The Wavefunction of the Universe, Cosmological Polytopes and the Emergence of Lorentz Invariance and Unitarity

Paolo Benincasa

Niels Bohr International Academy

August the 14th – Current Themes in High Energy Physics and Cosmology 2018, NBI

based on: N. Arkani-Hamed, **P.B.**, A. Postnikov – 1709.02813
N. Arkani-Hamed, **P.B.** – 18xx.xxxxx

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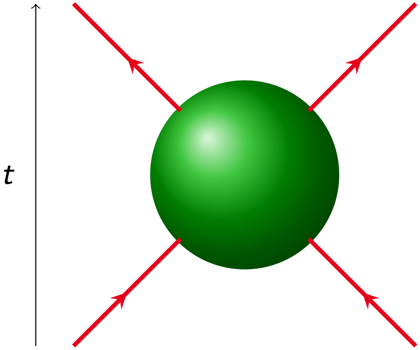
Observables and boundary data

Observables and boundary data

In flat space-time



Observables and boundary data

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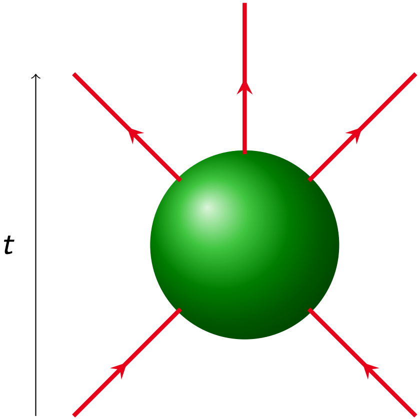

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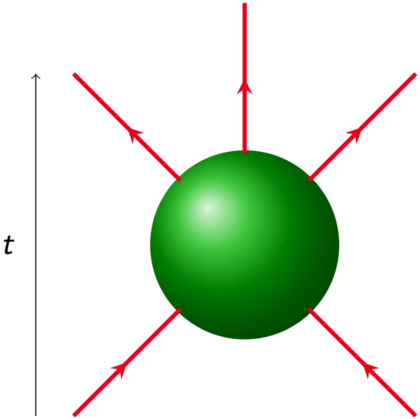
Observables and boundary data

In flat space-time



Experiment repeated ∞ times

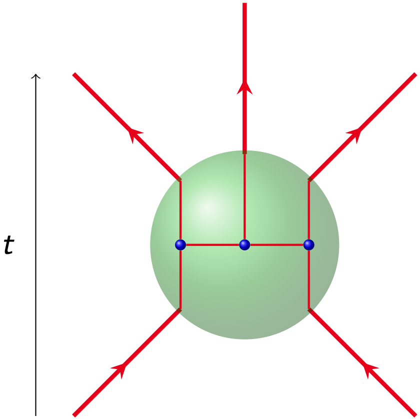
S-Matrix

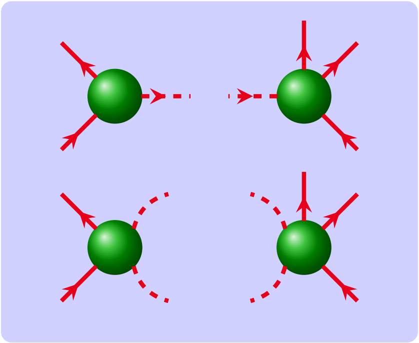
Probability of scattering processes

Relative frequencies

Observables and boundary data

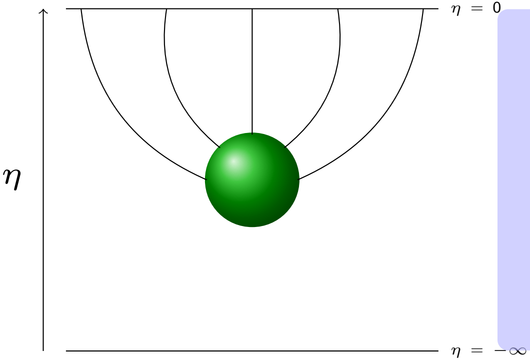
In flat space-time





Observables and boundary data

In cosmology



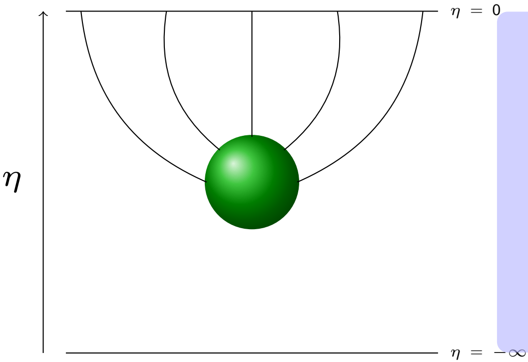
Experiment non-repeatable

Accelerated expansion

Observables and boundary data

In cosmology

No Q.M. observable!



