# Is the Universe Isotropic?

SUBIR SARKAR

None of us can understand why there is a Universe at all, why anything should exist; that's the ultimate question. But while we cannot answer this question, we can at least make progress with the next simpler one, of what the Universe as a whole is like.
**Dennis Sciama (1978)**

*… the Universe must appear to be the same to all observers wherever they are. This **'cosmological principle'** …*
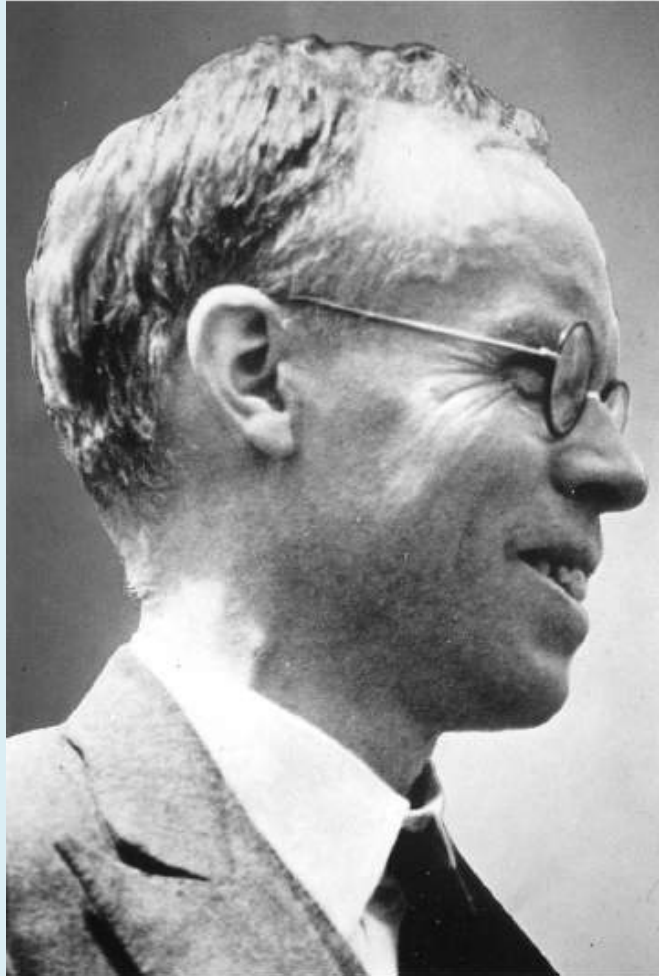
Edward Arthur Milne (1896-1950)

Rouse Ball Professor of Mathematics and Fellow of Wadham College, Oxford, 1928-

All we can *ever* learn about the universe is contained within our past light cone

our galaxy worldline

distant galaxy worldline

w=const

u

k

y=const
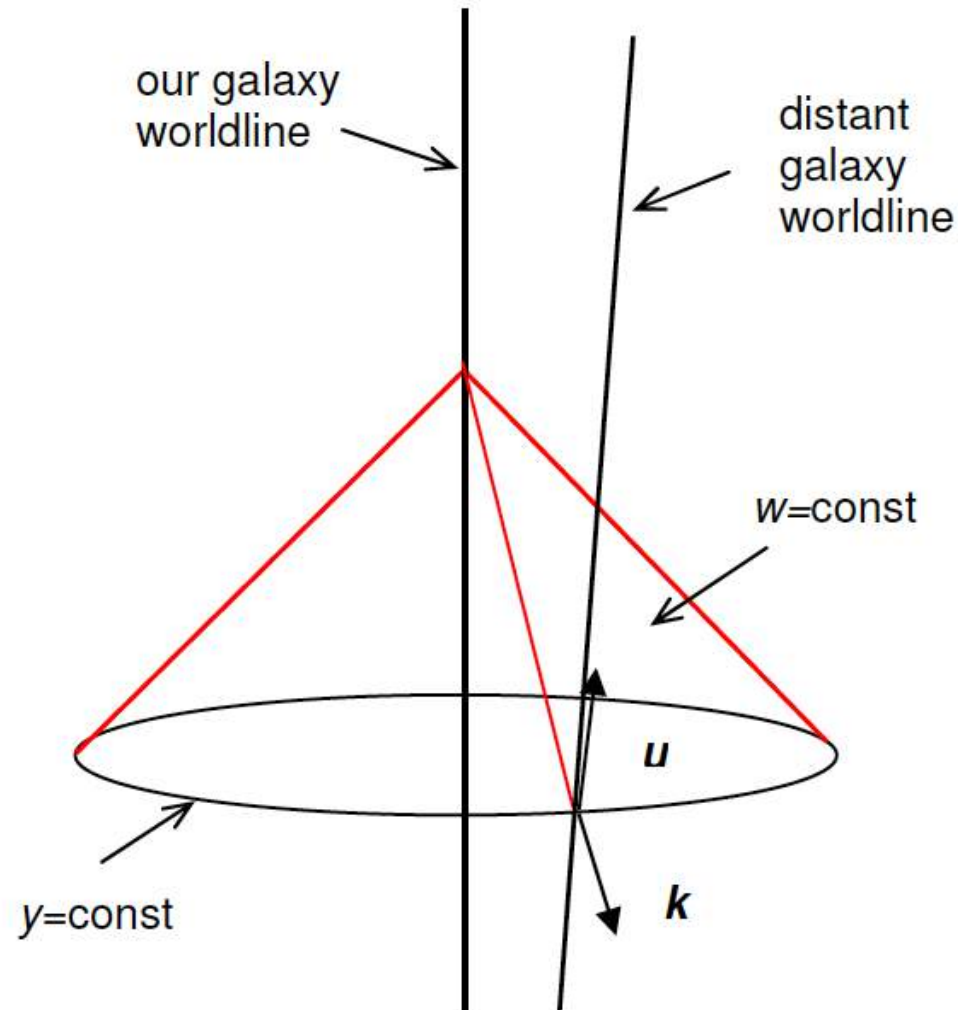
We *cannot* move over cosmological distances and check that the universe looks the same from 'over there' as it does from here ... so **there are *limits* to what we can know**

STANDARD COSMOLOGICAL MODEL

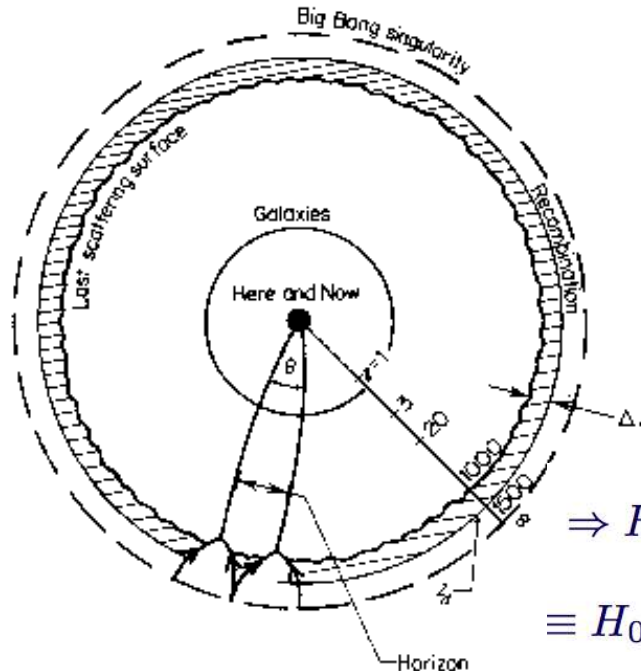Universe is isotropic + homogeneous (when averaged on large scales)
$\Rightarrow$ Maximally-symmetric space-time and ideal fluid energy-momentum tensor

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu$$
$$= a^2(\eta)\left[d\eta^2 - d\bar{x}^2\right]$$
$$a^2(\eta)d\eta^2 \equiv dt^2$$

Robertson-Walker

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda g_{\mu\nu}$$
$$= 8\pi G_N T_{\mu\nu}$$

Einstein

$$\Rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N \rho_m}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$
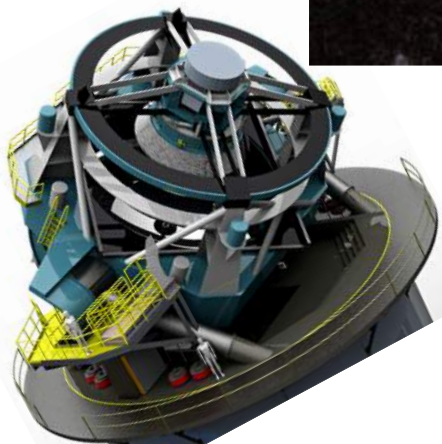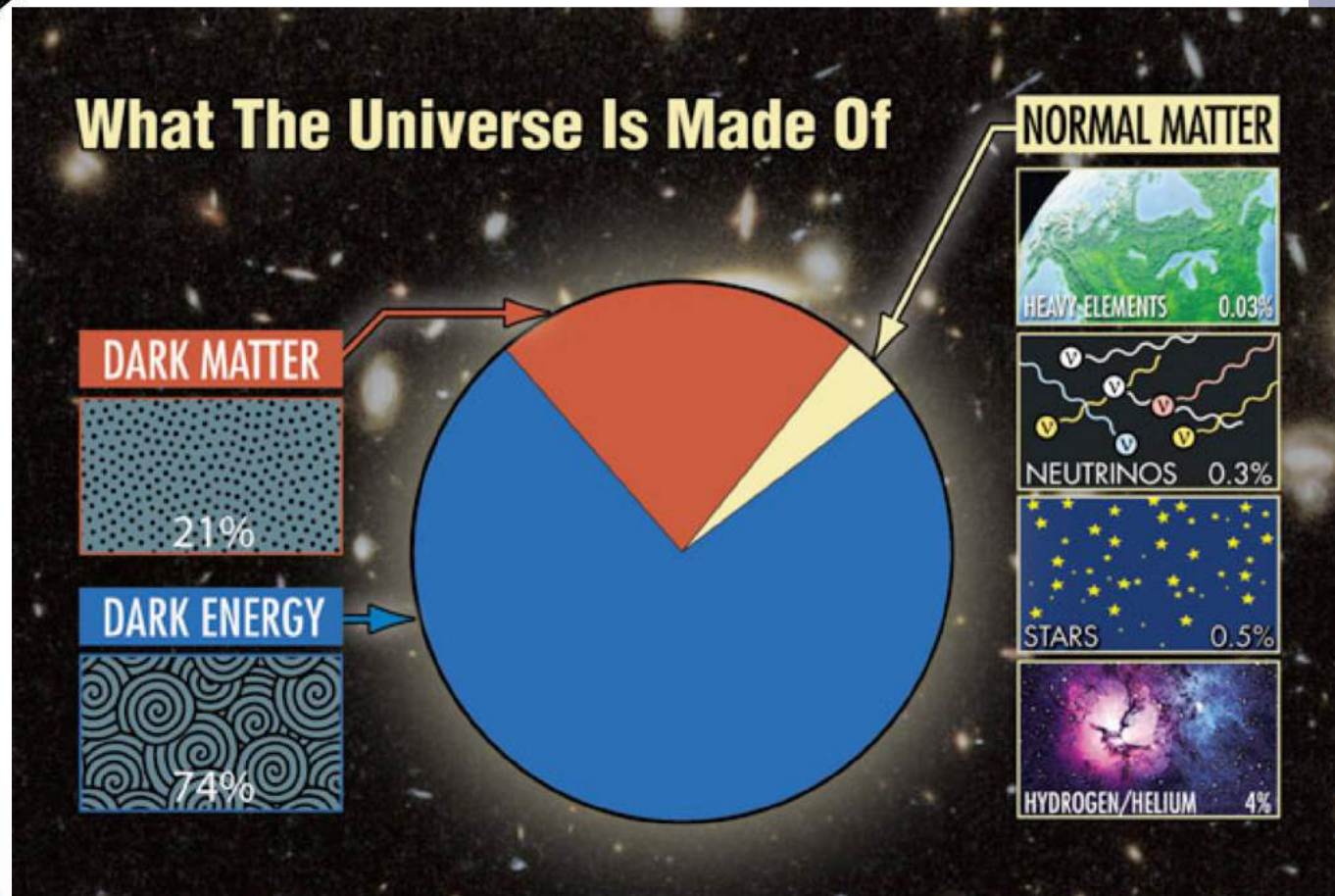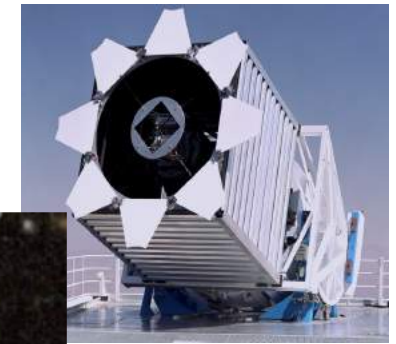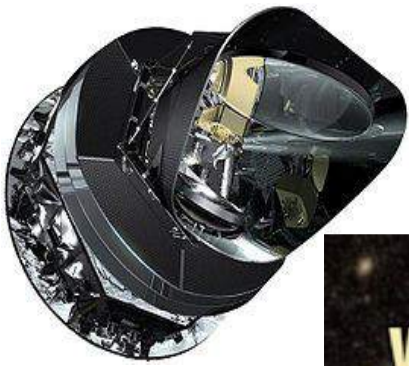$$\equiv H_0^2\left[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]$$

The **Friedmann-Lemaitre** equation $\Rightarrow$ cosmic 'sum rule': $\Omega_{matter} + \Omega_{curvature} + \Omega_\Lambda = 1$

Observe ~zero curvature (CMB fluctuations) + insufficient matter to make up critical density
$\rightarrow$ infer universe is dominated by **dark energy** with: $\Omega_\Lambda = 1 - \Omega_m - \Omega_k \sim 0.7 \Rightarrow \Lambda \sim 2H_0^2$

Since 1998 (Riess et al.[1], Perlmutter et al.[2]), surveys of cosmologically distant Type Ia supernovae (SNe Ia) have indicated an acceleration of the expansion of the Universe, distant SNe Ia being dimmer that expected in a decelerating Universe. With the assumption that the Universe can be described on average as isotropic and homogeneous, this acceleration implies either the existence of a fluid with negative pressure usually called "Dark Energy", a constant in the equations of general relativity or modifications of gravity on cosmological scales.

What The Universe Is Made Of

DARK MATTER
21%

DARK ENERGY
74%

NORMAL MATTER
HEAVY ELEMENTS 0.03%
NEUTRINOS 0.3%
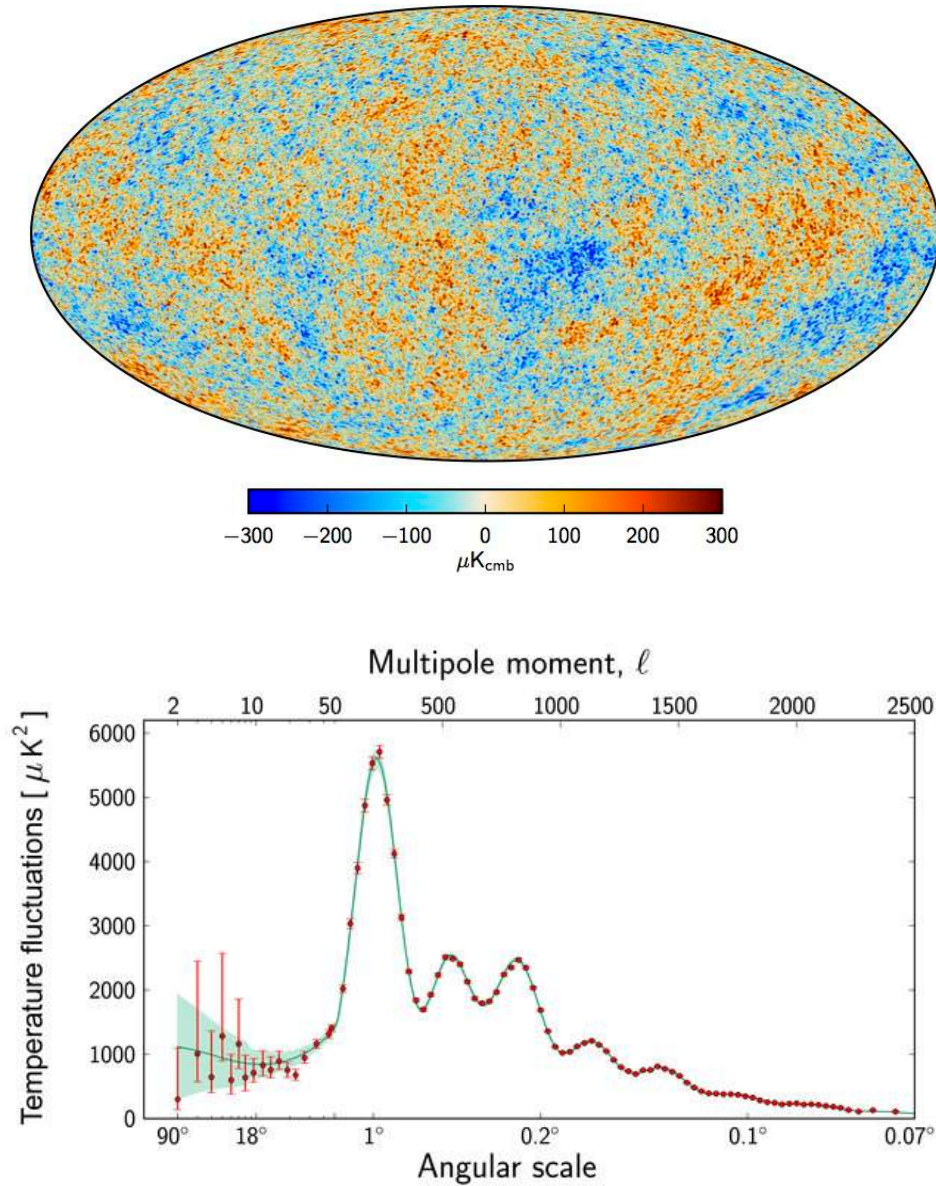STARS 0.5%
HYDROGEN/HELIUM 4%

There has been substantial investment in major satellites and telescopes to *measure the parameters* of the 'standard cosmological model' with increasing 'precision'... but surprisingly little interest in **testing its foundational assumptions**
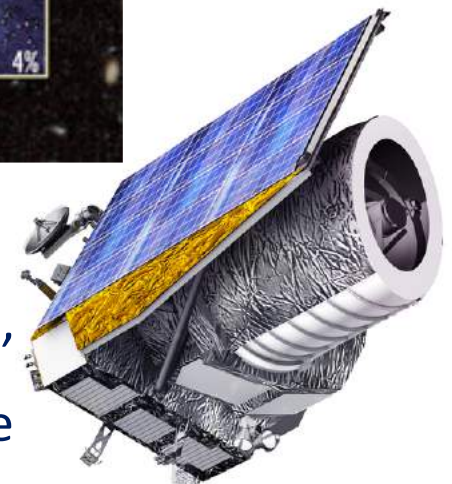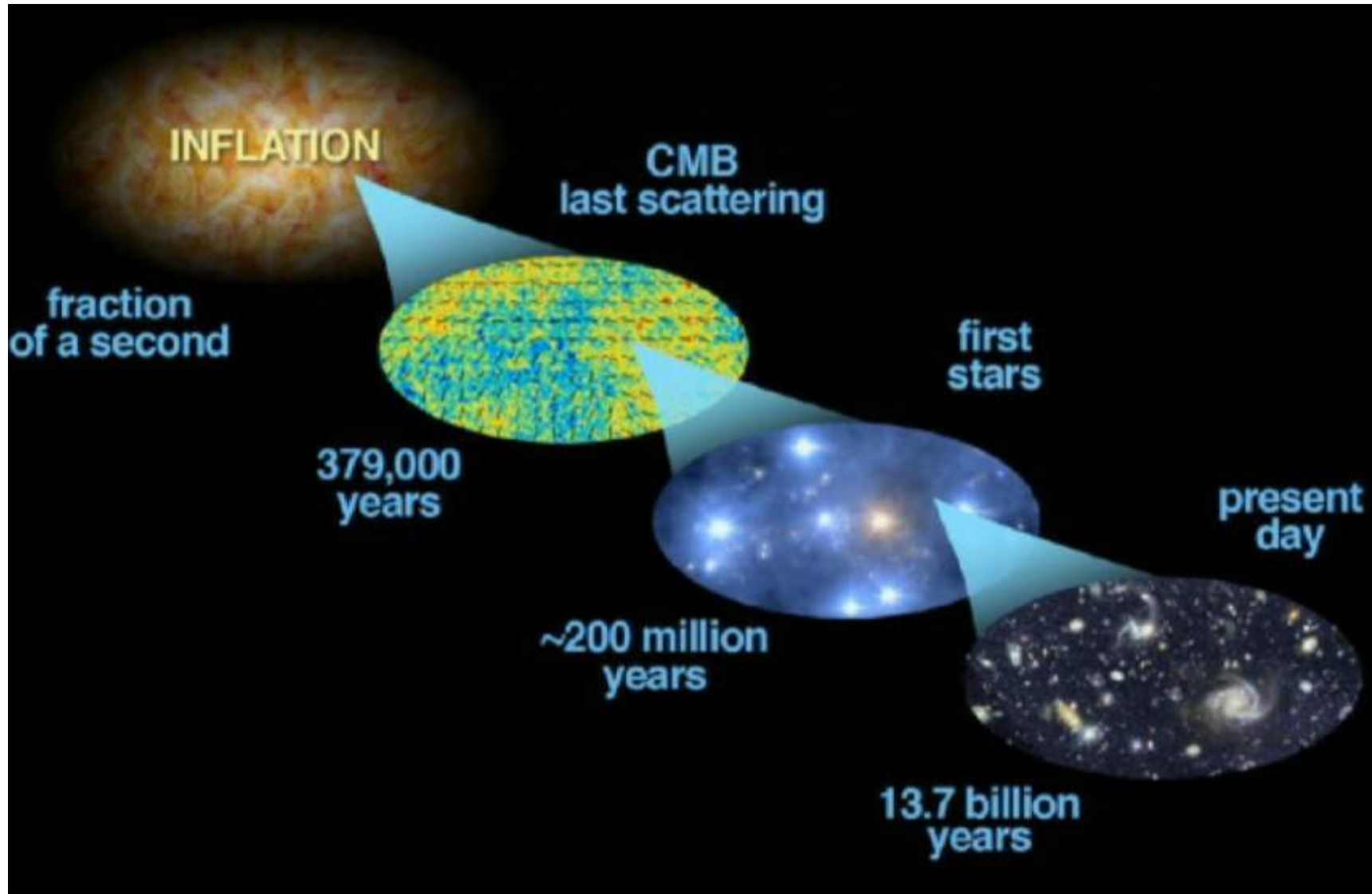
"Data from the Planck satellite show the universe to be highly isotropic" (Wikipedia)

