Gerard 't Hooft

The quantization of black holes

with or without strings.

Workshop on *"Current Themes in High Energy Physics and Cosmology"*
Niels Bohr Institute.

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For the Introduction to this lecture see next Friday's Sackler Lecture.

Our prototype is the Schwarzschild black hole, with $M_{\mathrm{BH}} \gg M_{\mathrm{Planck}}$

Metric: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$,

$$\mathrm{d}s^2 = \frac{1}{1-\frac{2GM}{r}}\mathrm{d}r^2 - \left(1-\tfrac{2GM}{r}\right)\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2 \ ; \quad \begin{cases} \Omega & \equiv \quad (\theta,\,\varphi)\ , \\ \mathrm{d}\Omega & \equiv \quad (\mathrm{d}\theta,\, \sin\theta\,\mathrm{d}\varphi)\ . \end{cases}$$

Replace r, t by Kruskal-Szekeres coordinates x, y :

$$\begin{aligned} x\,y &= \left(\frac{r}{2GM}-1\right)e^{r/2GM} \qquad ; \\ y/x &= e^{t/2GM} \qquad\qquad\qquad . \end{aligned}$$

Then $$\mathrm{d}s^2 = \frac{32(GM)^3}{r}\,e^{-r/2GM}\,\mathrm{d}x\,\mathrm{d}y + r^2\mathrm{d}\Omega^2\ .$$

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Note: *two* solutions, $(x,\ y)$ and $(-x,\ -y)$ for every $(r,\ t)$.

References:

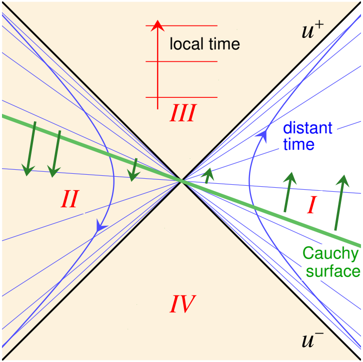
- What happens in a black hole when a particle meets its antipode, arXiv:1804.05744.
- The firewall transformation for black holes and some of its implications, Foundations of Physics, 47 (12) 1503-1542 (2017) DOI 10.1007/s10701-017-0122-3, e-Print: arxiv:1612.08640 [gr-qc]
- The Quantum Black Hole as a Hydrogen Atom: Microstates without Strings Attached, e-Print: arXiv:1605.05119 [gr-qc]
- Black hole unitarity and antipodal entanglement, Found. Phys., 49(9), 1185-1198 (2016), DOI 10.1007/s10701-016-0014-y; arXiv:1601.03447v4[gr-qc].
- A pedagogical treatment: http://www.staff.science.uu.nl/~hooft101/lectures/GtHBlackHole_2018.pdf

Everything happens in the region close to the horizon: $r \approx 2GM$. There:

$$x = \frac{\sqrt{e/2}}{2GM}\, u^+ \ ; \qquad y = \frac{\sqrt{e/2}}{2GM}\, u^- \ ; \qquad 2GM \equiv R$$

At $r = 2GM$, we have $x = 0$: past event horizon, and
$y = 0$: future event horizon.

$$\mathrm{d}s^2 \ \to \ 2\mathrm{d}u^+\mathrm{d}u^- + R^2\mathrm{d}\Omega^2 \ . \qquad \text{time} \ \ t/4GM = \tau$$



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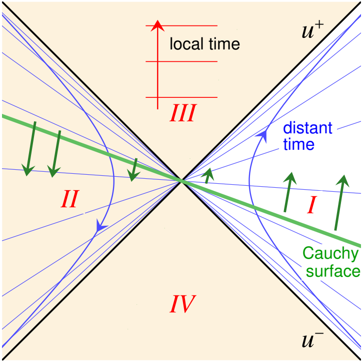
time $t/4GM = \tau$

$$u^-(\tau) = u^-(0)e^{\tau}$$
$$u^+(\tau) = u^+(0)e^{-\tau}$$

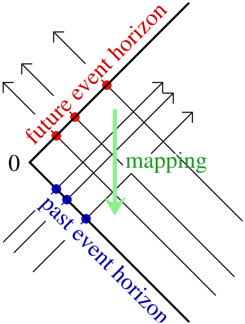


As time goes forwards, u^+ approaches the horizon asymptotically;

as time goes backwards, u^- approaches the past horizon asymptotically (tortoises).