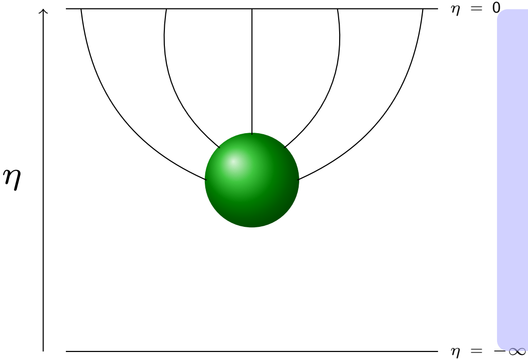
Experiment non-repeatable

Accelerated expansion

Observables and boundary data

In cosmology

Late-time correlators



Experiment non-repeatable

Spatial averages

Accelerated expansion

Universe ∞ large

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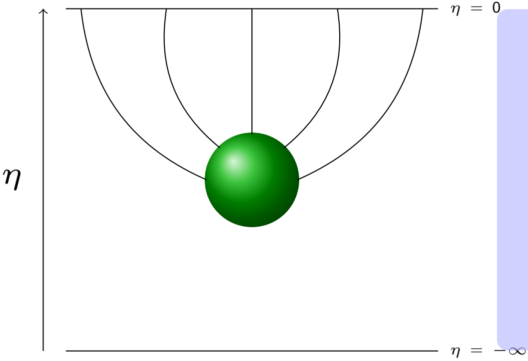
Observables and boundary data

In cosmology

Universe wavefunction



Experiment non-repeatable

Spatial averages

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Universe ∞ large

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Observables and boundary data

In cosmology: Late-time correlators

$$\langle \frac{d\rho}{\rho}(\vec{x}_1)\, \frac{d\rho}{\rho}(\vec{x}_2) \rangle$$

Observables and boundary data

In cosmology: Late-time correlators

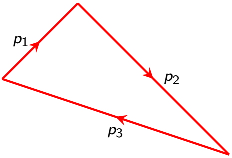
$$\langle \frac{d\rho}{\rho}(\vec{p}_1)\, \frac{d\rho}{\rho}(\vec{p}_2)\rangle \;=\; \delta^{(3)}\left(\vec{p}_1 + \vec{p}_2\right) \frac{10^{-10}}{|\vec{p}|^3}$$

Observables and boundary data

In cosmology: Late-time correlators

$$\langle \frac{d\rho}{\rho}(\vec{p}_1)\,\frac{d\rho}{\rho}(\vec{p}_2)\rangle \;=\; \delta^{(3)}\left(\vec{p}_1+\vec{p}_2\right)\frac{10^{-10}}{|\vec{p}\,|^3}$$

$$\langle\prod_{i=1}^{3}\frac{d\rho}{\rho}(\vec{p}_i)\rangle \;=\; \delta^{(3)}\left(\sum_{i=1}^{3}\vec{p}_i\right)\,f_3(p_i)$$



Observables and boundary data

In cosmology: Late-time correlators

$$\langle \frac{d\rho}{\rho}(\vec{p}_1)\,\frac{d\rho}{\rho}(\vec{p}_2)\rangle \;=\; \delta^{(3)}\left(\vec{p}_1+\vec{p}_2\right)\frac{10^{-10}}{|\vec{p}|^3}$$

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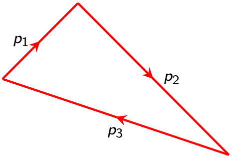


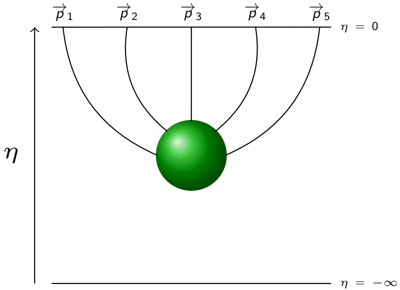
The wavefunction of the universe $\Psi[\phi]$ encodes these correlations

Observables and boundary data

In cosmology: Wavefunction of the universe

Observables and boundary data

In cosmology: Wavefunction of the universe



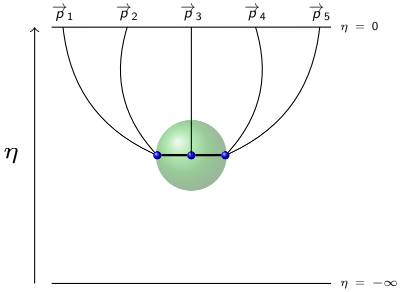
$$\Psi[\phi] = \int D\varphi \, e^{iS[\phi_0+\varphi]}$$

$$\begin{cases} \phi_0 = \text{free solution} \\ \varphi(\eta = 0) = 0 \end{cases}$$

$$\langle \prod_{i=1}^{n} \phi(p_i) \rangle = \int D\phi \prod_{i=1}^{n} \phi(p_i) \, |\Psi[\phi]|^2$$

Observables and boundary data

In cosmology: Wavefunction of the universe



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Observables and boundary data

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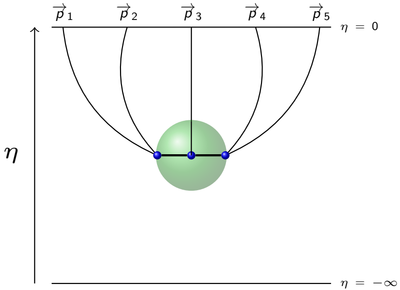


$$\Psi[\phi] = \int D\varphi \, e^{iS[\phi_0+\varphi]}$$

$$\begin{cases} \phi_0 = \text{free solution} \\ \varphi(\eta = 0) = 0 \end{cases}$$

Which are the invariant properties of Ψ making it consistent with a unitary evolution in cosmological space-times?

The wavefunction of the universe: In this talk

How can Ψ be represented without any reference to time evolution?