We observe a statistically isotropic Gaussian random field of small temperature fluctuations (fully quantified by the 2-point correlations ➤ angular power spectrum)

Standard model of structure formation

The tiny **CMB temperature fluctuations** are understood as due to **scalar density perturbations** with an ~scale-invariant spectrum which were generated during an early phase of inflationary expansion ... these perturbations have subsequently grown into the **large-scale structure** of galaxies observed today through **gravitational instability** in a sea of **dark matter**

But the CMB sky is in fact rather anisotropic

There is a ~100 times *bigger* anisotropy in the form of a dipole with $\Delta T/T \sim 10^{-3}$

$$T(\theta) = \frac{T_0 \sqrt{1 - \beta^2}}{1 - \beta \cos\theta}$$
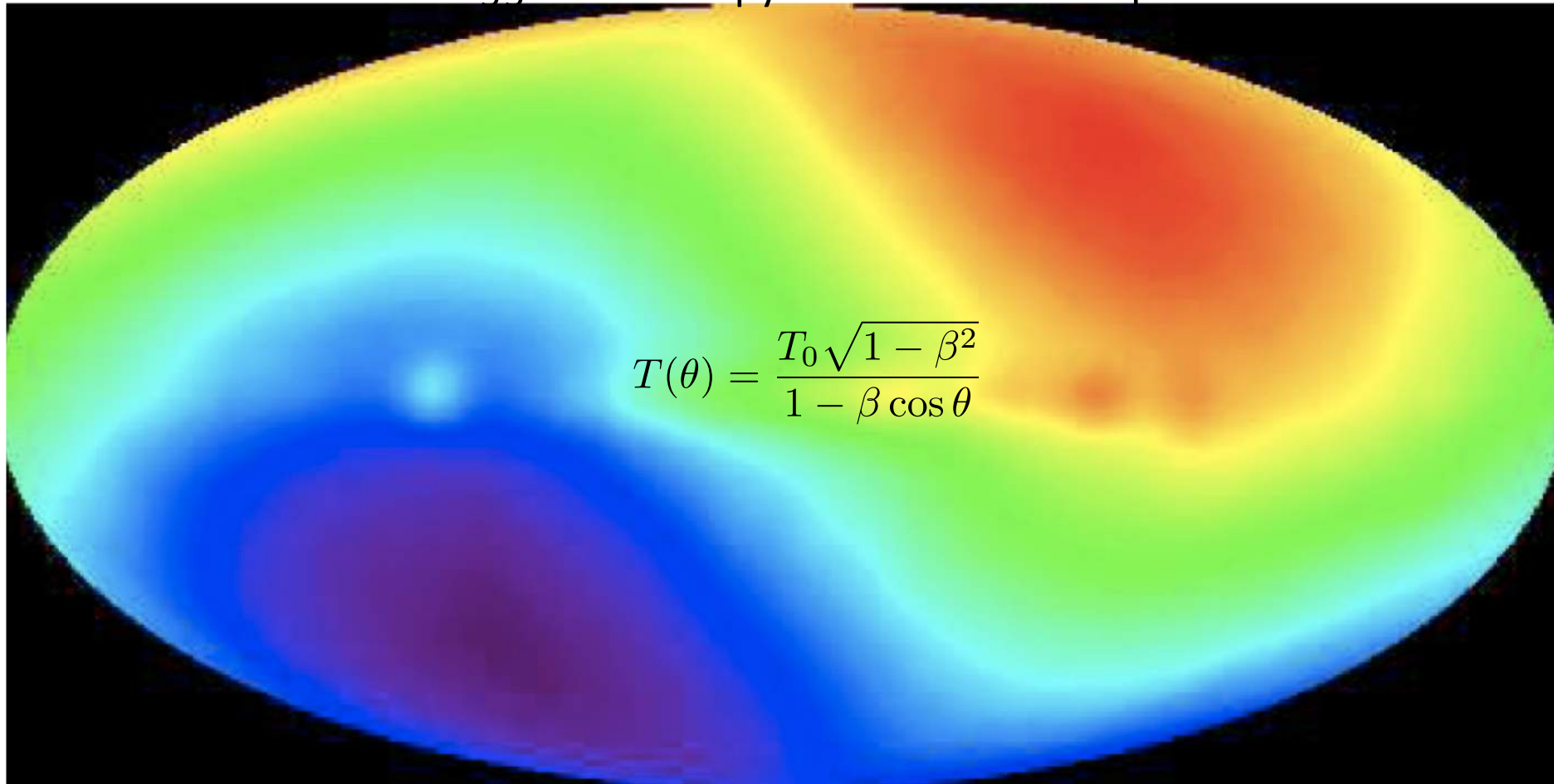
This is *interpreted* as due to our motion at 368 km/s wrt the frame in which the CMB is truly isotropic $\Rightarrow$ motion of the Local Group at 627 km/s towards l=263.85°, b=48.25°

This motion is presumed to be due to local *inhomogeneity* in the matter distribution
Its scale – beyond which we converge to the CMB frame – is supposedly of $O(100)$ Mpc
(Counts of galaxies in the SDSS & WiggleZ surveys said to scale as $\sim r^3$ on larger scales)

This is what our universe *actually* looks like locally (out to ~300 Mpc)
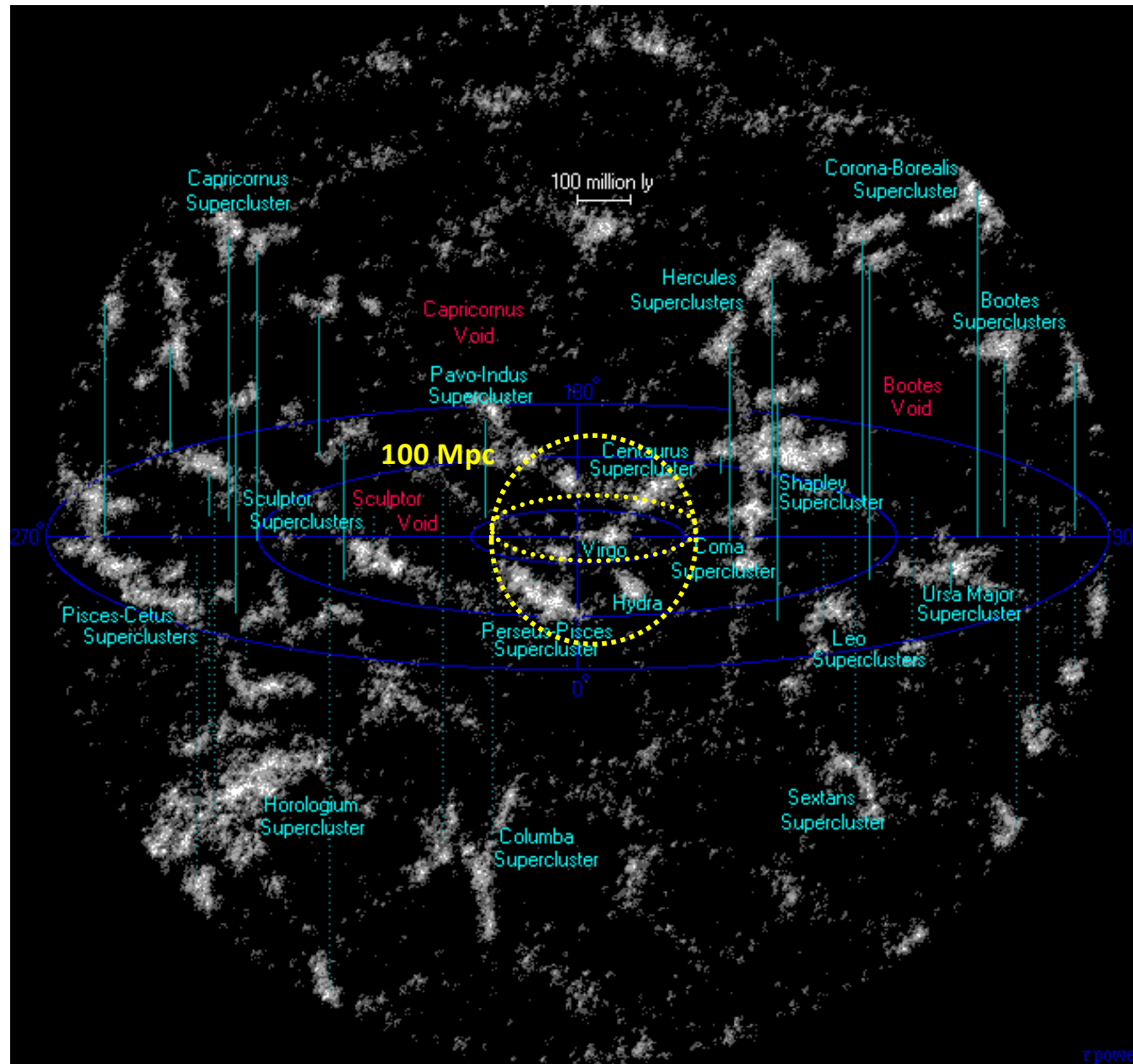
Our motion is towards the Shapley supercluster, supposedly due to a 'Great Attractor' beyond …
We are *not* `Copernican' (comoving) observers

Theory of peculiar velocity fields

In linear perturbation theory, the growth of the density contrast $\delta(x) = [\rho(x) - \bar{\rho}]/\bar{\rho}$ as a function of commoving coordinates and time is governed by:

$$\frac{\partial^2 \delta}{\partial t^2} + 2H(t)\frac{\partial \delta}{\partial t} = 4\pi G_{\mathrm{N}}\bar{\rho}\delta$$

We are interested in the 'growing mode' solution – the density contrast grows self-similarly and so does the perturbation potential and its gradient … so the direction of the acceleration (and its integral – the peculiar velocity) remains *unchanged*.

The peculiar velocity field is related to the density contrast as:

$$v(\mathbf{x}) = \frac{2}{3H_0}\int \mathrm{d}^3 y \frac{\mathbf{x} - \mathbf{y}}{|\mathbf{x} - \mathbf{y}|^3}\delta(\mathbf{y}),$$

So the peculiar Hubble flow, $\boldsymbol{\delta}H(\boldsymbol{x}) = H_{\mathrm{L}}(\boldsymbol{x}) - H_0$ ($\Rightarrow$ trace of the shear tensor), is:

$$\delta H(\mathbf{x}) = \int \mathrm{d}^3\mathbf{y}\ \mathbf{v}(\mathbf{y}) \cdot \frac{\mathbf{x} - \mathbf{y}}{|\mathbf{x} - \mathbf{y}|^2}W(\mathbf{x} - \mathbf{y}),$$

where $H_{\mathrm{L}}(\mathbf{x})$ is the **local** value of the Hubble parameter and $W(\mathbf{x} - \mathbf{y})$ is the 'window function' (e.g. $\theta(R - |\mathbf{x} - \mathbf{y}|)$ $(4\pi R^3/3)^{-1}$ for a volume-limited survey, out to distance R)

Theory of peculiar velocity fields (cont.)

Rewrite in terms of the Fourier transform $\delta(\mathbf{k}) \equiv (2\pi)^{3/2} \int \mathrm{d}^3 x \, \delta(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}}$:

$$\frac{\delta H}{H_0} = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \delta(k) \mathcal{W}_H(kR) e^{ik.x}, \ \mathcal{W}_H(x) = \frac{3}{x^3} \left(\sin x - \int_o^x \mathrm{d}y \frac{\sin y}{y} \right)$$
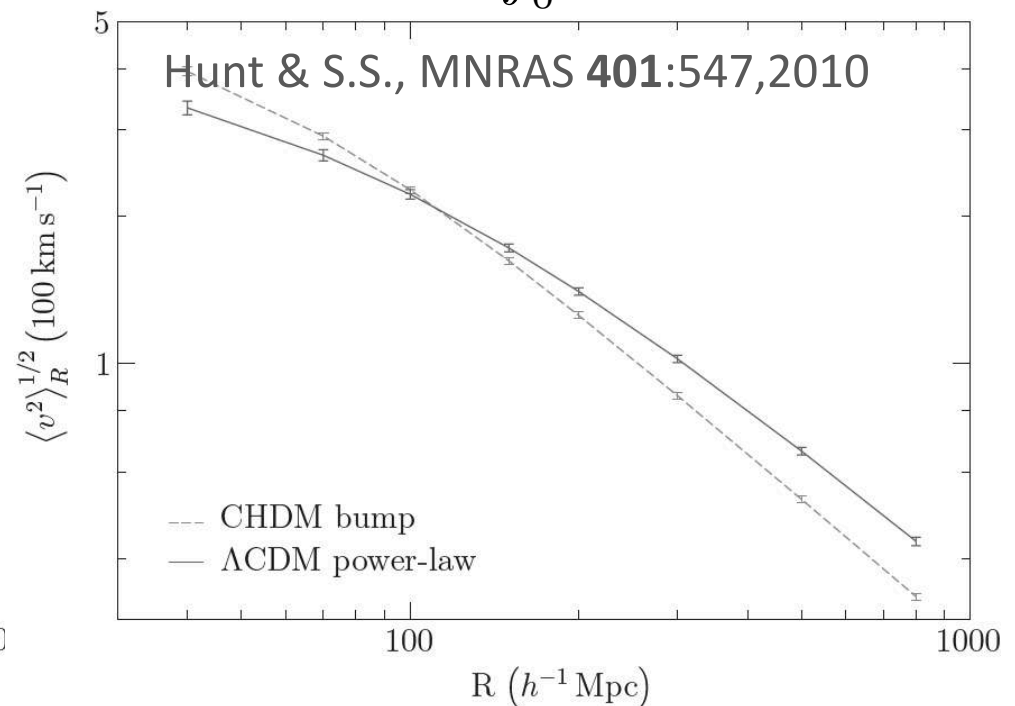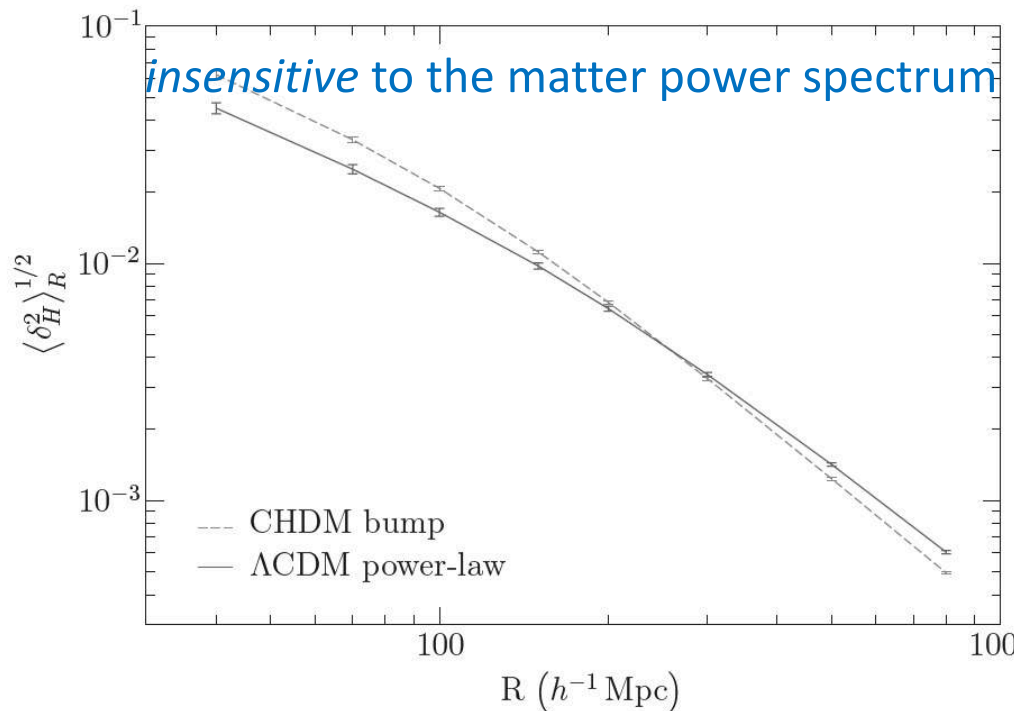
Window function

Then the RMS fluctuation in the local Hubble constant $\delta_H \equiv \langle (\delta H/H_0)^2 \rangle^{1/2}$ is:

$$\delta_H^2 = \frac{f^2}{2\pi^2} \int_0^\infty k^2 \mathrm{d}k \ P(k) \mathcal{W}^2(kR), \ P(k) \equiv |\delta(k)^2|, \ f \simeq \Omega_m^{4/7} + \frac{\Omega_\Lambda}{70} (1 + \frac{\Omega_m}{2})$$
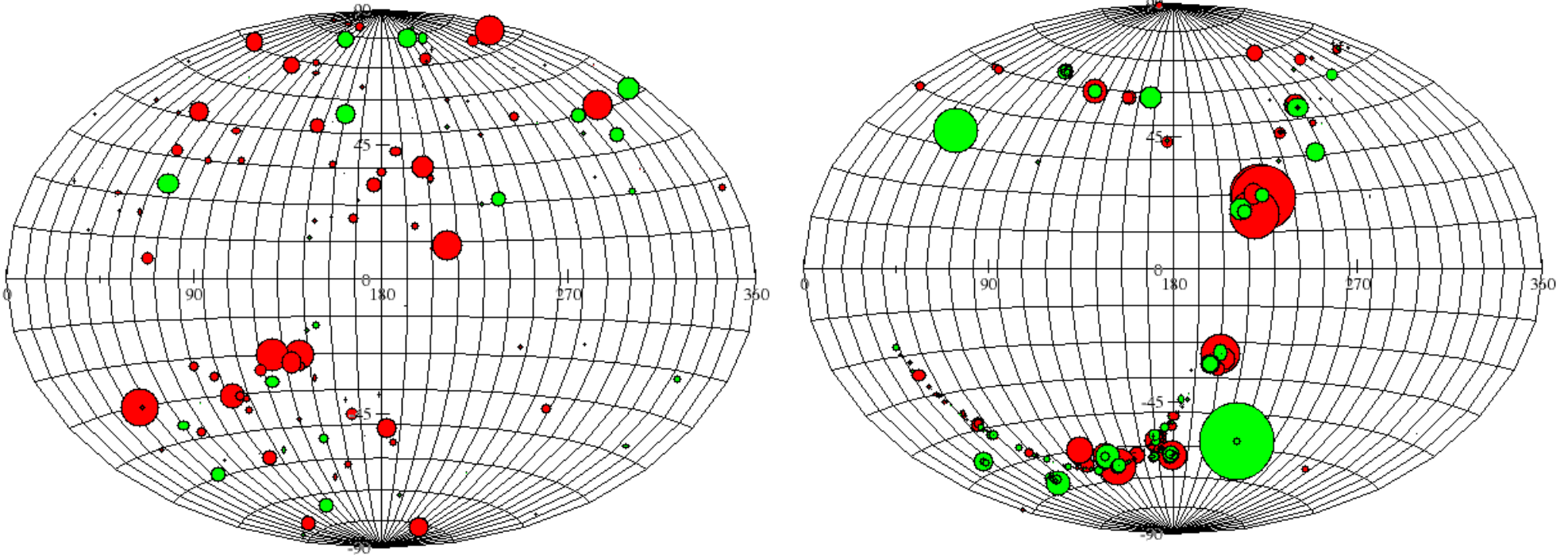
Power spectrum of matter fluctuations   Growth rate

Similarly the variance of the peculiar velocity is: $\langle v^2 \rangle_R = \frac{f^2 H_0^2}{2\pi^2} \int_0^\infty \mathrm{d}k P(k) \mathcal{W}^2(kR)$

insensitive to the matter power spectrum

Hunt & S.S., MNRAS **401**:547,2010

Union 2 compilation of 557 Sne Ia

Aitoff-Hammer plot, Galactic coordinates

Left panel: The red spots represent the data points for $z < 0.06$ with distance moduli μ_{data} *bigger* than the values μ_{CDM} predicted by ΛCDM, and the green spots are those with μ_{data} *less* than μ_{CDM}; the spot size is a relative measure of the discrepancy. A dipole anisotropy is visible around the direction $b = -30°$, $l = 96°$ (red points) and its opposite direction $b = 30°$, $l = 276°$ (small green points), which is the direction of the CMB dipole.   **Right panel**: Same plot for $z > 0.06$

Colin, Mohayaee, S.S. & Shafieloo, MNRAS **414**:264,2011

Use this to do *tomography* of the local Hubble flow by asking if the supernovae are at the expected distances: **Residuals ⇒ 'peculiar velocity' flow in local universe**

Method of Residuals and Smoothing

$$q_i(z_i, \theta_i, \phi_i) = \frac{\mu_i(z_i, \theta_i, \phi_i) - \tilde{\mu}_i(z_i, \theta_i, \phi_i)}{\sigma_i(z_i, \theta_i, \phi_i)}$$ Calculation of Residuals

