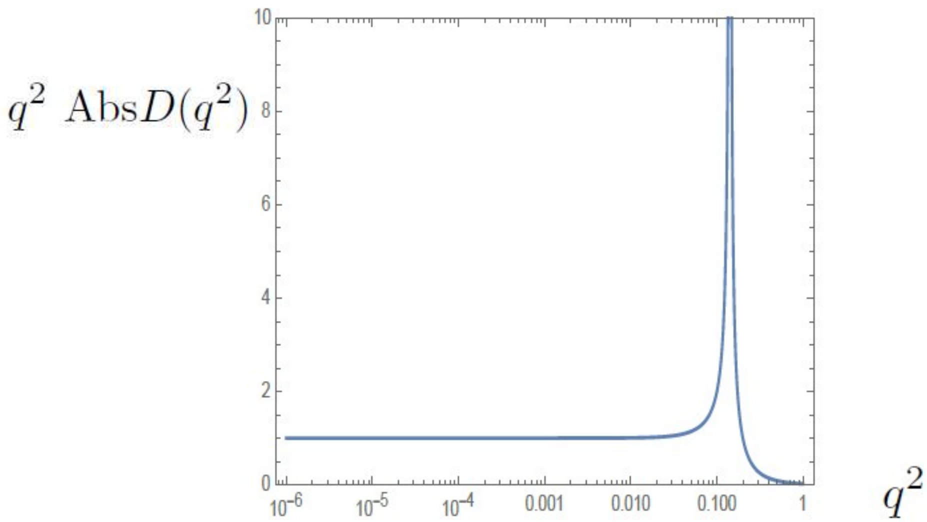


Figure units: $\kappa^2 = 1$

$\xi^2 = 0.1$

Narrow width pole position: $$M^2 = m_r^2 + i\gamma = \frac{2\xi^2(m_r)}{\kappa^2} + i\frac{2\xi^2 m_r^2 N_{eff}}{640\pi}$$

Written as "ghost-like" plus "opposite sign width"

$$D(q) = \left[\frac{\kappa^2}{1 + \frac{N_{\text{eff}}\xi^2(m_r)}{320\pi^2}}\right] \frac{1}{-\delta q^2 + i\gamma} = \left[\frac{\kappa^2}{1 + \frac{N_{\text{eff}}\xi^2(m_r)}{320\pi^2}}\right] \frac{-1}{\delta q^2 - i\gamma}$$

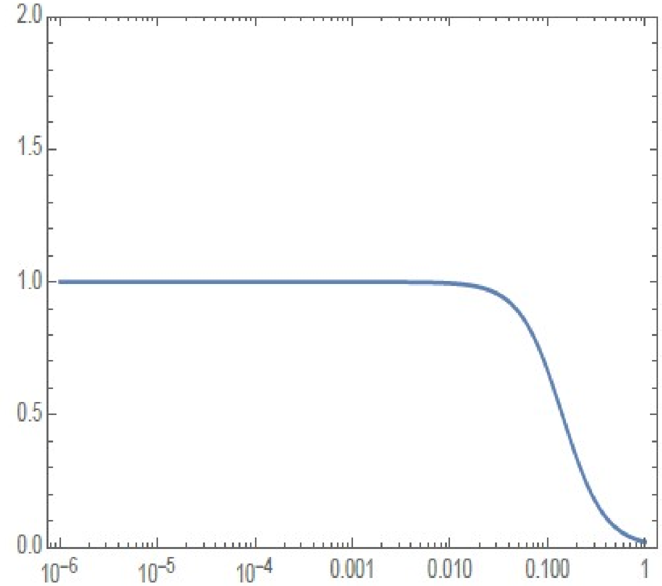
$$q^2 = m_r^2 + \delta q^2$$

vs

$$D(q) = \frac{1}{q^2 - (M - i\frac{\Gamma}{2})^2} = \frac{1}{\delta q^2 + iM\Gamma}$$

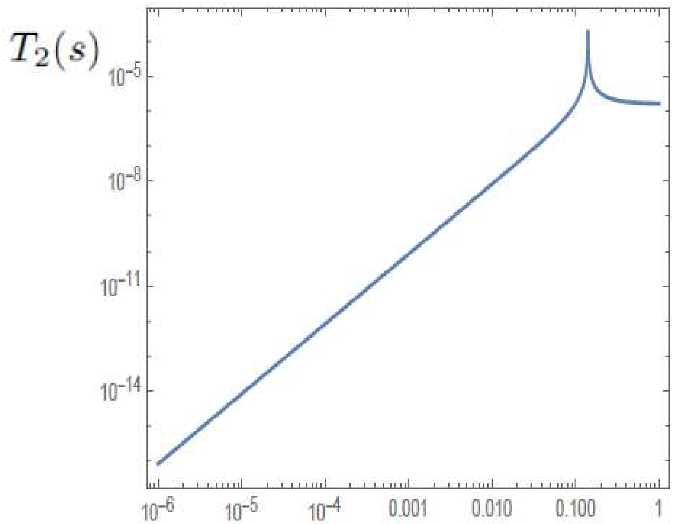
Euclidean/spacelike propagator:

$q^2 \ \mathrm{Abs}D(q^2)$



J= 2 scattering is unitary:

- and weakly coupled:



Elastic unitarity $$T_2(s) = \frac{A(s)}{f(s) - iA(s)} = \frac{A(s)[f(s) + iA(s)]}{f^2(s) + A^2(s)}$$

Satisfied

$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\left\{\frac{1}{\kappa^2} - \frac{s}{2\xi^2(\mu)} - \frac{sN_{\rm eff}}{640\pi^2}\left[\ln\left(\frac{s}{\mu^2}\right) - i\pi\right] - \frac{sN_q}{1280\pi^2}\ln\left(\frac{s^2}{\mu^4}\right)\right\}^{-1}.$$

Note for later: R^2 terms do not contribute to scattering

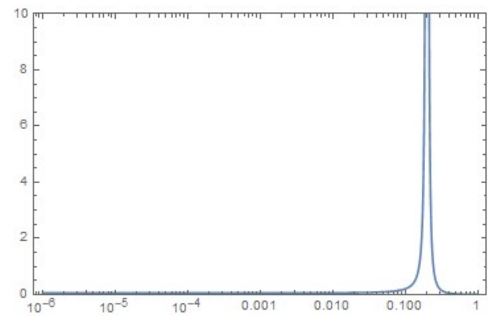
Modified Lehman representation:

$$D(q) = \frac{1}{q^2 + i\epsilon} \underbrace{- \frac{\beta}{q^2 - M^2} - \frac{\beta^*}{q^2 - M^{*2}}}_{\text{Sum is real}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

Imaginary parts live here

Spectral function has resonant behavior:

$$\rho(s) = \frac{\frac{N_{eff}\kappa^2}{640\pi}}{\left(1 - \frac{s}{m_r^2} - \frac{N_{eff}\kappa^2 s}{640\pi^2}\log\frac{s}{m_r^2}\right)^2 + \left(\frac{N_{eff}\kappa^2 s}{640\pi}\right)^2}$$



Currently studying loops with full propagator

- unitarity seems to work out using spectral integral
- new imaginary parts from resonant production (like G. O'C. W.)