Perturbative structure of Ψ

Combinatorics and geometry of Ψ

Ψ and the S-matrix

The wavefunction of the universe: Bulk representation

The model

$$S = \int d^d x \int_{-\infty}^{0} d\eta \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \left(\partial_\mu \phi\right) \left(\partial_\nu \phi\right) - \xi R \phi^2 - \sum_{k \geq 3} \frac{\lambda_k}{k!} \phi^k \right]$$

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) \left[-d\eta^2 + dx^i dx_i \right], \qquad \xi = \frac{d-1}{4d}$$

The wavefunction of the universe: Bulk representation

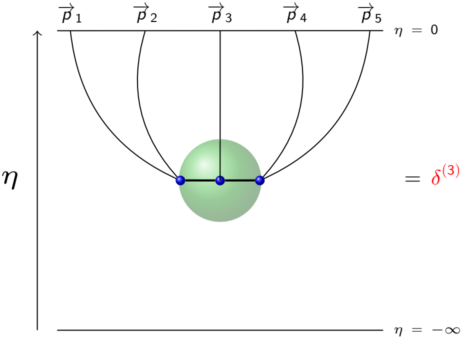
The model

$$S = \int d^d x \int_{-\infty}^{0} d\eta \left[\frac{1}{2} g_{\text{flat}}^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - \sum_{k \geq 3} \frac{\lambda_k(\eta)}{k!} \phi^k \right]$$

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) \left[-d\eta^2 + dx^i dx_i \right], \qquad \xi = \frac{d-1}{4d}$$

$$\lambda_k(\eta) = \int_0^{+\infty} d\varepsilon \, e^{i\varepsilon\eta} \tilde{\lambda}_k(\varepsilon)$$

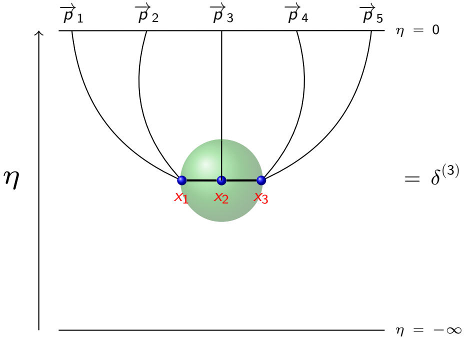
Path integration $\Longrightarrow$ Feynman Rules



$$= \delta^{(3)}\left(\sum_{i=1}^{5} \vec{p}_i\right)$$

The wavefunction of the universe: Bulk representation

Path integration $\Longrightarrow$ Feynman Rules



$$= \delta^{(3)}\left(\sum_{i=1}^{5} \vec{p}_i\right) \int_{-\infty}^{0} \prod_{j=1}^{3} d\eta_j \, e^{i x_i \eta_j}$$

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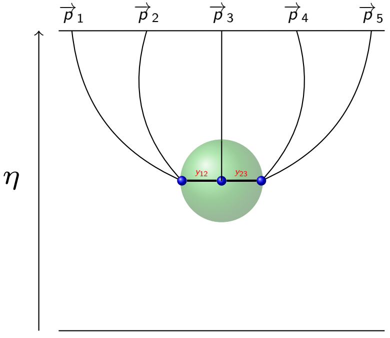
The wavefunction of the universe: Bulk representation

Path integration $\Longrightarrow$ Feynman Rules

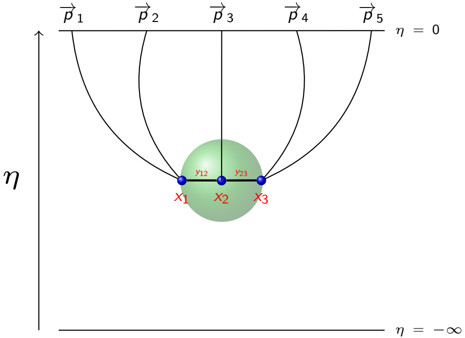


$$= \delta^{(3)}\left(\sum_{i=1}^{5} \vec{p}_i\right) \int_{-\infty}^{0} \prod_{j=1}^{3} d\eta_j \, e^{ix_i\eta_j} \, G(\eta_1, \eta_2) G(\eta_2, \eta_3)$$

$$G(\eta_j, \eta_k) = \frac{1}{y_{jk}} \left[e^{-iy_{jk}(\eta_j - \eta_k)} \vartheta(\eta_j - \eta_k) + e^{iy_{jk}(\eta_k - \eta_j)} \vartheta(\eta_j - \eta_k) - e^{iy_{jk}(\eta_k + \eta_j)} \right]$$

The wavefunction of the universe: Bulk representation

Path integration $\Longrightarrow$ Feynman Rules



What have we learnt?

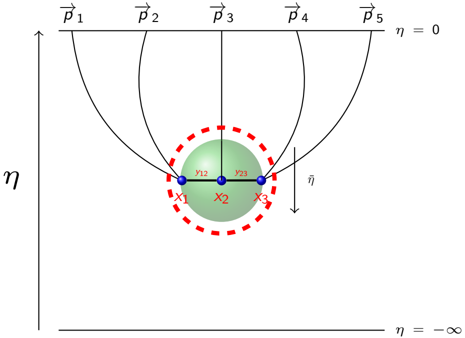
Explicit time flow: $\begin{cases} 3^{n_e} \text{ terms} \\ \text{spurious poles } y_{jk} \end{cases}$

Physical poles: sum of the energies

$$G(\eta_j, \eta_k) = \frac{1}{y_{jk}} \left[e^{-iy_{jk}(\eta_j - \eta_k)} \vartheta(\eta_j - \eta_k) + e^{iy_{jk}(\eta_k - \eta_j)} \vartheta(\eta_j - \eta_k) - e^{iy_{jk}(\eta_k + \eta_j)} \right]$$

The wavefunction of the universe: Bulk representation

Path integration $\Longrightarrow$ Feynman Rules



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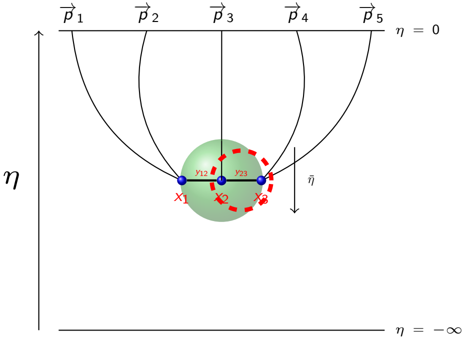
Physical poles: sum of the energies

$$\sim \frac{1}{x_1 + x_2 + x_3}$$

Flat-space scattering amplitude!

