UV structure of gravity loop integrands

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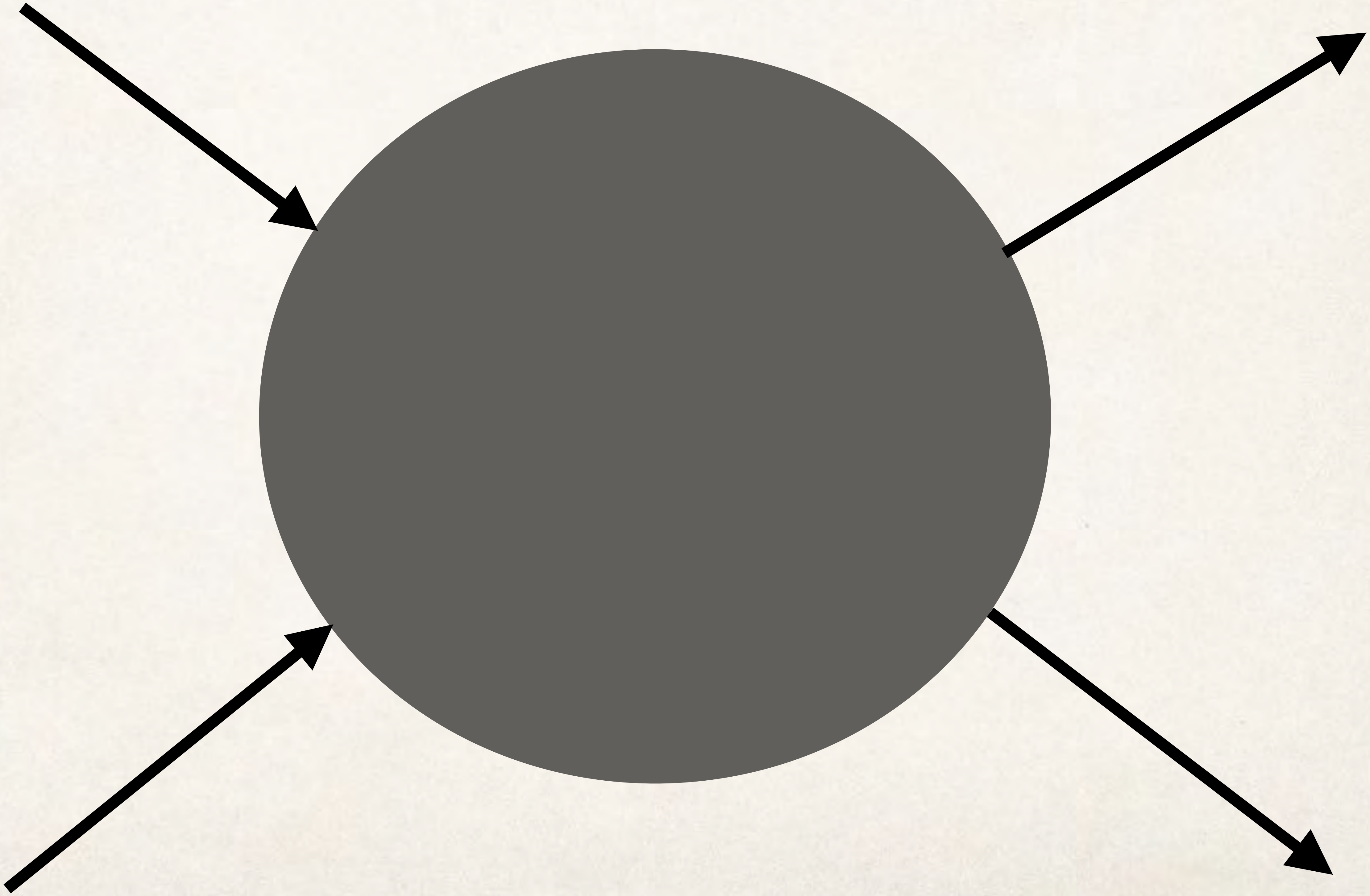
Current Themes in High Energy Physics and Cosmology, NBI, Aug 15, 2018

What happens during the scattering process of elementary particles?



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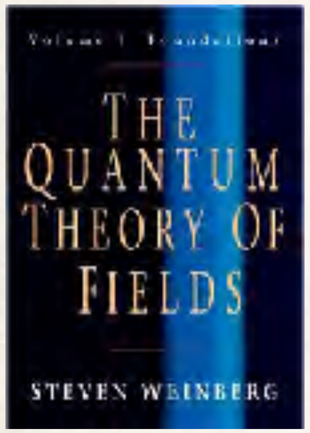

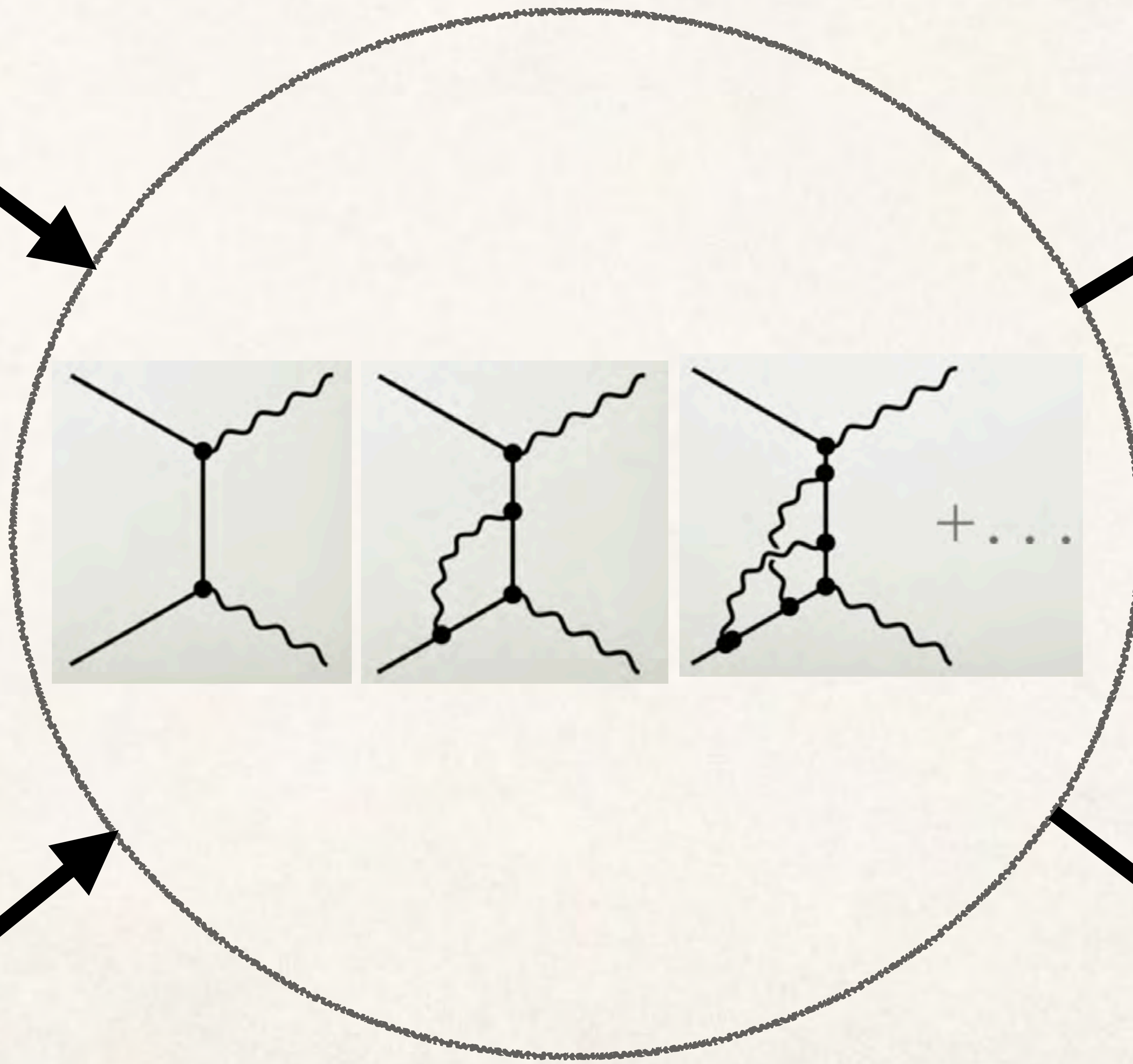

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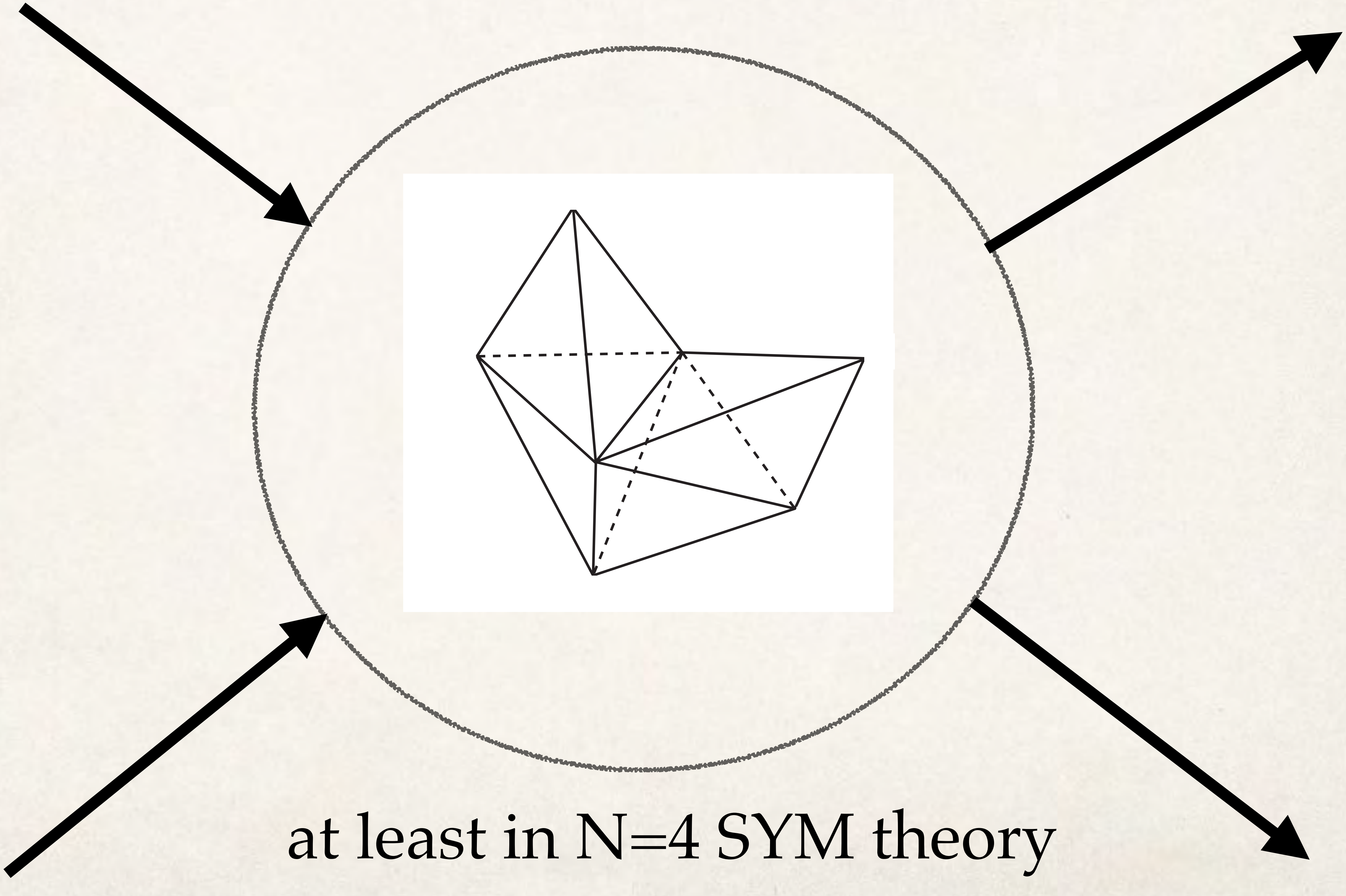
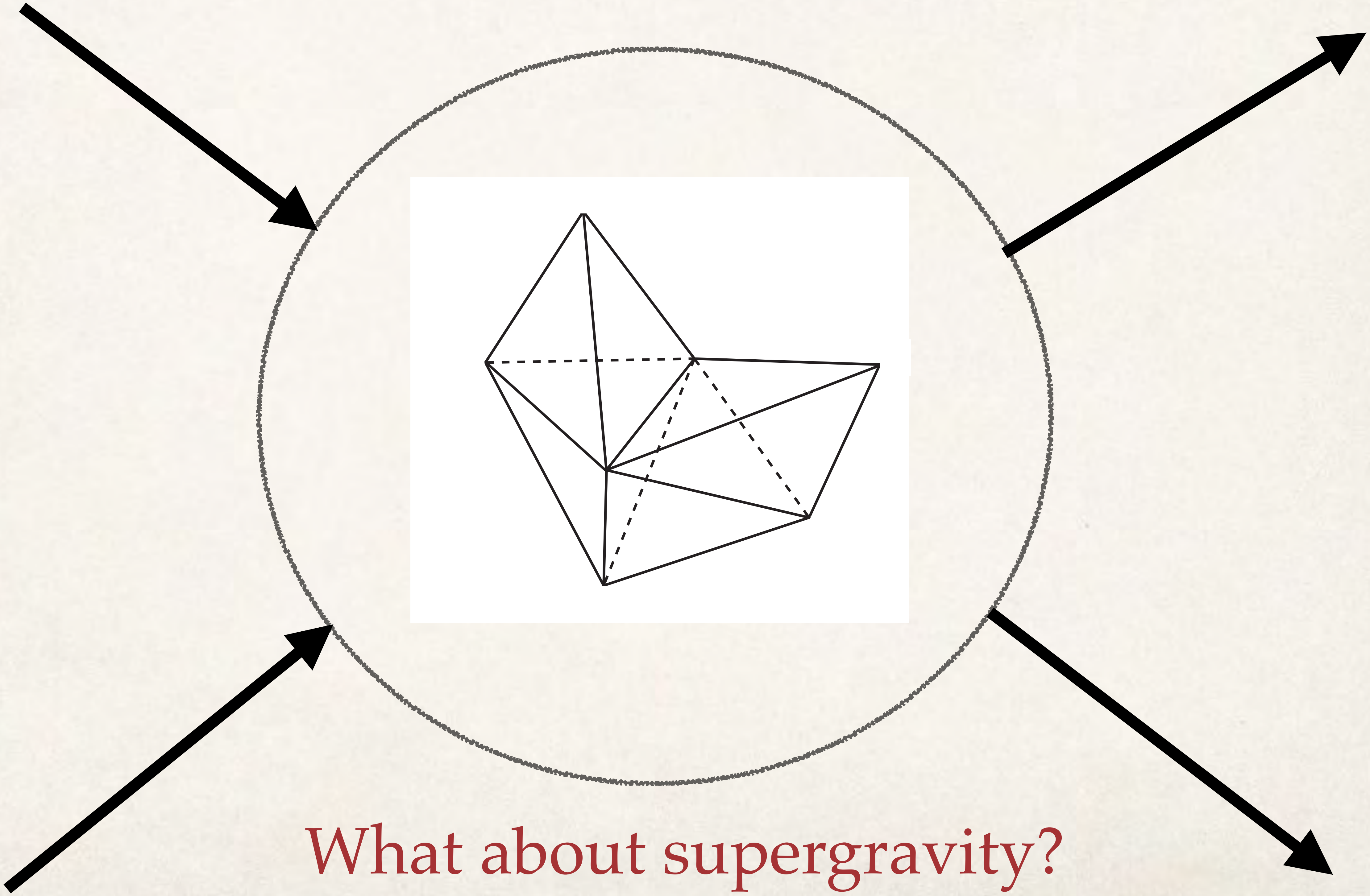


at least in N=4 SYM theory

What happens during the scattering process of elementary particles?