Now lets revisit these ingredients

Quartic propagators in general

Expectation of ghosts/ negative norms

From:

$$\frac{-i}{q^4} \sim \frac{-i}{q^2(q^2-\mu^2)} = \frac{1}{\mu^2}\left(\frac{i}{q^2} - \frac{i}{q^2-\mu^2}\right)$$

The Lee-Wick mechanism

~1969

Apparently consistent finite higher derivative theories
- extra state with negative norm
- modifies propagators

$$iD_{F\mu\nu}(q) = -ig_{\mu\nu}\left[\frac{1}{q^2} - \frac{1}{q^2-\Lambda^2}\right] = -ig_{\mu\nu}\frac{-\Lambda^2}{q^2(q^2-\Lambda^2)} = -ig_{\mu\nu}\frac{1}{q^2(1-\frac{q^2}{\Lambda^2})}$$

Key is that coupling to light states leads to unstable ghost
- does not appear in the asymptotic spectrum

$$G(q^2) = \frac{1}{(q^2+i\epsilon)\left[1+\Pi(q^2)-\frac{q^2}{\Lambda^2}\right]}$$

$$\Pi(q^2) = q^2\frac{\alpha}{3\pi}\int_{4m_e^2}^{\infty} ds\frac{1}{s(s-q^2-i\epsilon)}\sqrt{1-\frac{4m_e^2}{s}}\left(1+\frac{2m_e^2}{s}\right)$$

LW theories seem mostly healthy

$$G(q) = \frac{1}{(q^2 + i\epsilon)\left(1 - \frac{N\alpha}{3\pi}[\log|q^2|/m_r^2 - i\pi] - \frac{q^2}{m_r^2}\right)}$$

Unitarity seems satisfied – direct calculation

But “micro-causality” is violated
- non-causal propagation of order the ghost width

Modern overview of unitarity and causality
-Grinstein, O’Connell, Wise (2009)

Strong similarity to quadratic gravity

Loops do the same in the graviton propagator

Both matter and graviton loops

Tomboulis 1977

Effect of unitarity

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left[L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right]}{2q^2\left(1 - \frac{N_s G_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}\,.$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu/q^2.$$

Decay of high mass ghost states could be a common feature

Other issue: Kallen-Lehmann spectral representation:

Positivity of spectral density leads to propagator

$$\Delta(p) = \int_0^\infty d\mu^2 \rho(\mu^2) \frac{1}{p^2 - \mu^2 + i\epsilon},$$

Scalar propagator cannot fall faster than $1/p^2$ in UV

But not valid for gauge fields

Oehme
Zimmerman

$$\Delta(p^2) \sim \frac{1}{p^2}[\ln(\frac{p^2}{\mu^2})]^{-\alpha_0/2b}$$

$$\frac{\alpha_0}{2b} = \frac{13}{22}$$

Also not valid in LW theories

- microcausality violation
- indefinite metric

And: Ostrogradsky instability

With $\mathcal{L} = \mathcal{L}(\phi, \dot{\phi}, \ddot{\phi})$,

Woodard
Scolarpedia

Two coordinates ϕ and $\dot{\phi}$

Two momenta:
$$\pi_1 = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \ddot{\phi}}$$
$$\pi_2 = \frac{\partial \mathcal{L}}{\partial \ddot{\phi}} \quad .$$

Form Hamiltonian:

$$\mathcal{H} = \pi_1 \dot{\phi} + \pi_2 a(\phi, \dot{\phi}, \pi_2) - \mathcal{L}(\phi, \dot{\phi}, a(\phi, \dot{\phi}, \pi_2))$$

First term is signal of instability

But can be circumvented by modified quantization rules
- LW, Bender-Mannheim, Salvio-Strumia, Raidal-Veermae

This does not seem to appear in Lagrangian PI formulation

Simple example without instability:

Consider fourth order gauge theory:

$$\mathcal{L} = \frac{1}{2}(D^2\phi)^\dagger D^2\phi + ...$$

Doing PI yields:

$$\frac{1}{[\det(D^2D^2)]^{1/2}} = e^{-\frac{1}{2}Tr\log(D^2D^2)} = e^{-\frac{1}{2}\int d^4x<x|Tr\log(D^2D^2)|x>}$$

But we have:

$$\log(D^2D^2) = 2\log(D^2)$$

Up to constant, this is the usual gauge interaction

JFD
PRD 2017

Aside: Test case

Triplet “colored” scalar in SU(2) with quartic propagator

SU(2) valued field

$U = e^{i\frac{\tau^a \phi^a}{f}}$ with $U \to V(x) U V^\dagger(x)$ $V(x)$ in SU(2).

with scale invariant Lagrangian

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{f^2}{4} Tr\left[(D^\mu D_\mu U)^\dagger (D^\nu D_\nu U)\right] \\
&+ d_1 \left(Tr\left[D^\mu U^\dagger D_\mu U\right]\right)^2 + d_2 Tr\left[D^\mu U^\dagger D^\nu U\right] Tr\left[D_\mu U^\dagger D_\mu U\right] \\
&+ d_3 Tr\left[D^\mu U^\dagger D^\nu U D_\mu U^\dagger D_\mu U\right] + d_4 Tr\left[U^\dagger D^2 U\right] Tr\left[D_\mu U^\dagger D^\mu U\right] \\
&+ d_5 Tr\left[U^\dagger D^2 U D_\mu U^\dagger D^\mu U\right] + d_6 Tr\left[U^\dagger D^2 U\right] Tr\left[U^\dagger D^2 U\right] \\
&+ d_7 F^i_{\mu\nu} Tr\left[\tau^i D^\mu U^\dagger D^\nu U\right] + d_8 F^i_{\mu\nu} F^{j\mu\nu} Tr\left[\tau^i U^\dagger \tau^j U\right]
\end{aligned}
$$

Scalar will be confined – usual discussions of g.s. not relevant.
Should be possible to simulate on lattice

Background field renormalization can be accomplished

- for suitably general Lagrangian

With $\mathcal{L}(U) = \mathcal{L}(\bar{U}) + \Delta^a \mathcal{O}^{ab} \Delta^b + ...$

$$\mathcal{O}^{ab} = \left[D^2 D^2 + A^{\alpha\beta\gamma} D_\alpha D_\beta D_\gamma + B^{\alpha\beta} D_\alpha D_\beta + C^\alpha D_\alpha + E\right]^{ab}$$

The divergences are captured in the heat-kernel coefficient:

$$Tr < x| \log \mathcal{D} |x > |_{div} = \frac{i}{(4\pi)^{d/2}} \Gamma(2 - \frac{d}{2})\ Tr\ a_2(x)$$