The wavefunction of the universe: Bulk representation

Path integration $\Longrightarrow$ Feynman Rules



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Explicit time flow: $\begin{cases} 3^{n_e} \text{ terms} \\ \text{spurious poles } y_{jk} \end{cases}$

Physical poles: sum of the energies

$$\sim \frac{1}{y_{12} + x_2 + x_3}$$

Factorization: $\psi \otimes A$

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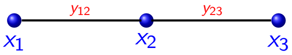
The wavefunction of the universe: Bulk representation

Path integration $\Longrightarrow$ Feynman Rules



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The wavefunction of the universe: Boundary representation

No time integrations: Directly in terms of *boundary* data

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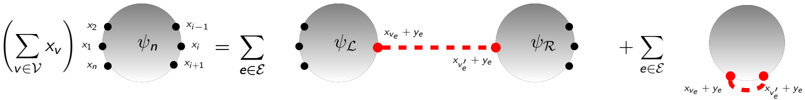

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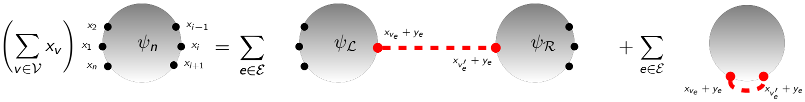

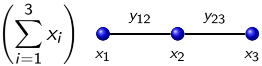

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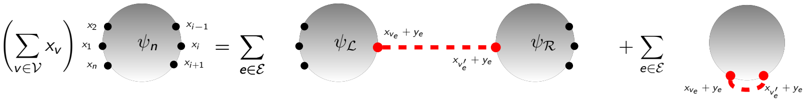
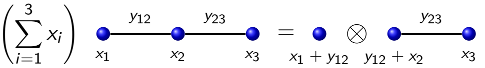
No time integrations: Directly in terms of *boundary* data

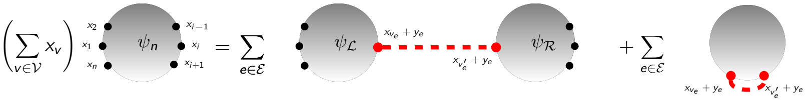


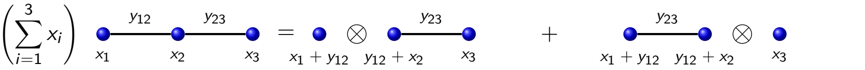


$$\frac{1}{x_1 + y_{12}} \otimes \frac{1}{(y_{12} + x_2 + x_3)(y_{12} + x_2 + y_{23})(y_{23} + x_3)}$$

The wavefunction of the universe: Boundary representation

No time integrations: Directly in terms of *boundary* data





$$\frac{1}{x_1+y_{12}} \otimes \frac{1}{(y_{12}+x_2+x_3)(y_{12}+x_2+y_{23})(y_{23}+x_3)} + \frac{1}{(x_1+x_2+y_{12})(x_1+y_{12})(y_{12}+x_2+y_{23})} \otimes \frac{1}{(y_{23}+x_3)}$$

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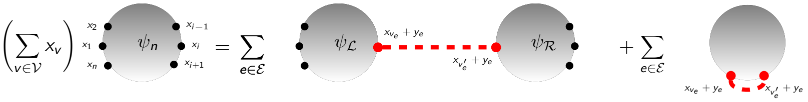
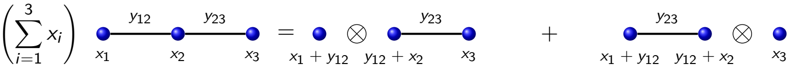
The wavefunction of the universe: Boundary representation

No time integrations: Directly in terms of **boundary** data





$$\frac{1}{x_1+y_{12}} \otimes \frac{1}{(y_{12}+x_2+x_3)(y_{12}+x_2+y_{23})(y_{23}+x_3)} + \frac{1}{(x_1+x_2+y_{12})(x_1+y_{12})(y_{12}+x_2+y_{23})} \otimes \frac{1}{(y_{23}+x_3)}$$

Combinatorial rules

Physical poles only

Recursive structure

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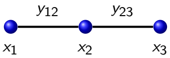
The wavefunction of the universe: Boundary representation

No time integrations: Directly in terms of *boundary* data



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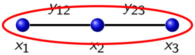
Combinatorial rules

The wavefunction of the universe: Boundary representation

No time integrations: Directly in terms of *boundary* data

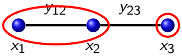


$$= \frac{1}{x_1 + x_2 + x_3}$$

Combinatorial rules

The wavefunction of the universe: Boundary representation

No time integrations: Directly in terms of ***boundary*** data

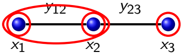


$$= \frac{1}{x_1 + x_2 + x_3} \left[\frac{1}{x_1 + x_2 + y_{23}} \times \frac{1}{y_{23} + x_3} \right.$$

Combinatorial rules

The wavefunction of the universe: Boundary representation

No time integrations: Directly in terms of ***boundary*** data



$$= \frac{1}{x_1 + x_2 + x_3} \left[\frac{1}{x_1 + x_2 + y_{23}} \times \frac{1}{y_{23} + x_3} \times \frac{1}{x_1 + y_{12}} \times \frac{1}{y_{12} + x_2 + y_{23}} + \right.$$