Particles going in generate wave functions on u^+, particles going out start as wave functions on u^-.

In a quantum mechanically pure black hole, we expect that all particles entering through the future even horizon, should, some time later, re-emerge from the past horizon; $\delta t = \mathcal{O}\big(8GM \log(M/M_{\rm Planck})\big)$.

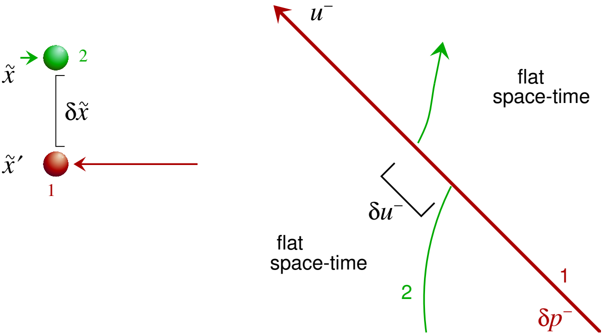


Whence this "boundary condition"?

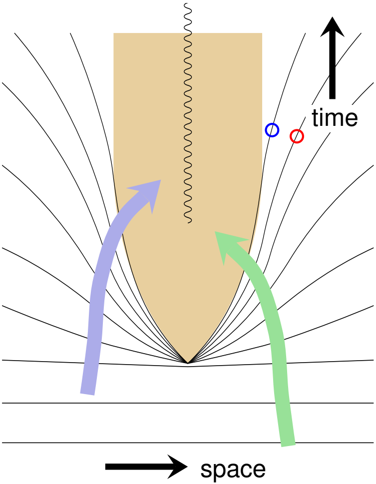
The gravitational force between in- and outgoing particles cannot be ignored; but can be calculated:

The gravitational backreaction:

Calculate the *Shapiro time delay* caused by the grav. field of a fast moving particle: simply Lorentz boost the field of a particle at rest:



$$\boxed{\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}\,')\, \log|\tilde{x} - \tilde{x}\,'|\ .}$$

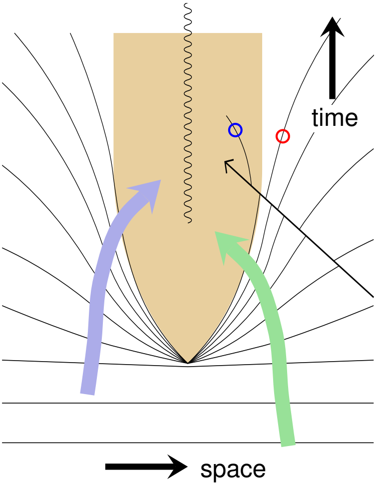


The gravitational back reaction has drastic consequences for the out-going particles.

The effect increases exponentially with time.

The in-particles leave their 'footprints' in the out-particles.

This changes everything!



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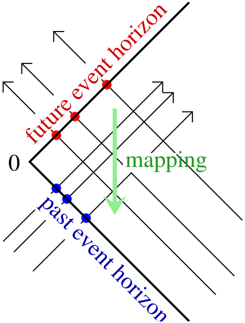

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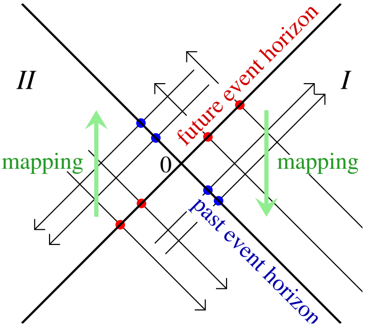
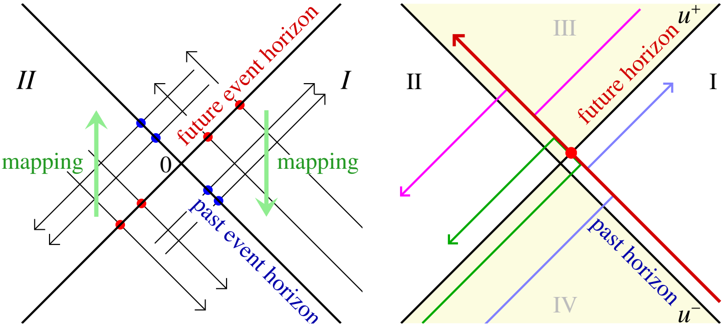


In a quantum mechanically pure black hole, we expect that all particles entering through the future even horizon, should, <u>some time later</u>, re-emerge from the past horizon; $\delta t = \mathcal{O}(8GM \log(M/M_{\rm Planck}).$



But the *gravitational back reaction* will move the data on the Cauchy surface to the left or to the right !