What about supergravity?

This talk

- Loop amplitudes in N=8 SUGRA
- Claim: there is a surprising behavior in the UV region not explained by known symmetries
- No claim about UV divergence but about certain unexpected cancelations at the level of integrand
- Motivation: find properties which fix gravity amplitudes uniquely and search for the geometric picture

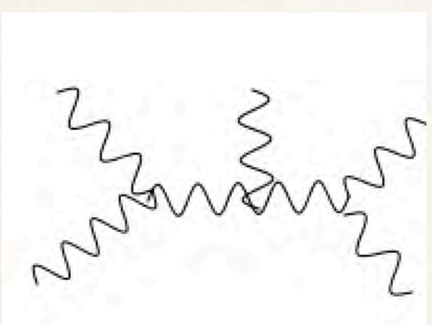
Prehistory: hidden simplicity

Gluon amplitudes

- Early 80s: plans for new "supercolliders" - need for new calculations of gluon amplitudes

- Leading order

$$gg \to ggg$$



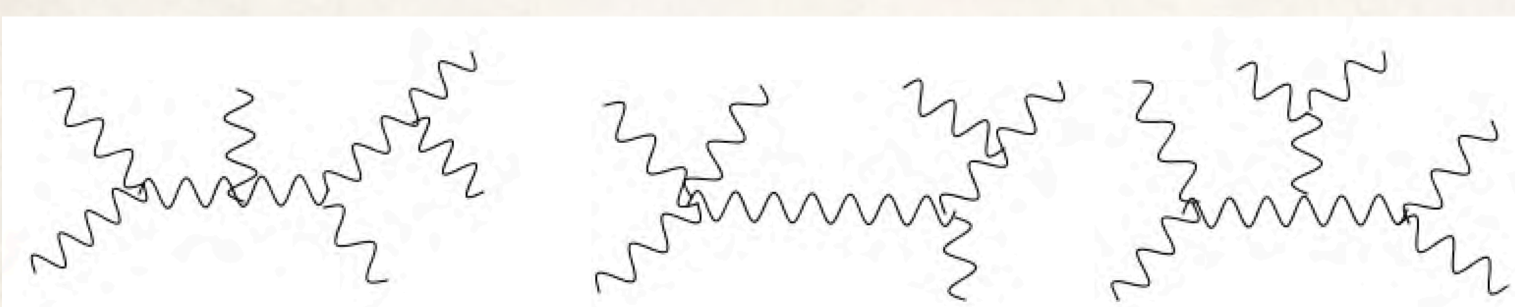
and many others

Brute force calculation 24 pages of result

$$(k_1 \cdot k_4)(\epsilon_2 \cdot k_1)(\epsilon_1 \cdot \epsilon_3)(\epsilon_4 \cdot \epsilon_5)$$

Parke-Taylor formula

- Next process on the list: $gg \to gggg$
- 220 Feynman diagrams ~100 pages of calculations
- Calculation finished in 1985
- Paper with 14 pages of result



GLUONIC TWO GOES TO FOUR

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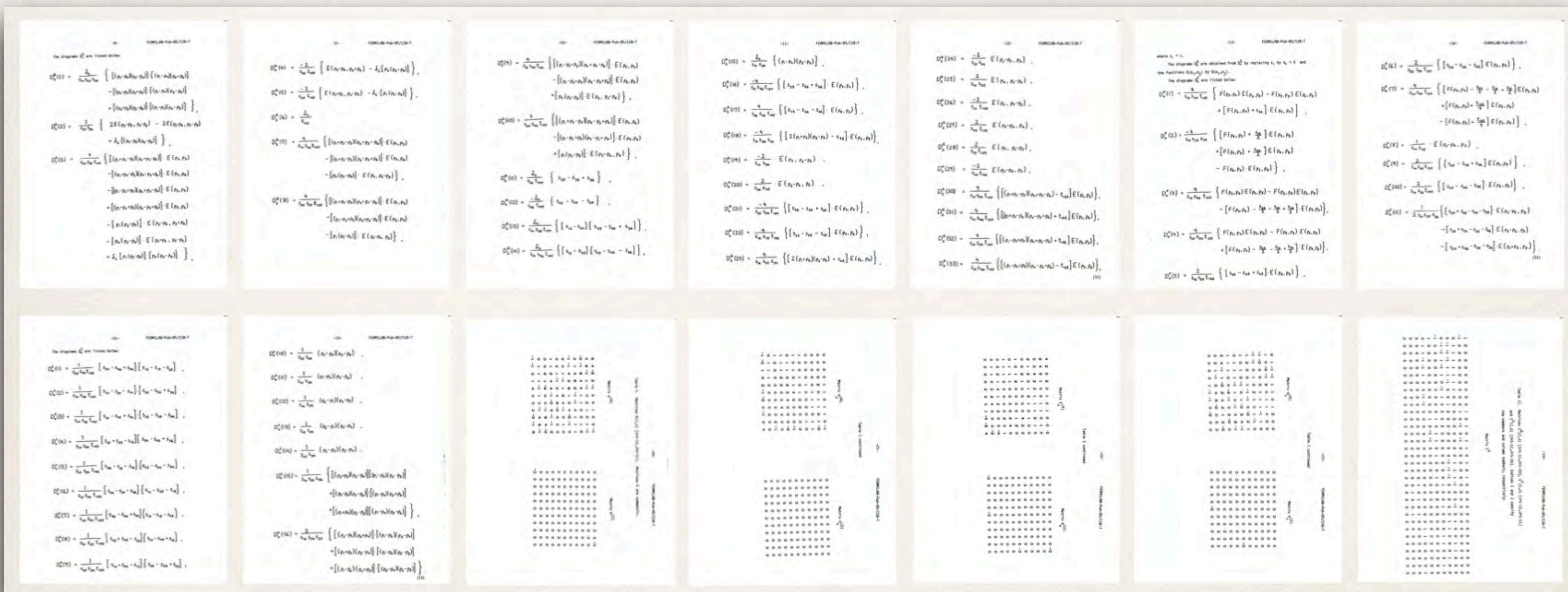
U.S.A.

ABSTRACT

The cross section for two gluon to four gluon scattering is given in a form suitable for fast numerical calculations.

Parke-Taylor formula

- Next process on the list: $gg \to gggg$
- 220 Feynman diagrams ~100 pages of calculations



Parke-Taylor formula

Our result has succesfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

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- Within a year they realized

$$A_6 = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}$$

Spinor-helicity variables

$$p^\mu = \sigma^\mu_{a\dot{a}}\lambda_a\tilde{\lambda}_{\dot{a}}$$

$$\langle 12\rangle = \epsilon_{ab}\lambda^{(1)}_a\lambda^{(2)}_b$$

$$[12] = \epsilon_{\dot{a}\dot{b}}\tilde{\lambda}^{(1)}_{\dot{a}}\tilde{\lambda}^{(2)}_{\dot{b}}$$

Parke-Taylor formula

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$$|A_6|^2 \sim \frac{(p_1 \cdot p_2)^3}{(p_2 \cdot p_3)(p_3 \cdot p_4)(p_4 \cdot p_5)(p_5 \cdot p_6)(p_6 \cdot p_1)}$$

Parke-Taylor formula

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- Within a year they realized

$$A_n = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\dots\langle n1\rangle}$$

AN AMPLITUDE FOR n GLUON SCATTERING

STEPHEN J. PARKE and T. R. TAYLOR

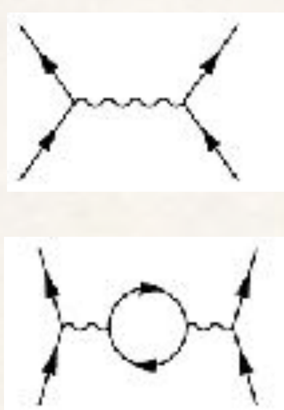
Fermi National Accelerator Laboratory

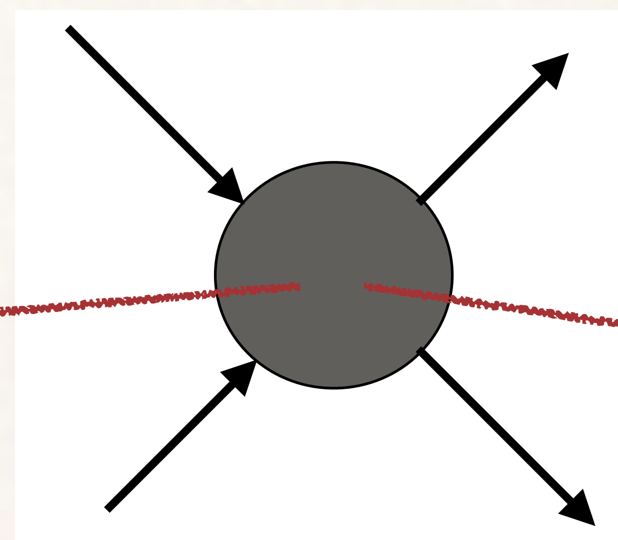
P.O. Box 500, Batavia, IL 60510.

Change of strategy

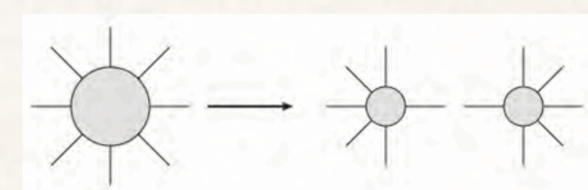
What is the scattering amplitude?

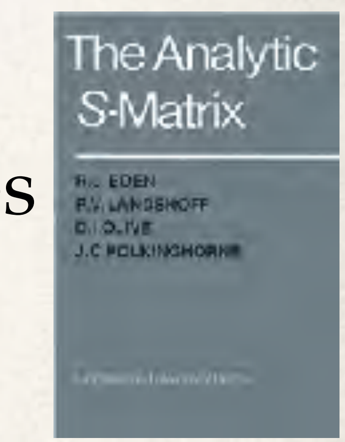
Feynman diagrams





Unique object fixed by physical properties





Modern methods use both:

- Calculate the amplitude directly
- Use perturbation theory

Lesson from Parke-Taylor:

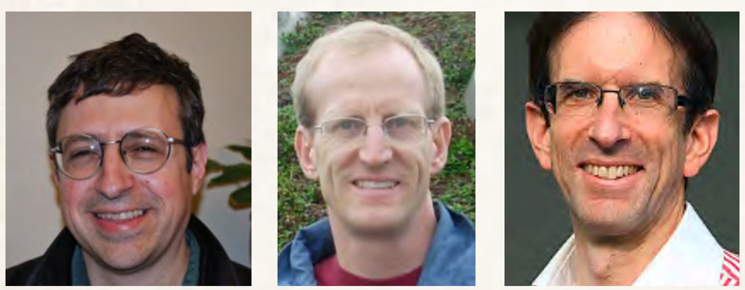
- On-shell gauge invariant objects
- Helicity amplitudes $A_{n,k}$

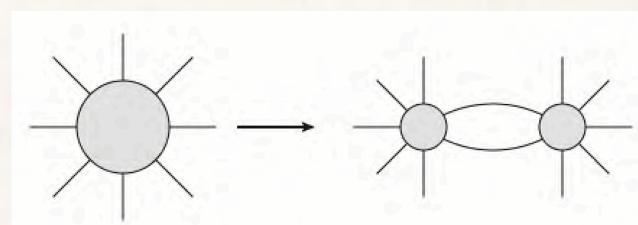
e.g. $k=2:\ 1^-2^-3^+4^+5^+\ldots n^+$

Parke-Taylor formula

New methods for amplitudes

- New efficient methods of calculations
  - Unitarity methods



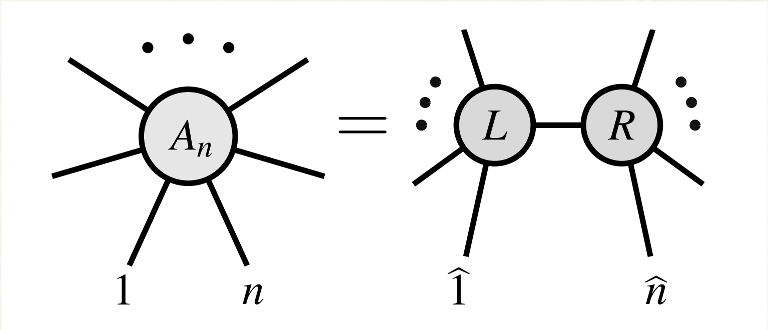


BlackHat collaboration
QCD background for LHC

(Bern, Dixon, Kosower, 1993-today)

  - Recursion relations

Build amplitude recursively from simpler amplitudes



(Britto, Cachazo, Feng, Witten, 2005)

	$gg \to 4g$	$gg \to 5g$	$gg \to 6g$
Feynman diagrams	220	2485	34300
Recursion relations	3	6	20