Combinatorial rules

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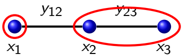

$$= \frac{1}{x_1+x_2+x_3} \Bigg[\frac{1}{x_1+x_2+y_{23}} \times \frac{1}{y_{23}+x_3} \times \frac{1}{x_1+y_{12}} \times \frac{1}{y_{12}+x_2+y_{23}} +$$

$$+ \frac{1}{x_1+y_{12}} \times \frac{1}{y_{12}+x_2+x_3}$$

Combinatorial rules

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The wavefunction of the universe: Boundary representation

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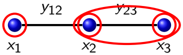


$$= \frac{1}{x_1+x_2+x_3}\left[\frac{1}{x_1+x_2+y_{23}}\times\frac{1}{y_{23}+x_3}\times\frac{1}{x_1+y_{12}}\times\frac{1}{y_{12}+x_2+y_{23}}+\right.$$

$$\left.+\frac{1}{x_1+y_{12}}\times\frac{1}{y_{12}+x_2+x_3}\times\frac{1}{y_{12}+x_2+y_{23}}\times\frac{1}{y_{23}+x_3}\right]$$

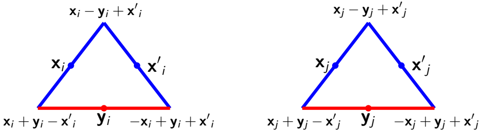
Combinatorial rules

Cosmological Polytopes

n_e triangles

$\mathbb{P}^{3n_e-1}$

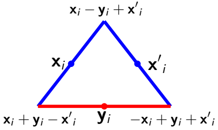
Cosmological Polytopes

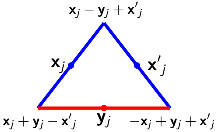


n_e triangles

$$\mathbb{P}^{3n_e-1}$$

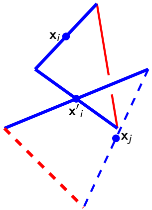
Cosmological Polytopes

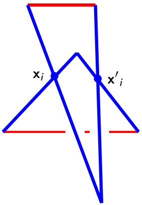




n_e triangles

$\mathbb{P}^{3n_e-1}$





$\mathbb{P}^{3n_e-r-1}$

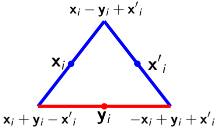
$\mathbf{x}'_i = \mathbf{x}'_j$

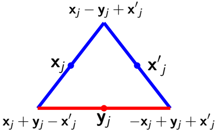
$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x}'_i = \mathbf{x}'_j$

$\mathbb{P}^4$

$\mathbb{P}^3$

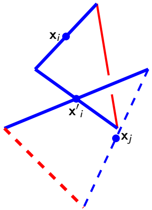
Cosmological Polytopes





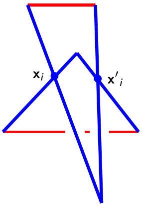
n_e triangles

$\mathbb{P}^{3n_e - r - 1}$



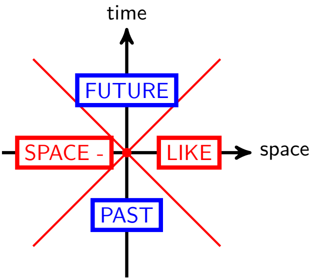
$x'_i = x'_j$

$\mathbb{P}^4$

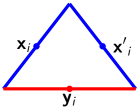


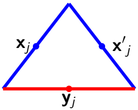
$x_i = x_j, \quad x'_i = x'_j$

$\mathbb{P}^3$



Cosmological Polytopes





n_e triangles

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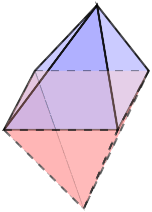

$\mathbf{x}'_i = \mathbf{x}'_j$

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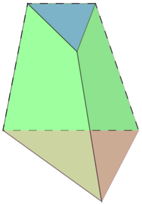

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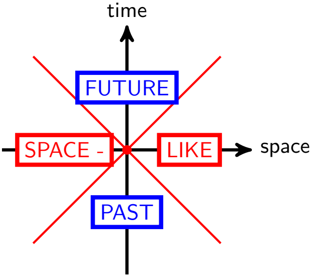

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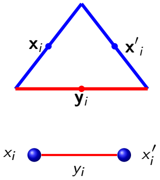
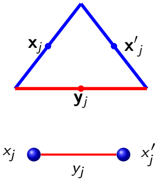
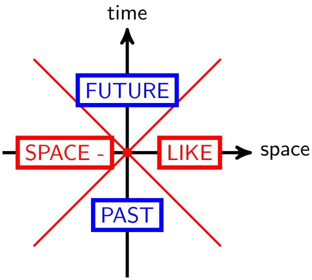
Cosmological Polytopes

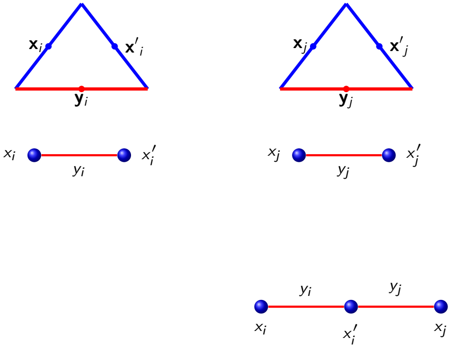




n_e triangles

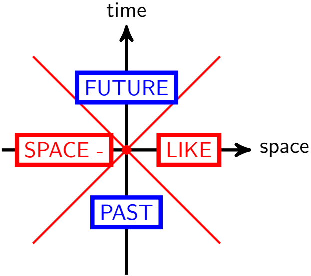
$$\mathbb{P}^{3n_e-r-1}$$



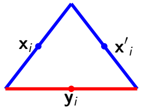


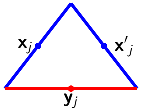
n_e triangles

$$\mathbb{P}^{3n_e - r - 1} \equiv \mathbb{P}^{n_e + n_v - 1}$$



Cosmological Polytopes





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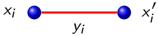