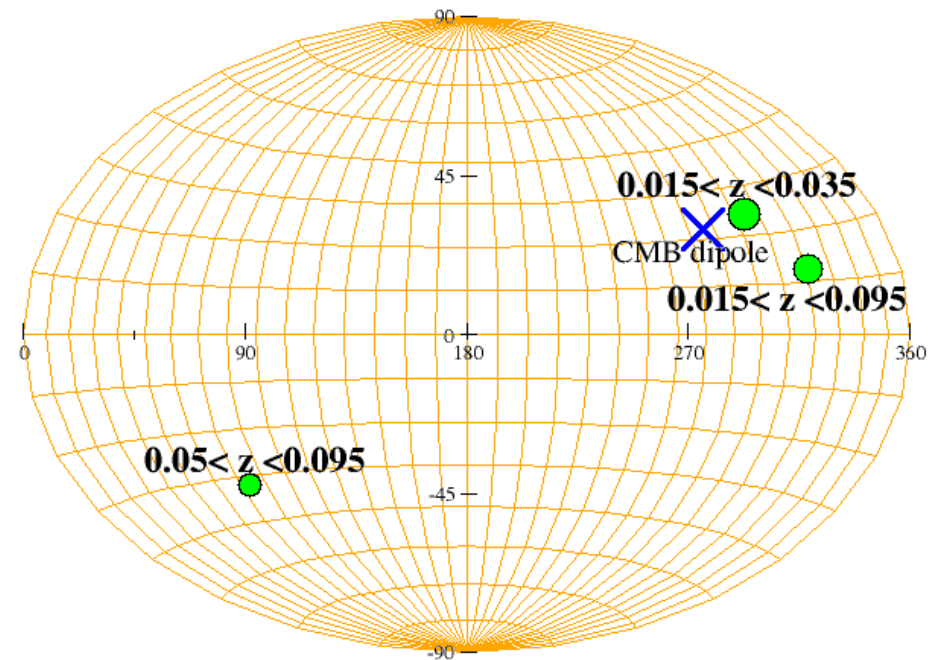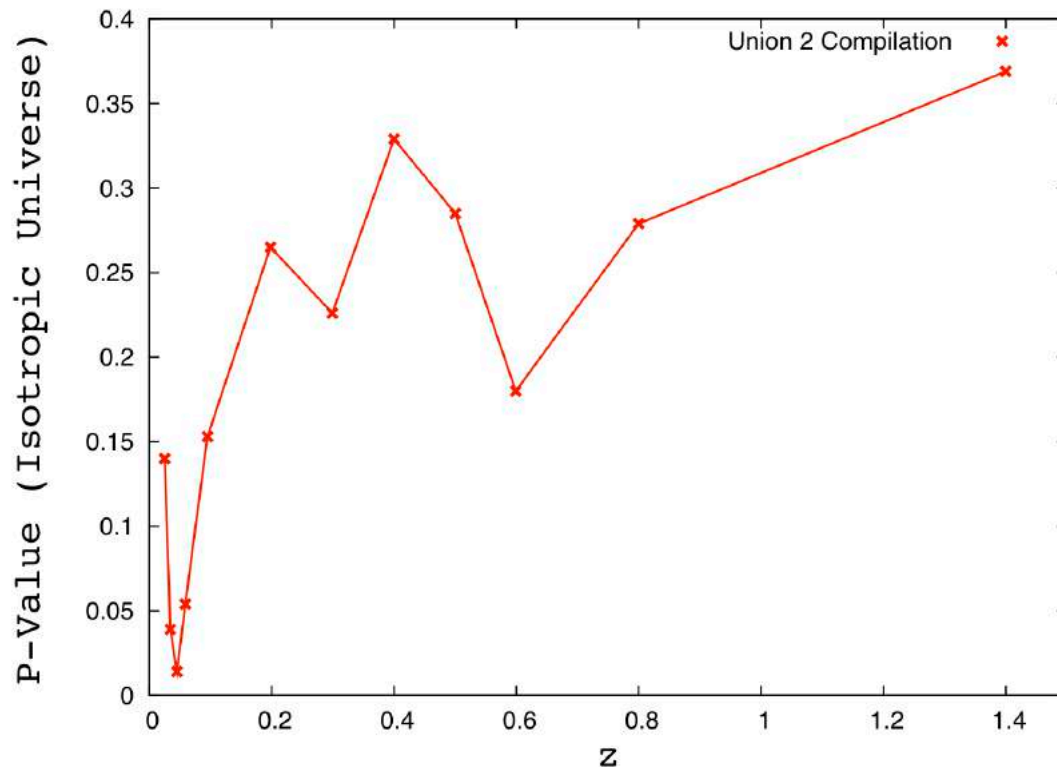$$Q(\theta, \phi) = \sum_{i=1}^{N} q_i(z_i, \theta_i, \phi_i) W(\theta, \phi, \theta_i, \phi_i)$$ 2D smoothing on unit sphere

$$W(\theta, \phi, \theta_i, \phi_i) = \frac{1}{\sqrt{2\pi}\delta} \exp\left[-\frac{L(\theta, \phi, \theta_i, \phi_i)^2}{2\delta^2}\right]$$ Window function

$$L(\theta, \phi, \theta_i, \phi_i) = 2\arcsin\frac{R}{2}, \quad R = \left([\sin(\theta_i)\cos(\phi_i) - \sin(\theta)\cos(\phi)]^2 \right.$$
$$\left. + [\sin(\theta_i)\sin(\phi_i) - \sin(\theta)\sin(\phi)]^2 + [\cos(\theta_i) - \cos(\theta)]^2\right)^{1/2}$$

$$\Delta Q_{\text{data}} = Q(\theta_{\max}, \phi_{\max}) - Q(\theta_{\min}, \phi_{\min})$$ Statistical measure

Calculate for the data (as well as for Monte Carlo simulations of isotropic distribution, in order to obtain P-value) using a ratio method to minimise systematic uncertainties
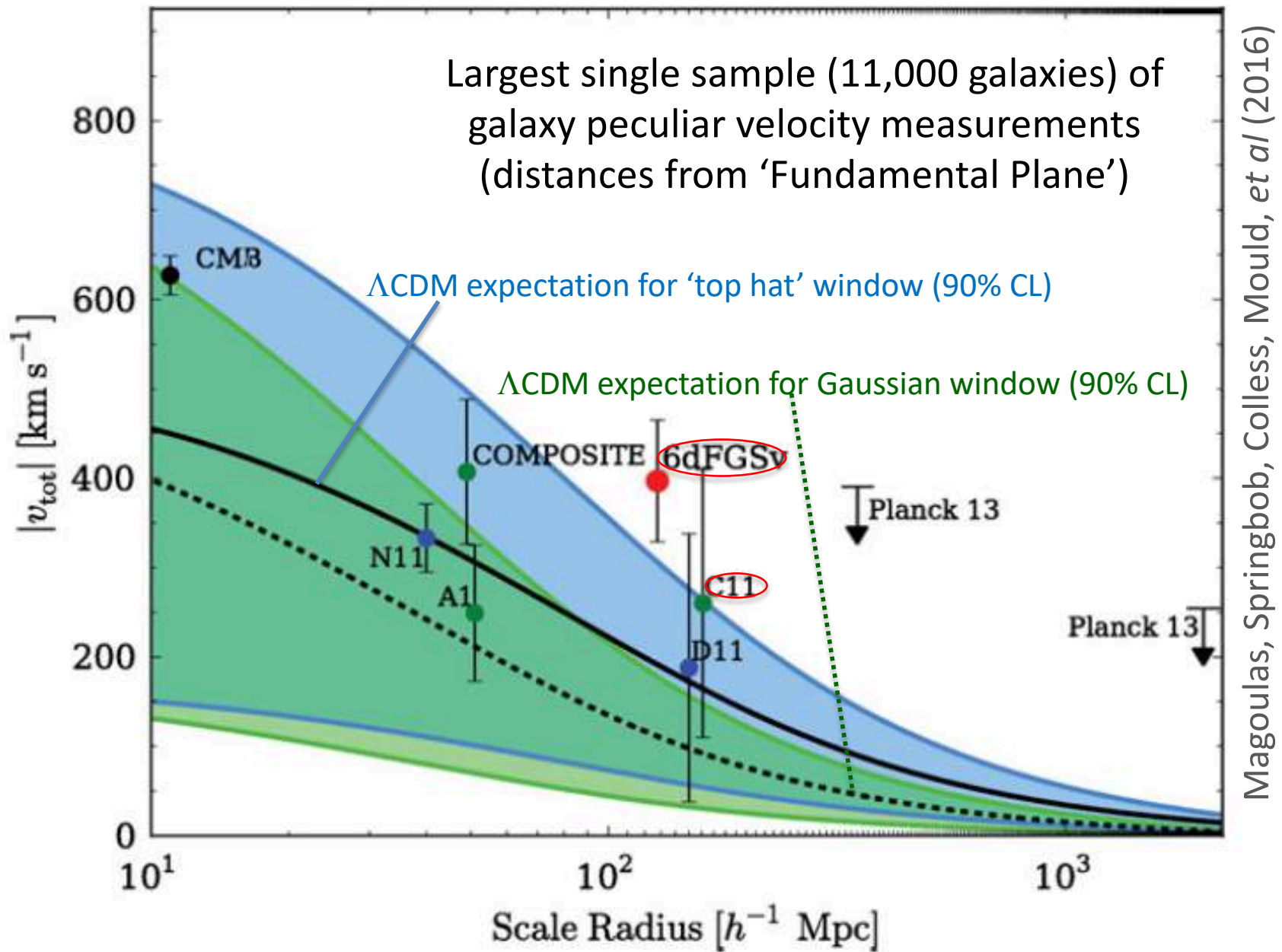
IS THE UNIVERSE ISOTROPIC?

Left panel: P-value for the consistency of the isotropic universe with the data versus redshift. At $z \approx 0.05$ (~200 Mpc) the P-value drops to 0.014 showing that isotropy is *excluded* at 3σ … so we have *not* converged to the CMB rest frame

Right panel: the cumulative analysis shows that at small redshift isotropy is excluded at 2–3 σ with P = 0.054 for $0.015 < z < 0.06$; however at higher redshift agreement is achieved within 1σ, with P = 0.594 for $0.15 < z < 1.4$

Largest single sample (11,000 galaxies) of galaxy peculiar velocity measurements (distances from 'Fundamental Plane')

ΛCDM expectation for 'top hat' window (90% CL)

ΛCDM expectation for Gaussian window (90% CL)

COMPOSITE   6dFGSv

CMB

Planck 13

N11

A1

C11

D11

Planck 13

Magoulas, Springbob, Colless, Mould, *et al* (2016)

$|v_{tot}|$ [km s^{-1}]

Scale Radius [h^{-1} Mpc]

According to the 'Dark Sky' ΛCDM Hubble Volume simulations, only ≲1% of Milky Way–like observers should experience a bulk flow as large as observed, extending out as far as is seen …

Do we infer acceleration even though the expansion is actually decelerating ... because we are inside a local 'bulk flow'?
(Tsagas 2010, 2011, 2012; Tsagas & Kadiltzoglou 2015)
... if so there should be a dipole asymmetry in the inferred deceleration parameter in the same direction – i.e. towards the CMB dipole
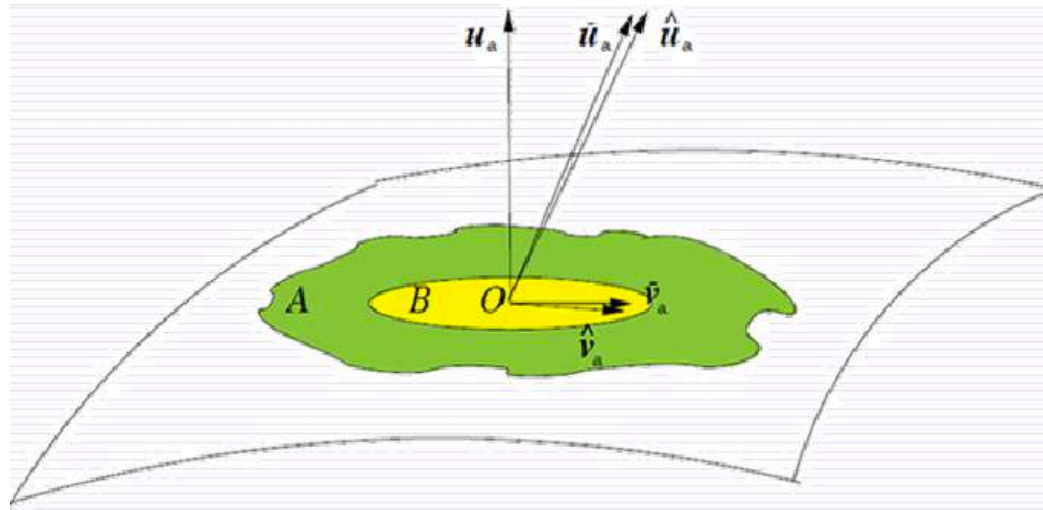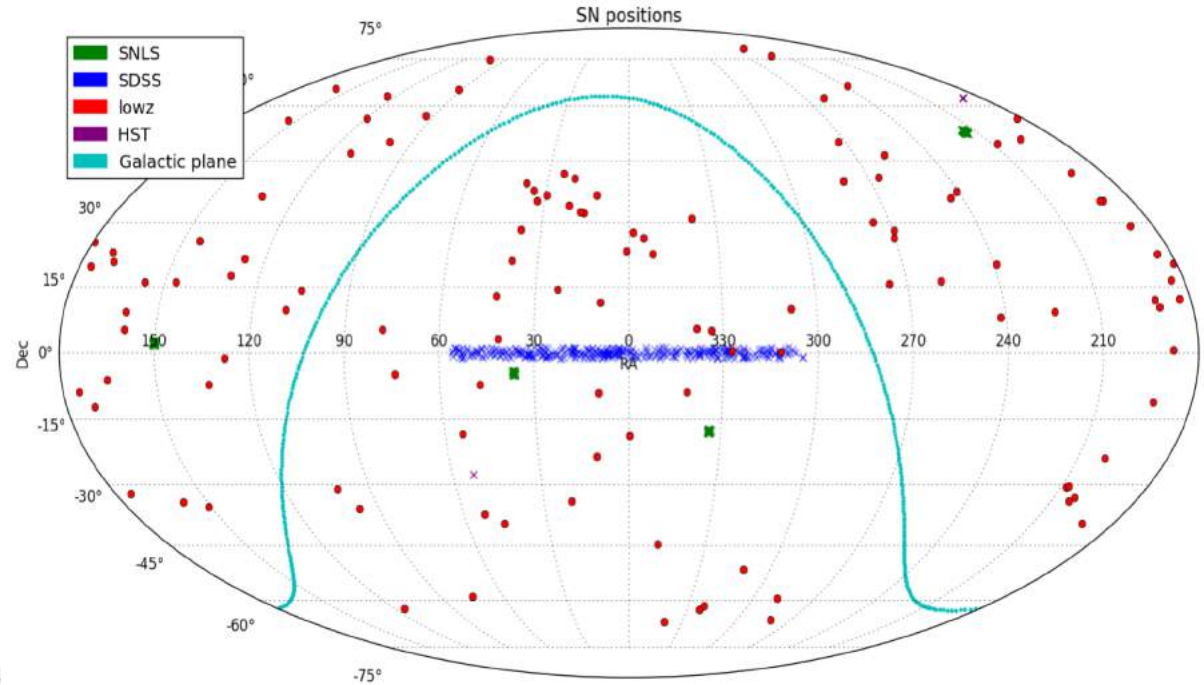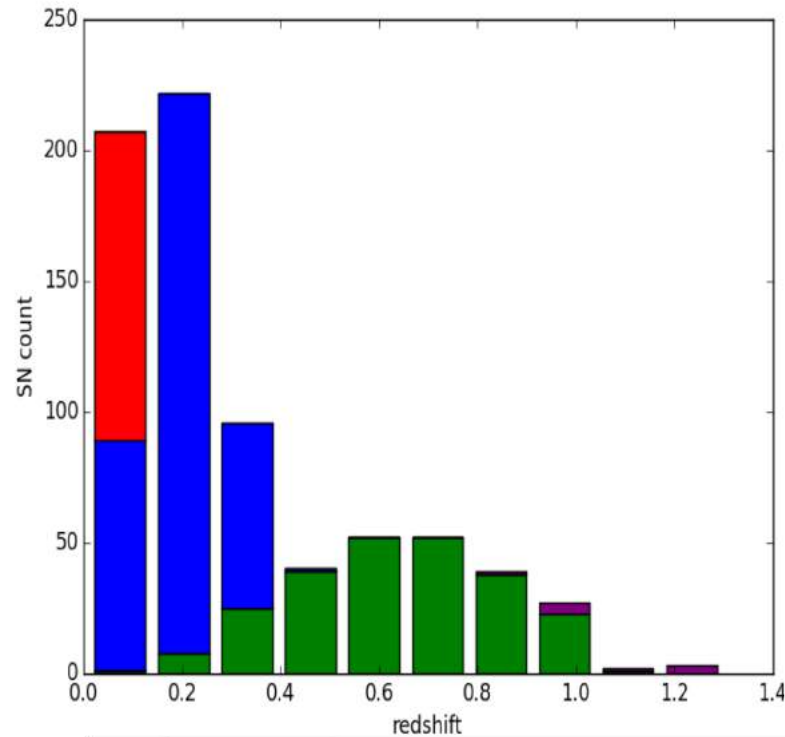

The patch A has mean peculiar velocity $\tilde{v}_a$ with $\vartheta = \tilde{\mathrm{D}}^a v_a \gtrless 0$ and $\dot{\vartheta} \gtrless 0$
(the sign depending on whether the bulk flow is faster or slower than the surroundings)

Inside region B, the r.h.s. of the expression

$$1 + \tilde{q} = (1 + q)\left(1 + \frac{\vartheta}{\Theta}\right)^{-2} - \frac{3\dot{\vartheta}}{\Theta^2}\left(1 + \frac{\vartheta}{\Theta}\right)^{-2}, \quad \tilde{\Theta} = \Theta + \vartheta,$$

drops below 1 and the comoving observer 'measures' *negative* deceleration parameter

Joint Lightcurve Analysis data (740 Sne Ia)

Betoule *et al*, A&A **568**:A22,2014

This page contains links to data associated with the SDSS-II/SNLS3 Joint Light-Curve Analysis (Betoule et al. 2014, submitted to A&A).

The release consists in:

1. Release history
   V1 (January 2014, paper submitted):
   V2 (March 2014):
   V3 (April 2014, paper accepted):
   V4 (June 2014):
   V5 (March 2015):
   V6 (March 2015):
2. Installation of the C++ likelihood code
   Installation of the cosmomc plugin
3. SALT2 model
4. Error propagation
   Error decomposition
   SALT2 light-curve model uncertainties

1. The end products of the analysis and a C++ code to compute the likelihood of this data associated to a cosmological model. The code enables both evaluations of the *complete* likelihood, and *fast* evaluations of an *approximate* likelihood (see Betoule et al. 2014, Appendix E).
2. The version 2.4 of the SALT2 light-curve model used for the analysis plus 200 random realizations usable for the propagation of model uncertainties.
3. The exact set of Supernovae light-curves used in the analysis.

We also deliver presentation material.

Since March 2014, the JLA likelihood plugin is included in the official release of cosmomc. For older versions, the plugin is still available (see below: *Installation of the cosmomc plugin*).

To analyze the JLA sample with SNANA, see $SNDATA_ROOT/sample_input_files/JLA2014/AAA_README.

1 Release history

V1 (January 2014, paper submitted):

First arxiv version.

V2 (March 2014):

Same as v1 with additionnal information (R.A., Dec. and bias correction) in the file of light-curve parameters.

V3 (April 2014, paper accepted):

Same as v2 with the addition of a C++ likelihood code in an independant archive (jla_likelihood_v3.tgz).