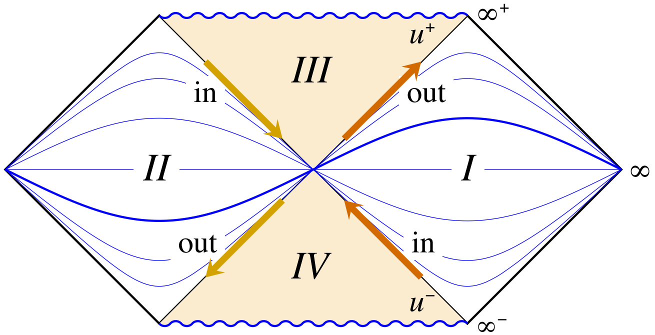
How to take this back reaction into account correctly $(G = \hbar = 1)$:

- We describe the microstates in terms of the QFT of soft-relativistic particles populating the rigid background metric.
- If we include soft-relativistic gravitons then we are also taking mild excitations of this metric itself into account.
- We must be aware of the fact that, as time proceeds (for an external observer), the momenta of the particles in any fixed frame will diverge exponentially: $p^{\pm}(\tau) \to e^{\mp\tau} p^{\pm}(0)$, $\quad \tau \equiv t/4M_{\rm BH}$; so the gravitational shift *diverges with time*.
- Therefore, we first only consider *small* time intervals $\mathcal{O}(M_{\rm BH} \log M_{\rm BH})$ in natural units.
- In these time units, the lifetime of the black hole, $\mathcal{O}(M_{\rm BH}^3)$, is close to *eternal*.
- Therefore, we first concentrate on the metric of an eternal black hole. Its metric is fixed (apart from the fluctuations effectuated by the gravitons)
- The local observer sees no particles from the implosion, and no Hawking particles (they all are too far away in time).

Let's redefine $u^+ \to f(u^+)$, $u^- \to g(u^-)$, then metric keeps the form $\mathrm{d}s^2 = 2A(u)\mathrm{d}u^+\mathrm{d}u^- + r^2(u)\mathrm{d}\Omega^2$. $\to$ *Map $u^\pm$ on compact domains*:

The Penrose diagram for the eternal black hole.

For pure Schwarzschild (without matter either responsible for the formation of a black hole, or representing its final decay):



Hard and soft particles

defined differently in other publications (!)

Particles near the horizon(s). Mass shell: $2p^+p^- + \tilde{p}^2 + \mu^2 = 0.$
$\tilde{p}$ is the transverse part of the momentum, $|\tilde{p}| \approx L/R$, $\mu =$ mass;
$p^-(\tau) = p^-(0)e^{\tau}$, $p^+(\tau) = p^+(0)e^{-\tau}$. $\tilde{p}$ and μ are constant.
Note: for Hawking particles that escape to ∞ , $|\tilde{p}|$ and μ are always small.

Define *soft* particles: $|\vec{p}|,\ \mu \ll M_{\rm Planck}$ Negligible effect on space-time.
Define hard particles as particles that do cause space-time curvature.

We claim that all black hole microstates can be represented *exclusively* in terms of soft particles on an eternal background metric.

This will be confirmed a posteriori.

As $\tau \to \infty, \quad p^+ \to 0, \quad p^- \to \infty$:

As $\tau \gg 1$, the in-particles become hard. Their interactions with other in-particles are negligible (they basically move in parallel orbits), but they do interact with the out-particles. The interaction through QFT forces stay weak, but the *gravitational forces* make that (early) in-particles interact strongly with (late) out-particles.

The fact that the mutual interactions between hard particles, at the Planck mass or beyond, will not be needed, is a very important aspect of this work. As we shall see.

Thus, we start with *only soft particles* on a Cauchy surface of the Penrose diagram. These will define *all* quantum microstates of the black hole *at a given time*.

Now, the question is how do these evolve with time.

The soft particles won't stay soft; their longitudinal momenta will quickly explode. To see what happens, calculate δu^- (the Shapiro shift).