New methods for amplitudes

- Many new approaches and discoveries
    - String amplitudes
    - Amplitudes/Wilson loops duality
    - Hexagon bootstrap
    - Scattering equations
    - Color-kinematics duality
    - Ambitwistor strings
    - Integrability methods
    - On-shell diagrams, Amplituhedron and beyond
    - ……
- Not a single “amplitudes method”

Loop integrand

Loop amplitude

- There are deep mysteries about tree-level amplitudes
- In this talk I will talk about loops

$$\mathcal{A} = \sum_{FD} \int \mathcal{I}_j \, d^4\ell_1 \ldots d^4\ell_L$$

Obtain from Feynman rules

- We can rewrite it as:

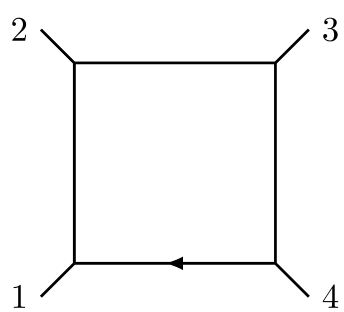
$$\mathcal{A} = \sum_k c_k \int \mathcal{I}_k \, d^4\ell_1 \ldots \ell_L$$

Kinematical coefficients

Basis integrals

One loop example

- Box integral

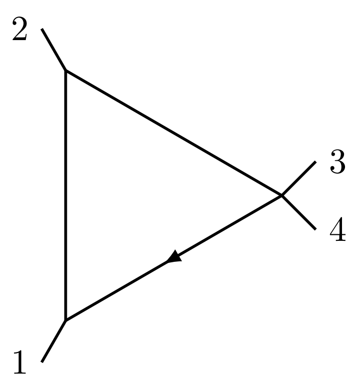


$$I = \frac{d^4\ell \; st}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2(\ell-k_4)^2}$$

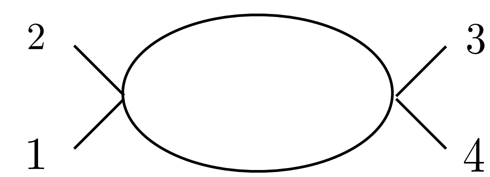
Tadpoles and other integrals

Vanish in dim reg

- Triangle and box integrals



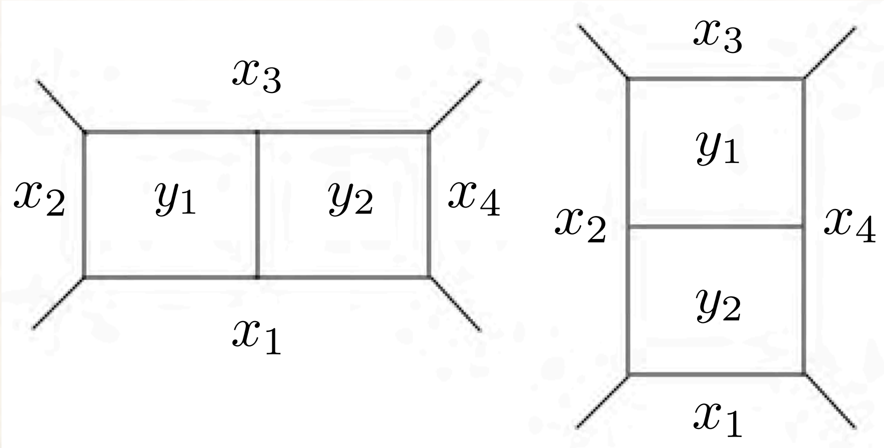
$$I = \frac{d^4\ell \; s}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2}$$



$$I = \frac{d^4\ell \, s}{\ell^2(\ell+k_1+k_2)^2}$$

Planar integrand

- Planar (large N) limit: we can define global variables



$$k_1 = (x_1 - x_2)$$
$$\ell_1 = (x_3 - y_1)$$

etc

Dual variables

- Switch integral and the sum:

$$\mathcal{A} = \sum_k c_k \int \mathcal{I}_k \, d^4\ell_1 \dots \ell_L = \int \mathcal{I} \, d^4\ell_1 \dots d^4\ell_L$$

Loop integrand

Planar integrand

- Loop integrand is a rational function of momenta
  - Get the final amplitude: still want to integrate

$$A^{L-loop} = \int d^4\ell_1 \dots d^4\ell_L \, \mathcal{I}$$

$$A^{1-loop} \sim \mathrm{Li}_2, \log, \zeta_2$$

$$A^{L-loop} \sim \, ?$$

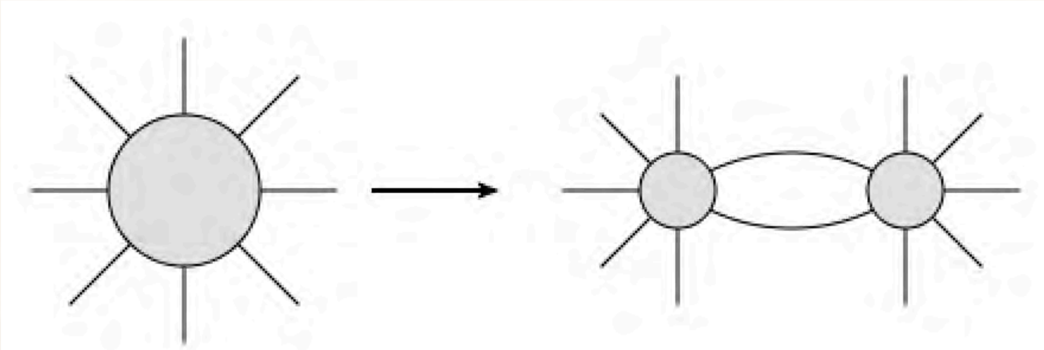
polylogs
elliptic polylogs
beyond

Study the integrand instead

- simpler (rational) function
- many variables (loop momenta)
- properties of the amplitude non-trivially encoded in the integrand

Cuts of the integrand

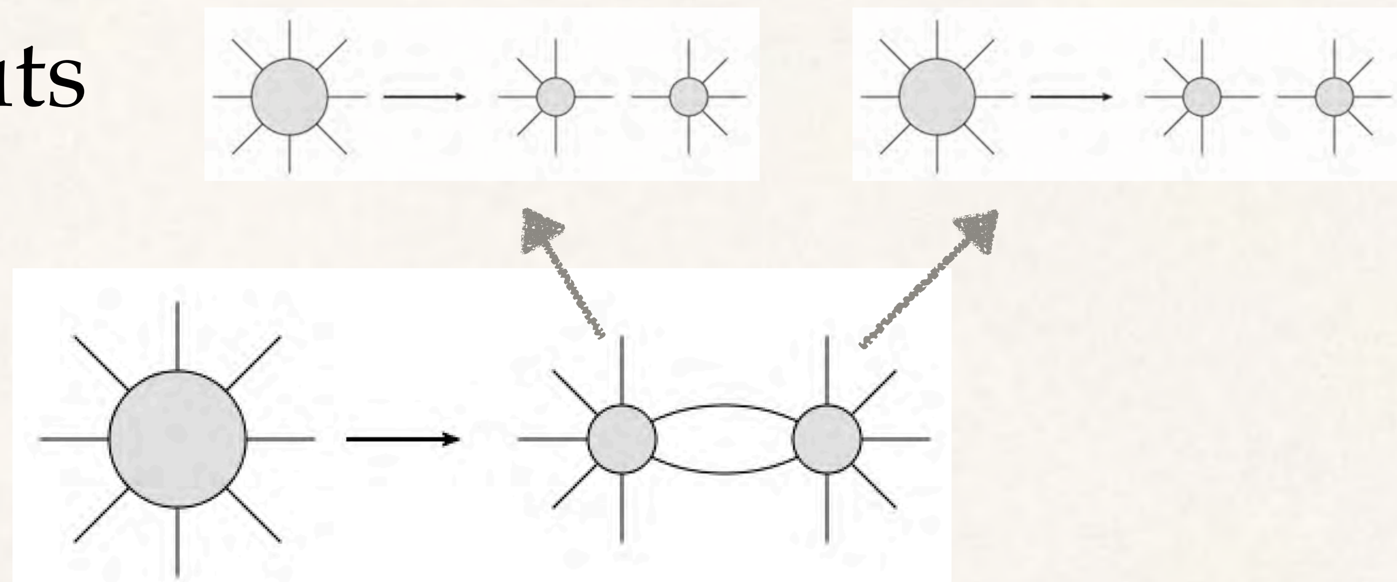
- Once we have the integrand we can take residues on poles: $\text{Cut} \leftrightarrow \ell^2 = 0$
- Unitarity cut: $\ell^2 = (\ell + Q)^2 = 0$



$$\mathcal{M}^{1-loop} \xrightarrow[\ell^2=(\ell+Q)^2=0]{} \mathcal{M}_L^{tree} \frac{1}{\ell^2(\ell+Q)^2} \mathcal{M}_R^{tree}$$

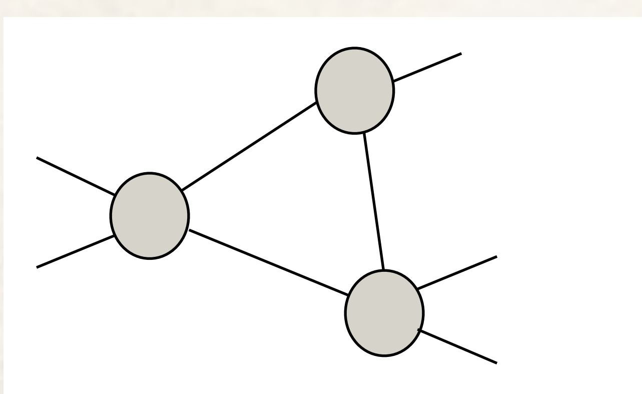
One-loop unitarity

- Higher cuts



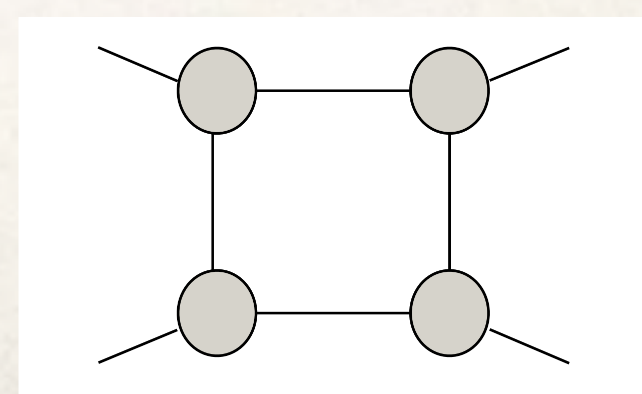
Triple cut

$$\ell^2 = (\ell + Q_1)^2 = (\ell + Q_2)^2 = 0$$



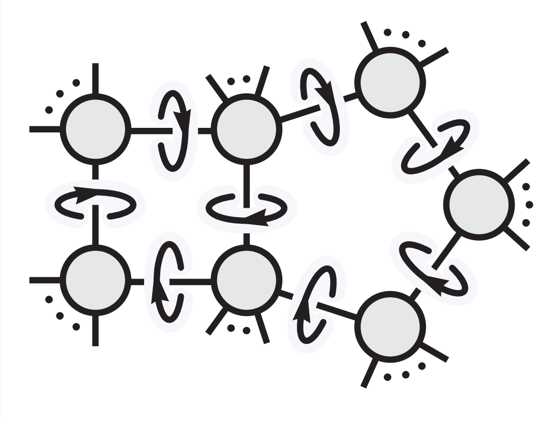
Quadruple cut

$$\ell^2 = (\ell + Q_1)^2 = (\ell + Q_2)^2 = (\ell + Q_3)^2 = 0$$



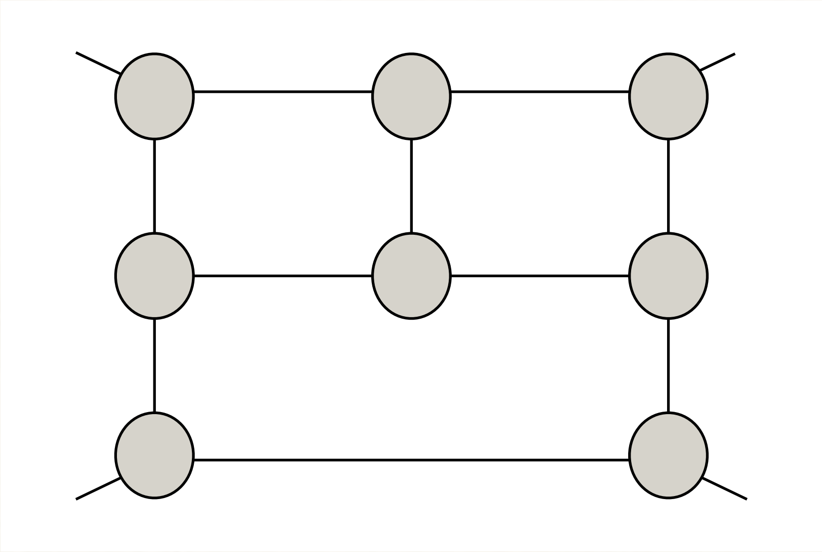
Generalized unitarity

- Generalized cuts



Cut more propagators
- complex on-shell momenta
- product of tree amplitudes

- On-shell diagrams: products of 3pt amplitudes



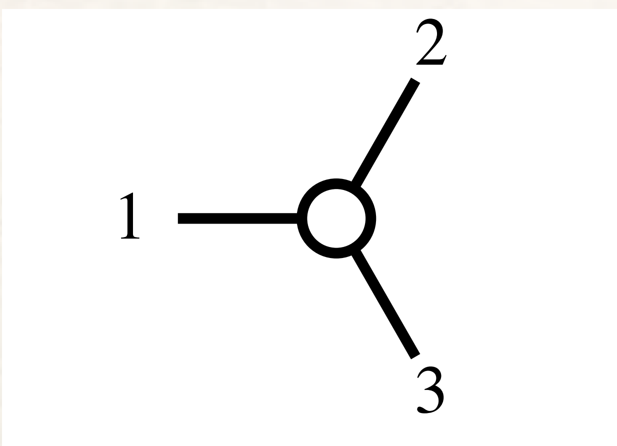
3pt on-shell kinematics
very restrictive