Barvinsky and Vilkovisky have worked this out

$$\begin{aligned}
a_2 \ = \ & \frac{1}{6} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8} F_{\mu\nu} [D^\mu, A^\nu] + \frac{9}{80} F_{\mu\nu} A^{\mu\alpha\beta} A^\nu{}_{\alpha\beta} + \frac{9}{160} F_{\mu\nu} A^\mu A^\nu + \frac{1}{8} C_\mu A^\mu + \frac{1}{24} B_{\mu\nu} B^{\mu\nu} + \frac{1}{48} B^2 \\
- \ & \frac{1}{16} B D_\mu A^\mu + \frac{1}{8} B^{\mu\nu} D_\mu A_\nu - \frac{1}{8} B^{\mu\nu} D^\alpha A_{\mu\nu\alpha} + \frac{9}{80} A^{\mu\nu\alpha} D_\mu D_\nu A_\alpha - \frac{3}{80} A^\mu D_\mu D_\nu A^\nu - \frac{3}{160} A^\mu D^2 A_\mu \\
- \ & \frac{3}{40} A^{\mu\nu\alpha} D_\mu D_\beta A^\beta{}_{\nu\alpha} - \frac{1}{80} A^{\mu\nu\alpha} D^2 A_{\mu\nu\alpha} - \frac{1}{640} B (2A^{\mu\nu\alpha} A_{\mu\nu\alpha} + 3A^\mu A_\mu) \\
- \ & \frac{3}{320} B^{\mu\nu} (2A_{\mu\alpha\beta} A_\nu{}^{\alpha\beta} + A_\mu A_\nu + A_{\mu\nu\alpha} A^\alpha + A^\alpha A_{\mu\nu\alpha}) - \frac{1}{640} A^\beta D_\beta (2A^{\mu\alpha\beta} A_{\mu\alpha\beta} + 3A^\mu A_\mu) \\
- \ & \frac{1}{640} A^\beta [2(D_\beta A^{\mu\nu\alpha}) A_{\mu\nu\alpha} + 3(D_\beta A^\mu) A_\mu] - \frac{3}{160} A^{\mu\nu\alpha} D_\mu (A_\nu A_\alpha + 2A_{\nu\beta\gamma} A_\alpha^{\ \beta\gamma} + A_{\nu\alpha\beta} A^\beta + A^\beta A_{\nu\alpha\beta}) \\
+ \ & \frac{3}{320} A^{\mu\nu\alpha} (A_\nu D_\mu A_\alpha + 2A_{\nu\beta\gamma} D_\mu A_\alpha^{\ \beta\gamma} + A_{\nu\alpha\beta} D_\mu A^\beta + A^\beta D_\mu A_{\nu\alpha\beta}) \\
+ \ & \frac{1}{960 \times 32 \times 42} A^{\mu\nu\alpha} A^{\beta\gamma\delta} A^{\epsilon\sigma\rho} A^{\lambda\omega\eta} g_{\mu\nu\alpha\beta\gamma\epsilon\sigma\rho\lambda\omega\eta}
\end{aligned}$$

Starting point for us:

Minkowski path integral

$$\int [dh_{\mu\nu}]\ e^{i\int d^4x \sqrt{-g}\mathcal{L}_g}$$

Perturbative for now

Perhaps we will need to adjust along the way

Analogy: Dirac field

Quadratic gravity

$$S_{\text{quad}} = \int d^4x\sqrt{-g}\left[\frac{1}{6f_0^2}R^2 - \frac{1}{2\xi^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} - \eta G\right]$$

Free-field mode decomposition depends on gauge fixing.

-all contain a scalar mode and tensor mode

Scalar has massive non-ghost and massless ghost – due to R^2

$$D^{(0)}_{\mu\nu\alpha\beta}(q^2) = \left(\frac{q^4}{f_0^2} - \frac{2q^2}{\kappa^2}\right)^{-1}\mathcal{P}^{(0)}_{\mu\nu\alpha\beta} = \frac{\kappa^2}{2}\left(\frac{1}{q^2 - M_0^2} - \frac{1}{q^2}\right)\mathcal{P}^{(0)}_{\mu\nu\alpha\beta}$$

Spin 2 mode has massive ghost

$$D^{(2)}_{\mu\nu\alpha\beta}(q^2) = \left(\frac{q^2}{\kappa^2} - \frac{q^4}{2\xi^2}\right)^{-1}\mathcal{P}^{(2)}_{\mu\nu\alpha\beta} = \kappa^2\left(\frac{1}{q^2} - \frac{1}{q^2 - M_2^2}\right)\mathcal{P}^{(2)}_{\mu\nu\alpha\beta}$$

Renormalizeable QFT:

Stelle,
Julve-Tonin
Fradkin - Tseytlin

Dimensionless coupling constants

Background field methods preserve symmetry

Weyl term is asymptotically free

R^2 term is not AF if f_0^2 is positive
- but it is renormalizeable
- perhaps revisit this sector with “unstable ghosts” paradigm

Now look at effect of "helper" YM gauge interaction

Keep R^2 terms weakly coupled at the Planck scale

Helper YM will become strong at some scale
- call this the Planck scale
- will generate non-scale invariant terms in gravity action

Most particularly, can generate G

Low energy gravity turns into Einstein gravity
- also weakly coupled (for different reason)

Calculating the induced G for QCD

Adler-Zee formula:

$$\frac{1}{16\pi G} = \frac{i}{96} \int d^4x \; x^2 < 0|T \; T(x)T(0)|0 >$$

where:

$$T(x) = \eta_{\mu\nu} T^{\mu\nu}(x)$$

Quick derivation:

Zee

Weak field limit:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Gravitaional coupling:

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} T_{\mu\nu}$$

To second order in the gravitational field:

$$i \int d^4x \mathcal{L}_{eff} = \frac{1}{2} \left(\frac{-i}{2} \right)^2 \int d^4x \; d^4y \; h_{\mu\nu}(x) h_{\alpha\beta}(y) < 0|T \; T^{\mu\nu}(x) T^{\alpha\beta}(y)|0 >$$

For convenience, consider special case

(see also Brown, Zee)

$$h_{\mu\nu}(x) = \frac{1}{4} \eta_{\mu\nu} h(x)$$

Slowly varying fields (on QCD scale):

$$h(y) = h(x) + (y-x)^\mu \partial_\mu h(x) + \frac{1}{2}(y-x)^\mu (y-x)^\nu \partial_\mu \partial_\nu h(x) + \ldots.$$

This results in the effective Lagrangian:

$$
\begin{aligned}
i\mathcal{L}_{eff}(x) &= -\frac{1}{128}h^2(x)\int d^4z \ <0|T\ T(z)T(0)|0> \\
&+ \frac{1}{1024}(\partial_\mu h(x))^2\int d^4z\ z^2 \ <0|T\ T(z)T(0)|0>
\end{aligned}
$$

The first term is part of the cosmological constant, the second is the Einstein action. Identify via:

$$\sqrt{-g}R = -\frac{3}{32}(\partial_\mu h(x))^2$$

This gives the Adler-Zee formula:

$$\frac{1}{16\pi G} = \frac{i}{96}\int d^4x\ x^2 <0|T\ T(x)T(0)|0>$$

QCD

For QCD (with no quarks or massless quarks)

$$T^{\mu}_{\ \mu} = \frac{\beta(g)}{2g} F^{a}_{\mu\nu} F^{a\mu\nu}$$

From trace anomaly

F^2 creates a scalar glueball from the vacuum

The scalar correlator

$$< 0|F^2(x)\ F^2(0)|0 >$$

has been well studied in QCD.