V4 (June 2014):

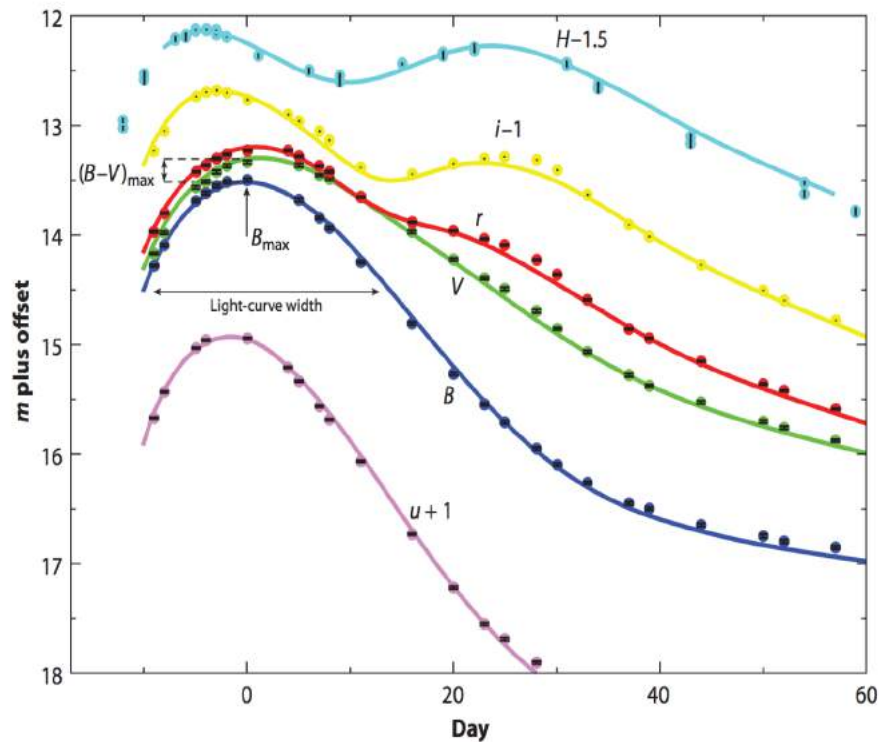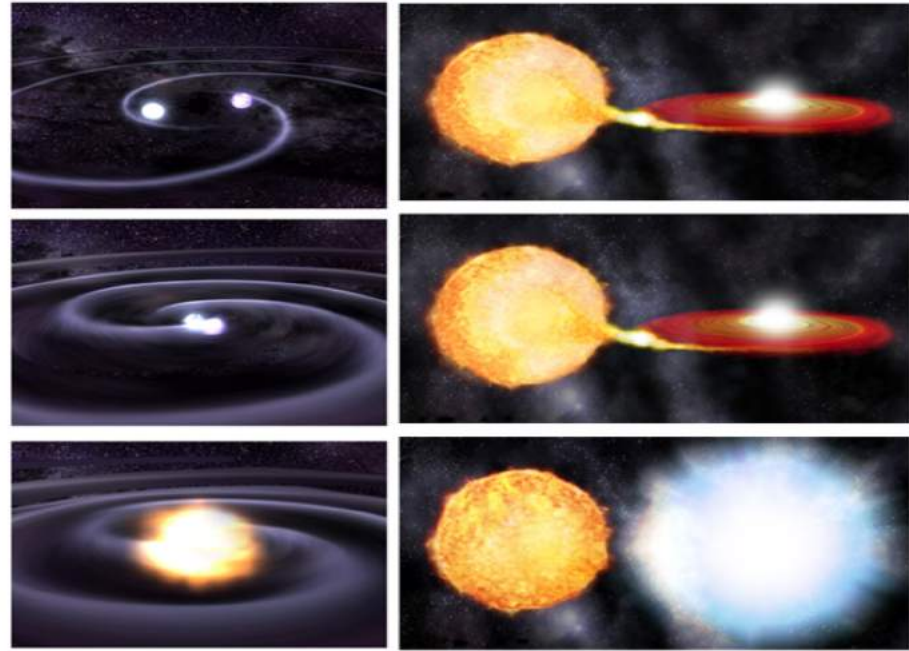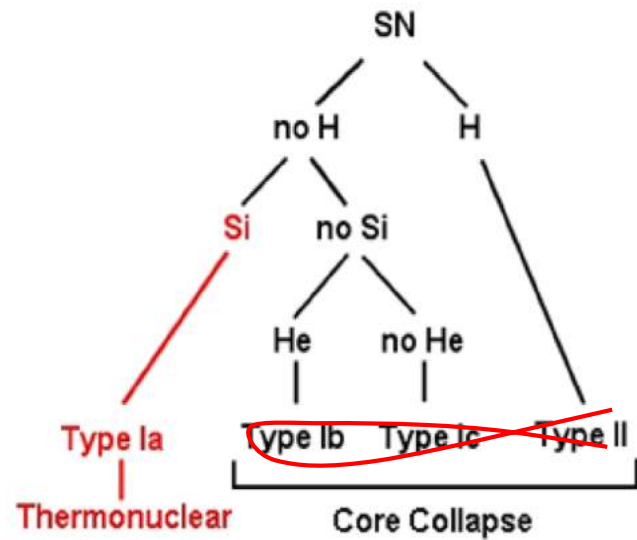Data *publicly* available
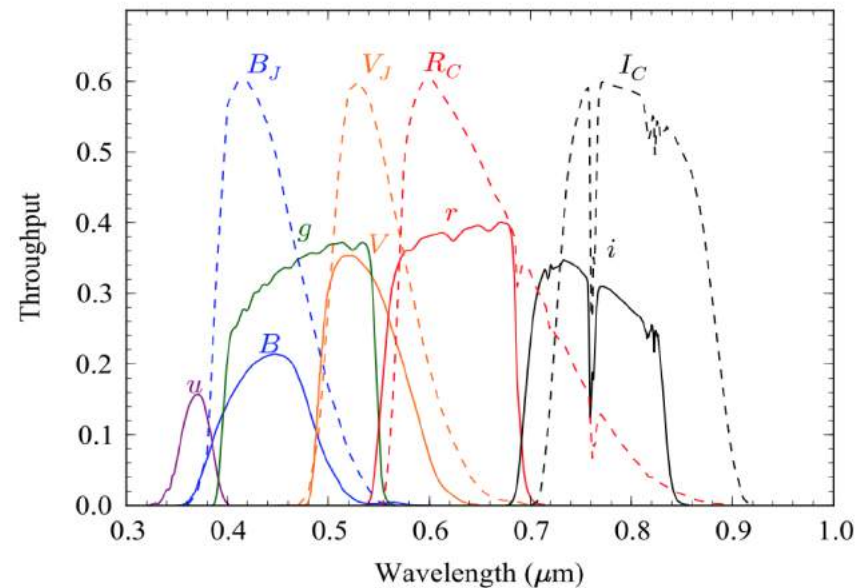http://supernovae.in2p3.fr/sdss_snls_jla/

We apply a *principled* statistical analysis: Maximal Likelihood
Nielsen, Guffanti & S.S.,
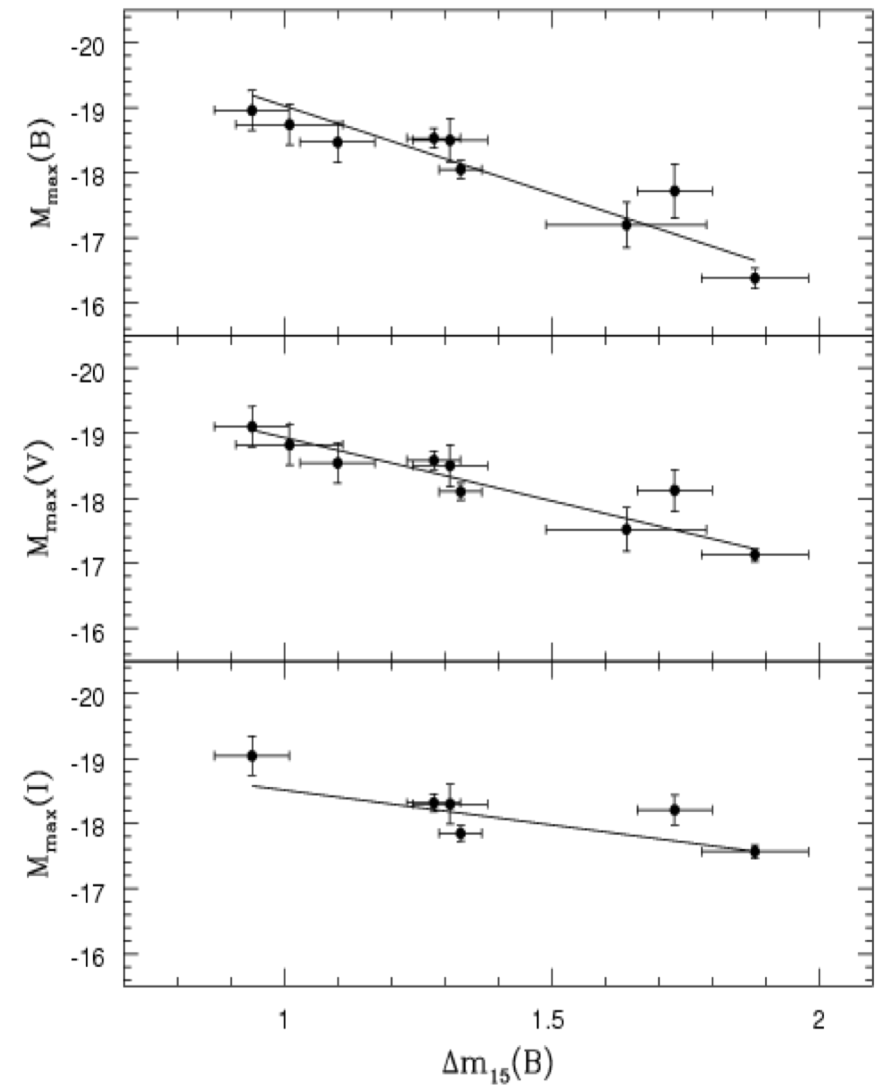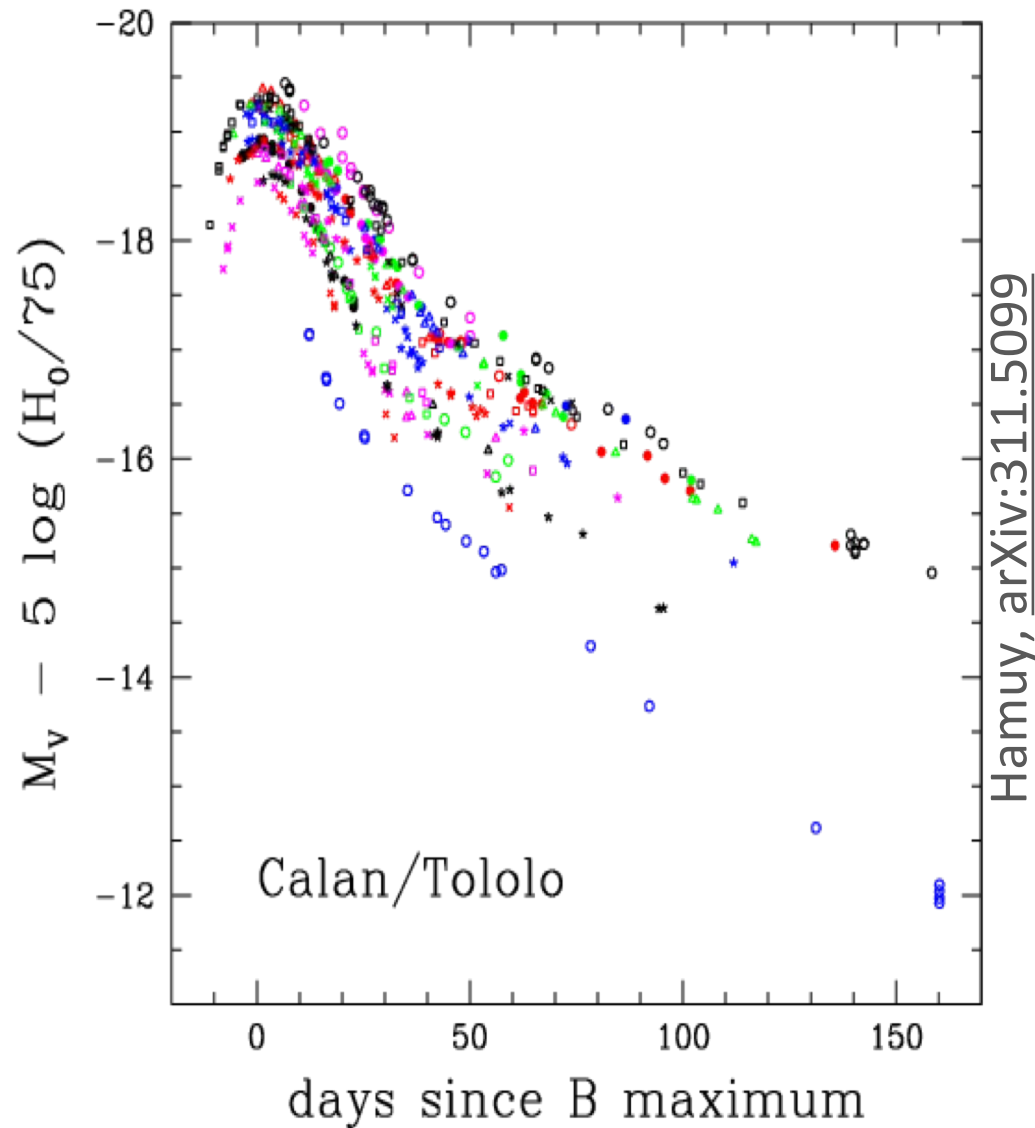Sci.Rep. **6**:35596,2016

What are Type Ia supernovae?
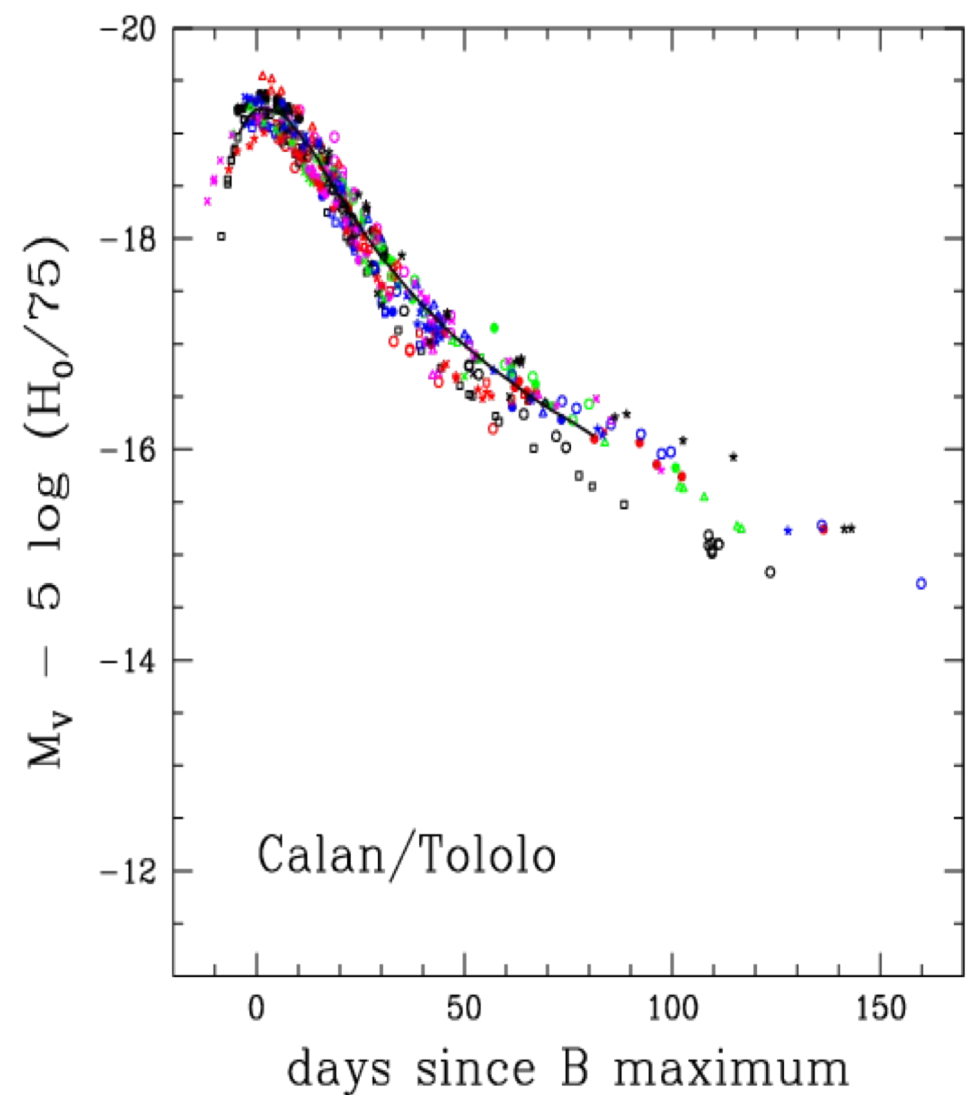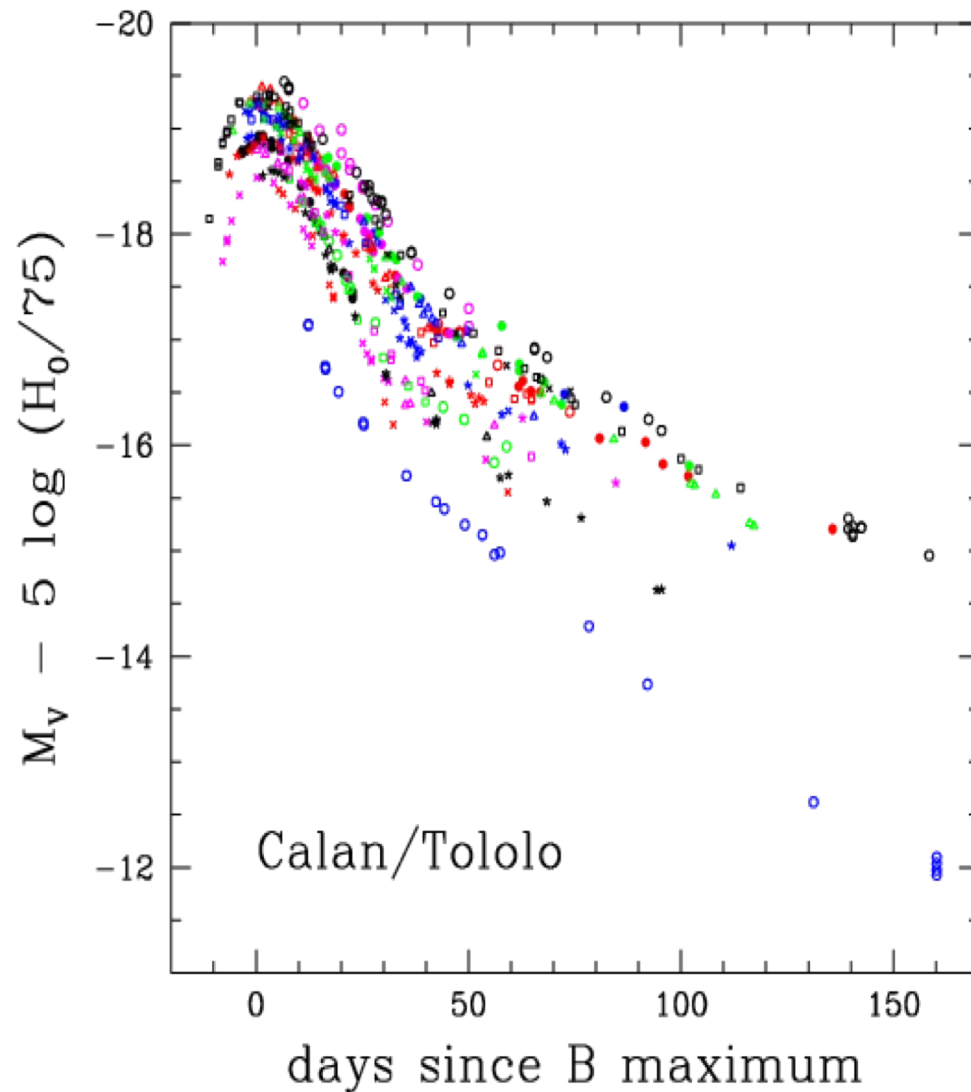

$$m = -2.5 \log(F/F_{\rm ref})$$

THEY ARE CERTAINLY *NOT* 'STANDARD CANDLES'

Hamuy, arXiv:311.5099

Phillips, ApJ **413**:L105, 1993

But they can be 'standardised' using the observed correlation between their peak magnitude and light-curve width (NB: this is *not* understood theoretically)

Type Ia supernovae as 'standardisable candles'

Hamuy, 1311.5099

$$\mu_B = m_B^* - M + \alpha X_1 - \beta \mathcal{C}$$

Use a standard template (e.g. SALT 2) to make 'stretch' and 'colour' corrections …

Spectral Adaptive Lightcurve Template

(For making 'stretch' and 'colour' corrections to the observed lightcurves)

$$\mu_B = m_B^* - M + \alpha X_1 - \beta \mathcal{C}$$

B-band

SALT 2 parameters

Name	z_{cmb}	$m_B^\star$	X_1	$\mathcal{C}$	$M_{stellar}$
03D1ar	0.002	23.941 ± 0.033	-0.945 ± 0.209	0.266 ± 0.035	10.1 ± 0.5
03D1au	0.503	23.002 ± 0.088	1.273 ± 0.150	-0.012 ± 0.030	9.5 ± 0.1
03D1aw	0.581	23.574 ± 0.090	0.974 ± 0.274	-0.025 ± 0.037	9.2 ± 0.1
03D1ax	0.495	22.960 ± 0.088	-0.729 ± 0.102	-0.100 ± 0.030	11.6 ± 0.1
03D1bp	0.346	22.398 ± 0.087	-1.155 ± 0.113	-0.041 ± 0.027	10.8 ± 0.1
03D1co	0.678	24.078 ± 0.098	0.619 ± 0.404	-0.039 ± 0.067	8.6 ± 0.3
03D1dt	0.611	23.285 ± 0.093	-1.162 ± 1.641	-0.095 ± 0.050	9.7 ± 0.1
03D1ew	0.866	24.354 ± 0.106	0.376 ± 0.348	-0.063 ± 0.068	8.5 ± 0.8
03D1fc	0.331	21.861 ± 0.086	0.650 ± 0.119	-0.018 ± 0.024	10.4 ± 0.0
03D1fq	0.799	24.510 ± 0.102	-1.057 ± 0.407	-0.056 ± 0.065	10.7 ± 0.1
03D3aw	0.450	22.667 ± 0.092	0.810 ± 0.232	-0.086 ± 0.038	10.7 ± 0.0
03D3ay	0.371	22.273 ± 0.091	0.570 ± 0.198	-0.054 ± 0.033	10.2 ± 0.1
03D3ba	0.292	21.961 ± 0.093	0.761 ± 0.173	0.116 ± 0.035	10.2 ± 0.1
03D3bl	0.356	22.927 ± 0.087	0.056 ± 0.193	0.205 ± 0.030	10.8 ± 0.1

? ? ? ? ? ?