On-shell diagrams

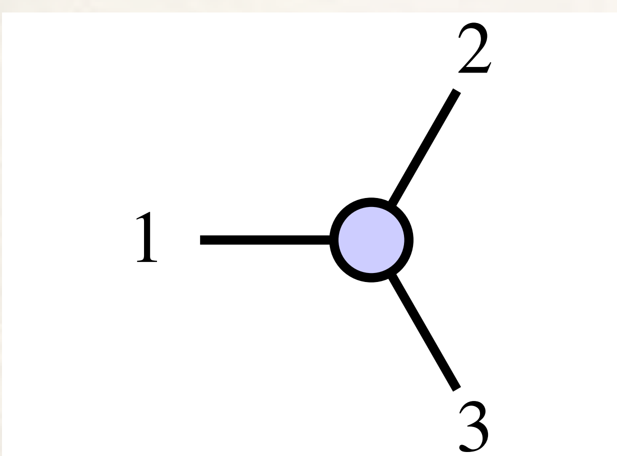
(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

Three point kinematics

- Two options



$\lambda_1 \sim \lambda_2 \sim \lambda_3$



$\widetilde{\lambda}_1 \sim \widetilde{\lambda}_2 \sim \widetilde{\lambda}_3$

<u>Spinor helicity variables</u>

$$p^\mu = \sigma^\mu_{a\dot{a}} \lambda_a \widetilde{\lambda}_{\dot{a}}$$

$$\langle 12 \rangle = \epsilon_{ab} \lambda_{1a} \lambda_{2b}$$

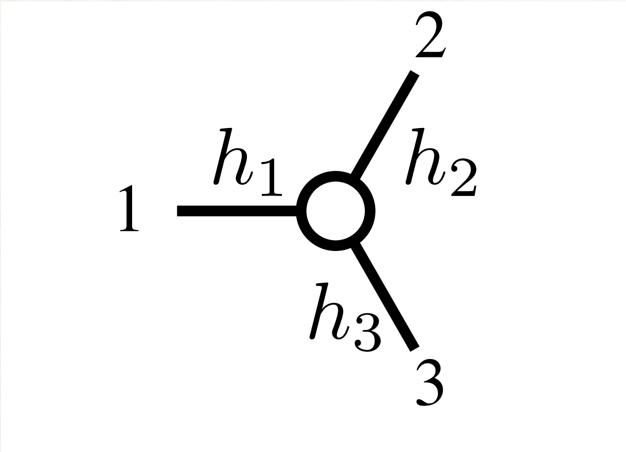
$$[12] = \epsilon_{\dot{a}\dot{b}} \lambda_{1\dot{a}} \lambda_{2\dot{b}}$$

Two solutions for 3pt kinematics

$$p_1^2 = p_2^2 = p_3^2 = (p_1 + p_2 + p_3) = 0$$

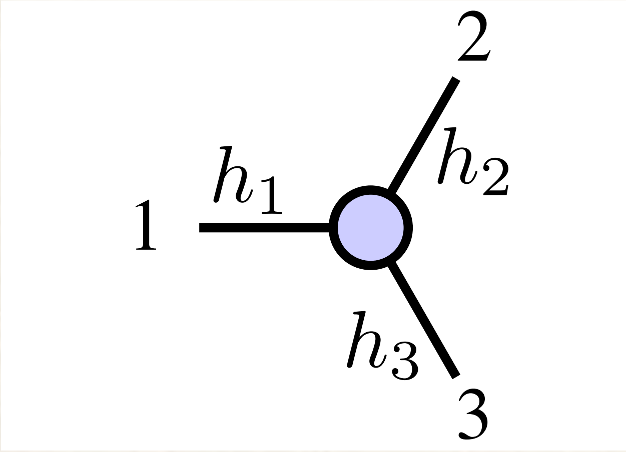
Three point amplitudes

- Two solutions for amplitudes



$$A_3 = [12]^{+h_1+h_2-h_3}[23]^{-h_1+h_2+h_3}[31]^{+h_1-h_2+h_3}$$

$$h_1 + h_2 + h_3 \geq 0$$



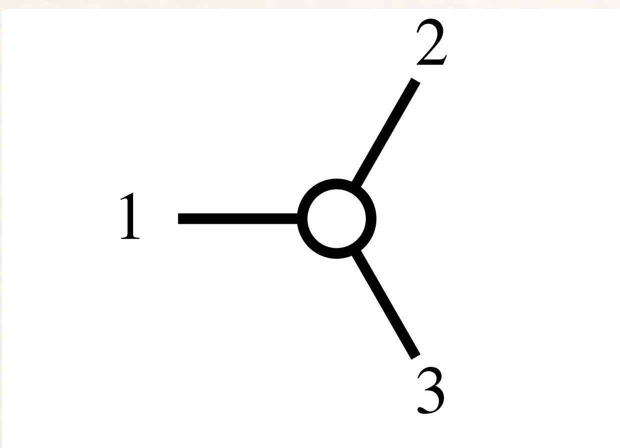
$$A_3 = \langle 12\rangle^{-h_1-h_2+h_3}\langle 23\rangle^{+h_1-h_2-h_3}\langle 31\rangle^{-h_1+h_2-h_3}$$

$$h_1 + h_2 + h_3 \leq 0$$

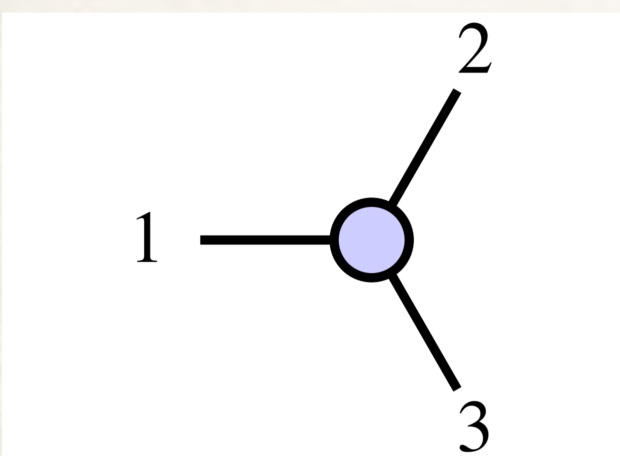
Supersymmetry: amplitudes of super-fields
(all component fields included)

Three point amplitudes

- In N=4 SYM: no need to specify helicities



$$\mathcal{A}_3^{(1)} = \frac{\delta^4(p_1+p_2+p_3)\delta^4([23]\widetilde{\eta}_1+[31]\widetilde{\eta}_2+[12]\widetilde{\eta}_3)}{[12][23][31]}$$

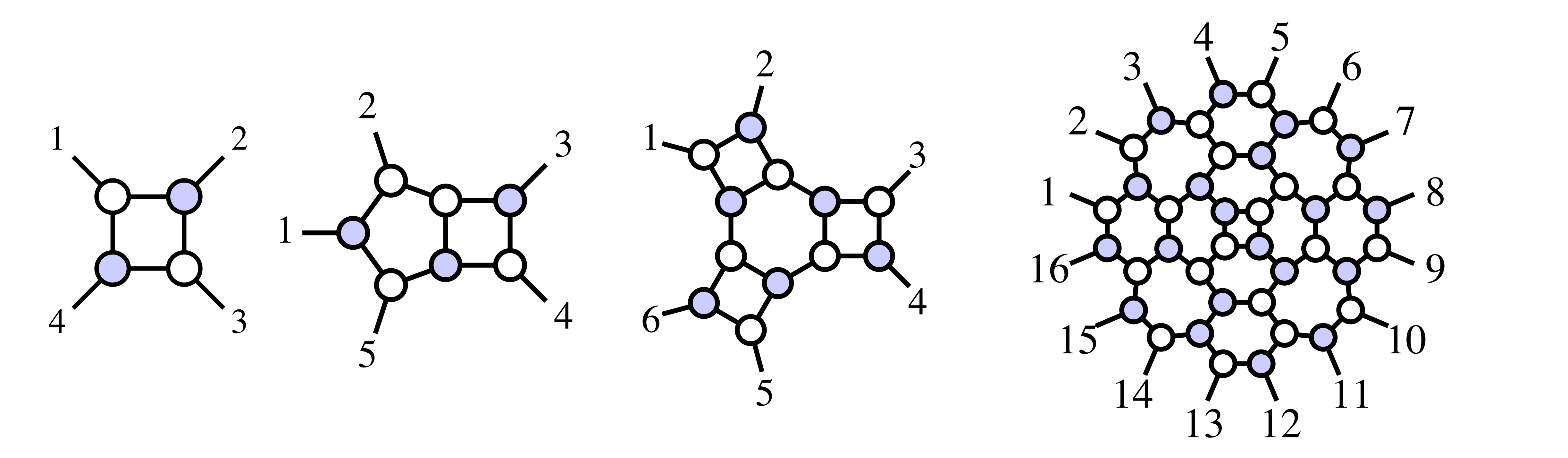


$$\mathcal{A}_3^{(2)} = \frac{\delta^4(p_1+p_2+p_3)\delta^8(\lambda_1\widetilde{\eta}_1+\lambda_2\widetilde{\eta}_2+\lambda_3\widetilde{\eta}_3)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

Easy book-keeping

On-shell diagrams

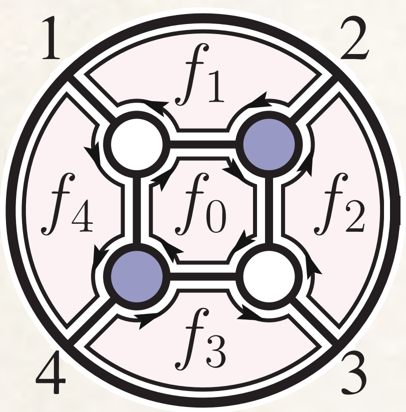
- Draw arbitrary graph with three point vertices

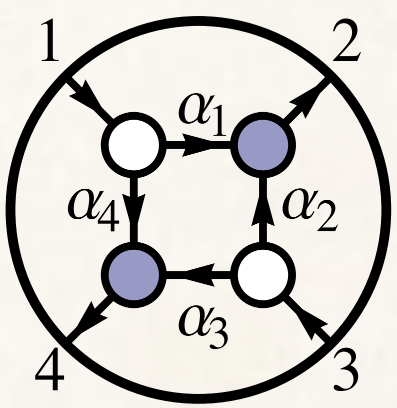


- All legs are on-shell: gauge invariant objects
- Cuts of loop integrands: products of 3pt amplitudes

Same diagrams in mathematics

- Building matrix with positive minors



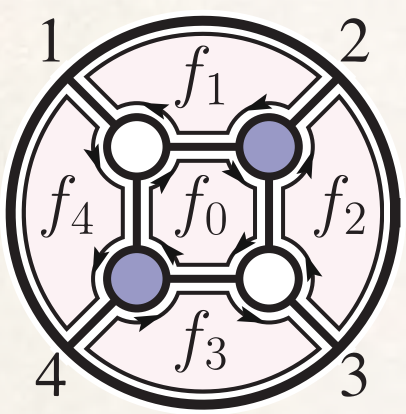


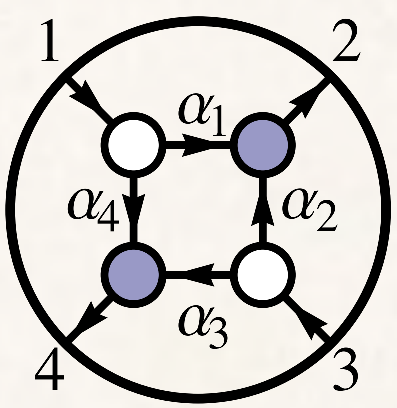
$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix} \qquad \alpha_k > 0$$

- Positive Grassmannian
  - Active area of research in algebraic geometry and combinatorics
  - Connection to cluster algebras, KP equations,…

Surprising connection

- Building matrix with positive minors





$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- For N=4 SYM the value of the diagram is equal to

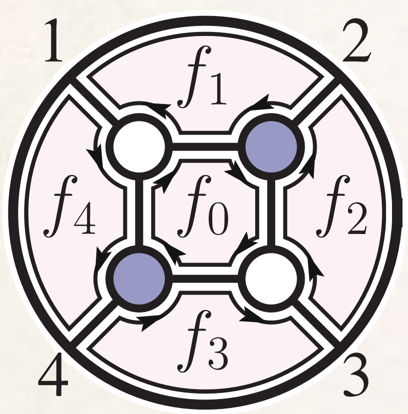
$$\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\delta(C\cdot Z)$$

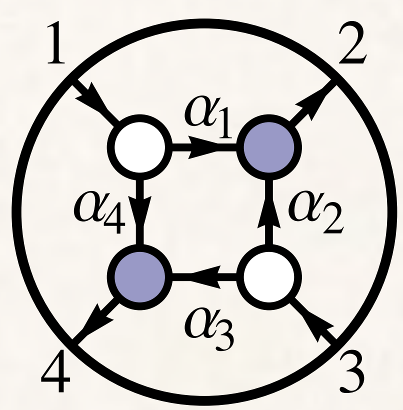
→ Solves for α_i in terms of $\lambda_i, \widetilde{\lambda}_i$ and gives $\delta(P)\delta(Q)$

$$\delta(C\cdot Z) = \delta(C\cdot\widetilde{\lambda})\delta(C^{\perp}\cdot\lambda)$$

Surprising connection

- Building matrix with positive minors





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(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

Surprising connection

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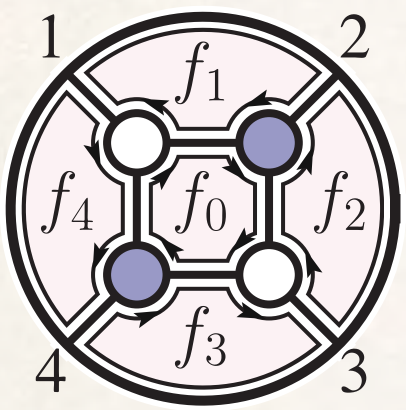
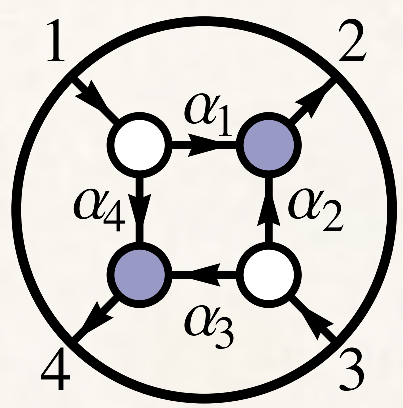




