Coordinate space approach to double copy

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based on

[arXiv:1301.4176 arXiv:1309.0546 arXiv:1312.6523 arXiv:1402.4649 arXiv:1408.4434 arXiv:1602.08267 arXiv:1610.07192 arXiv:1707.03234 arXiv:1711.08476 arXiv:1807.02486
A. Anastasiou, L. Borsten, M. J. Duff, M. Hughes, A. Marrani, S. Nagy and M. Zoccali]

Copenhagen August 2018

Basic idea

- Strong nuclear, Weak nuclear and Electromagnetic forces described by Yang-Mills gauge theory (non-abelian generalisation of Maxwell). Gluons, W, Z and photons have spin 1.

- Gravitational force described by Einstein's general relativity. Gravitons have spin 2.

- But maybe $(spin\ 2) = (spin\ 1)^2$. If so:

  1) Do global gravitational symmetries follow from flat-space Yang-Mills symmetries?

  2) Do local gravitational symmetries and Bianchi identities follow from flat-space Yang-Mills symmetries?

  3) What about twin supergravities with same bosonic lagrangian but different fermions?

  4) Are all supergravities Yang-Mills squared?

Gravity as square of Yang-Mills

- A recurring theme in attempts to understand the quantum theory of gravity and appears in several different forms:
- Closed states from products of open states and KLT relations in string theory [Kawai, Lewellen, Tye:1985, Siegel:1988],
- On-shell $D = 10$ Type IIA and IIB supergravity representations from on-shell $D = 10$ super Yang-Mills representations [Green, Schwarz and Witten:1987],
- Vector theory of gravity [Svidzinsky 2009]
- Supergravity scattering amplitudes from those of super Yang-Mills in various dimensions, [Bern, Carrasco, Johanson:2008, 2010; Bern, Huang, Kiermaier, 2010: Bjerrum-Bohr, Damgaard, Monteiro, O'Connell 2012,Montiero, O'Connell, White 2011, 2014, Bianchi:2008, Elvang, Huang:2012, Cachazo:2013, Dolan:2013]
- Ambitwistor strings [Hodges:2011, Mason:2013, Geyer:2014]
- See talks by [Goldberger, Montiero, O'Connell]

Gravity from Yang-Mills

- LOCAL SYMMETRIES: general covariance, local lorentz invariance, local supersymmetry, local p-form gauge invariance [arXiv:1408.4434, Physica Scripta 90 (2015)] [A. Anastasiou, L. Borsten, M. J. Duff, L. J. Hughes and S. Nagy]
- BRST SQUARED[NEW] [arXiv:1807.02486] [A. Anastasiou, L. Borsten, M.J. Duff, S. Nagy, M. Zoccali]
- GLOBAL SYMMETRIES eg $G = E_7$ in $D = 4, \mathcal{N} = 8$ supergravity [arXiv:1301.4176 arXiv:1312.6523 arXiv:1402.4649 arXiv:1502.05359] [A. Anastasiou, L. Borsten, M. J. Duff, L. J. Hughes and S. Nagy]
- TWIN SUPERGRAVITIES FROM (YANG-MILLS)^2 [arXiv:1610.07192] [A. Anastasiou, L. Borsten, M. J. Duff, L. J. Hughes, A.Marrani, S. Nagy and M. Zoccali]
- TWIN CFTs[NEW] [to appear] [L. Borsten, M. J. Duff, A.Marrani,]

Local symmetries

- LOCAL SYMMETRIES

Product?

- Most of the literature is concerned with products of momentum-space scattering amplitudes, but we are interested in products of off-shell left and right Yang-Mills field in coordinate-space
$$A_\mu(x)(L) \otimes A_\nu(x)(R)$$
so it is hard to find a conventional field theory definition of the product.
- Where do the gauge indices go?
- Does it obey the Leibnitz rule
$$\partial_\mu(f \otimes g) = (\partial_\mu f) \otimes g + f \otimes (\partial_\mu g)$$
If not, why not?