Induced G – in SU(N) theories

JFD, Menezes

Construct the sum rule for QCD – lattice data, OPE and pert. theory

Separate long and short distance techniques at $x=x_0$

$$\psi(x) = <0|T\,T(x)T(0)|0> = [\psi_{pert}(x) + \psi_{OPE}(x)]\,\Theta(x_0 - x) + \psi_{lattice}(x)\Theta(x - x_0)$$

Perturbative:

$$\psi_{pert} = \frac{C_\psi}{x^8(\log(1/\Lambda^2 x^2))^2} \quad , \qquad C_\psi = \frac{96}{\pi^4}$$

(Adler)

OPE:

$$\psi_{OPE} = \left(\frac{b}{8\pi}\right)^2 \left[\frac{\alpha_s^2 b}{\pi^3 x^4} < \alpha_s G^2 > + \frac{2\alpha_s^2}{\pi^2 x^2} < gG^3 > + \frac{29\alpha_s^3 \log(\mu^2 x^2)}{2\pi^2 x^2} < gG^3 > \right]$$

(NSVZ, Bagan, Steele)

Lattice: $\psi_{lattice} = \frac{t_0^2 M_g}{4\pi^2 x} K_1(M_g x)$ (Wightman function)

$t_0 = 1.1 \pm 0.22$ GeV^3 $\qquad M_g = 1.71 \pm 0.05 \pm 0.08$ GeV

(Chen, et al.2006)

There are subtle features

$$v = -\log(\Lambda^2 x^2)$$

The perturbative part needs to be regularized:
- Adler contour (dim. reg.)
- We have a second regularization (agrees)

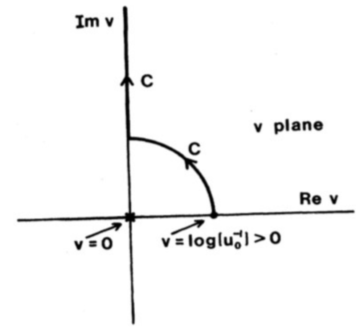


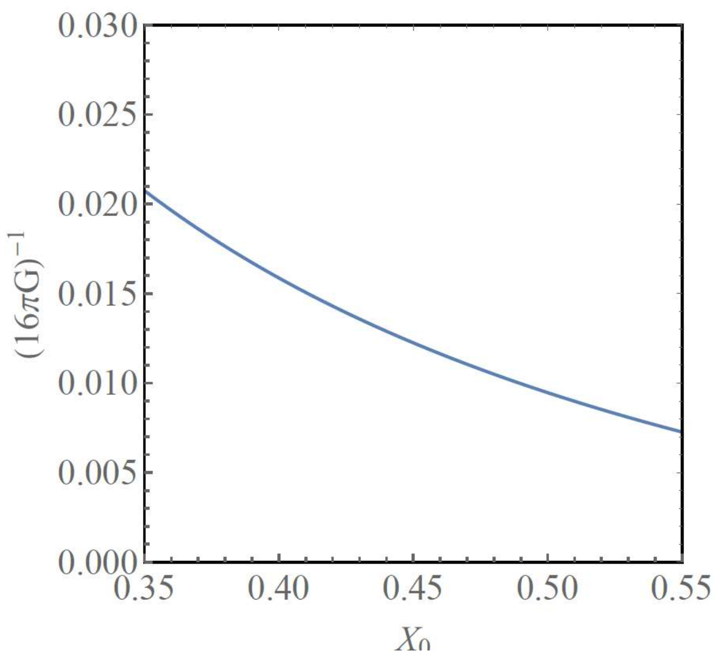
FIG. 5. Contour of integration C to be used in evaluating Eq. (5.56). The contour begins at $v=\log u_0^{-1}=\log(\mathscr{M}^2 t_0)^{-1}$ and must avoid the singularity at $v=0$.

Result is forced to be real

Matching at x=x_0 is not perfect
- scalar glueball sum rule analysis historically difficult

Results

Pure QCD – **Induced G is positive**



Matching at $X_0^{-1} = 2$ GeV:

$$\frac{1}{16\pi G_{\rm ind}} = 0.095 \pm 0.030 \ \mathrm{GeV}^2$$

Note also:

Cosmological constant:

$$\Lambda_{\text{ind}} = \frac{1}{4}\langle 0|T^{\mu}_{\ \mu}|0\rangle = \frac{1}{4}\left\langle 0\left|\frac{\beta(g)}{2g}F^{a}_{\mu\nu}F^{a\mu\nu}\right|0\right\rangle$$

With standard values, Λ = -0.0044 GeV^4 (return to this later)

Induced R^2 term

Brown-Zee

$$\frac{1}{6f_{\text{ind}}^2} = \frac{i}{13824}\int d^4z\,(z^2)^2\langle T\{\bar{T}(z)\bar{T}(0)\}\rangle.$$

We find (low energy only):

$$\frac{1}{6f_{\text{ind}}^2} = 0.00079 \pm 0.00030.$$

Intermediate summary #1

Dimensional transmutation may allow a scale invariant and renormalizeable model to lead to Einstein-Hilbert action

Could scale a QCD-like theory up to the Planck scale

The Spin 2 propagator

Projection operators

$$\mathcal{P}^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}$$
$$\mathcal{P}^{(1)}_{\mu\nu\rho\sigma} = \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \omega_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \omega_{\mu\sigma}\theta_{\nu\rho})$$
$$\mathcal{P}^{(0)}_{\mu\nu\rho\sigma} = \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}$$
$$\bar{\mathcal{P}}^{(0)}_{\mu\nu\rho\sigma} = \omega_{\mu\nu}\omega_{\rho\sigma}$$

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$
$$\omega_{\mu\nu} = \frac{q_\mu q_\nu}{q^2}.$$

The spin 2 part of the propagator

$$\tilde{D}_{\mu\nu\alpha\beta}(q^2) = \mathcal{P}^{(2)}_{\mu\nu\alpha\beta} D(q^2),$$

First: Vacuum polarization of normal light fields

- the EFT regime

$$\Pi_{\mu\nu,\alpha\beta}(q^2) = -\frac{N_{\text{eff}}^{(0)}}{640\pi^2} q^4 \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right) \mathcal{P}^{(2)}_{\mu\nu\alpha\beta}.$$

- with usual prescription

$$\ln(-q^2 - i\epsilon) = \ln(|q^2|) - i\pi\theta(q^2)$$

Leads to propagator

$$D^{-1}(q^2) = \frac{q^2 + i\epsilon}{\bar{\kappa}^2} - \frac{q^4}{2\xi^2(\mu)} - \frac{N_{\text{eff}}}{640\pi^2} q^4 \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right)$$

Next, the very high energy regime

- non-gravitational light fields are the same
- gravity now quartic propagators not quadratic

No imaginary part generated from quadratic gravity

-on shell triple graviton coupling vanishes in Minkowski

$$\sqrt{-g}\ R^2 = \ldots\ + \frac{1}{2} h^\lambda_\lambda R^{(1)} R^{(1)} + 2R^{(1)} R^{(2)}\ +\ \ldots$$

-on shell condition is R=0

Or regularize with $D \sim \dfrac{1}{q^4 + \epsilon^2}$

Logs then come with $-\dfrac{q^4 N_q}{1280\pi^2} \ln\left(\dfrac{(q^2)^2}{\mu^4}\right)$ $\qquad N_q = 199/3$