An in-particle with momentum p^- at solid angle $\Omega' = (\theta', \varphi')$ causes a shift δu^- at solid angle $\Omega = (\theta, \varphi)$:

$$\delta u^-(\Omega) = 8\pi G\, f(\Omega, \Omega')p^- \; ; \qquad (1 - \Delta_\Omega)f(\Omega, \Omega') = \delta^2(\Omega, \Omega') \; .$$

If there are many in-particles:

$$p^-(\Omega) = \sum_i p_i^- \delta^2(\Omega, \Omega_i)$$

$$\delta u^-(\Omega) = 8\pi G \int \mathrm{d}^2\Omega' f(\Omega, \Omega')p^-(\Omega') \; .$$

The *distant* observer will see unending streams of in- and out-particles with given positions u^+ or u^-. Suppose $p^-(\Omega)$ represents *all* in-particles needed to describe any black hole in a given quantum state.

Later, we will see how imploding matter may be included in the description starting with momentum distributions p^- at the distant past.

Then the out-particles will be at positions $u^-(\Omega)$ given by

$$u^-_{\rm out}(\Omega)=8\pi G\int {\rm d}^2\Omega' f(\Omega,\Omega')p^-_{\rm in}(\Omega')\quad .$$

Spherical harmonics expansion:

$$u^\pm(\Omega)=\sum_{\ell,m}u_{\ell m}Y_{\ell m}(\Omega)\ ,\qquad p^\pm(\Omega)=\sum_{\ell,m}p^\pm_{\ell m}Y_{\ell m}(\Omega)\ ;$$

$$[u^\pm(\Omega),\ p^\mp(\Omega')]=i\delta^2(\Omega,\ \Omega')\ ,\qquad [u^\pm_{\ell m},\ p^\mp_{\ell' m'}]=i\delta_{\ell\ell'}\delta_{mm'}\ ;$$

$$u^-_{\rm out}=\frac{8\pi G}{\ell^2+\ell+1}p^-_{\rm in}\ ,\qquad u^+_{\rm in}=-\frac{8\pi G}{\ell^2+\ell+1}p^+_{\rm out}\ ,$$

$p^\pm_{\ell m}$ = total momentum in of ${}^{\rm out}_{\rm in}$-particles in (ℓ,m)-wave ,

$u^\pm_{\ell m}$ = (ℓ,m)-component of c.m. position of ${}^{\rm in}_{\rm out}$-particles .

Because we have linear equations, all different ℓ, m waves decouple; for each (ℓ, m)-mode we have just one set of variables $u^{\pm}$ and $p^{\pm}$. They represent only one independent coordinate u^{+}, with $p^{-} = -i\partial/\partial u^{+}$, while $u^{-} \propto p^{-}$ and $p^{+} \propto u^{+}$.

The dynamics completely factorises in (ℓ, m) spherical harmonics –

Probably this remains true in the harmonics of a Kerr black hole, but that generalization is not considered here.

3 more steps to be taken:

1. Starting with the QFT states of the particles entering the future event horizon, we must calculate their $p^-(\theta, \varphi)$ distribution there. The high momentum cut-offs must be chosen such that this mapping is unitary.

Comparable to vertex insertions as in string theory

2. $p^-(\theta, \varphi)$ on the future event horizon generates $u^-(\theta, \varphi)$ on the past event horizon. But its support is $[-\infty, +\infty]$, so it is spread over both regions I and II. What is the physical interpretation of region II ?

Postulate that region II refers to the same black hole as region I, but *not at the same solid angle* $\Omega = (\theta, \varphi)$. Only <u>one</u> possibility:

The antipodal identification: $\Omega \to \tilde{\Omega} = (\pi - \theta, \varphi + \pi)$

3. "Firewalls". Soft particles become hard particles. Must be 'removed'.

Suggestion: all information carried by the in- particles is now present in the out-particles. The in-particles are redundant ("quantum clones"). Leave hard particles out. Hilbert space is now completely specified by the coordinates $u^-(\Omega)$ of soft out-particles. As soon as $|u^-| > L_{\rm Planck}$, these out-particles are soft.

This is the "firewall transformation"; it removes firewalls.

The basic, explicit, calculation

The algebra, 3 slides ago, generates the scattering matrix, by giving us the *boundary condition* that replaces $|\mathrm{in}\rangle$-states by $|\mathrm{out}\rangle$-states. This boundary condition replaces the old *brick wall model* and, in the spherical harmonics expansion, it is embarassingly easy to derive.