There may well be other variables that the magnitude correlates with …

Cosmology

$$\mu \equiv 25 + 5\log_{10}(d_{\mathrm{L}}/\mathrm{Mpc}), \quad \text{where:}$$

$$d_{\mathrm{L}} = (1+z)\frac{d_{\mathrm{H}}}{\sqrt{\Omega_k}}\,\mathrm{sinn}\left(\sqrt{\Omega_k}\int_0^z \frac{H_0\,\mathrm{d}z'}{H(z')}\right),$$

$$d_{\mathrm{H}} = c/H_0, \quad H_0 \equiv 100h\ \mathrm{km\,s^{-1}Mpc^{-1}},$$

$$H = H_0\sqrt{\Omega_{\mathrm{m}}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda},$$

$$\mathrm{sinn} \to \mathrm{sinh} \text{ for } \Omega_k > 0 \text{ and } \mathrm{sinn} \to \mathrm{sin} \text{ for } \Omega_k < 0$$

Distance modulus

$$\mu_C = m - M = -2.5\log\frac{F/F_{\mathrm{ref}}}{L/L_{\mathrm{ref}}} = 5\log\frac{d_L}{10\mathrm{pc}}$$

Acceleration is a *kinematic* quantity so the data can also be analysed without any dynamical model by expanding the time variation of the scale factor in a Taylor series
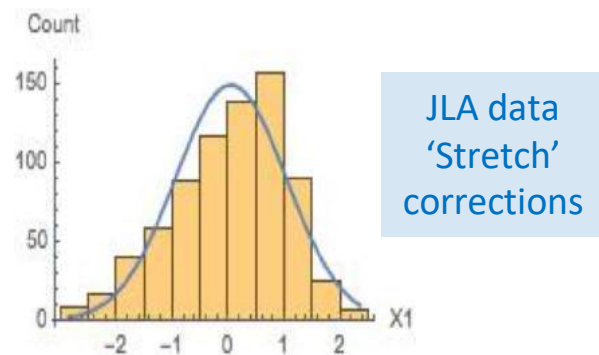(e.g. Visser, CQG **21**:2603,2004)

$$d_L(z) = \frac{c\,z}{H_0}\left\{1 + \frac{1}{2}[1-q_0]\,z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2\,a_0^2}\right]z^2 + O(z^3)\right\}$$

Construct a Maximum Likelihood Estimator

$$\mathcal{L} = \text{probability density}(\text{data}|\text{model})$$

$$\mathcal{L} = p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|\theta]$$

$$= \int p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta_{\text{cosmo}}]$$

$$\times p[(M, x_1, c)|\theta_{\text{SN}}]dM\,dx_1\,dc$$

Well-approximated as Gaussian

JLA data 'Stretch' corrections

JLA data 'Colour' corrections
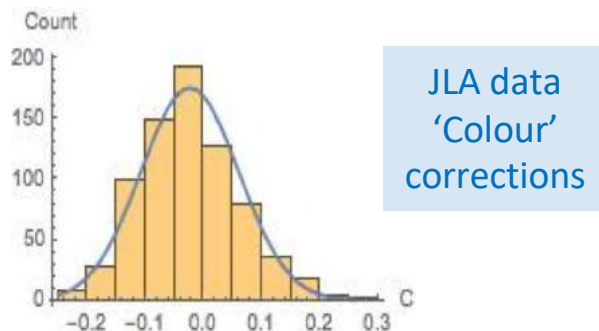
$$p[(M, x_1, c)|\theta] = p(M|\theta)p(x_1|\theta)p(c|\theta),$$

$$p(M|\theta) = \frac{1}{\sqrt{2\pi\sigma_M^2}} \exp\left(-\left[\frac{M - M_0}{\sigma_{M0}}\right]^2 \Big/ 2\right)$$

$$p(x_1|\theta) = \frac{1}{\sqrt{2\pi\sigma_{x0}^2}} \exp\left(-\left[\frac{x_1 - x_{10}}{\sigma_{x0}}\right]^2 \Big/ 2\right)$$

$$p(c|\theta) = \frac{1}{\sqrt{2\pi\sigma_{c0}^2}} \exp\left(-\left[\frac{c - c_0}{\sigma_{c0}}\right]^2 \Big/ 2\right)$$

Nielsen, Guffanti & S.S., Sci.Rep. **6**:35596,2016

Likelihood

$$p(Y|\theta) = \frac{1}{\sqrt{|2\pi\Sigma_l|}} \exp\left[-\frac{1}{2}(Y - Y_0)\Sigma_l^{-1}(Y - Y_0)^{\mathrm{T}}\right]$$

$$p(\hat{X}|X,\theta) = \frac{1}{\sqrt{|2\pi\Sigma_d|}} \exp\left[-\frac{1}{2}(\hat{X} - X)\Sigma_d^{-1}(\hat{X} - X)^{\mathrm{T}}\right]$$

intrinsic distributions

$$\mathcal{L} = \frac{1}{\sqrt{|2\pi(\Sigma_d + A^{\mathrm{T}}\Sigma_l A)|}}$$
$$\times \exp\left(-\frac{1}{2}(\hat{Z} - Y_0 A)(\Sigma_d + A^{\mathrm{T}}\Sigma_l A)^{-1}(\hat{Z} - Y_0 A)^{\mathrm{T}}\right)$$

cosmology

SALT2

Confidence regions

$$p_{\mathrm{cov}} = \int_0^{-2\log\mathcal{L}/\mathcal{L}_{\max}} \chi^2(x;\nu)\,dx$$

$$\mathcal{L}_p(\theta) = \max_\phi \mathcal{L}(\theta,\phi)$$

1,2,3-sigma

solve for Likelihood value

Data consistent with uniform expansion @3σ!

Opens up interesting possibilities e.g. could the cosmic fluid be *viscous* – perhaps associated with structure formation (e.g. Floerchinger *et al*, PRL **114**:091301,2015)

profile likelihood

MLE, *best fit*

Ω_M	0.341
Ω_Λ	0.569
α	0.134
x_0	0.038
σ^2_{x0}	0.931
β	3.058
c_0	-0.016
σ^2_{c0}	0.071
M_0	-19.05
σ^2_{M0}	0.108

Nielsen, Guffanti & S.S., Sci.Rep.**6**:35596,2016
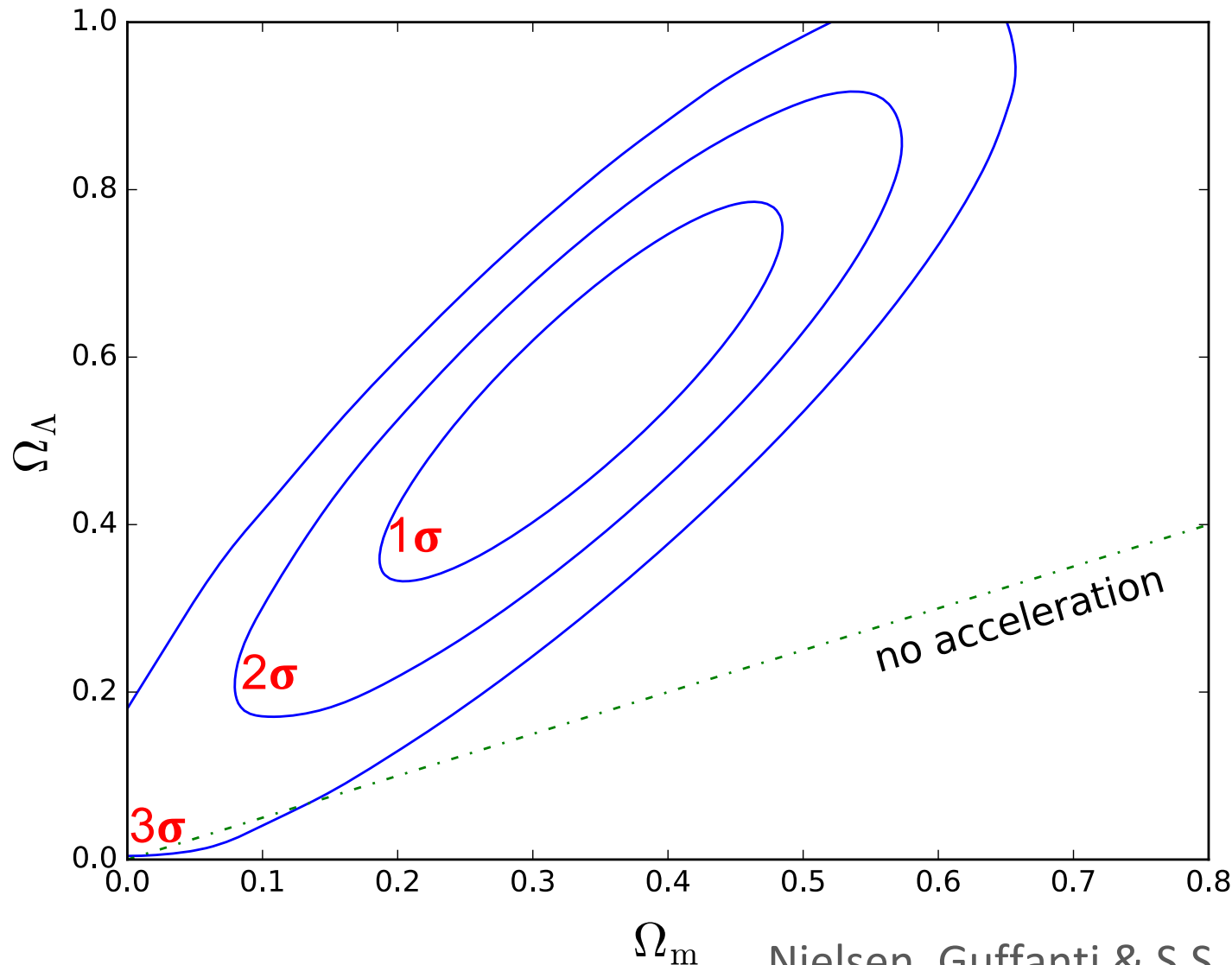
Rubin & Hayden (ApJ **833**:L30,2016) say that our model for the distribution of the light curve fit parameters should have included a dependence on redshift (to allow for 'Malmqvist bias' which JLA had in fact already corrected for)
... they add 12 more parameters to our (10 parameter) model to describe this
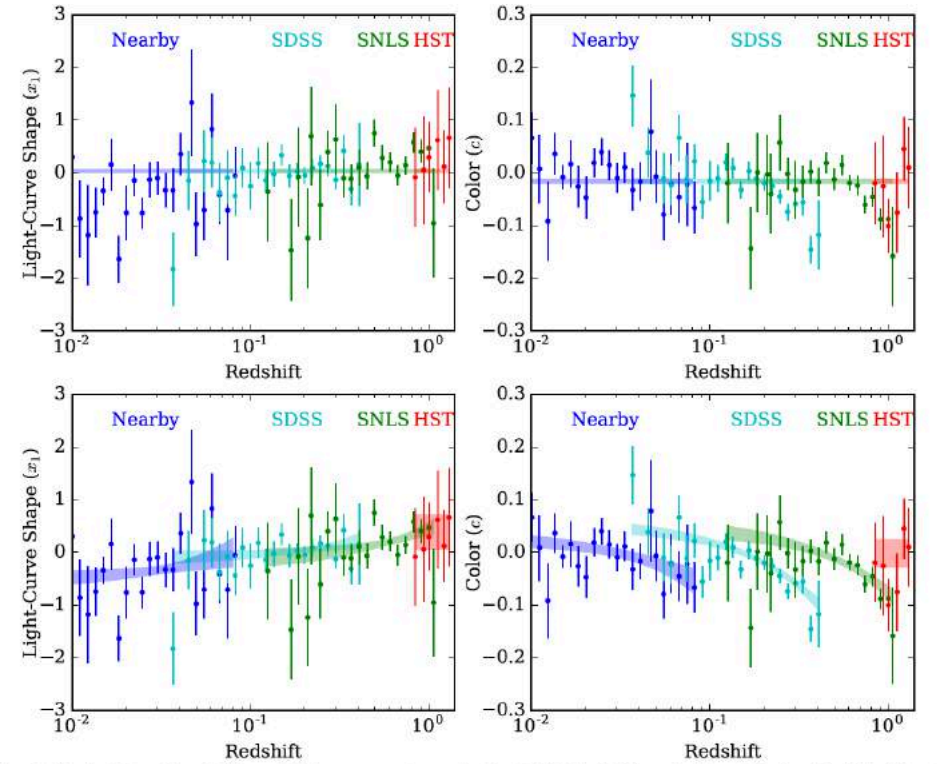
Figure 1. Binned x_1 (left panels) and c (right panels) light curve parameters as a function of redshift for the JLA sample. The trend of color with redshift within each ground-based sample is expected due to the combination of the color-luminosity relation combined with redshift-dependent luminosity detection limits. The top panels show the 68% credible constraints on a constant-in-redshift model, as was used in N16. The bottom panels show our proposed revision. Failing to model the drift in the mean observed distributions demonstrated by the bottom panels will tend to cause high-redshift SNe to appear brighter on average, therefore reducing the significance of accelerating expansion.
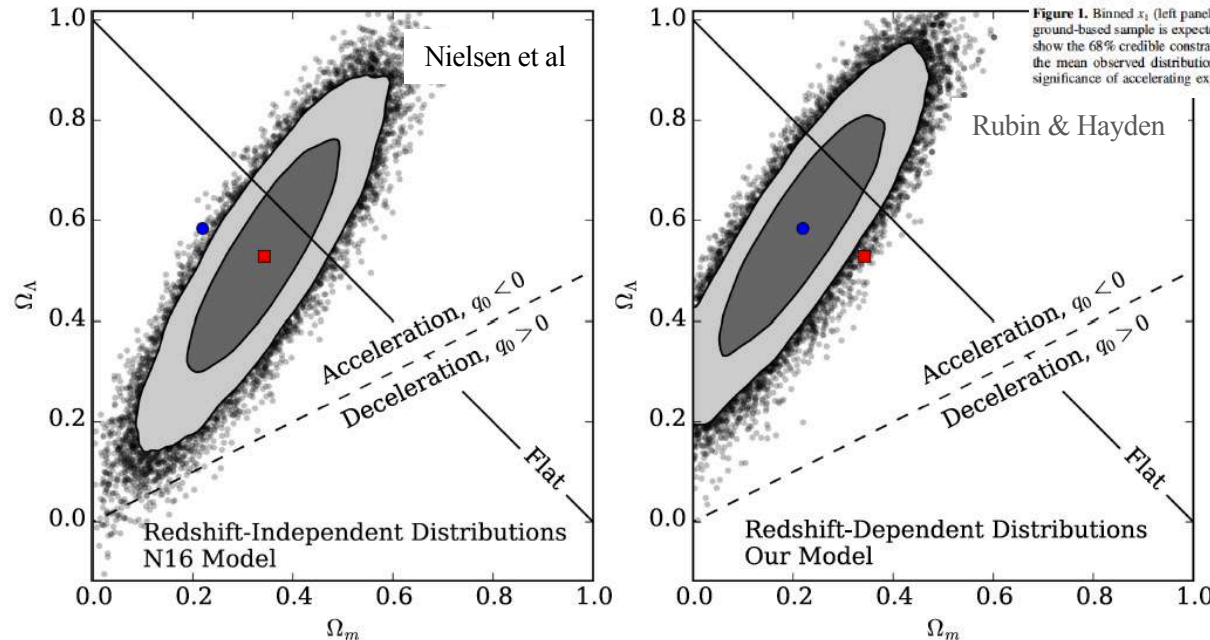
Figure 2. Ω_m–Ω_Λ constraints enclosing 68.3% and 95.4% of the samples from the posterior. Underneath, we plot all samples. The left panel shows the constraints obtained with x_1 and c distributions that are constant in redshift, as in the N16 analysis; the right panel shows the constraints from our model. The red square and blue circle show the location of the median of the samples from the respective posteriors.

Even if this is justified, the significance with which a non-accelerating universe is rejected rises only to $\lesssim 4\sigma$ ... still inadequate to claim a 'discovery' (even though the dataset has increased from ~50 to 740 SNe Ia in 20 yrs)!

The data can be analysed by expanding the time variation of the scale factor in a Taylor series, *without* reference to the ΛCDM model
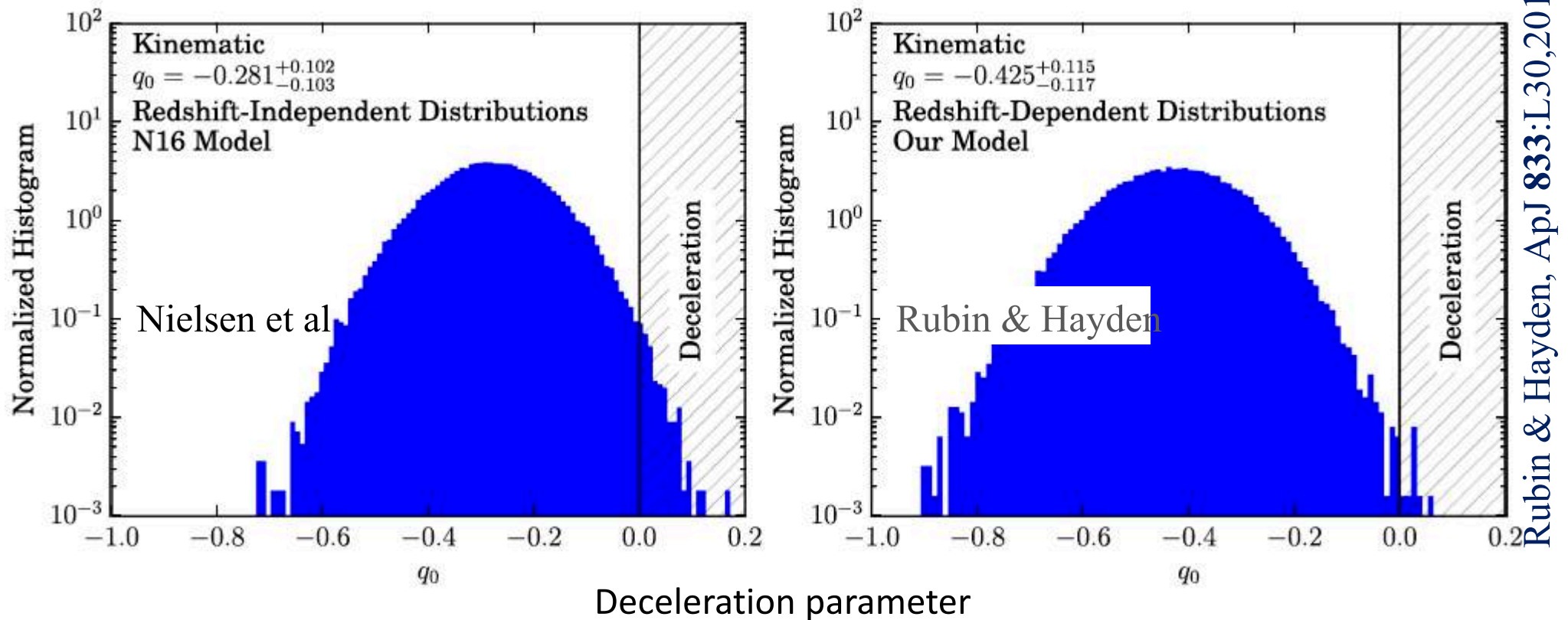
This yields 2.8σ evidence for acceleration in our approach … increasing to 3.7σ when an *ad-hoc* redshift-dependence is allowed in the light-curve parameters

Moreover allowing *z*-dependence in the lightcurve fitting parameters raises the question whether the absolute magnitude of SNe Ia might also be *z*-dependent?!

Such luminosity evolution would completely undermine their use as 'standard candles'!

When we analyse the JLA catalogue allowing for a dipole, we find that there *is* one
.... of comparable magnitude to the monopole (although with smaller significance)
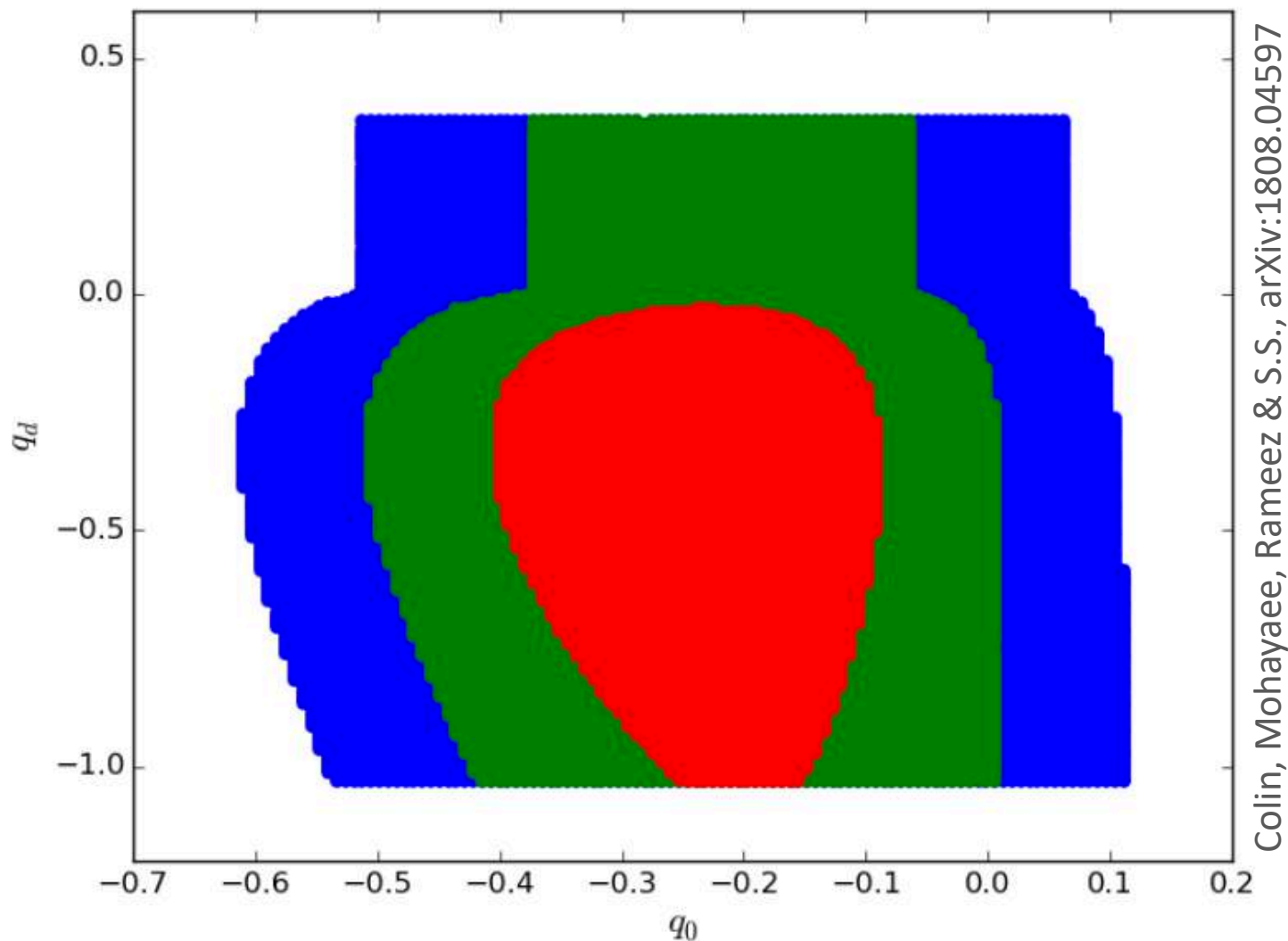
The significance of q_o being negative has now decreased to only **2σ!**

Cosmic acceleration may simply be an artefact of our being located inside a 'bulk flow'!