Verified by complete calculation

$$
\begin{aligned}
\mathcal{T}^{\mu\nu}{}_{\alpha\beta\gamma\delta}(k,q) \;=\;& -\frac{i}{2\xi^2}\Bigg\{\Big[\eta^{\mu\nu}q_\lambda q_\kappa + I^{\mu\nu}{}_{\lambda\kappa}q^2 - q^\sigma\Big(I^{\mu}{}_{\sigma\lambda\kappa}q^\nu + I^{\nu}{}_{\sigma\lambda\kappa}q^\mu\Big)\Big]\bar{\mathcal{R}}^{\lambda\kappa}{}_{\alpha\beta\gamma\delta}(k,q)\\
&- \frac{2}{3}\Big(\eta^{\mu\nu}q^2 - q^\mu q^\nu\Big)\bar{\mathcal{R}}_{\alpha\beta\gamma\delta}(k,q) - 2\mathcal{P}^{\mu\nu\sigma\tau}I_\sigma{}^\lambda{}_{\gamma\delta}I_{\tau\lambda\alpha\beta}p^2k^2\\
&+ \Big(\mathcal{P}^{\mu\nu}{}_{\sigma\lambda} - \frac{1}{4}\eta^{\mu\nu}\eta_{\sigma\lambda}\Big)q^\sigma\Big(p^\lambda k^2 + k^\lambda p^2\Big)I_{\alpha\beta\gamma\delta} - \Big(p^\mu p^\nu k^2 + k^\mu k^\nu p^2\Big)I_{\alpha\beta\gamma\delta}\\
&- \frac{1}{2}q^2\Big[I_\lambda{}^\nu{}_{\gamma\delta}I^{\lambda\mu}{}_{\alpha\beta}k^2 + I_\lambda{}^\nu{}_{\alpha\beta}I^{\lambda\mu}{}_{\gamma\delta}p^2 + (\mu\leftrightarrow\nu)\Big]\\
&+ q_\rho\Big[k^2\Big(I_\lambda{}^\nu{}_{\gamma\delta}I^{\lambda\mu}{}_{\alpha\beta}p^\rho + p^\mu(I^{\nu\lambda}{}_{\gamma\delta}I^{\rho}{}_{\lambda\alpha\beta} - I^{\rho\lambda}{}_{\gamma\delta}I^{\nu}{}_{\lambda\alpha\beta}) + \frac{1}{2}q^\mu\Big(I^{\rho\lambda}{}_{\gamma\delta}I^{\nu}{}_{\lambda\alpha\beta} - I^{\nu\lambda}{}_{\gamma\delta}I^{\rho}{}_{\lambda\alpha\beta}\Big)\Big)\\
&+ p^2\Big(I_\lambda{}^\nu{}_{\alpha\beta}I^{\lambda\mu}{}_{\gamma\delta}k^\rho + k^\mu(I^{\nu\lambda}{}_{\alpha\beta}I^{\rho}{}_{\lambda\gamma\delta} - I^{\rho\lambda}{}_{\alpha\beta}I^{\nu}{}_{\lambda\gamma\delta}) + \frac{1}{2}q^\mu\Big(I^{\rho\lambda}{}_{\alpha\beta}I^{\nu}{}_{\lambda\gamma\delta} - I^{\nu\lambda}{}_{\alpha\beta}I^{\rho}{}_{\lambda\gamma\delta}\Big)\Big) + (\mu\leftrightarrow\nu)\Big]\\
&+ q_\lambda q_\sigma\Big[k^2\Big(I^{\lambda\nu}{}_{\gamma\delta}I^{\mu\sigma}{}_{\alpha\beta} - \frac{1}{2}\Big(I^{\mu\nu}{}_{\gamma\delta}I^{\sigma\lambda}{}_{\alpha\beta} + I^{\mu\nu}{}_{\alpha\beta}I^{\lambda\sigma}{}_{\gamma\delta}\Big)\Big) + p^2\Big(I^{\lambda\nu}{}_{\alpha\beta}I^{\mu\sigma}{}_{\gamma\delta} - \frac{1}{2}\Big(I^{\mu\nu}{}_{\alpha\beta}I^{\sigma\lambda}{}_{\gamma\delta} + I^{\mu\nu}{}_{\gamma\delta}I^{\lambda\sigma}{}_{\alpha\beta}\Big)\Big)\\
&+ \frac{1}{2}q^\sigma\Big(k_\kappa\Big(I^{\mu\nu}{}_{\gamma\delta}I^{\kappa\lambda}{}_{\alpha\beta} - I^{\lambda\nu}{}_{\gamma\delta}I^{\mu\kappa}{}_{\alpha\beta}\Big) + p_\kappa\Big(I^{\mu\nu}{}_{\alpha\beta}I^{\kappa\lambda}{}_{\gamma\delta} - I^{\lambda\nu}{}_{\alpha\beta}I^{\mu\kappa}{}_{\gamma\delta}\Big)\Big)\\
&+ \frac{1}{2}q^\nu\Big(k_\kappa\Big(I^{\lambda\sigma}{}_{\gamma\delta}I^{\mu\kappa}{}_{\alpha\beta} - I^{\mu\lambda}{}_{\gamma\delta}I^{\kappa\sigma}{}_{\alpha\beta}\Big) + p_\kappa\Big(I^{\lambda\sigma}{}_{\alpha\beta}I^{\mu\kappa}{}_{\gamma\delta} - I^{\mu\lambda}{}_{\alpha\beta}I^{\kappa\sigma}{}_{\gamma\delta}\Big)\Big) + (\mu\leftrightarrow\nu)\Big]\Bigg\} \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\mathcal{R}}^{\lambda\kappa}{}_{\alpha\beta\gamma\delta}(k,q) \;=\;& -I_{\alpha\beta\gamma\delta}q^\lambda q^\kappa + q^\sigma\Big[I_{\sigma\tau\gamma\delta}\Big(I^{\tau\kappa}{}_{\alpha\beta}k^\lambda - I^{\lambda\kappa}{}_{\alpha\beta}k^\tau\Big) + I_{\sigma\tau\alpha\beta}\Big(I^{\tau\kappa}{}_{\gamma\delta}p^\lambda - I^{\lambda\kappa}{}_{\gamma\delta}p^\tau\Big)\Big]\\
&+ \Big(I^{\tau\lambda}{}_{\gamma\delta}p_\sigma + I^{\tau}{}_{\sigma\gamma\delta}p^\lambda - I^{\lambda}{}_{\sigma\gamma\delta}p^\tau\Big)\Big(I^{\sigma\kappa}{}_{\alpha\beta}k_\tau + I^{\sigma}{}_{\tau\alpha\beta}k^\kappa - I_\tau{}^\kappa{}_{\alpha\beta}k^\sigma\Big) + k^2 I^{\kappa}{}_{\tau\gamma\delta}I^{\lambda\tau}{}_{\alpha\beta} + p^2 I^{\kappa}{}_{\tau\alpha\beta}I^{\lambda\tau}{}_{\gamma\delta}\\
&- 2k_\sigma k_\tau\Big(\frac{1}{3}I^{\lambda\kappa}{}_{\gamma\delta}I^{\sigma\tau}{}_{\alpha\beta} + \frac{1}{2}I^{\kappa\sigma}{}_{\gamma\delta}I^{\tau\lambda}{}_{\alpha\beta}\Big) - 2p_\sigma p_\tau\Big(\frac{1}{3}I^{\lambda\kappa}{}_{\alpha\beta}I^{\sigma\tau}{}_{\gamma\delta} + \frac{1}{2}I^{\kappa\sigma}{}_{\alpha\beta}I^{\tau\lambda}{}_{\gamma\delta}\Big)\\
&- q^\lambda\Big(I^{\kappa\sigma}{}_{\gamma\delta}I_{\tau\sigma\alpha\beta}k^\tau + I^{\kappa\sigma}{}_{\alpha\beta}I_{\tau\sigma\gamma\delta}p^\tau\Big) \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\mathcal{R}}_{\alpha\beta\gamma\delta}(k,q) \;=\;& -q^2 I_{\alpha\beta\gamma\delta} + \Big(I^{\lambda}{}_{\tau\gamma\delta}p_\sigma + I^{\lambda}{}_{\sigma\gamma\delta}p_\tau - I_{\tau\sigma\gamma\delta}p^\lambda\Big)\Big(I_\lambda{}^\tau{}_{\alpha\beta}k^\sigma + I^{\tau\sigma}{}_{\alpha\beta}k_\lambda - I_\lambda{}^\sigma{}_{\alpha\beta}k^\tau\Big)\\
&+ I^{\lambda}{}_{\sigma\gamma\delta}k_\tau\Big(I^{\tau}{}_{\lambda\alpha\beta}k^\sigma + I^{\tau\sigma}{}_{\alpha\beta}k_\lambda - I^{\sigma}{}_{\lambda\alpha\beta}k^\tau\Big) + I^{\lambda}{}_{\sigma\alpha\beta}p_\tau\Big(I^{\tau}{}_{\lambda\gamma\delta}p^\sigma + I^{\tau\sigma}{}_{\gamma\delta}p_\lambda - I^{\sigma}{}_{\lambda\gamma\delta}p\\
&+ \frac{1}{2}q^\lambda\Big(I_{\lambda\sigma\gamma\delta}I^{\kappa\sigma}{}_{\alpha\beta}k_\kappa + I_{\lambda\sigma\alpha\beta}I^{\kappa\sigma}{}_{\gamma\delta}p_\kappa\Big).
\end{aligned}
$$