$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

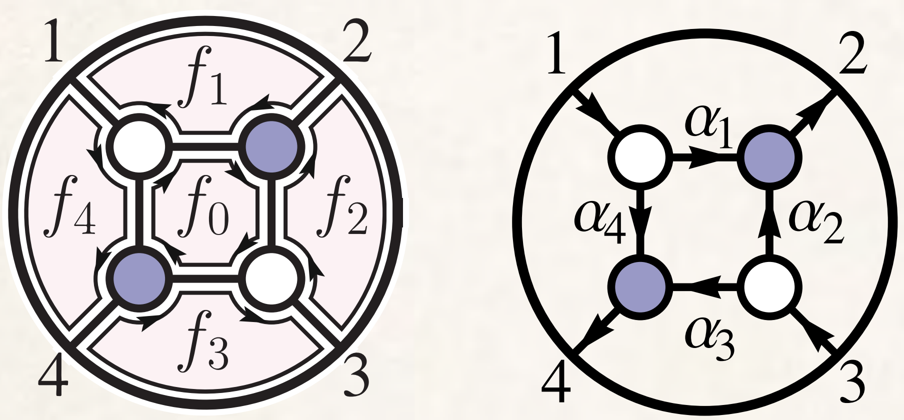
- For N<4 SYM the value of the diagram is equal to

$$\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\dots\frac{d\alpha_n}{\alpha_n}\cdot \mathcal{J}(\alpha)\delta(C\cdot Z)$$

(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

Surprising connection

- Building matrix with positive minors



$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

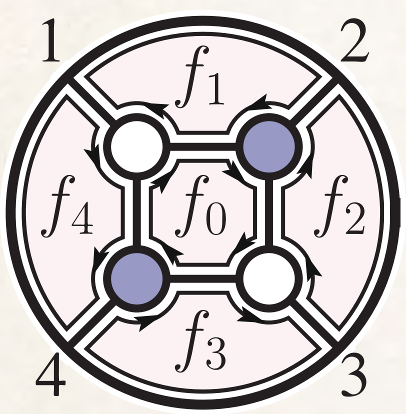
- For N=8 SUGRA the value of the diagram is equal to

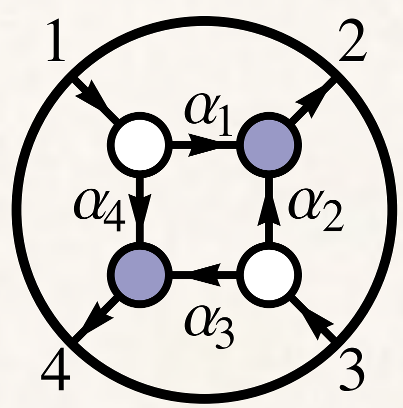
$$\Omega = \frac{d\alpha_1}{\alpha_1^3}\frac{d\alpha_2}{\alpha_2^3}\dots\frac{d\alpha_m}{\alpha_m^3}\prod_v \Delta_v \cdot \delta(C\cdot Z)$$

(Herrmann, JT 2016)

Surprising connection

- Building matrix with positive minors





$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

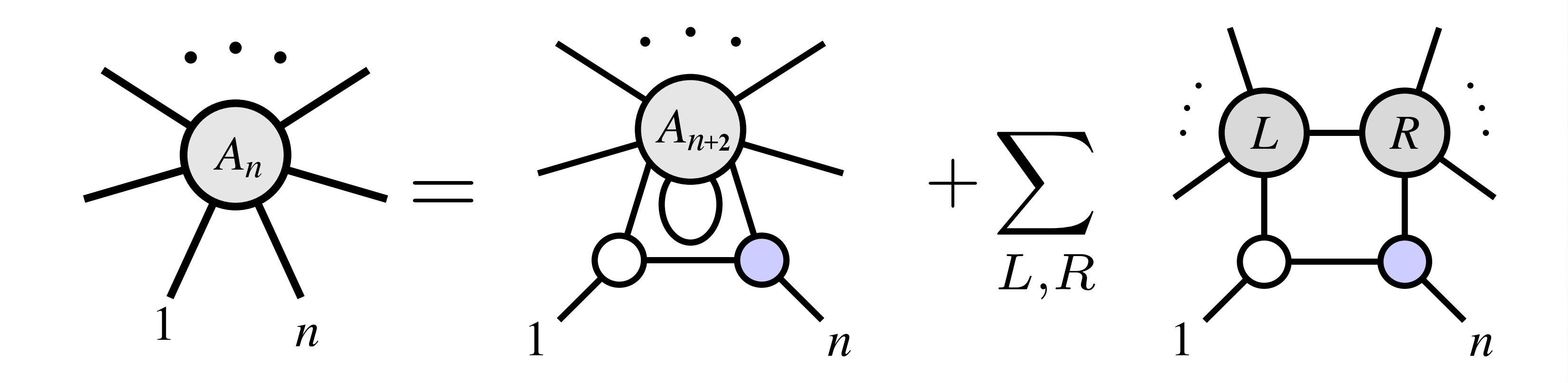
- For general QFT the value of the diagram is equal to

$$\Omega = F(\alpha)\,\delta(C\cdot Z)$$

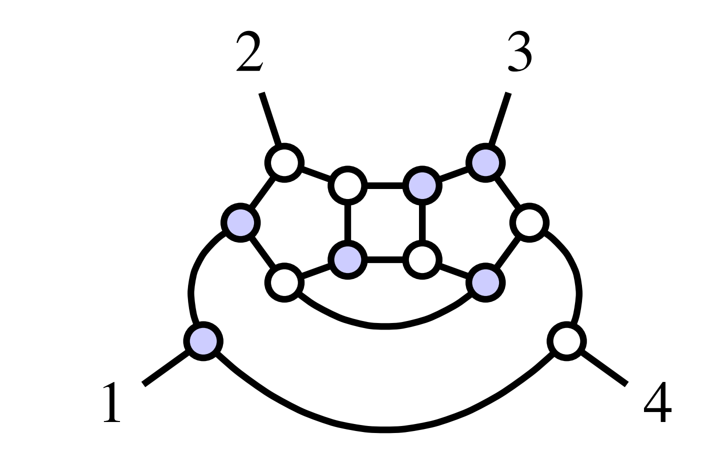
- In a sense $F(\alpha)$ defines a theory (as Lagrangian does)

Amplitude from recursion relations

- In any theory: on-shell diagrams = cuts of the amplitude
  - We learn about properties of the amplitude
- In planar N=4 SYM theory we have recursion relations



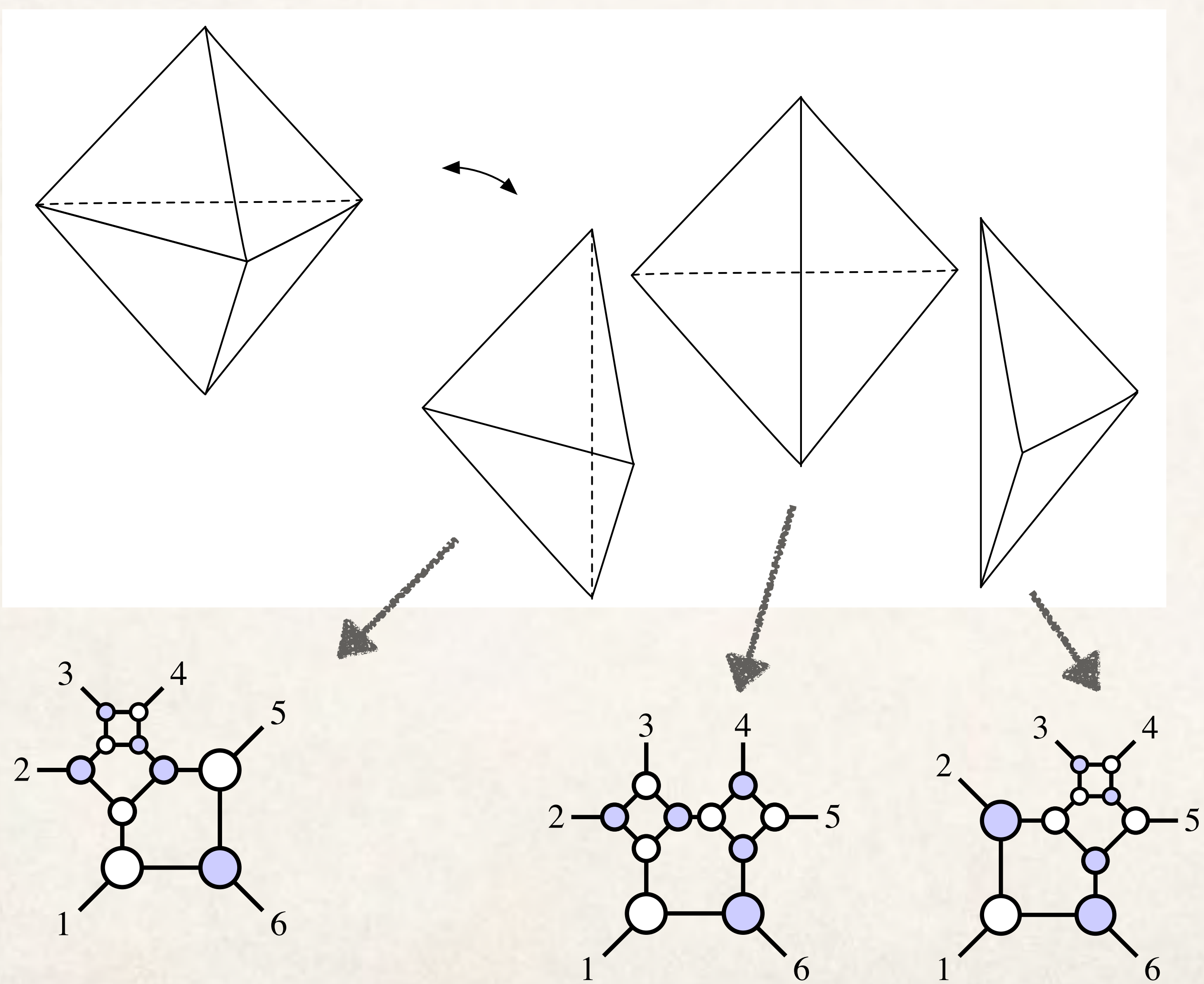
4pt 1-loop



Amplituhedron

(Arkani-Hamed, JT 2013)

- Pieces in the recursion glue together



$$Y = C \cdot Z$$

Logarithmic volume form

$$\Omega(Y, Z_i)$$

Tree-level + loop integrand

Uniqueness

- Amplitudes in planar N=4 SYM are completely fixed
  - IR properties: logarithmic singularities $\Omega \sim \frac{dx}{x}$
  - UV properties: no poles at infinity

    never a singularity at $\ell \to \infty$

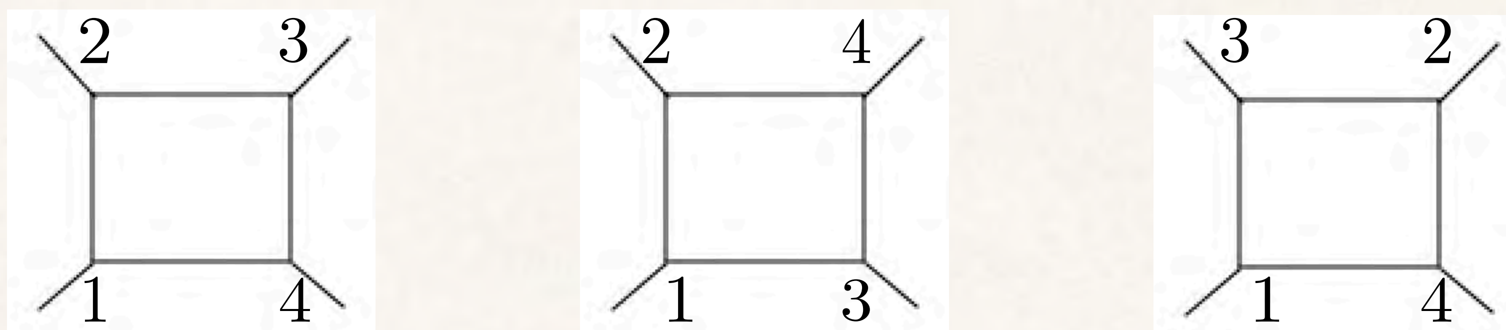
↓ + absence of unphysical singularities

Reproduce it by unique geometry
with the same properties

Non-planar amplitudes

Problem with labels

- No planarity - no labels, no unique integrand



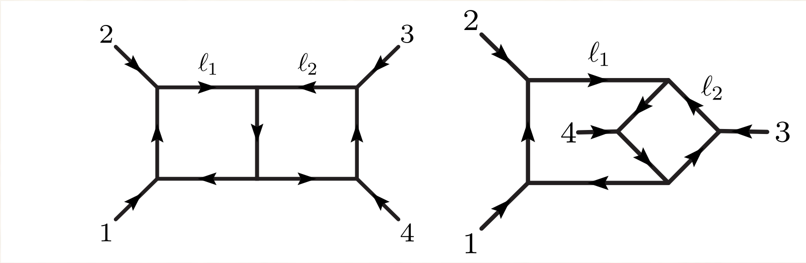
What is ℓ ?