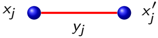

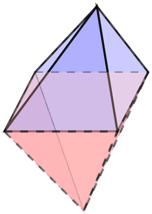

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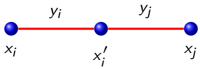
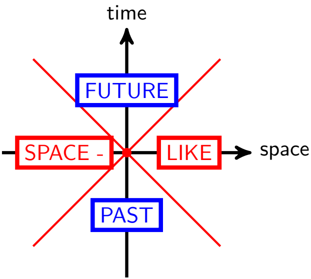
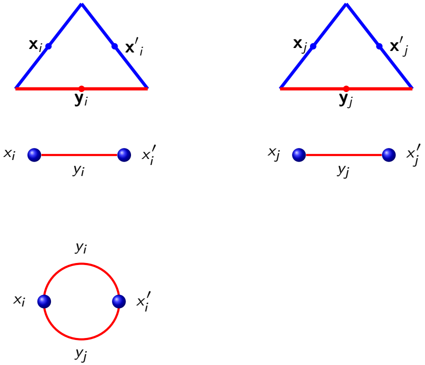




Cosmological Polytopes



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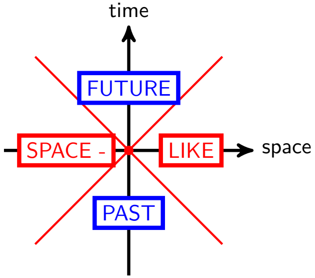
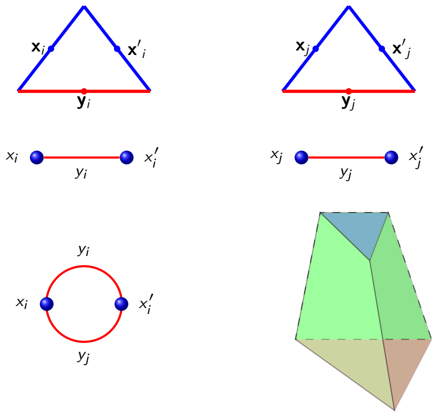


Cosmological Polytopes



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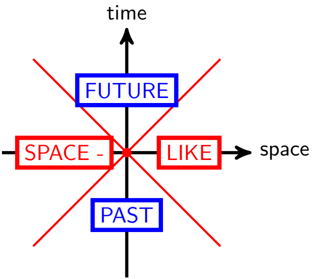
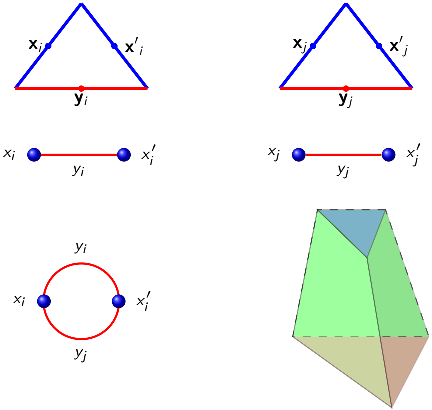


Cosmological Polytopes



n_e triangles

$$\mathbb{P}^{3n_e - r - 1} \equiv \mathbb{P}^{n_e + n_v - 1}$$

$$\mathcal{Y} = \sum_{v \in \mathcal{V}} x_v \mathbf{X}_v + \sum_{e \in \mathcal{E}} y_e \mathbf{Y}_e$$

$$\Omega = \left(\prod_{v,e} \frac{dx_v \, dy_e}{\text{Vol}\{GL(1)\}} \right) \Psi_{\mathcal{G}}(x_v, y_e)$$

$$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x}'_i = \mathbf{x}'_j$$

$\mathbb{P}^3$

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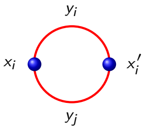
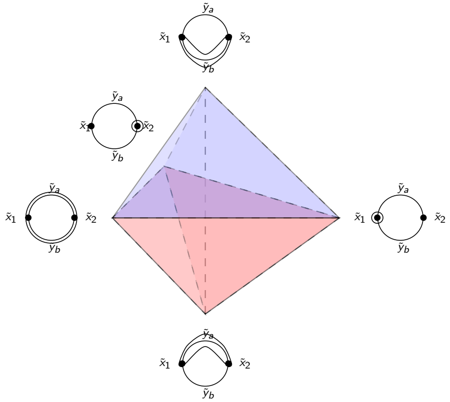
Cosmological Polytopes: The Dual Polytope $\tilde{\mathcal{P}}$

Each subgraph $\Longleftrightarrow$ Vertex in the space $\mathcal{Y} = (x's,\, y's)$