Overall:

$$D_2^{-1}(q) = \frac{q^2 + i\epsilon}{\tilde{\kappa}^2(q)} - \frac{q^4}{2\xi^2(\mu)} - \frac{q^4 N_{\rm eff}}{640\pi^2} \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right) - \frac{q^4 N_q}{1280\pi^2} \ln\left(\frac{(q^2)^2}{\mu^4}\right)$$

Pole in propagator

- in weak coupling, occurs well below Planck mass

$$\tilde{\kappa} = \kappa \qquad N_q = 0$$

Expand $q^2 = m_r^2 + \delta q^2$

Real part vanishes for $m_r^2 = \dfrac{2\xi^2(m_r)}{\kappa^2}$

Near this point $D^{-1}(q) = -\dfrac{\delta q^2}{\kappa^2}\left(1 + \dfrac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}\right) + i\dfrac{2\xi^2 m_r^2}{\kappa^2}\dfrac{N_{\rm eff}}{640\pi}$.

Imaginary part is down by two more powers of the coupling

$$\gamma = 2\xi^2 m_r^2 \frac{N_{\rm eff}}{640\pi\left(1 + \frac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}\right)}$$

Unusual features:

Norm and sign of imaginary part are opposite normal

$$D(q) = \left[\frac{\kappa^2}{1+\frac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}}\right] \frac{1}{-\delta q^2 + i\gamma} = \left[\frac{\kappa^2}{1+\frac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}}\right] \frac{-1}{\delta q^2 - i\gamma}$$

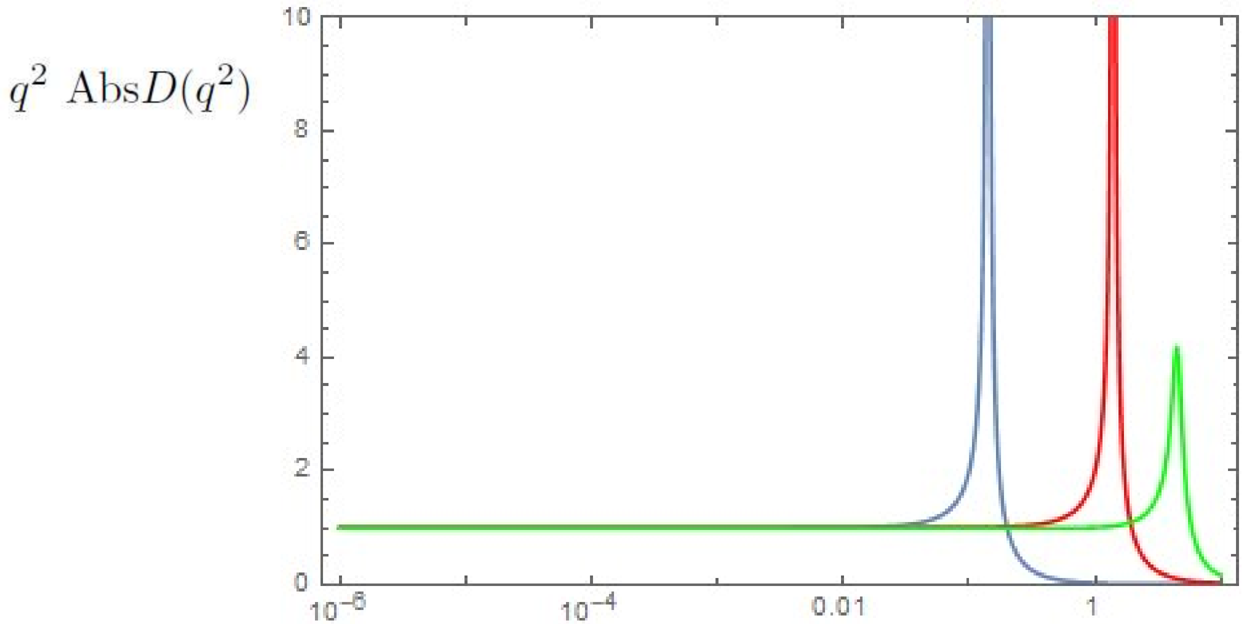
vs

$$D(q) = \frac{1}{q^2 - (M - i\frac{\Gamma}{2})^2} \qquad\qquad D(q) = \frac{1}{\delta q^2 + iM\Gamma} \ .$$

The two signs are important:

$$\frac{1}{i\gamma} = \frac{1}{iM\Gamma} \qquad\qquad iD(q^2) \sim \frac{iZ}{q^2 - m_r^2 - iZ\gamma}$$

Resonance in propagator:



$\xi^2 = 0.1,\ 1,\ 10$

The propagator at long distances/times

Low momentum corrections are tiny ~ q^4

$$\frac{1}{\frac{q^2+i\epsilon}{\kappa^2} - \frac{q^4}{2\xi^2(\mu)} - \frac{N_{\rm eff}}{640\pi^2} q^4 \ln\left(\frac{-q^2-i\epsilon}{\mu^2}\right)} \sim \kappa^2 \left[\frac{1}{q^2+i\epsilon} + \frac{\kappa^2}{2\xi^2(\mu)} + \frac{\kappa^2 N_{\rm eff}}{640\pi^2} \ln\left(\frac{-q^2}{\mu^2}\right) + \ldots\right]$$