- No planar limit of gravity amplitudes
- Same problem in full N=4 SYM amplitudes
  - We have to work with diagrams
  - In addition we have to include color factors

Non-planar N=4 SYM amplitudes

- Conservative approach

$$A = \sum_i a_i \cdot C_i \cdot I_i$$



$$f^{1ab} f^{bcd} \dots f^{4ef}$$

- Integrals same properties as in the planar limit:
  - Logarithmic singularites
  - No poles at infinity

→ This suggests there is a hidden symmetry in the full theory

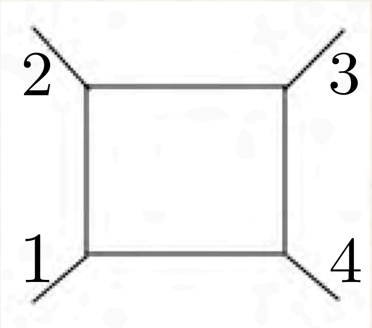
(Arkani-Hamed, Bourjaily, Cachazo, JT 2014)

(Bern, Herrmann, Litsey, Stankowicz, JT 2014, 2015)

Non-planar labels

- The lack of labels does not allow us to formulate the amplitude geometrically like Amplituhedron

- Some attempts to solve the labeling problems
  - Sum over all labels - overcounting
  - Linearized propagators



(Baadsgaard, Bjerrum-Bohr, Bourjaily, Caron-Huot, Damgaard, Feng 2015)

$$I = \frac{1}{\ell^2(\ell \cdot p_1)(\ell \cdot (p_1 + p_2))(\ell \cdot p_4)}$$

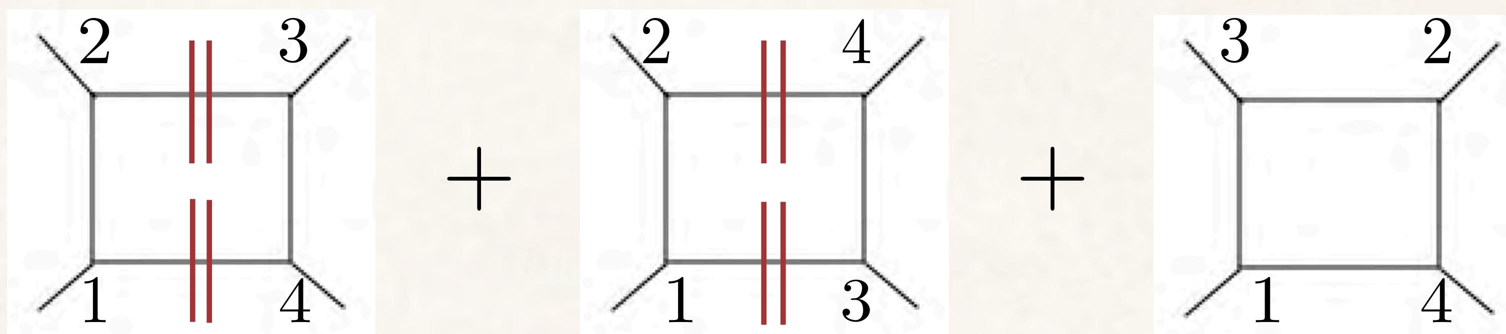
spurious poles do not cancel

- Well-defined are **cuts** of amplitudes: **products of trees**

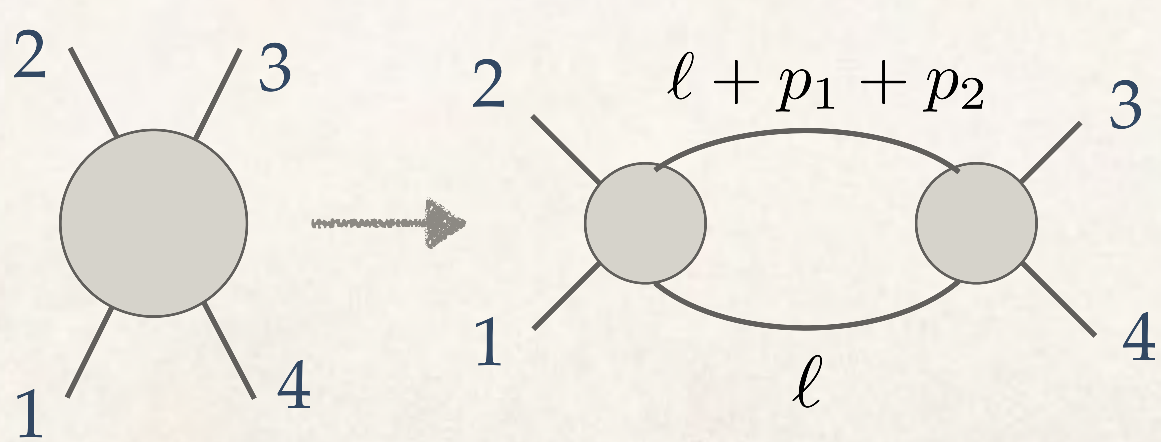
Unique labels from cuts

- We can cut the "non-planar integrand"

$A_4^{1-loop} =$



- Set two propagators to zero: $\ell^2 = (\ell + p_1 + p_2)^2 = 0$

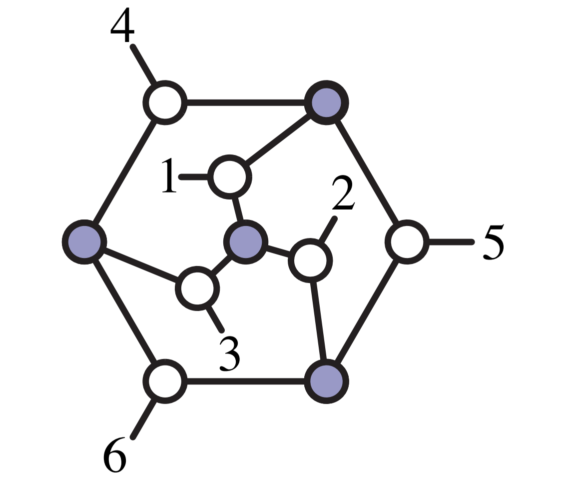


Residue is the product of trees

$$\text{Cut } I = A_4(1, 2, \ell, \ell + p_1 + p_2) A_4(3, 4, -\ell, -\ell - p_1 - p_2)$$

Non-planar cuts

- We can cut more via generalized unitarity
- If we cut everything into 3pt vertices: on-shell diagrams



Non-planar N=4 SYM theory

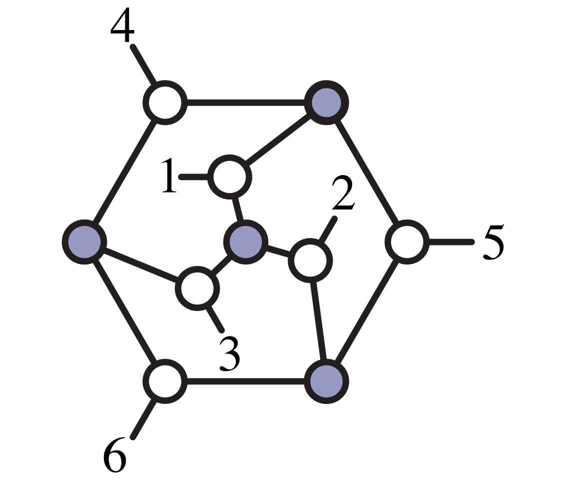
- Connection to Grassmannian
- Logarithmic form

$$\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\dots\frac{d\alpha_m}{\alpha_m}\cdot\delta(C\cdot Z)$$

- Precise geometry not known in general

Non-planar cuts

- We can cut more via generalized unitarity
- If we cut everything into 3pt vertices: on-shell diagrams



N=8 supergravity

- Connection to Grassmannian
- Non-logarithmic form

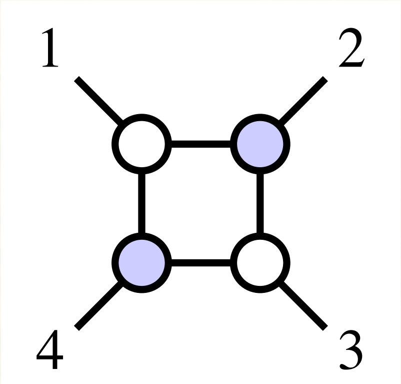
$$\Omega = \frac{d\alpha_1}{\alpha_1^3}\frac{d\alpha_2}{\alpha_2^3}\dots\frac{d\alpha_m}{\alpha_m^3}\prod_v \Delta_v \cdot \delta(C\cdot Z)$$

- New features different from YM

Cuts in N=8 supergravity

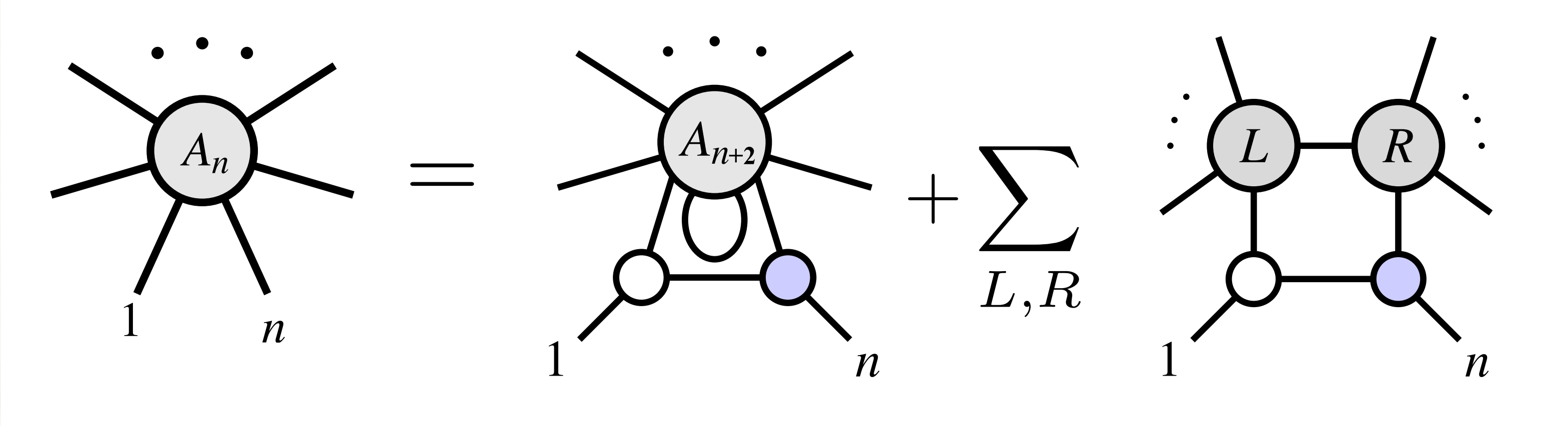
Singularities in IR

- On-shell diagrams and cuts are all about singularities in the IR



$$\ell_1^2 = \ell_2^2 = \ell_3^2 = \ell_4^2 = 0$$

- For N=4 SYM: this is full story, knowing on-shell diagrams is enough to fix the amplitude — there is no UV region
  - Recursion relations



Singularities in IR

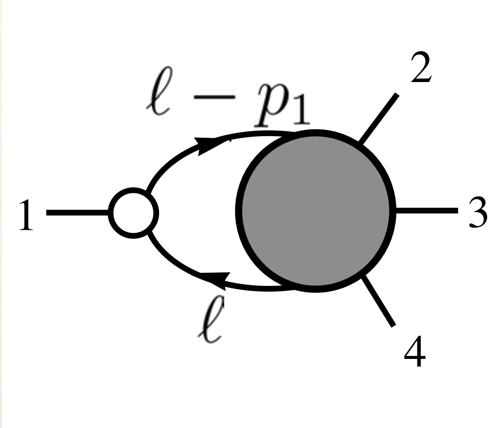
- For N=8 there are both IR and UV regions due to a different powercounting: on-shell diagrams are not enough

- In IR the amplitudes behaves very mildly

$$A_{YM}^{L-loop} \sim \frac{1}{\epsilon^{2L}} \qquad A_{GR}^{L-loop} \sim \frac{1}{\epsilon^{L}}$$

- This behavior can be nicely seen from a particular cut of the integrand

$$\ell^2 = (\ell - p_1)^2 = 0$$



collinear region

$$\ell \sim p_1$$

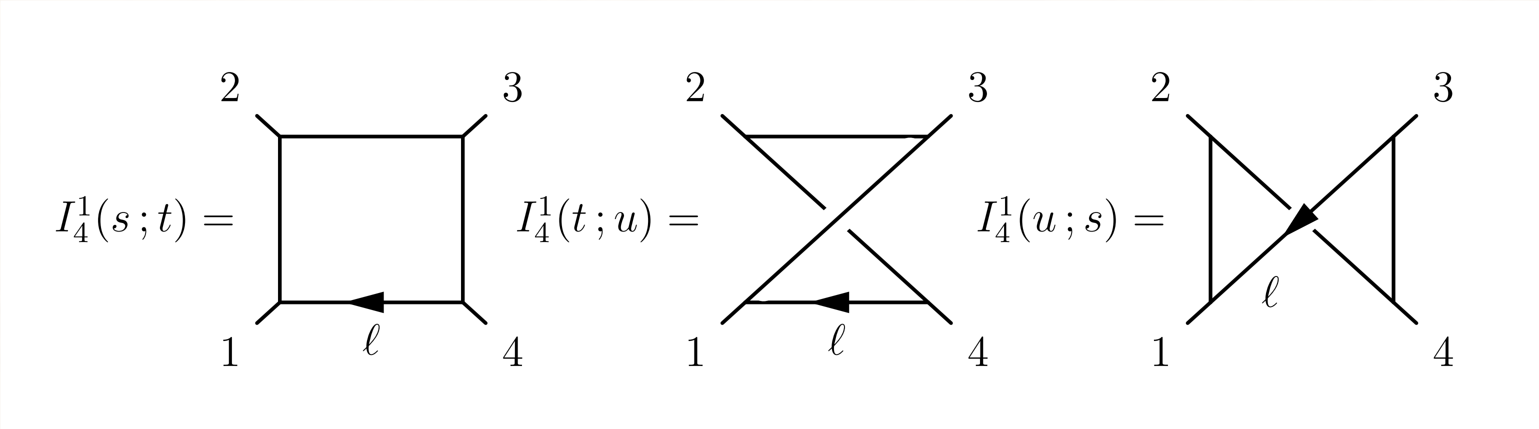
cut of the amplitude cancels

Singularities in IR

(Herrmann, JT 2016)

- Requires cancelation between diagrams even at 1-loop

$$\ell = \alpha p_1$$



Integrand cut (On-shell diagram)	$C_1(\alpha)$	$+$	$C_2(\alpha)$	$+$	$C_3(\alpha)$	$=$	0
Integral	$\frac{1}{\epsilon^2}$	$+$	$\frac{1}{\epsilon^2}$	$+$	$\frac{1}{\epsilon^2}$	$=$	$\frac{1}{\epsilon}$

- There is more detailed version of this cancelation suggesting there is still something to learn in IR

UV from integrand

- Simple scaling check

$$I = \int \frac{d^4\ell}{\ell^2(\ell+p_1)^2(\ell-p_2)^2} \qquad I = \int \frac{d^4\ell}{\ell^2(\ell+p_1+p_2)^2}$$

$$\ell \to \infty$$

$$I \sim \int \frac{d\ell}{\ell^3} \qquad I \sim \int \frac{d\ell}{\ell} \sim \ln\Lambda$$

pole at infinity

UV from integrand

- We can repeat the same exercise for higher loops

$$I = \int \frac{d^4\ell_1\, d^4\ell_2}{\ell_1^2(\ell_1+p_1)^2(\ell_1+\ell_2)^2\ell_2^2(\ell_2+p_3)^2} \sim \int \frac{d\ell}{\ell^3}$$