All of this is NOT a model, or a theory, or an assumption, but a calculation:

Apart from the most basic assumption of unitary evolution,
which forces us to fold the Penrose diagram along the antipodes,
this is nothing more than applying GR and quantum mchanics !

At every (ℓ, φ), we have just one variable p^-, proportional to one u^-, with the Fourier transform p^+, proportional to u^+.

So we have 1 dimensional quantum mechanics !

Let there be two operators, u and p, obeying the commutator equation

$$[u,\, p] = i \ , \qquad \text{so that} \qquad \langle u|p\rangle = \tfrac{1}{\sqrt{2\pi}} e^{ipu} \ .$$

and a wave function $|\psi\rangle$, defined by $\psi(u) \equiv \langle u|\psi\rangle$. Its Fourier transform is

$$\hat\psi(p) \equiv \langle p|\psi\rangle = \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}u\, e^{-ipu}\psi(u) \ .$$

Now introduce tortoise coordinates, and split both u and p in a positive part and a negative part:

$$u \equiv \sigma_u\, e^{\varrho_u} \ , \quad p = \sigma_p\, e^{\varrho_p} \ ; \quad \sigma_u = \pm 1 \ , \quad \sigma_p = \pm 1 \ , \quad \text{and}$$

$$\tilde\psi_{\sigma_u}(\varrho_u) \equiv e^{\frac12 \varrho_u}\, \psi(\sigma_u\, e^{\varrho_u}) \ , \quad \tilde{\hat\psi}_{\sigma_p}(\varrho_p) \equiv e^{\frac12 \varrho_p}\, \hat\psi(\sigma_p\, e^{\varrho_p}) \ ;$$

normalisation requires: (1)

$$|\psi|^2 = \sum_{\sigma_u=\pm} \int_{-\infty}^{\infty} \mathrm{d}\varrho_u |\tilde\psi_{\sigma_u}(\varrho_u)|^2 = \sum_{\sigma_p=\pm} \int_{-\infty}^{\infty} \mathrm{d}\varrho_p |\tilde{\hat\psi}_{\sigma_p}(\varrho_p)|^2 \ .$$

What is the Fourier transform in these tortoise coordinates?

$$\tilde{\hat{\psi}}_{\sigma_p}(\varrho_p) = \sum_{\sigma_u=\pm 1} \int_{-\infty}^{\infty} \mathrm{d}\varrho_u \, K_{\sigma_u \sigma_p}(\varrho_u + \varrho_p) \, \tilde{\psi}_{\sigma_u}(\varrho_u) \ ,$$

$$\text{with} \quad K_\sigma(\varrho) \equiv \tfrac{1}{\sqrt{2\pi}} e^{\frac{1}{2}\varrho} \, e^{-i\sigma \, e^{\varrho}} \ .$$

Notice the symmetry under $\varrho_u \to \varrho_u + \lambda$, $\varrho_p \to \varrho_p - \lambda$, which is simply the symmetry $u \to u \, e^{\lambda}$, $p \to p \, e^{-\lambda}$, a property of the Fourier transform, a consequence of time translation invariance for the external observer, and thus an invariance of our algebra.

We now use this symmetry to write plane waves:

$$\tilde{\psi}_{\sigma_u}(\varrho_u) \equiv \check{\psi}_{\sigma_u}(\kappa) \, e^{-i\kappa\varrho_u} \quad \text{and} \quad \tilde{\hat{\psi}}_{\sigma_p}(\varrho_p) \equiv \check{\hat{\psi}}_{\sigma_p}(\kappa) \, e^{i\kappa\varrho_p} \quad \text{with}$$

$$\check{\hat{\psi}}_{\sigma_p}(\kappa) = \sum_{\sigma_p=\pm 1} F_{\sigma_u\sigma_p}(\kappa) \check{\psi}_{\sigma_u}(\kappa) \ ; \quad F_\sigma(\kappa) \equiv \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} K_\sigma(\varrho) e^{-i\kappa\varrho} \mathrm{d}\varrho \ .$$

Thus, we see left-going waves produce right-going waves. One finds (just do the integral):