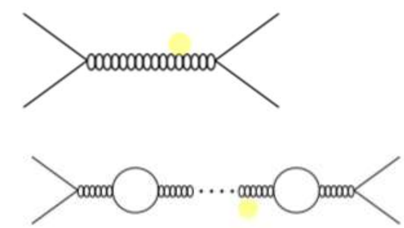
Long distance effects are just the EFT logarithms

$$D(x-y) = \kappa^2 \left[D_0(x-y) + \frac{\kappa^2}{2\xi^2(\mu)} \delta^4(x) + \frac{\kappa^2 N_{\rm eff}}{640\pi^2} L(x)\right]$$

$$L(x) = \int \frac{d^4q}{(2\pi)^4} e^{-q\cdot(x-y)} \ln(-q^2)$$

Unitarity in the spin two channel

Do these features cause trouble in scattering?
- consider scattering in spin 2 channel



First consider single scalar at low energy:

$$i\mathcal{M} = \left(\frac{1}{2}V_{\mu\nu}(q)\right)\left[iD^{\mu\nu\alpha\beta}(q^2)\right]\left(\frac{1}{2}V_{\alpha\beta}(-q)\right)$$

$$\mathcal{M} = 16\pi\sum_{J=0}^{\infty}(2J+1)T_J(s)P_J(\cos\theta)$$

Results in

$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\bar{D}(s).$$

$N_{\rm eff} = 1/6$ for a single scalar field

$$\bar{D}^{-1}(s) \;=\; \frac{1}{\tilde{\kappa}^2}\left\{1 - \frac{\tilde{\kappa}^2 s}{2\xi^2(\mu)} - \frac{\tilde{\kappa}^2 s N_{\rm eff}}{640\pi^2}\ln\left(\frac{s}{\mu^2}\right) + \frac{i\tilde{\kappa}^2 s N_{\rm eff}}{640\pi}\right\}$$

Satisfies elastic unitarity:

$$\mathrm{Im}T_2 = |T_2|^2.$$

This implies the structure

$$T_2(s) = \frac{A(s)}{f(s) - iA(s)} = \frac{A(s)[f(s) + iA(s)]}{f^2(s) + A^2(s)}$$

for any real *f(s)*

Signs and magnitudes work out for $A(s) = -\frac{N_{\mathrm{eff}} s}{640\pi}.$

Multi-particle problem:

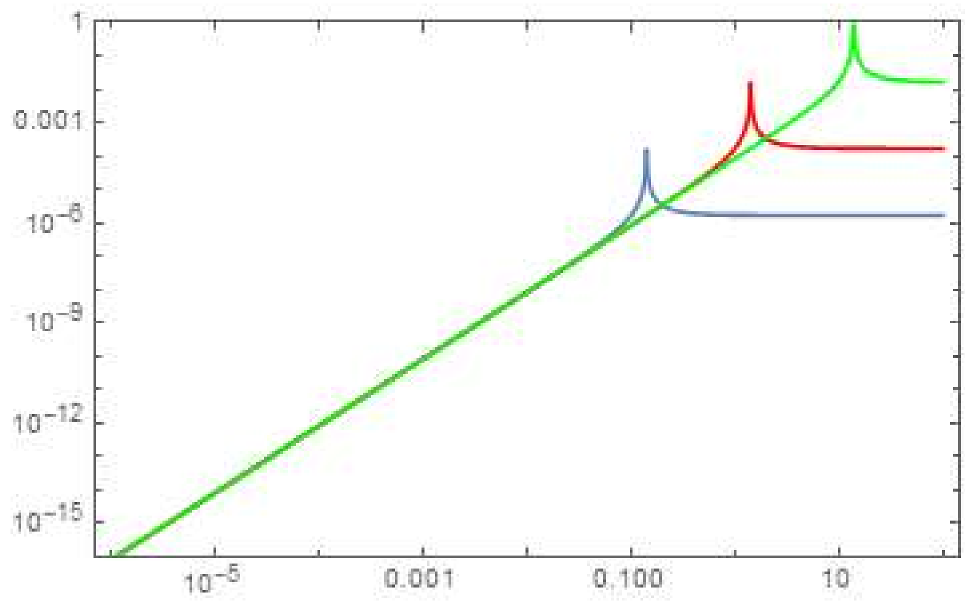
- just diagonalize the J=2 channel
- same result but with general N

Very high energy region:

- quartic propagators for gravity
- does not couple to on-shell gravitons
- also does not contribute to imaginary parts
- unitarity still works

$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\left\{\frac{1}{\kappa^2} - \frac{s}{2\xi^2(\mu)} - \frac{sN_{\rm eff}}{640\pi^2}\left[\ln\left(\frac{s}{\mu^2}\right) - i\pi\right] - \frac{sN_q}{1280\pi^2}\ln\left(\frac{s^2}{\mu^4}\right)\right\}^{-1}.$$

Scattering amplitude:



The cosmological constant problem

Embarrassment for general picture

- vacuum energy generated from scale invariant start

Some options:

1) Unimodular gravity – vacuum energy is not relevant

2) Caswell-Banks-Zaks – beta function vanishes in IR

3) Strong coupled gravity – perhaps gravity cancels gauge

4) Spin connection as helper gauge theory – no vev JFD'16

5) Supersymmetric helper gauge theory – no c.c. at this scale

6) Abandon our principles – just add a constant to cancel it

Intermediate summary #2

Interactions make the ghosts unstable

Signs in propagator unusual

But these signs are important for consistency, unitarity

Unstable ghosts in loops

Note: above gravity loops done with quadratic **or** quartic propagators

Ghost pole not included in the gravity loops so far

Will the high mass unstable ghosts cause trouble in loops?

Unstable particles in loops is its own research subfield
- but here the bigger issues are the norms and signs

Approximation for here:
- look at main effects near region of the pole

$$\tilde{\kappa} = \kappa \qquad N_q = 0$$

Modified Lehman and higher order loops

Physical propagator evaluated with $q^2 + i\epsilon$

$$D^{-1}(q^2) = \frac{q^2 + i\epsilon}{\tilde{\kappa}^2} - \frac{q^4}{2\xi^2(\mu)} - \frac{N_{\rm eff}}{640\pi^2} q^4 \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right)$$

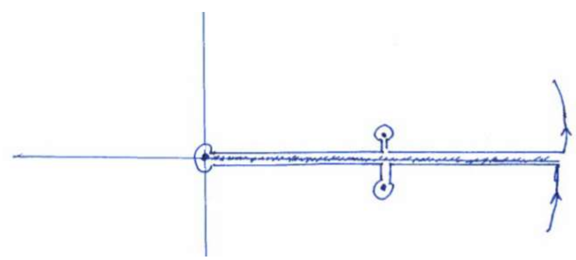
Cut along real axis

Another pole found on other side of cut using $q^2 - i\epsilon$

Then

$$f(a) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{z-a}\, dz$$

plus



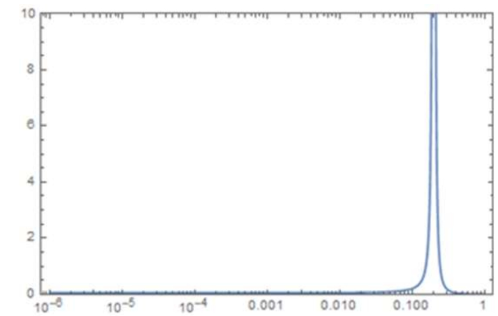
leads to a representation

$$D(q) = \frac{1}{q^2 + i\epsilon} - \frac{\beta}{q^2 - M^2} - \frac{\beta^*}{q^2 - M^{*2}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