A MOVING OBSERVER → KINEMATIC DIPOLE

$$\sigma(\theta)_{obs} = \sigma_{rest}\left[1 + [2 + x(1+\alpha)]\frac{v}{c}\cos(\theta)\right]$$

Aberration

Rest frame

Moving frame

$$\tan\phi = \frac{\sin\theta}{\gamma * cos\theta - \frac{v}{c}}$$

θ

ϕ

Observer, velocity v

+

Doppler boosting

$\phi \propto E^{-x}$ -ve power law

Differential flux

Energy

Flux limited catalog ➜ more sources in direction of motion

Ellis & Baldwin (1984)
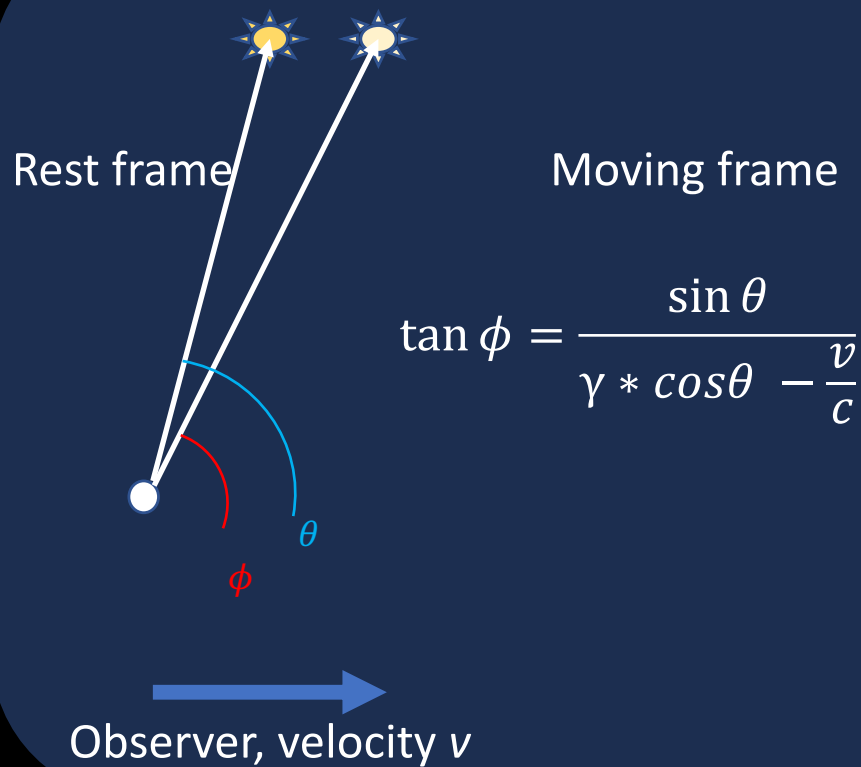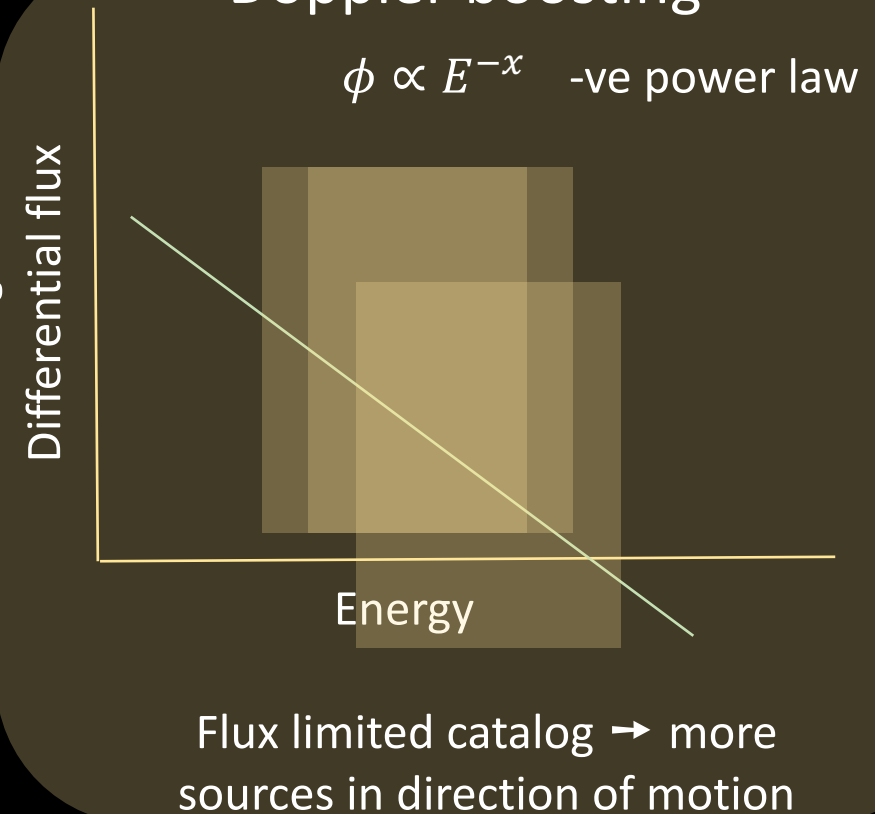
DIPOLES IN A CATALOGUE OF GALAXIES

All-sky catalogue with N sources with redshift distribution $D(z)$ from a directionally unbiased survey
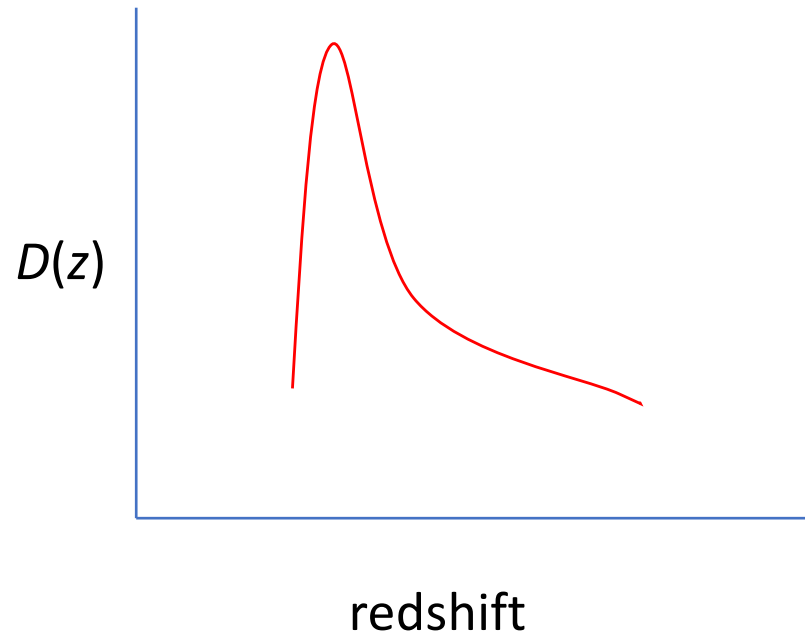
$$\vec{\delta} = \overrightarrow{\mathcal{K}}\,(\vec{v}_{obs}, x, \alpha) + \overrightarrow{\mathcal{R}}\,(N) + \vec{\mathcal{S}}\,(D(z))$$

$\overrightarrow{\mathcal{K}} \rightarrow$ The kinematic dipole: *independent* of source distance, but depends on source spectrum, source flux function, observer velocity

$\overrightarrow{\mathcal{R}} \rightarrow$ The random dipole: $\propto 1/\sqrt{N}$ isotropically distributed

$\vec{\mathcal{S}} \rightarrow$ The dipole component of an actual anisotropy in the distribution of sources in the cosmic rest frame (significant for shallow surveys)

$D(z)$

redshift

Radio sources: NVSS + SUMSS, 600,000 galaxies $z \sim 1$, $\vec{\mathcal{S}}\,(D(z)) \rightarrow 0$
Colin, Mohayaee, Rameez & Sarkar, MNRAS **471**:1045,2017

Wide Field Infrared Survey Explorer, 2,400,000 galaxies, $z \sim 0.14$, $\vec{\mathcal{S}}\,(D(z))$ significant
Rameez, Mohayaee, Sarkar & Colin MNRAS **477**:1722,2018

ESTIMATORS FOR THE DIPOLE

$$\vec{D}_H = \hat{z} * \frac{N_{UH} - N_{LH}}{N_{UH} + N_{LH}}$$

$$\vec{D}_C = \frac{1}{N} \sum_{i=1}^{N} \hat{r}_i$$
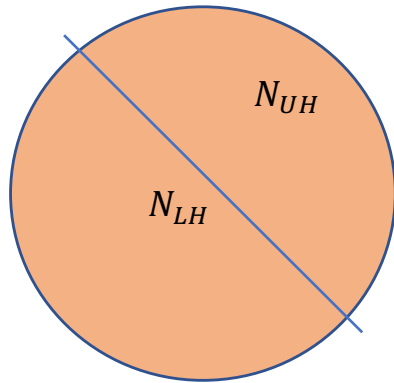
Vary the direction of the hemispheres until maximum asymmetry is observed

Add up unit vectors corresponding to directions in the sky for every source

Easy visualisation

Relatively lower bias and statistical error $1/\sqrt{N}$

High bias and statistical error $\sim 1/\sqrt{N}$

$$\vec{D}_C = \frac{\hat{z}}{N} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} \sigma(\theta) \cos\theta \sin\theta \mathrm{d}\theta \mathrm{d}\phi$$

(Rubart & Schwarz 2013)

$$\vec{D}_H = \frac{\hat{z}}{N} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} \sigma(\theta) \frac{|\cos\theta|}{\cos\theta} \sin\theta \mathrm{d}\theta \mathrm{d}\phi$$

N_{UH}

N_{LH}

The NRAO VLA Sky Survey (NVSS)

Sydney University Molonglo Sky Survey (SUMSS)
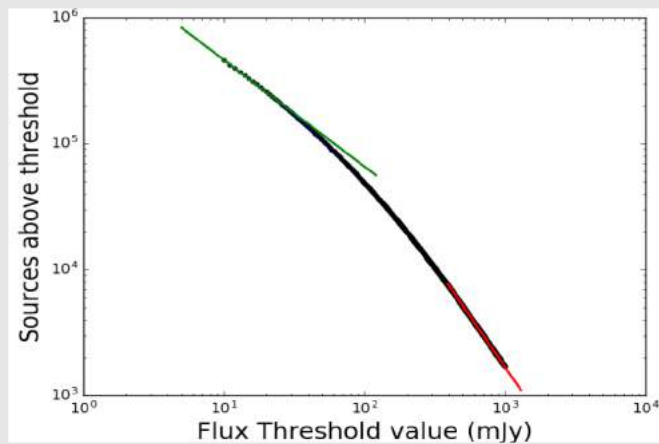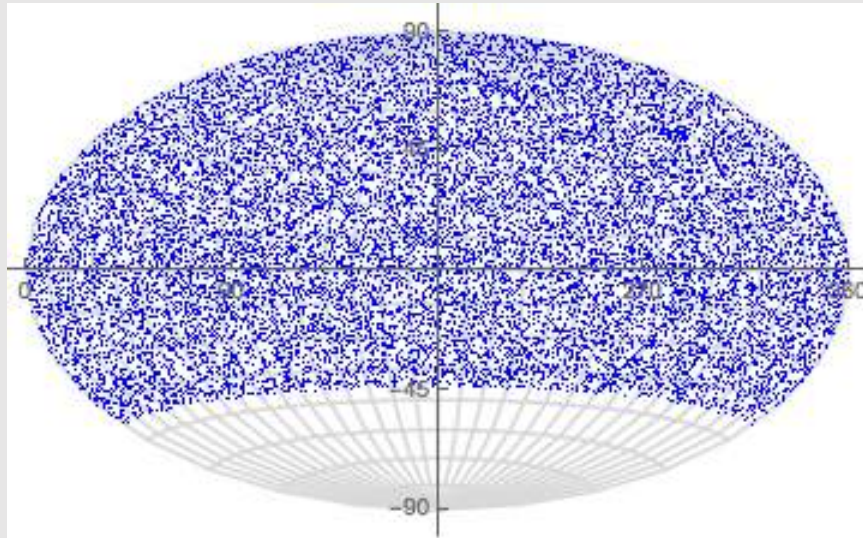
1.4 GHz survey (down to Dec = -40.4º)
National Radio Astronomy Observatory

1,773,488 sources >2.5 mJy
(complete above 10 mJy)
Most are believed to be at $z \gtrsim 1$
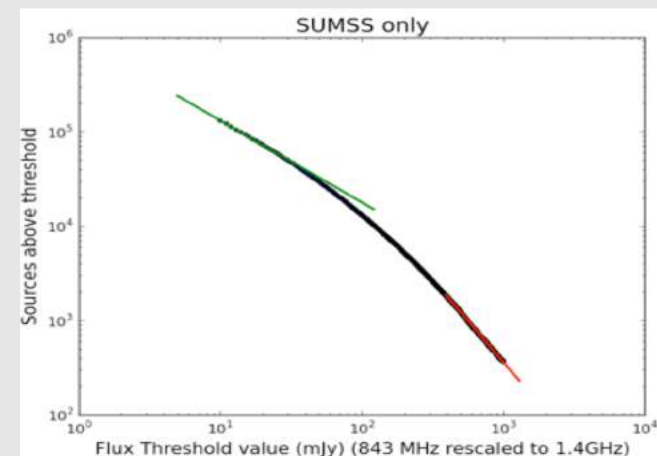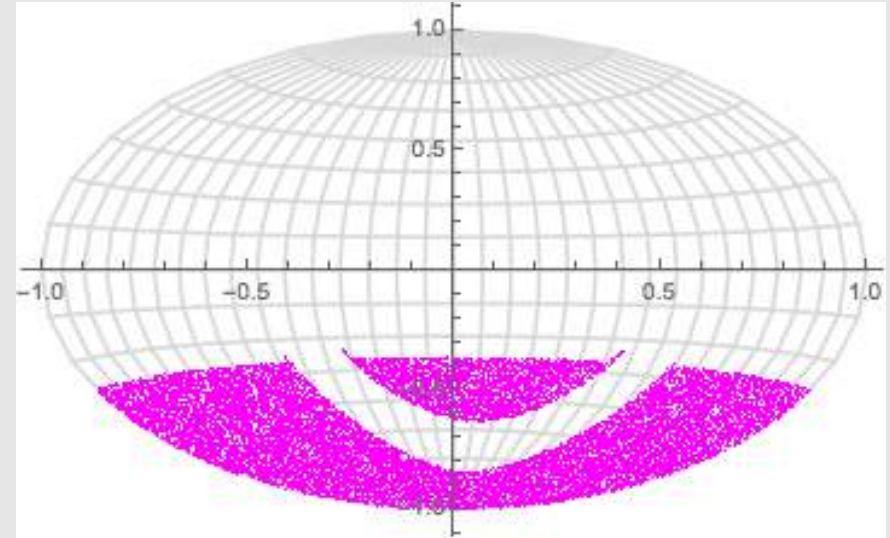
843 MHz survey (Dec < -30.0º)
Molonglo Observatory Synthesis telescope

211,050 sources (with similar sensitivity and resolution to NVSS catalogue)
... Similar expected redshift distribution

THE NVSUMSS-COMBINED ALL SKY CATALOG

- Rescale SUMSS fluxes by $(843/1400)^{-0.75} \sim 1.46$ to match with NVSS (within ~1%)

- Remove Galactic Plane at $\pm 10°$ (also super-galactic plane)

- Remove NVSS sources below, and SUMSS sources above, dec -30 (or -40)

- Apply common threshold flux cut to both samples

- Remove *any* nearby sources (common with 2MRS & LRS)

Our Peculiar velocity wrt radio sources

Colin et al, MNRAS **471**:1045,2017
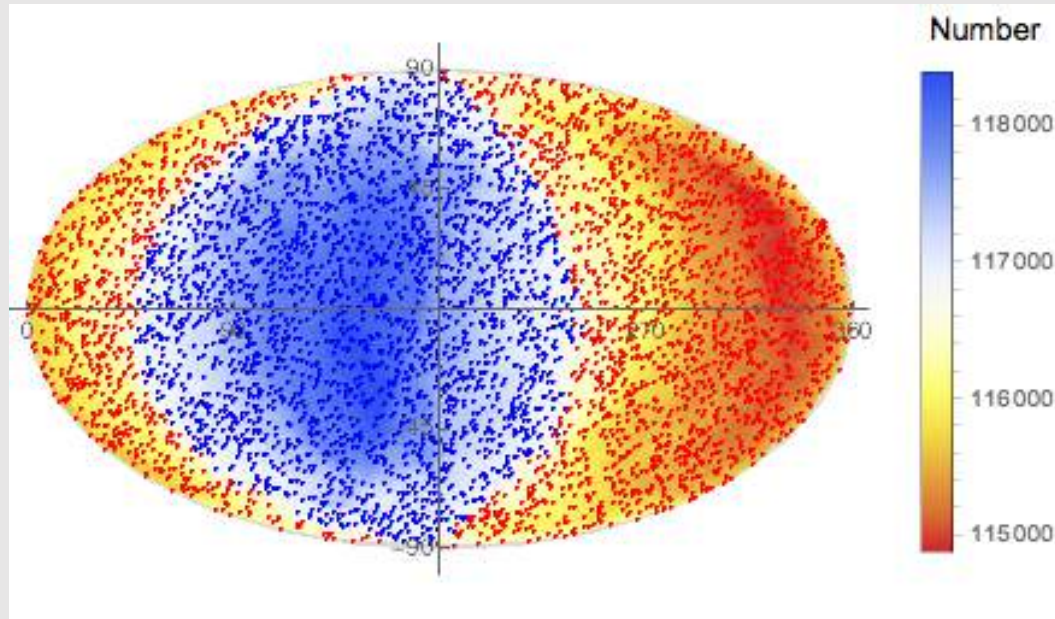
Velocity ~ 1355 ± 174 km/s
(with the 3D linear estimator)

Direction within 10° of CMB
dipole (but **4 times *faster***)!

Statistical significance: 99.75%
⇒ 2.81**σ** (by Monte Carlo)
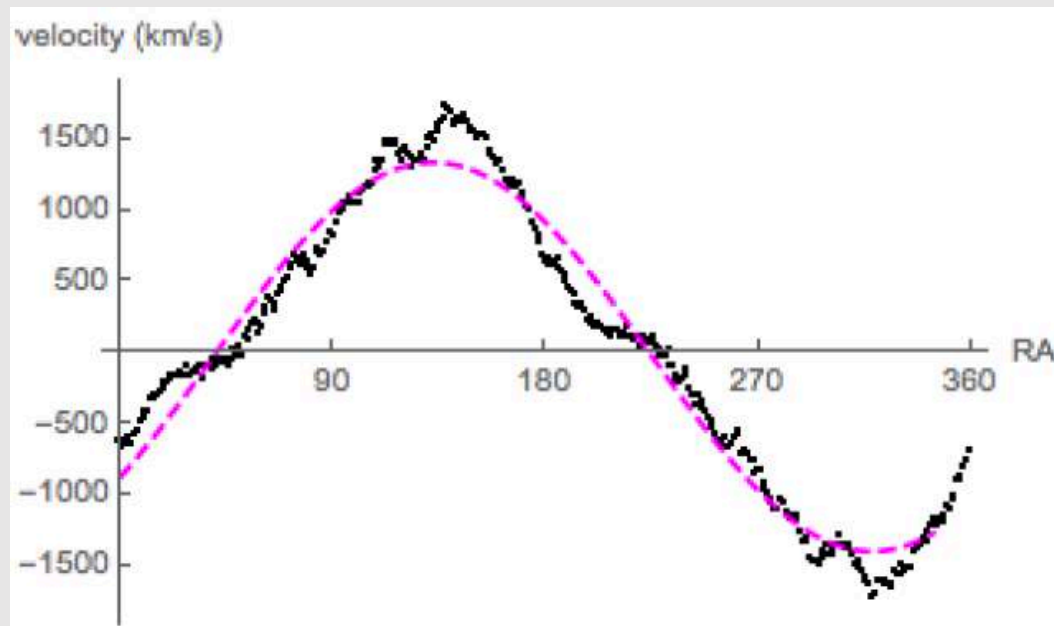
Confirms claim by Singal (2011)
which was criticized subsequently
(Gibelyou & Huterer 2012, Rubart &
Schwarz 2013, Nusser & Tiwari 2015)