The Fourier transform in $x,\ p$ space is non-local:

$$\langle x|p\rangle = \tfrac{1}{\sqrt{2\pi}}\, e^{ipx}$$

But if we write $x = \sigma_x\, e^{\varrho_x}$ and $p = \sigma_p\, e^{\varrho_p}$, where σ_x and σ_p are signs $\pm$, then the relation becomes:

$$\begin{aligned} \langle \varrho_x, \sigma_x|\varrho_p, \sigma_p\rangle &= \tfrac{1}{\sqrt{2\pi}}\, e^{\frac{1}{2}(\varrho_x+\varrho_p) + i\sigma_x\sigma_p\, e^{\varrho_x+\varrho_p}} \\ &= K_{-\sigma_x\sigma_p}(\varrho_x+\varrho_p)\ . \end{aligned}$$

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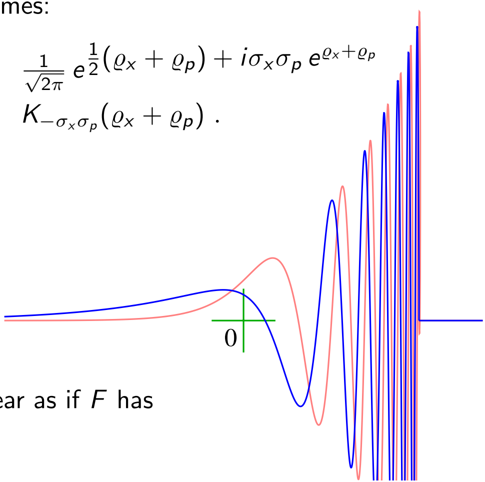
$K_+(x)$:



In practice it will appear as if F has a finite support.

Look at how our soft particle wave functions evolve with time τ, slide # 10 or 13.

Their Hamiltonian is the dilaton operator. Let κ be the *energy:*

$$H = -\tfrac{1}{2}(u^+ p^- + p^- u^+) \;=\; \tfrac{1}{2}(u^- p^+ + p^+ u^-) =$$
$$i\frac{\partial}{\partial \varrho_{u^+}} \;=\; -i\frac{\partial}{\partial \varrho_{u^-}} \;=\; -i\frac{\partial}{\partial \varrho_{p^-}} \;=\; i\frac{\partial}{\partial \varrho_{p^+}} \;=\; \kappa \;.$$

The energy eigen states are $C\,(p^-)^{i\kappa} \;=\; C\,e^{i\kappa\,\varrho_{p^-}}$,

The fourier operator on these states is:

$$F_\sigma(\kappa) = \tfrac{1}{\sqrt{2\pi}} \int_0^\infty \frac{\mathrm{d}y}{y} y^{\frac{1}{2}-i\kappa}\, e^{-i\sigma y} \;=\; \tfrac{1}{\sqrt{2\pi}}\,\Gamma(\tfrac{1}{2}-i\kappa)\, e^{-\frac{i\sigma\pi}{4}\,-\,\frac{\pi}{2}\kappa\sigma} \;.$$

Matrix $\begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix}$ is unitary: $F_+F_-^* = -F_-F_+^*$ and $|F_+|^2 + |F_-|^2 = 1$.

The scattering matrix

Add the scale factor $\frac{8\pi G}{\ell^2+\ell+1}$, to get, if $u^\pm = \sigma_\pm e^{\varrho^\pm}$,

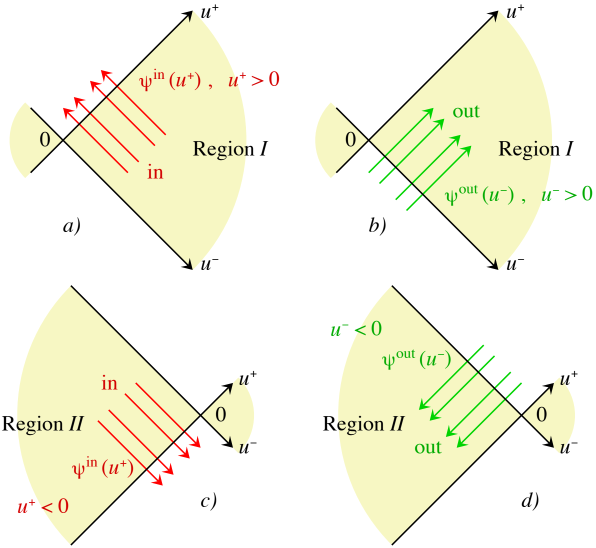
$$\psi^{\rm in}_{\sigma_+}\, e^{-i\kappa\varrho^+} \to \psi^{\rm out}_{\sigma_-} e^{i\kappa\varrho^-} \ ,$$