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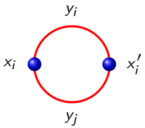
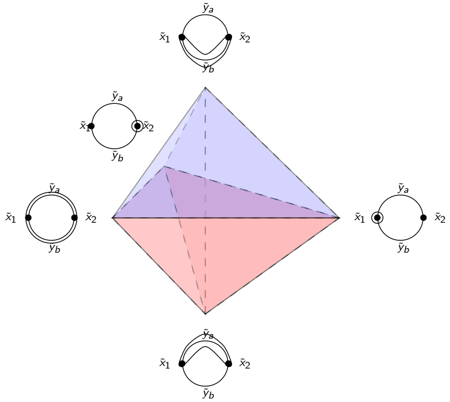




$$\Omega = \langle \mathcal{Y}, d\mathcal{Y} \rangle \text{Vol}\,\{\tilde{\mathcal{P}}\}$$

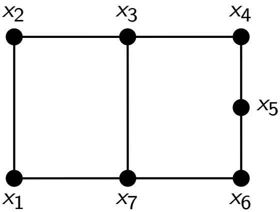
$$\boxed{\Psi \equiv \text{Vol}\,\{\tilde{\mathcal{P}}\}}$$

Cosmological Polytopes & Wavefunction: A dictionary

Cosmological Polytope $\mathcal{P}$	Universe Wavefunction Ψ
Canonical form	Ψ
Triangulations	Representations for Ψ
Boundaries	Residues of Ψ
Volume preserving transformations	Symmetries of Ψ

Cosmological Polytopes: The Face Structure

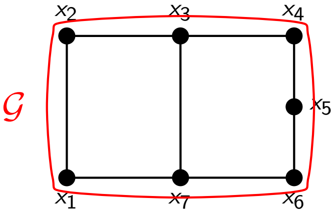
Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow$ 0



Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow 0$

$$\mathcal{W} \cdot \mathbf{V} = 0$$



$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

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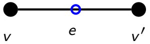
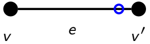
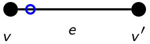


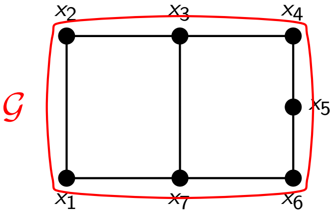
$$\mathcal{W}\cdot(\mathbf{x}_v + \mathbf{x}_{v'} - \mathbf{y}_e) = 0$$



$$\mathcal{W}\cdot(-\mathbf{x}_v + \mathbf{x}_{v'} + \mathbf{y}_e) = 0$$



$$\mathcal{W}\cdot(\mathbf{x}_v - \mathbf{x}_{v'} - \mathbf{y}_e) = 0$$



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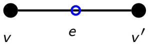
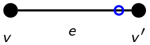


$$\mathcal{W}\cdot(\mathbf{x}_v+\mathbf{x}_{v'}-\mathbf{y}_e) = 0$$



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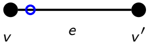
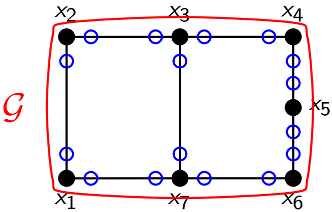


$$\mathcal{W}\cdot(\mathbf{x}_v-\mathbf{x}_{v'}-\mathbf{y}_e) = 0$$



$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

Vertices $(2n_e)$

$$\{\mathbf{x}_i-\mathbf{x}_j+\mathbf{y}_{ij},$$

$$-\mathbf{x}_i+\mathbf{x}_j+\mathbf{y}_{ij}\}$$

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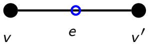

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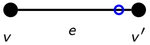

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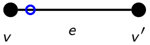
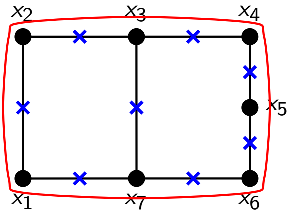


$\mathcal{W} \cdot (\mathbf{x}_v - \mathbf{x}_{v'} - \mathbf{y}_e) = 0$

$\mathcal{G}$



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Scattering Facet

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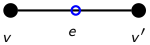

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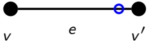

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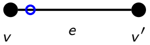

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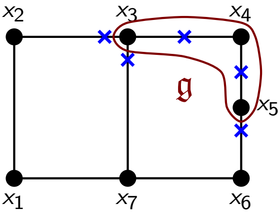
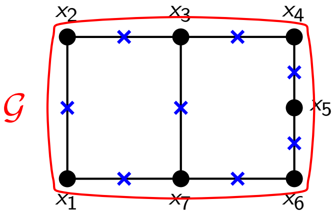


$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$

Scattering Facet: Emergent Unitarity

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



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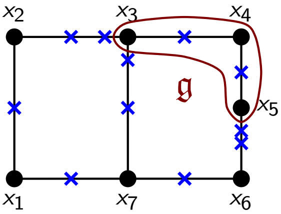

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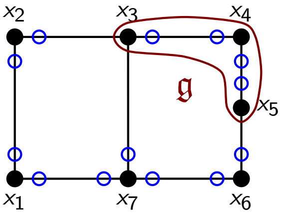

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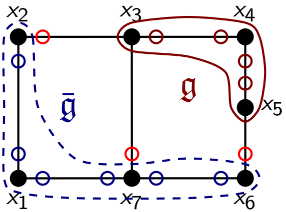

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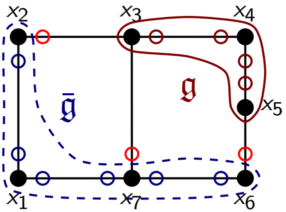


$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$

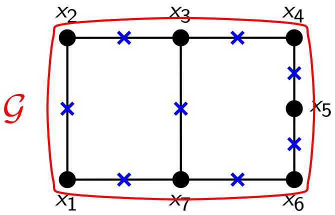
$$\Omega = \left(\prod_{e \in \mathcal{E}} \frac{1}{2y_e} \right) \mathcal{A}[\mathfrak{g}] \times \mathcal{A}[\bar{\mathfrak{g}}]$$

cutting rules!