We have addressed *all* the concerns
but this strange anomaly remains!
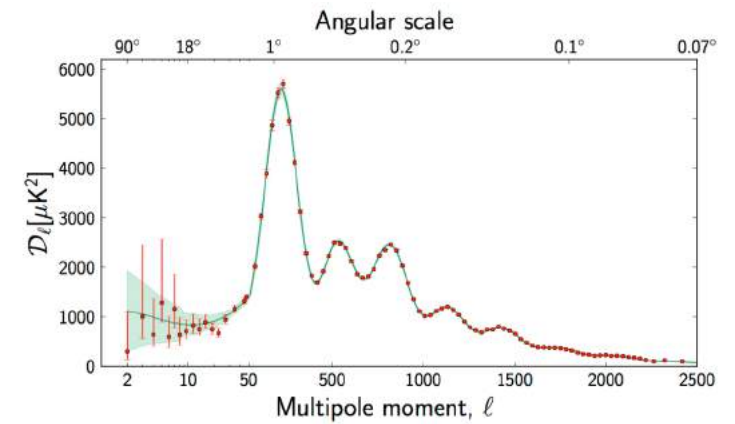
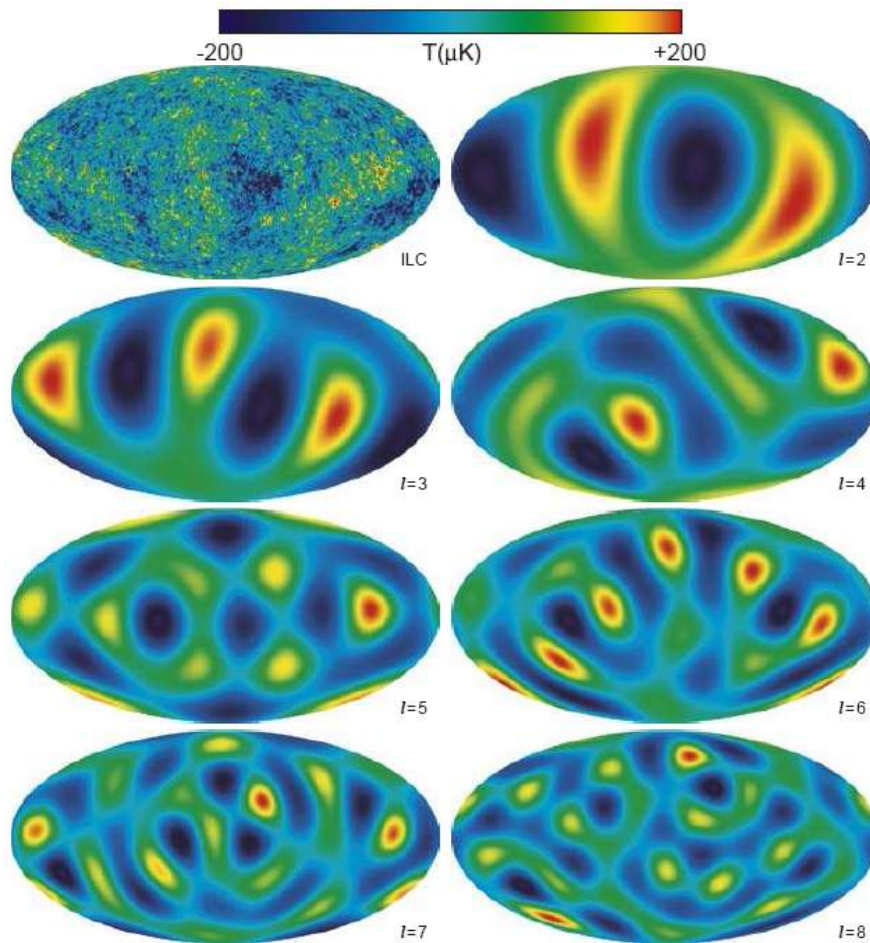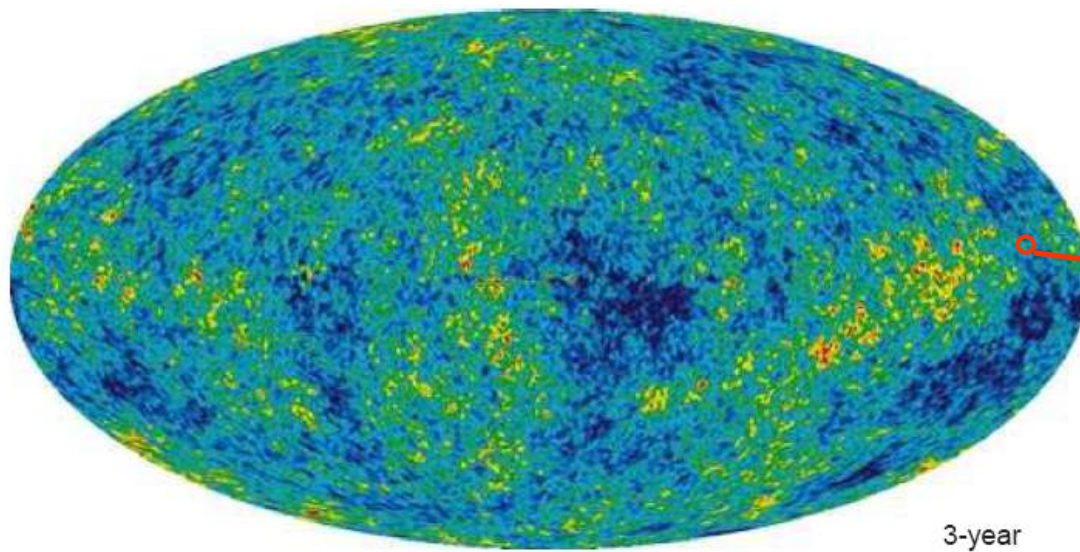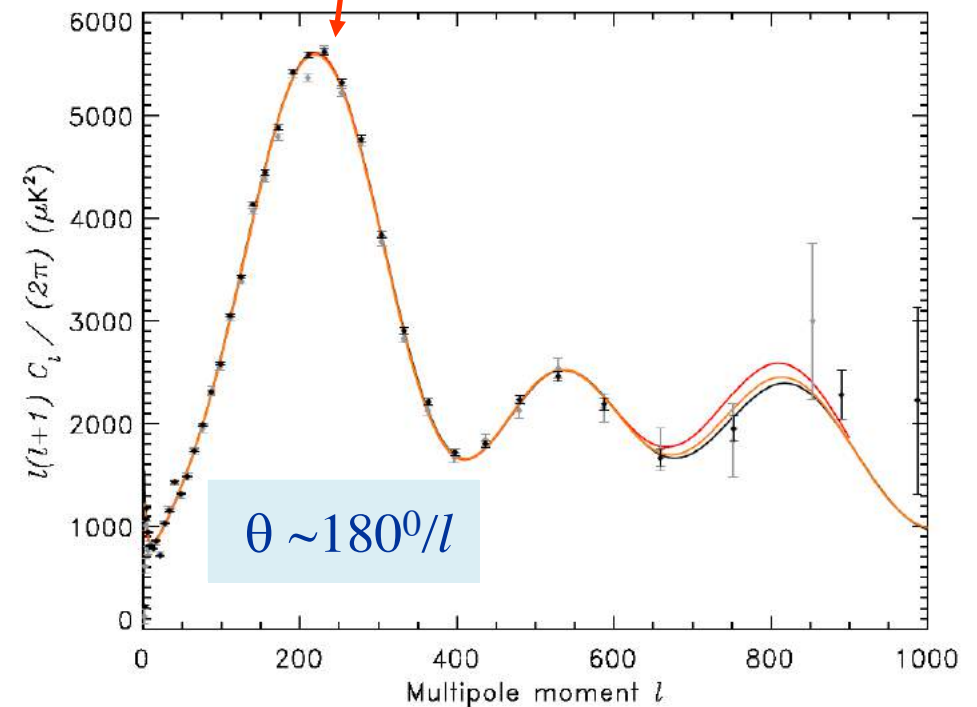# Progress in the study of the cosmic microwave background anisotropies
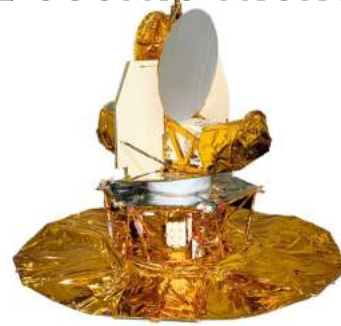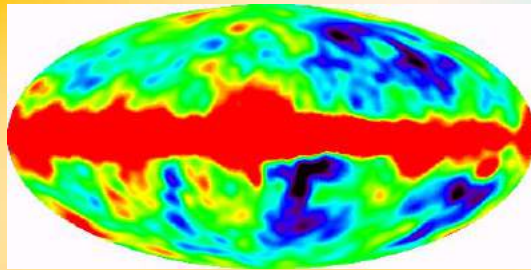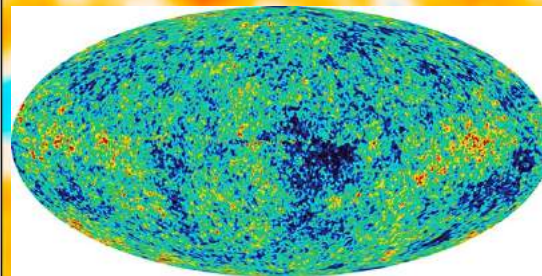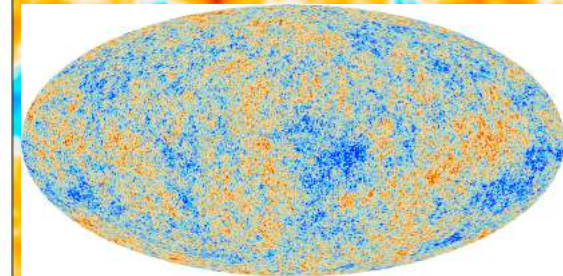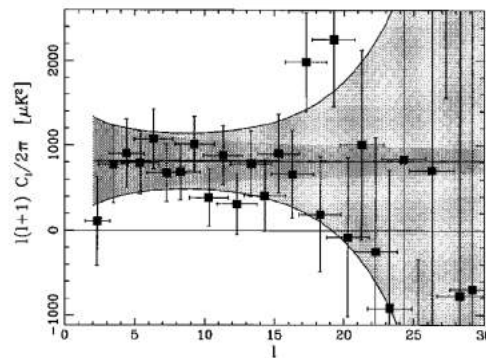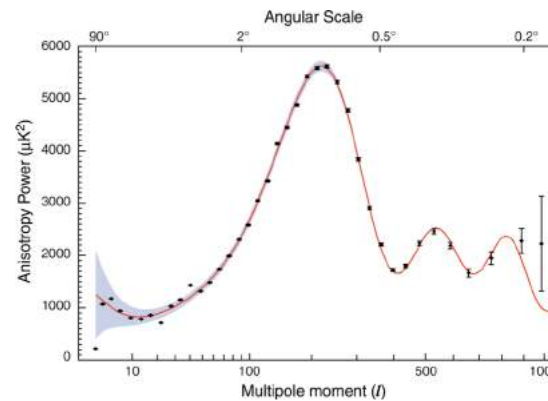
Coherent oscillations in photon-baryon plasma, excited by density perturbations on *super*-horizon scales …
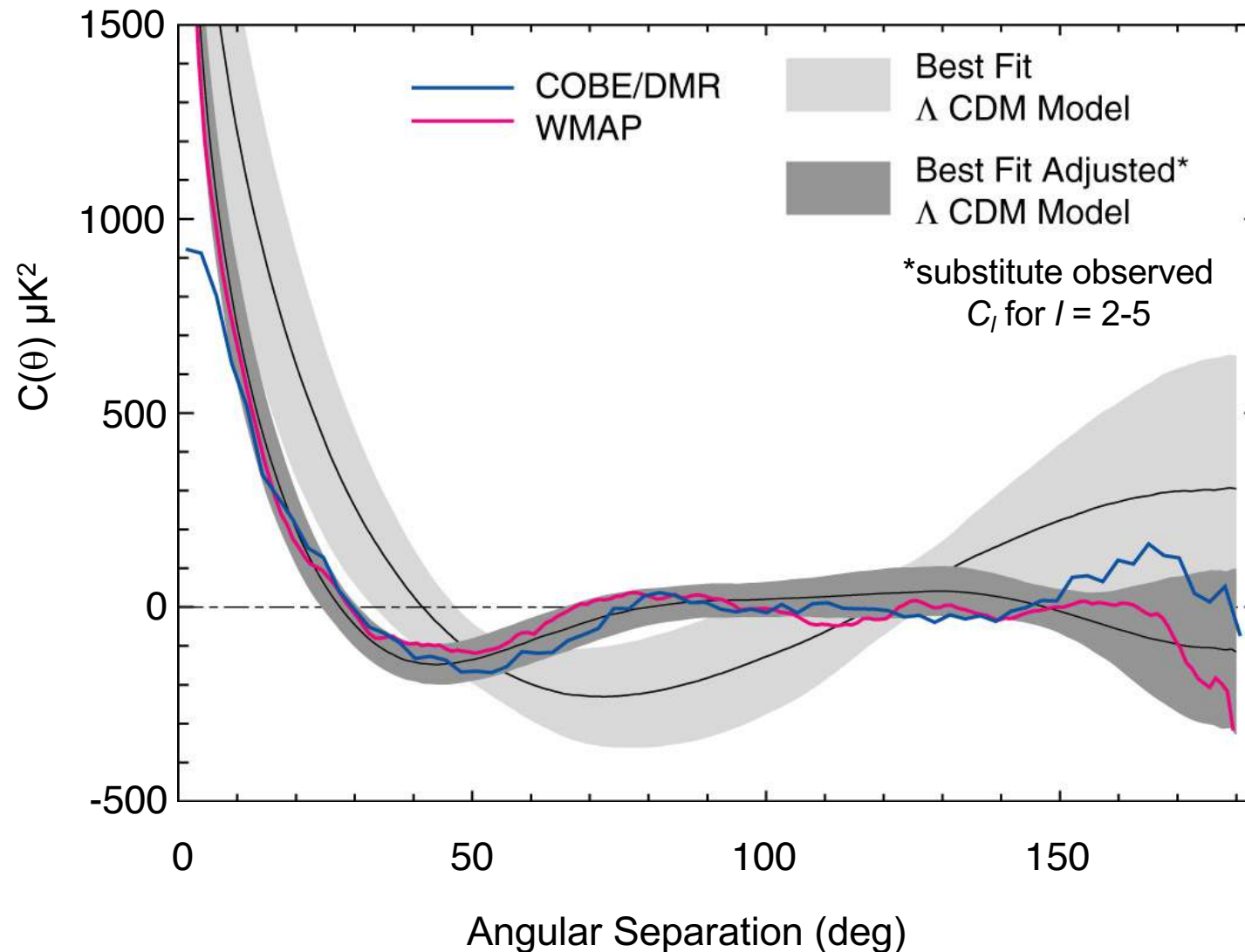
(Hubble radius at $t_{\rm rec}$)

$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$

3-year

-200 $T(\mu K)$ $+200$

ILC

$l=2$

$l=3$

$l=4$

$l=5$

$l=6$

$l=7$

$l=8$

$\theta \sim 180^0/l$

$l(l+1)\, C_l\, /\, (2\pi)\; (\mu K^2)$

Multipole moment l

The lack of power on large angular scales is most striking, although it is claimed to be *not* unlikely taking cosmic variance and foreground subtraction uncertainties into account
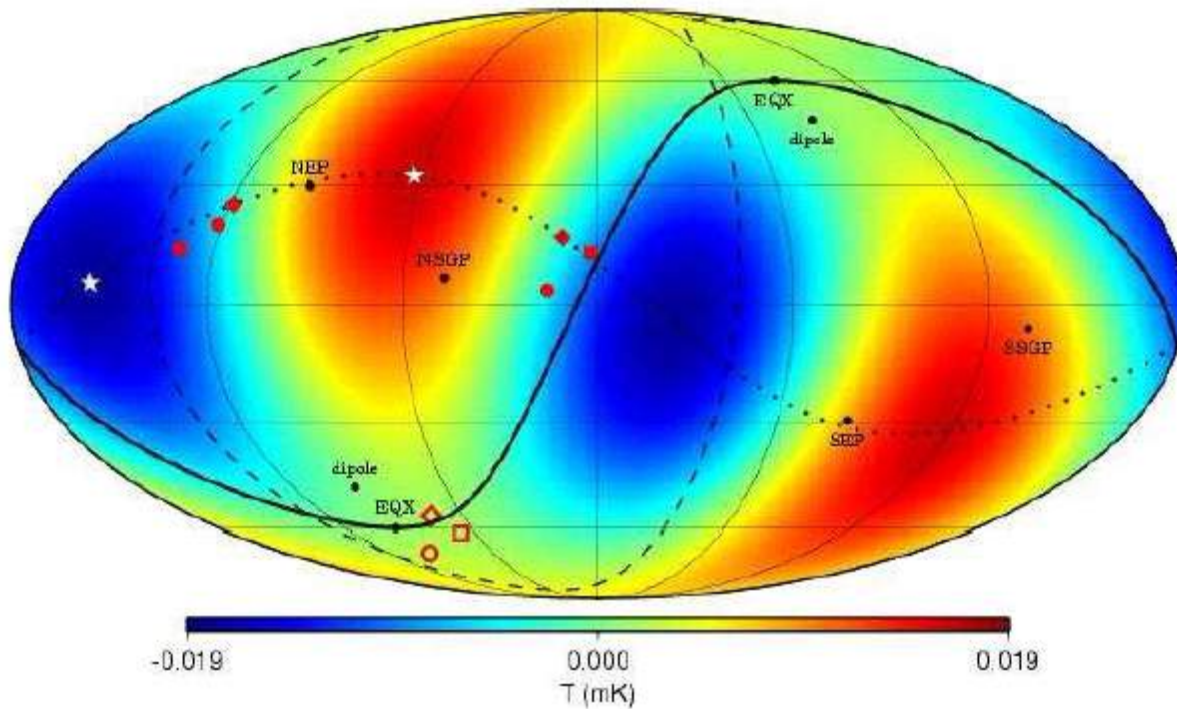➤ chance probability of $O(1\%)$?

$$C(\theta) = \left\langle T(n_i)T(n_j) \right\rangle$$

$$n_i \cdot n_j = \cos\theta$$

$$S = \int_{60°}^{180°} C(\theta)^2 d\theta$$

*A posteriori* likelihood of observed *S* is only (0.15 - 0.3) %

Moreover there is an unexpected alignment of low multipoles, a 'cold spot', and an asymmetry between the North and South ecliptic hemispheres

Curious alignment of quadrupole and octupole (along the ecliptic)
Power concentrated in plane tilted by ~30° from the Galactic plane
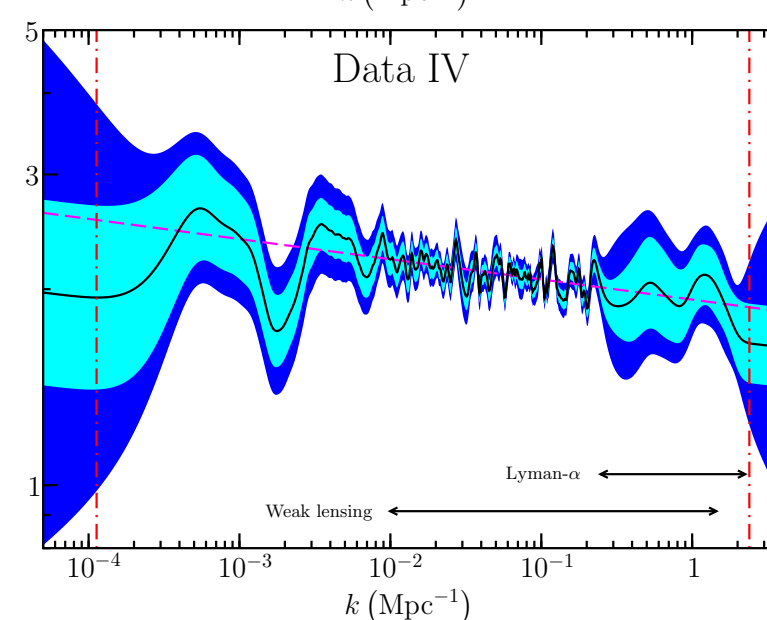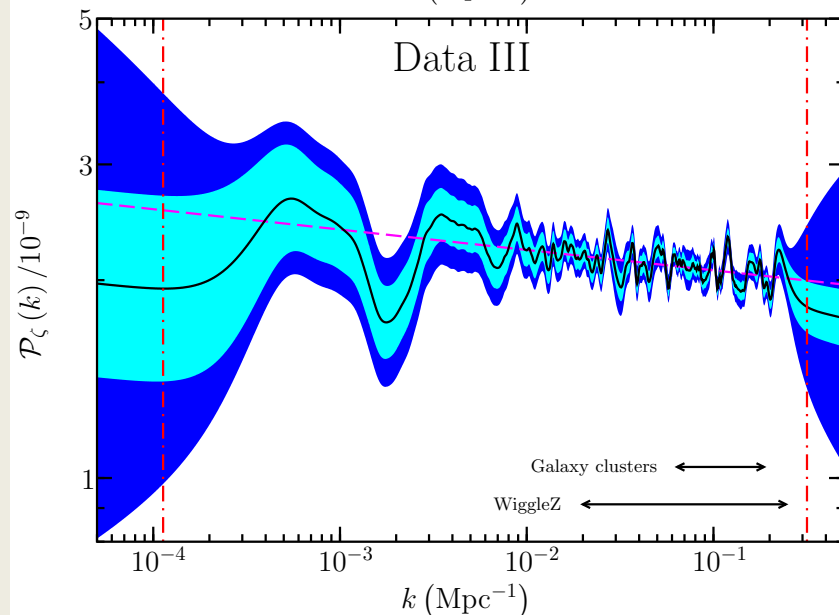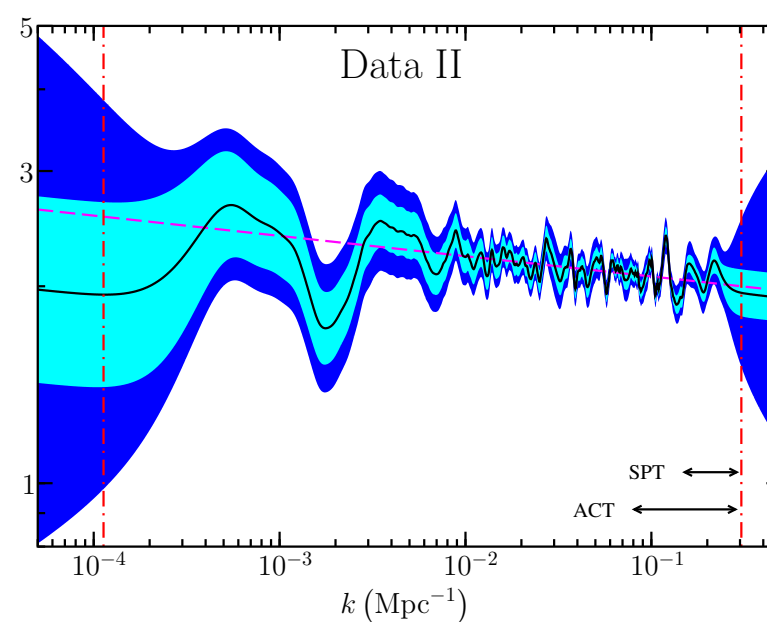($m = \pm l$ in suitable coord. system)
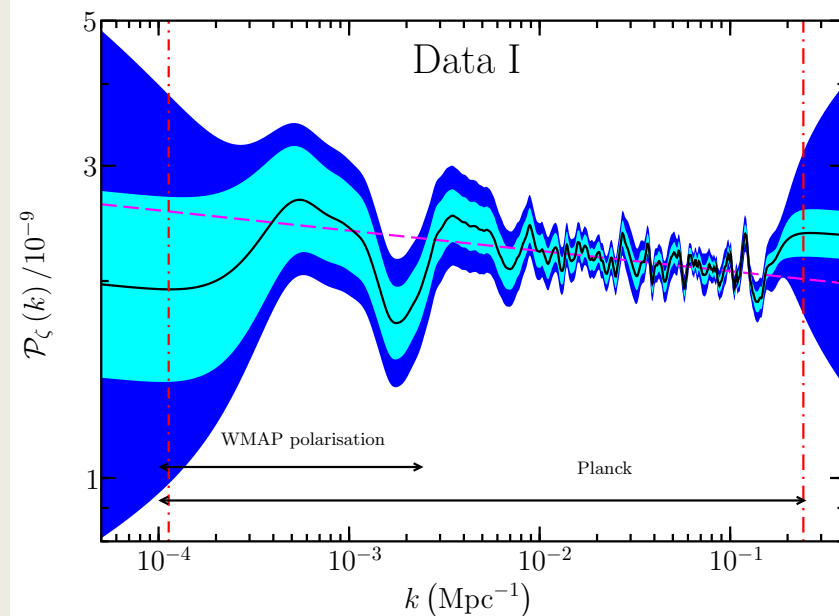
Probability of low quadrupole + alignment + "planarity": ~ 4 x 10⁻⁵