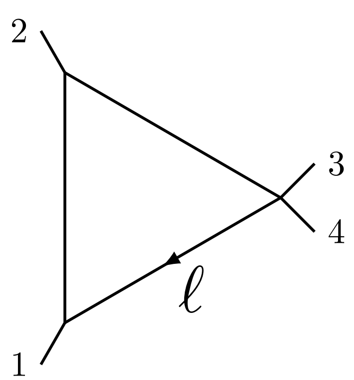
for $\ell_1, \ell_2 \sim \ell \to \infty$

- We can take this limit for the planar integrand but we can not do it for the non-planar amplitude

no good labels how to send $\ell \to \infty$

Poles at infinity

- This scaling is just a special example of the poles at infinity
- We can perform cuts first and then send $\ell \to \infty$



$$I = \frac{d^4\ell \; s}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2}$$

0 0 0

Cut all three propagators

Solution

$$\ell = \lambda_1(z\widetilde{\lambda}_2 - \widetilde{\lambda}_1)$$

Residue

$$\text{Cut}\, I = \frac{dz}{z}$$

Pole at

$$z \to \infty$$

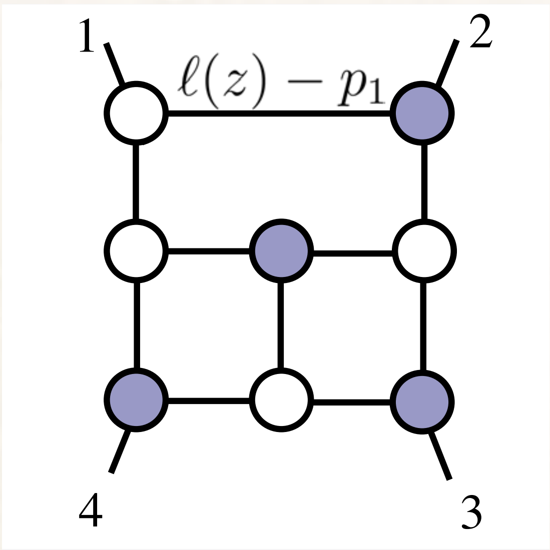
Special case: N=4 SYM

- Planar sector: there are no poles at infinity
  - Never generate in the cut structure a pole for $\ell \to \infty$
- Much stronger than UV finiteness, consequence of the hidden dual conformal symmetry
- All singularities are on the cuts when $\ell^2 = 0$
- Non-planar sector: evidence it is true as well
  - suggests a possible hidden symmetry in the full N=4 SYM theory

Cuts of N=8 supergravity

(Bern, Herrmann, Litsey, Stankowicz, JT 2014, 2015)

- We can check this property only on cuts
- Example: 3-loops



$$\sim \frac{dz}{z}$$

Pole at

$$z \to \infty$$
$$\ell(z) \to \infty$$

- Higher loops: higher poles $\sim z^{L-4}\, dz$

Cuts of N=8 supergravity

- This is an example of the maximal cut

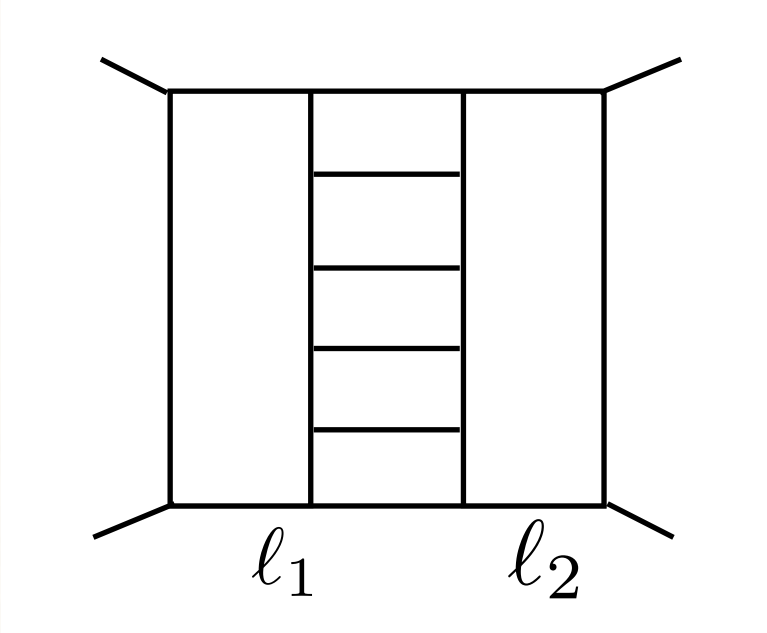
$$A = \sum_k c_k \int d^4\ell_1 \ldots d^4\ell_L \, \mathcal{I}_k$$

Cut all propagators in one of the integrals

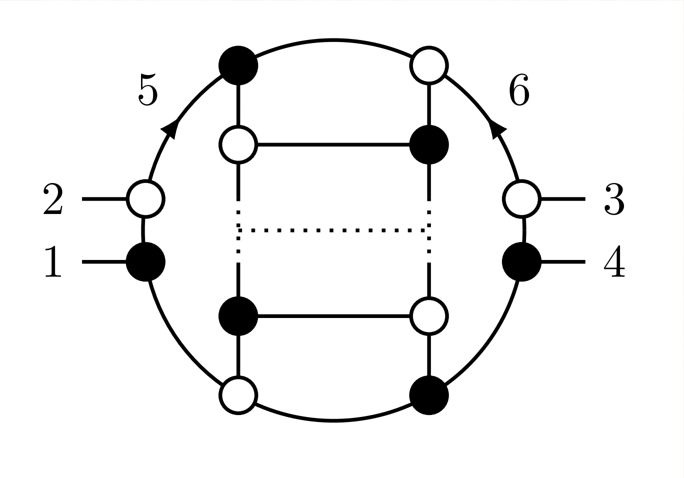
- Only one term in the sum contributes on the cut
- The numerator of $\mathcal{I}_k$ and the coefficient c_k given by the value of the cut (calculated as product of trees)

UV divergence in N=8

- Expected divergence at 7-loops from maximal cut



matches the on-shell diagram with pole at infinity



- The numerator of the diagram is fixed by the cut to be

$$\int d^4\ell_1 \ldots d^4\ell_7 \frac{(\ell_1 \cdot \ell_2)^8}{D} \to \int \frac{d\ell}{\ell} \qquad \text{for} \quad \ell_k \sim \ell \to \infty$$

UV divergence in N=8

- Unless there is some cancelation mechanism

$$A = \sum_k c_k \int d^4\ell_1 \ldots d^4\ell_L \, \mathcal{I}_k$$

UV of the amplitude given by the UV of the worst diagram

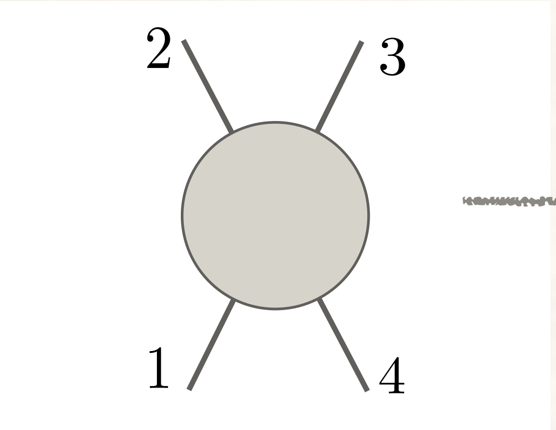
- Standard procedure: get the full amplitude, integrate, collect UV divergences and see if they cancel

- We are interested in a different question: is it possible to get improved behavior at infinity on the cut?

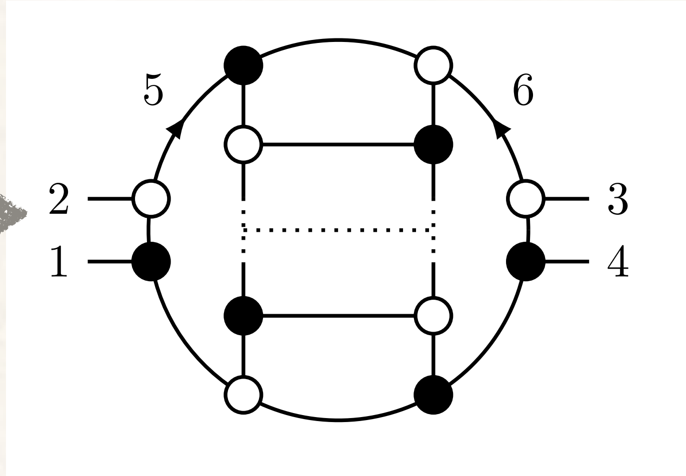
UV surprises in N=8 supergravity

(Herrmann, JT, to appear)

Cuts and poles at infinity



cut, cut, cut, cut,…

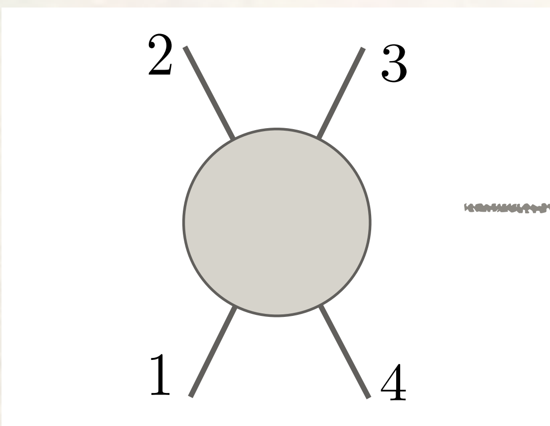


- Full amplitude
- All integrals contribute
- Can not check if there are poles at infinity

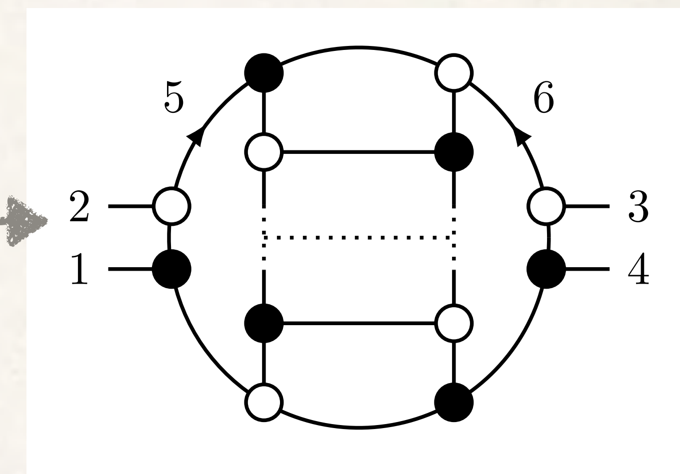
We want to do this but can not due to the lack of variables

- Maximal cut
- One integral contributes
- There are (higher) poles at infinity

Cuts and poles at infinity



cut, cut, cut, cut,…



- Full amplitude
- All integrals contribute
- Can not check if there are poles at infinity

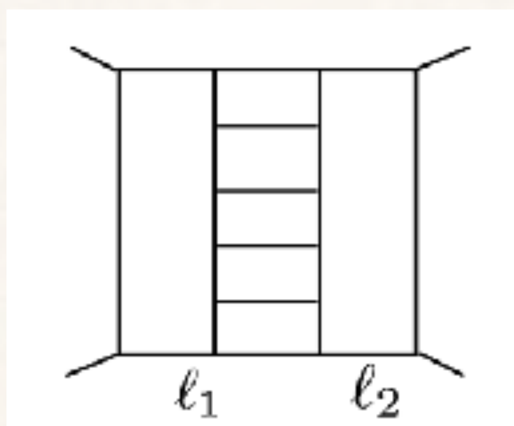
- Maximal cut
- One integral contributes
- There are (higher) poles at infinity

Stop half-way in the cut structure: allow for cancelations between diagrams

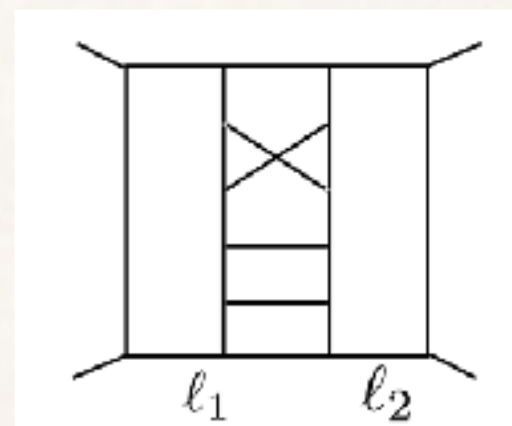
Non-trivial behavior at infinity

- We perform a cut where more diagrams contribute

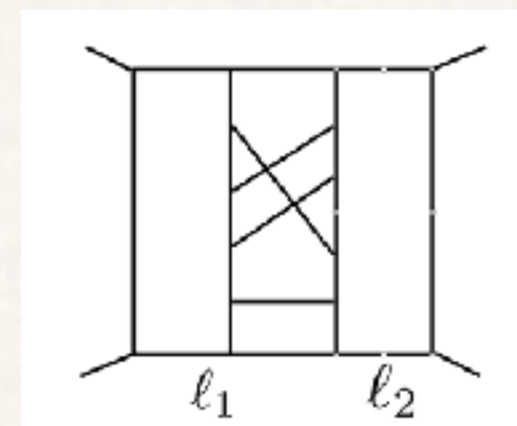
$A_4^{L-loop} =$



$+$



$+$



$+ \dots$

$$z^n = z^{m_1} + z^{m_2} + z^{m_3} + \dots$$

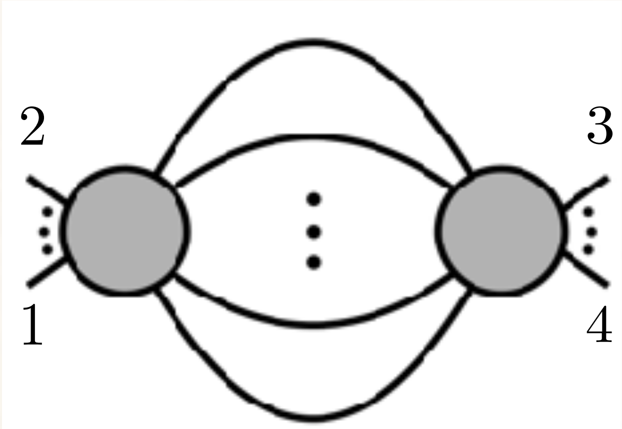
- Send loop momenta to infinity: $\ell_k \to \infty$ by sending $z \to \infty$