$\circ \implies$ *Energy flow!*

Scattering Facet: Emergent Lorentz Invariance

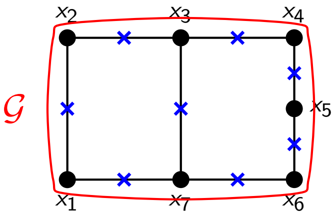
$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



Scattering Facet: Emergent Lorentz Invariance

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$$\Omega \sim \int_{\mathbb{R}^N} \prod_{j=1}^{\nu} \frac{dc_j}{c_j - i\varepsilon_j} \delta^{(N)} \left(\mathcal{Y} - \sum_{j=1}^{\nu} c_j \mathbf{V}^{(j)} \right)$$



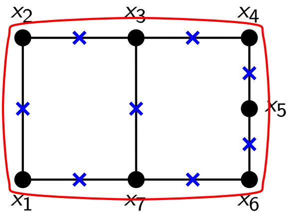
$$\mathbb{P}^{N-1}, \qquad N \equiv n_e + n_v - 1$$

$$\nu \equiv n_e + n_v - 1 + L$$

Scattering Facet: Emergent Lorentz Invariance

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$\mathcal{G}$



$$\Omega \sim \int_{\mathbb{R}^N} \prod_{j=1}^{\nu} \frac{dc_j}{c_j - i\varepsilon_j} \delta^{(N)}\left(\mathcal{Y} - \sum_{j=1}^{\nu} c_j \mathbf{V}^{(j)}\right)$$

$$\sim \int \prod_{j=1}^{L} \frac{dc_j}{\left(c_j - \frac{y_{e_j}}{2}\right)^2 - \left(\frac{y_{e_j}}{2} - i\varepsilon_j\right)^2} \times$$

$$\times \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} c_r - \frac{\mathfrak{y}_s}{2}\right)^2 - \left(\frac{y_s}{2} - i\varepsilon_s\right)^2}$$

$$\mathbb{P}^{N-1}, \qquad N \equiv n_e + n_v - 1$$

$$\nu \equiv n_e + n_v - 1 + L$$

Scattering Facet: Emergent Lorentz Invariance

$$\Omega \sim \int \prod_{j=1}^{L} \frac{dc_j}{\left(c_j - \frac{y_{e_j}}{2}\right)^2 - \left(\frac{y_{e_j}}{2} - i\varepsilon_j\right)^2} \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} c_r - \frac{\mathfrak{y}_s}{2}\right)^2 - \left(\frac{y_s}{2} - i\varepsilon_s\right)^2}$$

$$\mathcal{I} \sim \int \prod_{j=1}^{L} d\vec{l}^{(j)} dl_0^{(j)} \frac{1}{(l_0^{(j)})^2 - (|\vec{l}^{(j)}| - i\varepsilon_j)^2} \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} l_0^{(j)} - \mathfrak{p}_s\right)^2 - \left(|\vec{P}_s| - i\varepsilon_s\right)^2}$$

Lorentz invariant loop integrand!

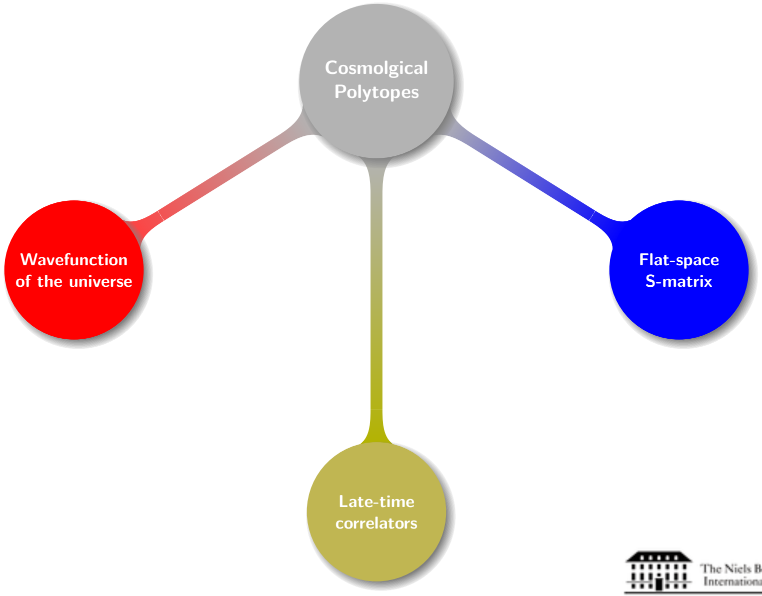
$c_j \sim l_0,$

$\Omega - i\varepsilon \sim \text{Feynman} - i\varepsilon$

Conclusion

- Cosmological polytope has an intrinsic definition and capture the singularity structure of the wavefunction.
- Triangulations of $\mathcal{P}$ and $\tilde{\mathcal{P}}$ return representations of Ψ: Feynman, OPFT, and many others.
- Face structure $\Longleftrightarrow$ Singularity structure.
- It contains the flat-space S-matrix.
- Rules for Ψ which *do not* refer to Lorentz invariance & unitarity but contains a Lorentz-invariant & unitary object.
- Conceptually transparent understanding of the cutting rules for *any* QFT without relying on the largest time equation.

Conclusion


Cosmolgical
Polytopes
Wavefunction
of the universe
Flat-space
S-matrix
Late-time
correlators