Tegmark *et al* (2003, 2004)
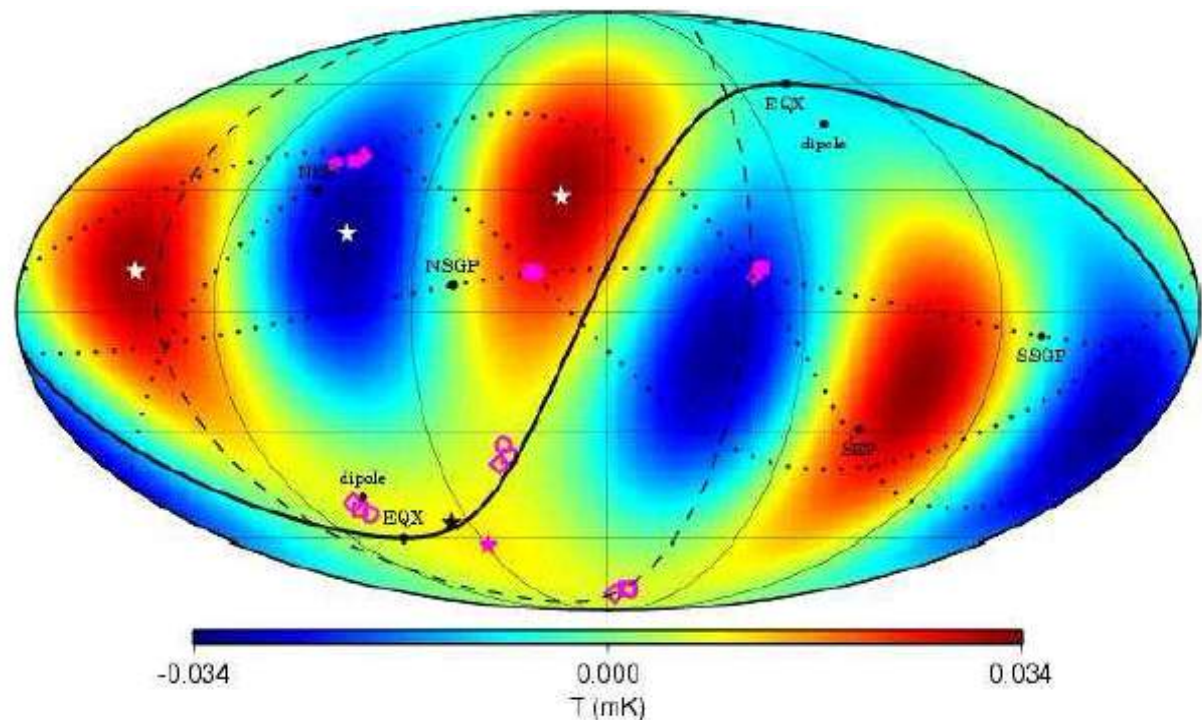
Copi, Schwarz, Starkman (2004)

The primordial spectrum of perturbations can be deconvoluted from CMB & LSS data *non*-parametrically, using 'Tikhonov regularisation' (Hunt & Sarkar, JCAP **12**:052,2015)

This is the *assumed* power-law spectrum which is supposedly the best-fit to data: $n_s = 0.969$

Comparison with Monte Carlo simulations shows ~2σ deviations from a power-law spectrum

We can also consider a **direction-dependent** component of the power spectrum of the CMB fluctuations, which is also allowed to vary with the scale (wave number):

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}(k) + \sqrt{4\pi} \sum_{LM} g_{LM}(k) Y_{LM}(\hat{\mathbf{k}})$$

... and focus on the **quadrupole** modulation (NB: Density field is *real*, hence symmetry requires *L* to be *even* – see Hajian & Souradeep 2005, Pullen & Kamionkowski 2007)
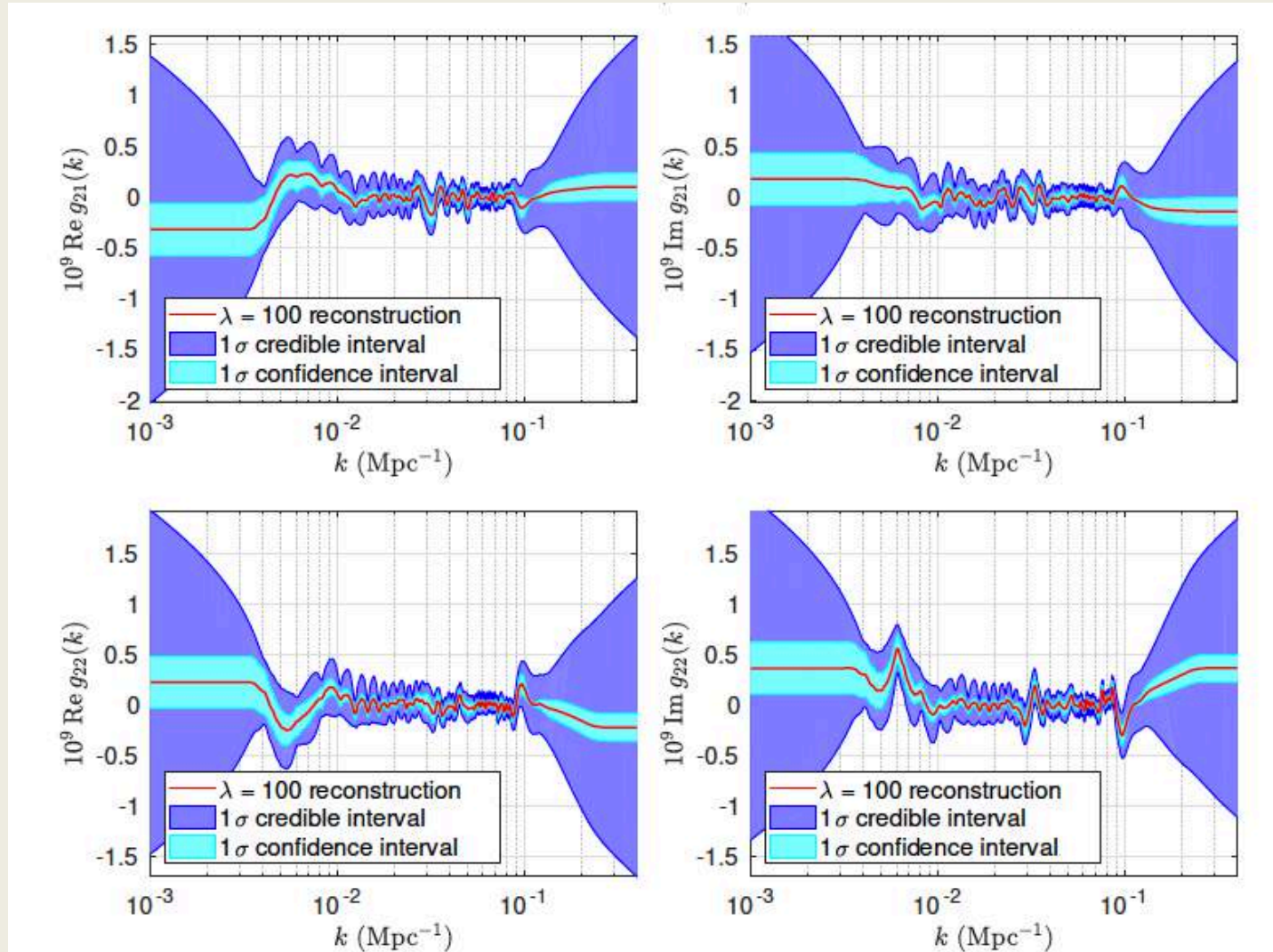
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}(k) + \sqrt{4\pi} \sum_{M=-2}^{2} g_{2M}(k) Y_{2M}(\hat{\mathbf{k}})$$

We compute these '**bipolar spherical harmonics**' for the Planck DR2-2015 SMICA map, and estimate the noise covariance from *Planck Full Focal Plane 9* simulations

Previous work by: Groeneboom & Eriksen (2009), Kim & Komatsu (2013); Theoretical models by: Ford (1989), Chibisov (1989), Ackerman *et al* (2007), Pitrou *et al* (2008), Himmetoglu *et al* (2009), Watanabe *et al* (2009), Bartolo *et al* (2013, 2018), ...
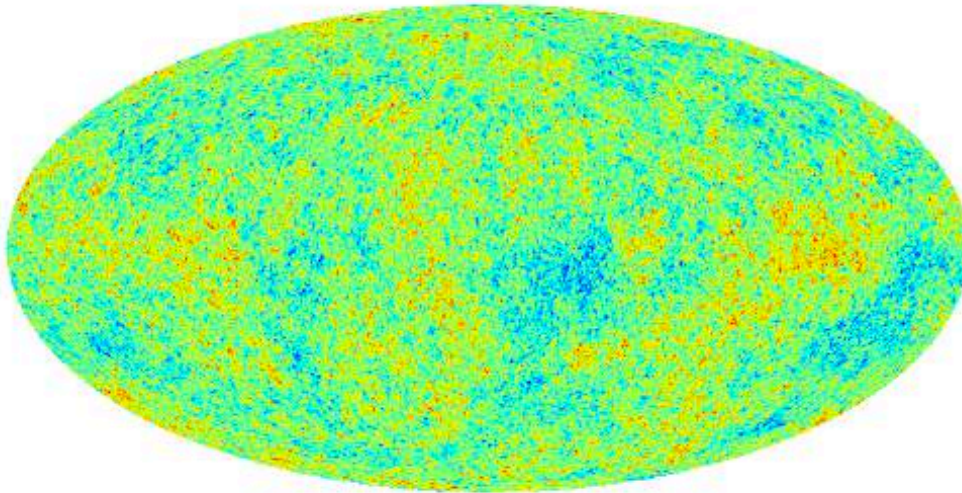
When a *constant* **quadrupolar modulation** is fitted to Planck data in the range 0.005 ≤ k/Mpc⁻¹ ≤ 0.008, its **preferred directions** are found to be *related* to the **cosmic hemispherical asymmetry**, and the **CMB dipole**

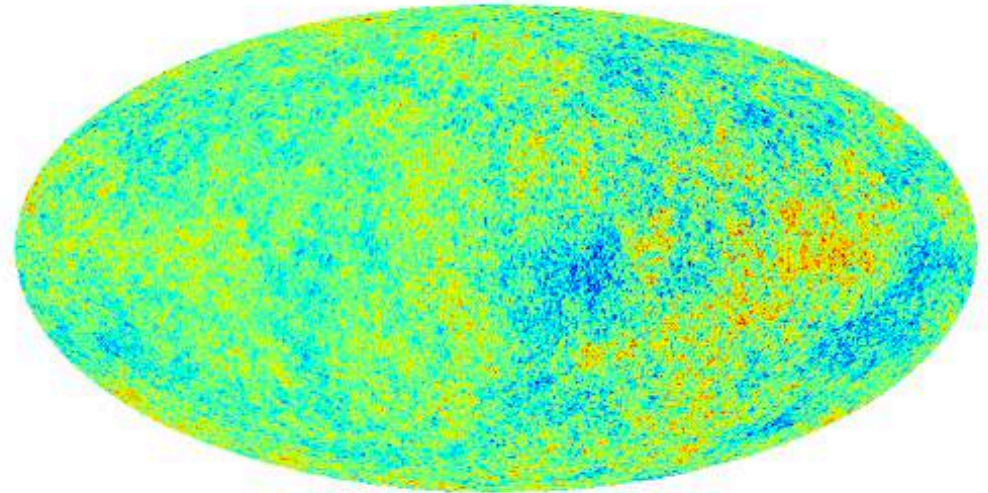The significance is 2.1σ with a test statistic sensitive only to the amplitude of the modulation … but with a statistic sensitive also to the direction, it rises to 6.9σ!
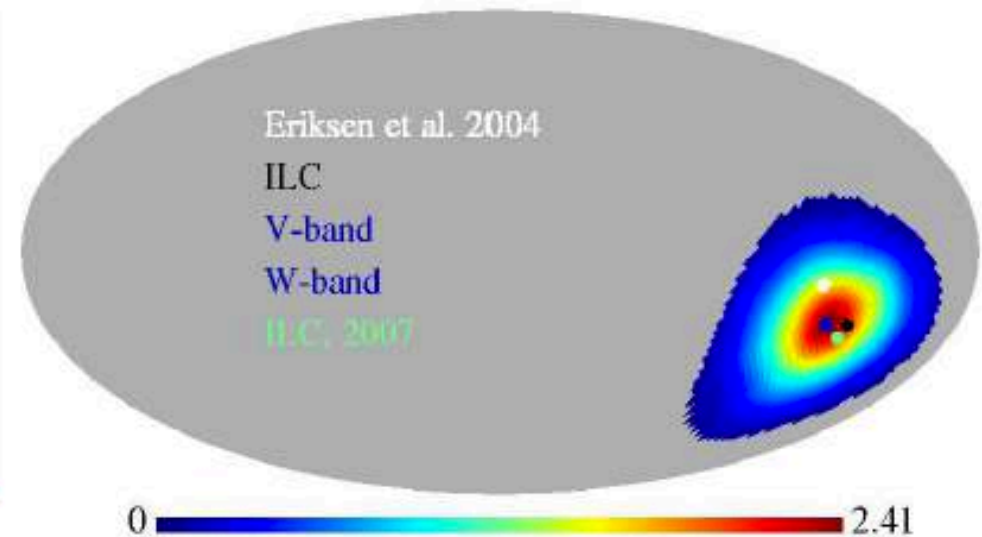
Eriksen *et al* (2004) found that the CMB fluctuations are stronger in one hemisphere of the sky than in the other (@3σ) ... as if the perturbations are modulated by a *dipole*

$$T_{\mathrm{iso}}(\hat{\mathbf{n}})$$

$$T_{\mathrm{iso}}(\hat{\mathbf{n}})(1 + A\hat{\mathbf{p}} \cdot \hat{\mathbf{n}})$$

ILC
V-band
W-band

Probability distribution

0   0.05   0.1
Modulation amplitude

Eriksen et al. 2004
ILC
V-band
W-band
ILC 2007

0   2.41
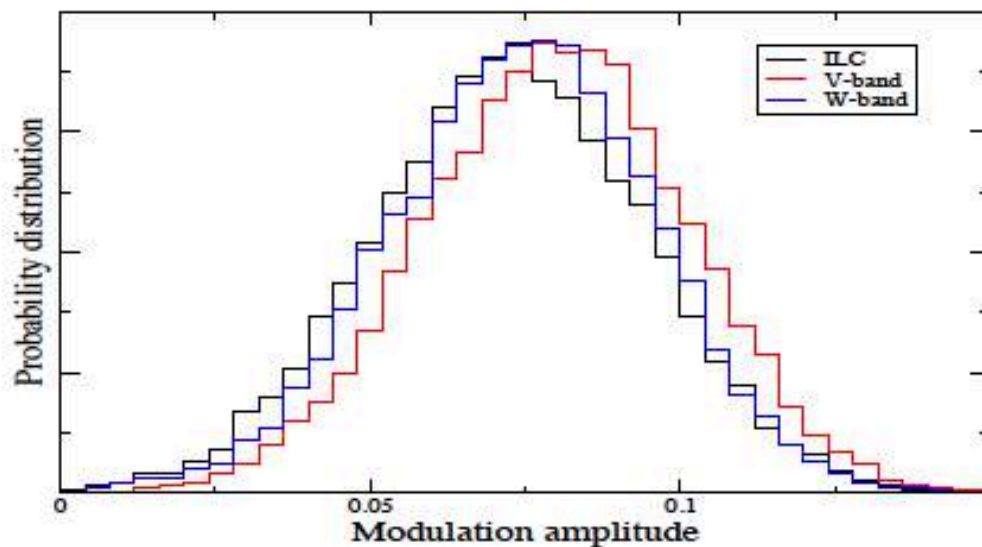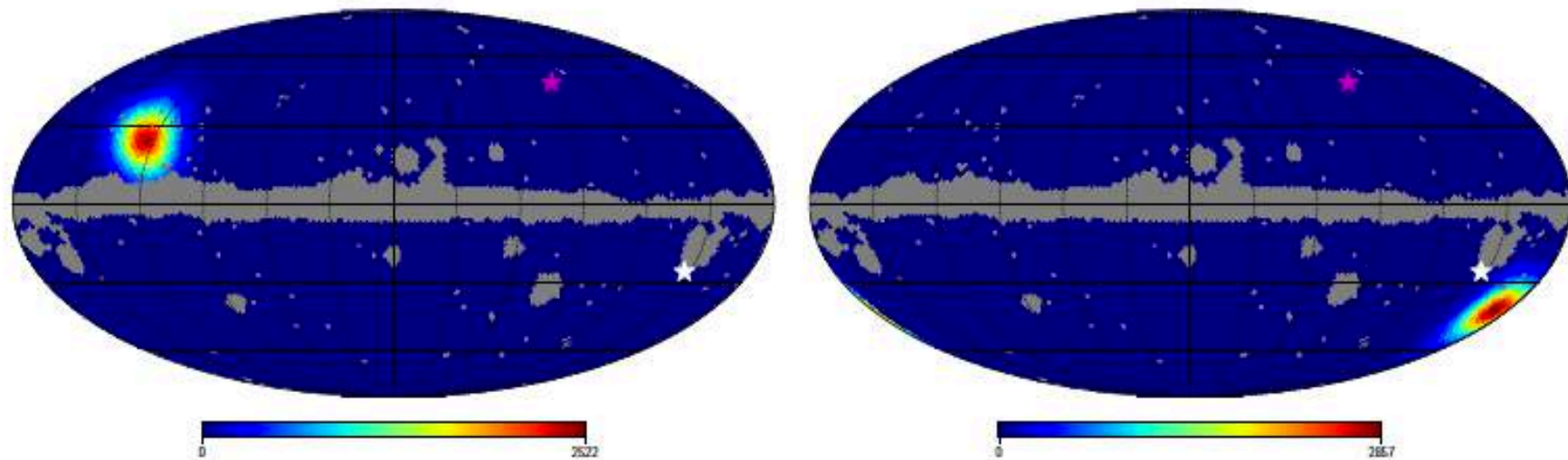
Hot ($10^9 g_A = 0.76 \pm 0.22$) quadrupole modulation (left panel), and cold ($10^9 g_A = -0.82 \pm 0.21$) modulation (right panel). The magenta and white stars indicate the direction of the CMB dipole and of the hemispherical asymmetry respectively.

For $k = 0.005$-$0.008\,\mathrm{Mpc}^{-1}$:		Angular distances to:	
Amp. $10^9 g_A$	Direction (l, b)	CMB dipole $(264^\circ, 48^\circ)$	Hemisph. asym. $(213^\circ, -26^\circ)$
0.76 ± 0.22	$(128^{\circ\,+14}_{-14}, 25^{\circ\,+11}_{-9})$	97°	97°
-0.82 ± 0.21	$(191^{\circ\,+15}_{-14}, -41^{\circ\,+10}_{-11})$	110°	24°

Durakovic et al, JCAP **02**:012,2018

Summary

- There is a dipole in the recession velocities of host galaxies of supernovae
  $\Rightarrow$ we are in a 'bulk flow' stretching out *well* beyond the expected scale (~100 Mpc)
  at which the universe is expected to become statistically homogeneous.

- The inference that the Hubble expansion rate is accelerating may be an artefact of
  the local bulk flow (there is indeed a dipole in q_0 in the same direction as the bulk
  flow, and the monopole in q_0 drops in significance to be consistent with zero at 2σ)

- The distribution of radio galaxies at $z \gtrsim 1$ also has a dipole in the same direction –
  but 4 times *bigger* than that in the CMB – so is at 2.8σ tension with it

- There is a scale-*dependent* quadrupolar modulation of CMB anisotropy   ... the
  direction is ~orthogonal to the CMB dipole

Could this be an indication of new horizon-scale physics? A tilted universe?

The 'standard' assumptions of exact isotropy and homogeneity are
questionable – data from forthcoming surveys (Euclid, LSST, SKA *etc*) will
hopefully provide sufficiently large datasets to enable definitive tests