<u>This is really three resonance structures</u>

Fix spectral function by matching

$$\rho(s) = \frac{\frac{N_{eff}\kappa^2}{640\pi}}{\left(1 - \frac{s}{m_r^2} - \frac{N_{eff}\kappa^2 s}{640\pi^2}\log\frac{s}{m_r^2}\right)^2 + \left(\frac{N_{eff}\kappa^2 s}{640\pi}\right)^2}$$



Previously we found a single resonance in propagator

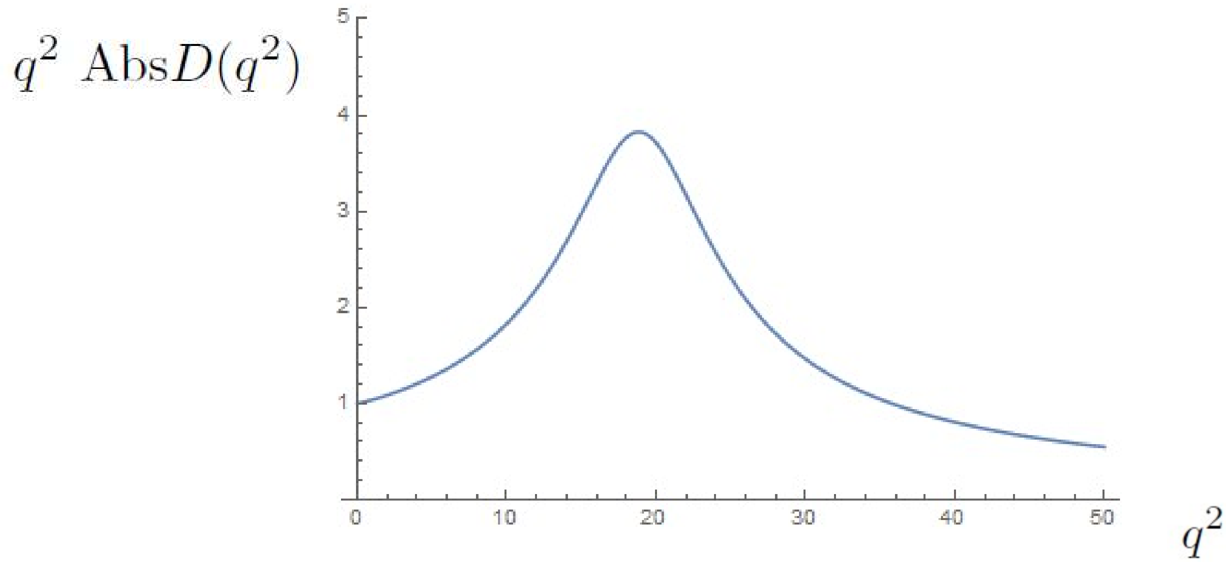
In Lehman representation we have three

The spectral portion and the cc pole essentially cancel

- exact in narrow width approximation

G.O'C.W

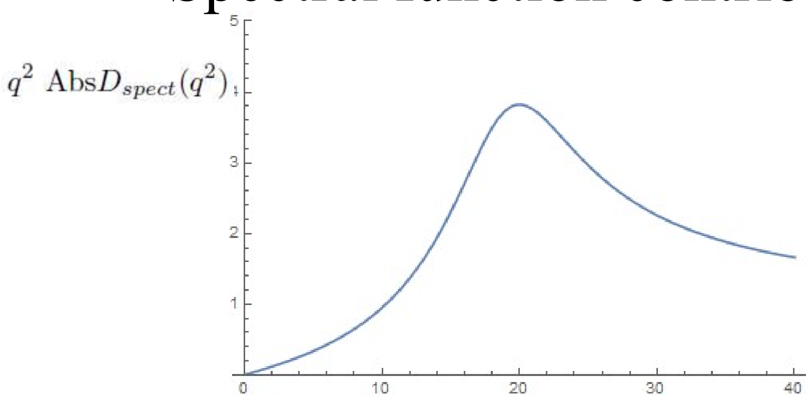
Beyond the narrow width approximation $\xi^2 = 10$



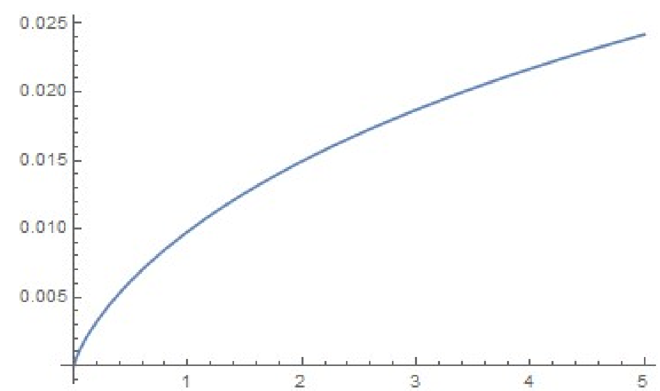
$M^2 = 18.9 + 4.7i$

$\beta = .925 + 0.17i$

Spectral function contribution



Diff. (spect. – cc pole)



Note change in scale

Lehman representation explains a lot of the physics

$$D(q) = \frac{1}{q^2 + i\epsilon} - \frac{\beta}{q^2 - M^2} - \frac{\beta^*}{q^2 - M^{*2}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

Imaginary parts only arise in “normal” parts of the propagator

Unitarity according to G. O’C.W

Imaginary parts from:

-massless x massless

- massless x spectral
- spectral x spectral

These are the normal particles in the theory

Two thresholds are new, but expected

- producing the heavy unstable ghost state

Gabriel Menezes

Alternative path:

Uses narrow width approximation

- but no unusual contour during integration

Steps:

$$D_2(q) \sim -\frac{1}{q^2 - m^2(m_r)}\theta(-q^2) - \frac{1}{q^2 - \widetilde{M}^2}\theta(q^2)$$

$$\Big(m^2(m_r) + i\gamma(m_r)\Big) \equiv \widetilde{M}^2,$$

Using narrow width

$$\frac{1}{z - i\epsilon} = \frac{1}{z + i\epsilon} + \frac{2i\epsilon}{z^2 + \epsilon^2}.$$

$$\frac{1}{q^2 - \widetilde{M}^2} \approx \frac{1}{q^2 - \widetilde{m}^2(m_r) + i\widetilde{\gamma}(m_r)} + 2\pi i\delta\big(q^2 - \widetilde{m}^2(m_r)\big).$$

and

$$\theta(x) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} du\, \frac{e^{iux}}{u - i\eta} = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} du\, \frac{e^{-iux}}{u + i\eta}$$

$$= \frac{\varepsilon}{2\pi i}\int_{-\infty}^{\infty} du\, \frac{e^{iu\varepsilon x}}{u - i\varepsilon\eta}$$

Reproduces imaginary parts from Cutkosky rules

-in scalar bubble diagram

Summary:

Gravity kept weakly coupled

Planck scale associated with helper gauge theory

Narrow, unstable ghost

Likelihood of micro-causality violation

Unitarity satisfied in simplest approximation

Loops of unstable ghosts being studied
- unitarity seems to be working

More exploration is warranted