- Any cancelation on any cut would be interesting

$$n < \max(m_1, m_2, \dots)$$

Multi-unitarity cut

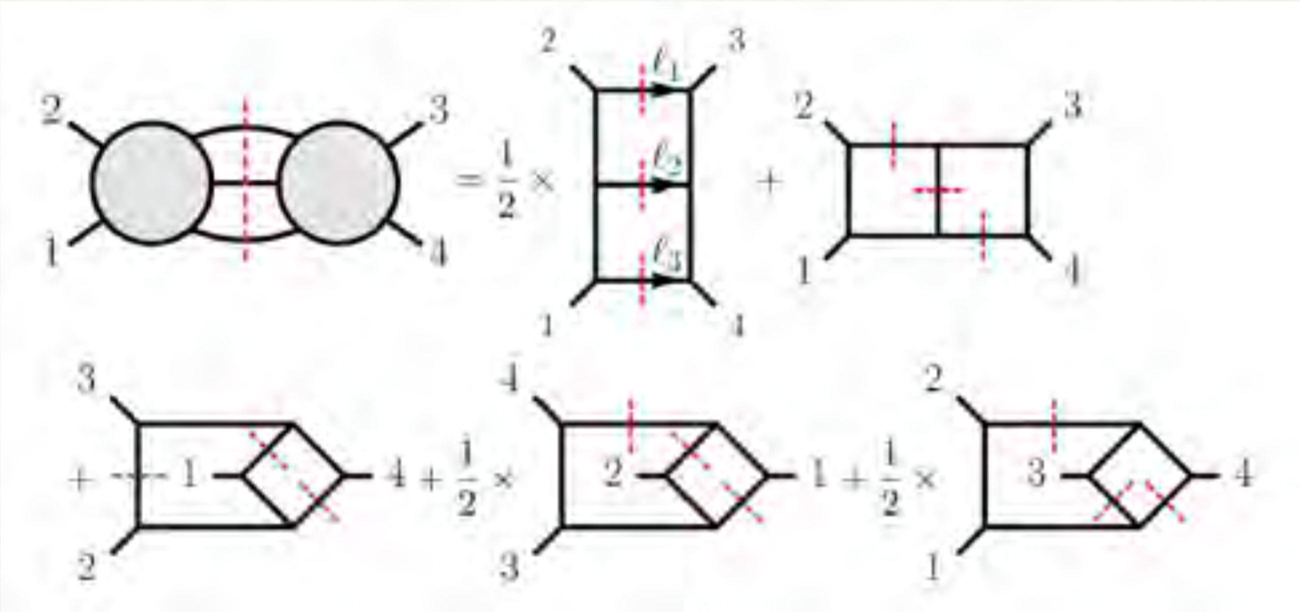
- Minimal cut which defines unique labels



$$\ell_k^2 = 0$$

$$\sum_k \ell_k = p_1 + p_2$$

2-loop check in N=4 SYM and N=8 SUGRA



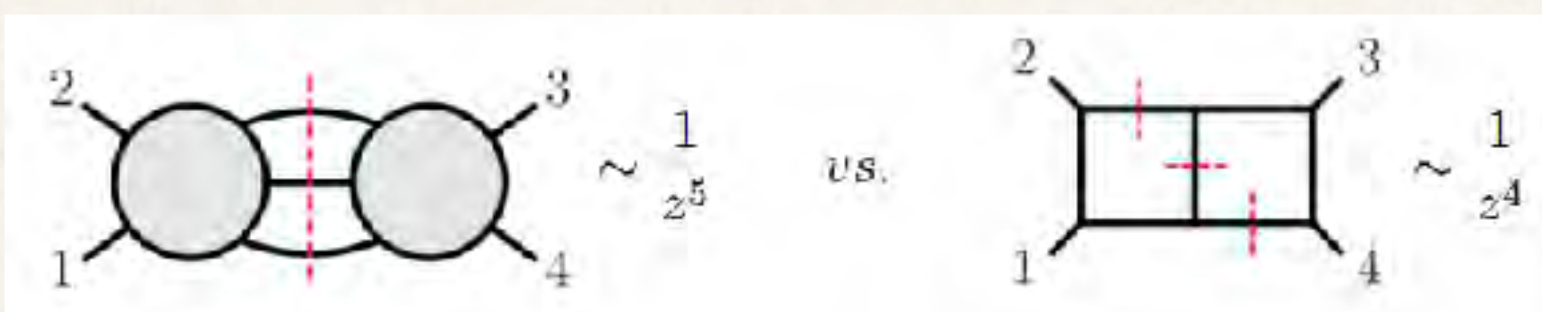
Multi-unitarity cut cancelations

- Send all loop momenta to infinity on the cut

$$\ell_k = \lambda_k \widetilde{\lambda}_k \qquad \ell_k \to \ell_k + z c_k \lambda_k \widetilde{\zeta} \qquad \sum_k c_k \lambda_k = 0$$

$$z \to \infty$$

Compare the cut to individual integrals in N=8 supergravity



N=4 SYM both $\sim \dfrac{1}{z^4}$

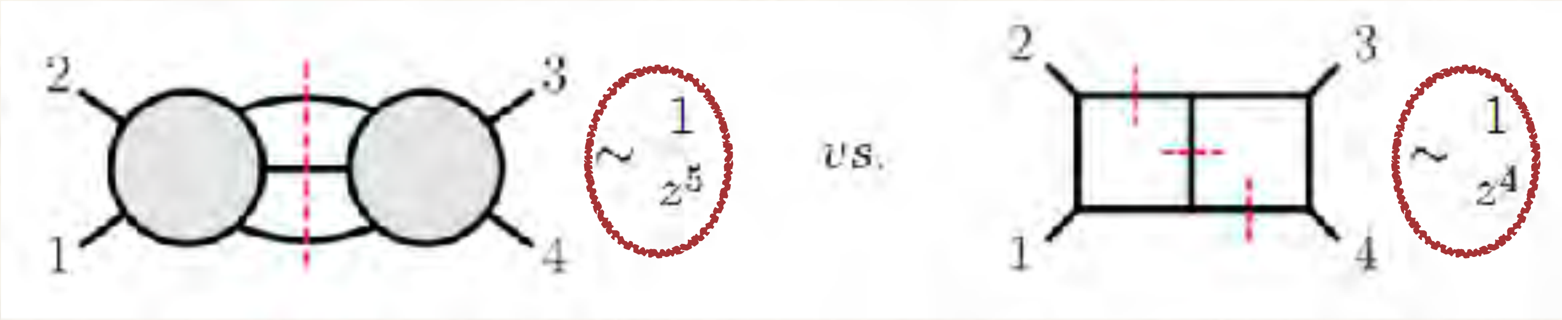
Multi-unitarity cut cancelations

- Send all loop momenta to infinity on the cut

$$\ell_k = \lambda_k \widetilde{\lambda}_k \qquad \ell_k \to \ell_k + z c_k \lambda_k \widetilde{\zeta} \qquad \sum_k c_k \lambda_k = 0$$

$$z \to \infty$$

Compare the cut to individual integrals in N=8 supergravity



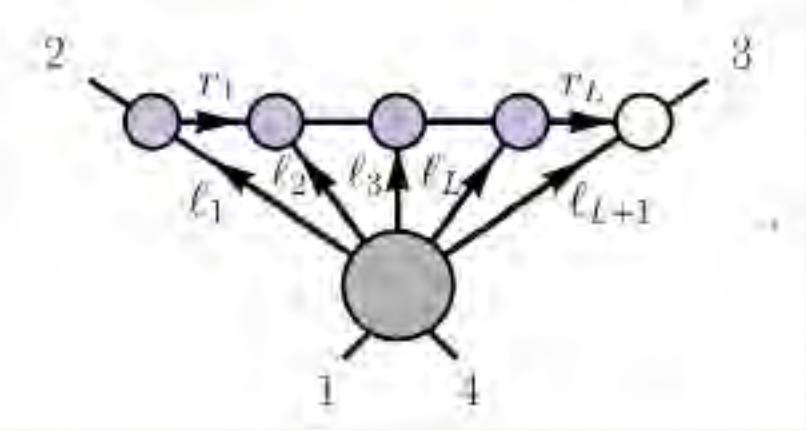
N=4 SYM both $\sim \dfrac{1}{z^4}$

Cancelation!

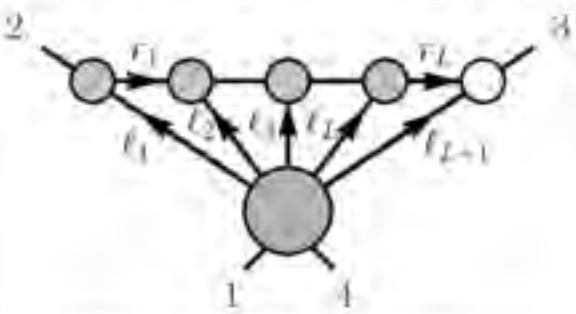
More cuts, more cancelations

- For practical purposes: to go to arbitrary loop order
  - cut more propagators
  - use parameter α
  - probe the pole at infinity

$$\alpha \to \infty$$



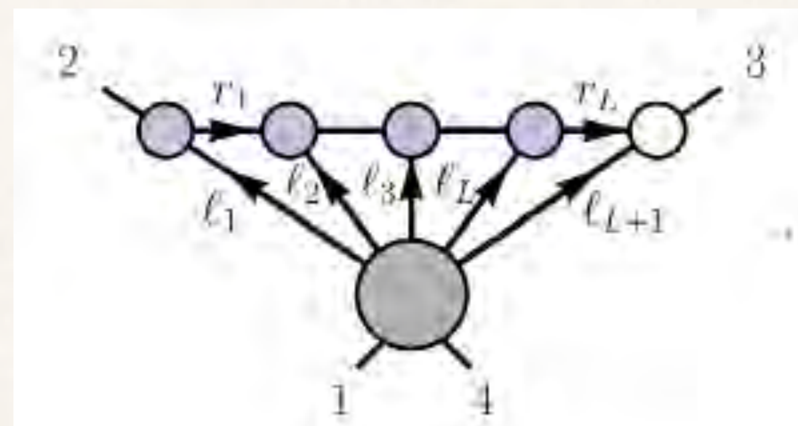
Compare to the cut of the explicit result for the N=8 amplitude in the literature

$L =$	2	3	4	L
	α^{-2}	α^{-3}	α^{-4}	α^{-L}
worst diagram	α^{-2}	α^{-1}	α^{0}	?

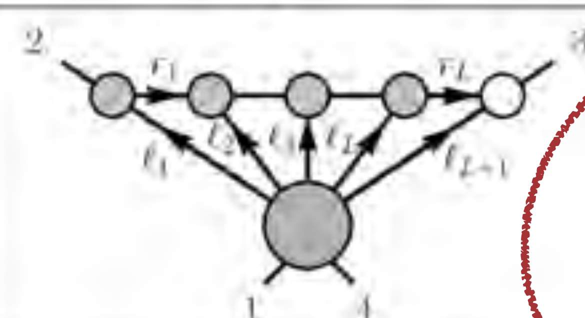
More cuts, more cancelations

- For practical purposes: to go to arbitrary loop order
  - cut more propagators
  - use parameter α
  - probe the pole at infinity

$$\alpha \to \infty$$



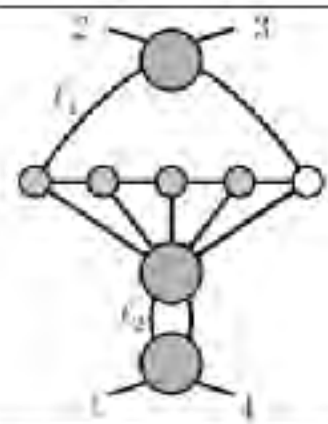
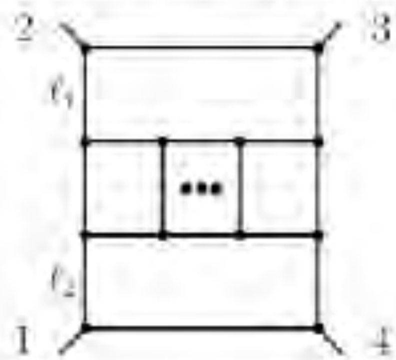
Compare to the cut of the explicit result for the N=8 amplitude in the literature

$L=$	2	3	4	L
	α^{-2}	α^{-3}	α^{-4}	α^{-L}
worst diagram	α^{-2}	α^{-1}	α^{0}	?

Cancelation!

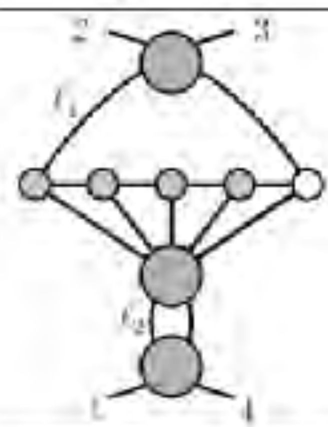
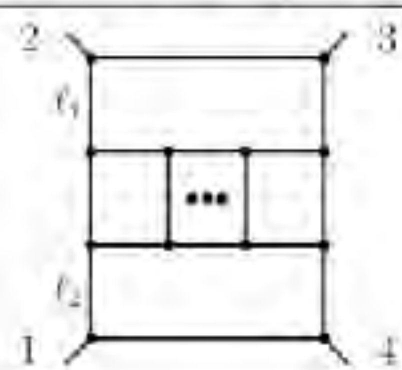
More cuts, more cancelations

- Another cut which hits the “worst behaved diagram”

$L =$	3	4	5	L
	α^{-10}	α^{-8}	α^{-8}	α^{-8}
	α^{-5}	α^{-4}	α^{-3}	α^{L-8}

More cuts, more cancelations

- Another cut which hits the "worst behaved diagram"

$L =$	3	4	5	L
	α^{-10}	α^{-8}	α^{-8}	α^{-8}
	α^{-5}	α^{-4}	α^{-3}	α^{L-8}

Cancelation!

Remarks

- In planar N=4 SYM: absence poles at infinity tight to dual conformal symmetry
- In N=8 SUGRA: poles at infinity present for maximal cuts but seem to disappear if we cut less, perhaps completely absent for "non-planar integrand"
- Examples we checked also work for pure GR, just overall shift by eight powers α^8

Outlook

- We have empirical evidence there is a surprising behavior of gravity integrands in the UV

- Explanation? Hidden property or symmetry? Relation to UV (e.g. controlling the divergence)? Explicit checks for N=8 but same mechanism seems to be there for GR

- Preliminary: using the behavior at infinity as a constraint to fix the amplitude uniquely

Amplituhedron for gravity?

Thank you for your attention