$$\begin{pmatrix}\psi^{\rm out}_+\\ \psi^{\rm out}_-\end{pmatrix} = \begin{pmatrix}F_+(\kappa) & F_-(\kappa)\\ F_-(\kappa) & F_+(\kappa)\end{pmatrix} e^{-i\kappa \log\left(8\pi G/(\ell^2+\ell+1)\right)} \begin{pmatrix}\psi^{\rm in}_+\\ \psi^{\rm in}_-\end{pmatrix} \ .$$

These equations generate the contributions to the scattering matrix from all (ℓ, m) sectors of the system, where $|m| \le \ell$. At every (ℓ, m), we have a contribution to the position operators $u^\pm(\theta,\varphi)$ and momentum operators $p^\pm(\theta,\varphi)$ proportional to the partial wave function $Y_{\ell m}(\theta,\varphi)$. The signs of $u^\pm(\theta,\varphi)$ tell us whether we are in region I or region II. The signs of $p^\pm(\theta,\varphi)$ tell us whether we added or sutracted a particle from region I or region II.

Considering all $(\ell,\ m)$ values with $\ell < \ell_0 \approx M_{\rm BH}/M_{\rm Planck}$ (the angular momentum limit), gives us $\approx 2^{\frac{1}{2}\ell_0^2}$ microstates, which is the right order of magnitude. But note that this black hole is not thermal.

a) Wave functions $\psi(u^+)$ of the in-particles live in region I, so $u^+ > 0$.
b) Out-particles in region I have $\psi(u^-)$ with $u^- > 0$.



The physical picture

c, d) In region II, in-particles have $u^+ < 0$ and out-particles $u^- < 0$.

Note that the in-particles will never get the opportunity to become truly hard particles.

Wave functions of soft particles going in are reflected as wave functions going out. These again emerge as soft particles.

Thus, there is no firewall, ever.

In the previous slide, the total of the in-particles in regions *I* and *II* are transformed (basically just a Fourier transform) into out-particles in the same two regions.

Note that the regions *III* and *IV* in the Penrose diagram (see slide 12) never play much of a role, even if an observer falling in region *III* would want to assure us that (s)he is still alive.

These regions are best to be seen as lying somewhere on the time-line *where time t is beyond infinity* (thus a mere repetition of the degrees of freedom we have seen before)

The antipodal identification

Regions I and II of the Penrose diagram are exact copies of one another. Often, it was thought that region II describes something like the 'inside' of a black hole. That cannot be right, since region II, like region I, has asymptotic regions. Hawking suggested that region II might be some other black hole, in an other universe, or far away in the same universe. However, our 2×2 scattering matrix implies that the two regions are in contact with each other quantum mechanically. In ordinary branches of physics, such long-distance communication can never take place.

Therefore it was natural to assume that region II describes the same black hole as region I. It must then represent some other part of the same black hole. Which other part? The local geometry stays the same, so we must be dealing with an $O(3)$ operator whose square is the identity.

There is exactly one possibility: This is the $O(3)$ operator $-\mathbb{I}$, being:

the antipodal mapping.

So this is *not* an element of $SO(3)$, which will give us constraints on the CP symmetry structure of the Standard Model interactions.

Antipodal *identification* only holds for the central point (origin) of the Penrose diagram. Regions *I* and *II* are different regions of the universe. But relating region *II* to region *I* by demanding that the angular coordinates are antipodes, means that now the mapping from Schwarzschild coordinates to Kruskal Szekeres coordinates is one-to-one. This now turns out to be an essential property of our coordinate transformations. Thus, we arrive at a new restriction for all general coordinate transformations:

> *In applying general coordinate transformations for quantized fields on a curved space-time background, to use them as a valid model for a physical quantum system, one must demand that the following constraint hold: the mapping must be one-to-one and differentiable. Every space-time point (r, t, θ, φ) now maps onto exactly one point $(x, y, \theta', \varphi')$, without the emergence of cusp singularities.*

The emergence of a non-trivial topology needs not be completely absurd, as long as no signals can be sent around. This is the case at hand here.

THE END