Convolution

- Here we present a $G_L \times G_R$ product rule :

$$[A_\mu{}^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R)](x)$$

where $\Phi_{ii'}$ is the "spectator" bi-adjoint scalar field introduced by Hodges [Hodges:2011] and Cachazo *et al* [Cachazo:2013] and where $\star$ denotes a convolution

$$[f \star g](x) = \int d^4y f(y) g(x-y).$$

Note $f \star g = g \star f, (f \star g) \star h = f \star (g \star h)$, and, importantly obeys

$$\partial_\mu (f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

and not Leibnitz

$$\partial_\mu (f \otimes g) = (\partial_\mu f) \otimes g + f \otimes (\partial_\mu g)$$

Gravity/Yang-Mills dictionary

For concreteness we focus on

- $\mathcal{N} = 1$ supergravity in $D = 4$, obtained by tensoring the $(4 + 4)$ off-shell $\mathcal{N}_L = 1$ Yang-Mills multiplet $(A_\mu(L), \chi(L), D(L))$ with the $(3 + 0)$ off-shell $\mathcal{N}_R = 0$ multiplet $A_\mu(R)$.
- Interestingly enough, this yields the new-minimal formulation of $\mathcal{N} = 1$ supergravity [Sohnius,West:1981] with its 12+12 multiplet $(h_{\mu\nu}, \psi_\mu, V_\mu, B_{\mu\nu})$
- The dictionary is,

$$\begin{aligned} Z_{\mu\nu} \equiv h_{\mu\nu} + B_{\mu\nu} &= A_\mu{}^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R) \\ \psi_\nu &= \chi^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R) \\ V_\nu &= D^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R), \end{aligned}$$

Yang-Mills symmetries

- The left supermultiplet is described by a vector superfield $V^i(L)$ transforming as

$$\delta V^i(L) = \Lambda^i(L) + \bar{\Lambda}^i(L) + f^i{}_{jk} V^j(L)\theta^k(L) \\ + \delta_{(a,\lambda,\epsilon)} V^i(L).$$

Similarly the right Yang-Mills field ${A_\nu}^{i'}(R)$ transforms as

$$\delta {A_\nu}^{i'}(R) = \partial_\nu \sigma^{i'}(R) + f^{i'}{}_{j'k'} {A_\nu}^{j'}(R)\theta^{k'}(R) \\ + \delta_{(a,\lambda)} {A_\nu}^{i'}(R).$$

and the spectator as

$$\delta\Phi_{ii'} = -f^j{}_{ik}\Phi_{ji'}\theta^k(L) - f^{j'}{}_{i'k'}\Phi_{ij'}\theta^{k'}(R) + \delta_a \Phi_{ii'}.$$

Plugging these into the dictionary gives the gravity transformation rules.

Gravitational symmetries

$$\begin{aligned} \delta Z_{\mu\nu} &= \partial_\nu \alpha_\mu(L) + \partial_\mu \alpha_\nu(R), \\ \delta \psi_\mu &= \partial_\mu \eta, \\ \delta V_\mu &= \partial_\mu \Lambda, \end{aligned}$$

where

$$\begin{array}{ccccccc} \alpha_\mu(L) & = & A_\mu{}^i(L) & \star & \Phi_{ii'} & \star & \sigma^{i'}(R), \\ \alpha_\nu(R) & = & \sigma^i(L) & \star & \Phi_{ii'} & \star & A_\nu{}^{i'}(R), \\ \eta & = & \chi^i(L) & \star & \Phi_{ii'} & \star & \sigma^{i'}(R), \\ \Lambda & = & D^i(L) & \star & \Phi_{ii'} & \star & \sigma^{i'}(R), \end{array}$$

illustrating how the local gravitational symmetries of general covariance, 2-form gauge invariance, local supersymmetry and local chiral symmetry follow from those of Yang-Mills.

Lorentz multiplet

New minimal supergravity also admits an off-shell Lorentz multiplet $(\Omega_{\mu ab}{}^{-}, \psi_{ab}, -2V_{ab}{}^{+})$ transforming as

$$\delta\mathcal{V}^{ab} = \Lambda^{ab} + \bar{\Lambda}^{ab} + \delta_{(a,\lambda,\epsilon)}\mathcal{V}^{ab}. \qquad (1)$$

This may also be derived by tensoring the left Yang-Mills superfield $V^{i}(L)$ with the right Yang-Mills field strength $F^{abi'}(R)$ using the dictionary

$$\mathcal{V}^{ab} = V^{i}(L) \star \Phi_{ii'} \star F^{abi'}(R),$$

$$\Lambda^{ab} = \Lambda^{i}(L) \star \Phi_{ii'} \star F^{abi'}(R).$$

Bianchi identities

- The corresponding Riemann and Torsion tensors are given by

$$R^{+}_{\mu\nu\rho\sigma} = -F_{\mu\nu}{}^{i}(L) \star \Phi_{ii'} \star F_{\rho\sigma}{}^{i'}(R) = R^{-}_{\rho\sigma\mu\nu}.$$

$$T^{+}_{\mu\nu\rho} = -F_{[\mu\nu}{}^{i}(L) \star \Phi_{ii'} \star A_{\rho]}{}^{i'}(R) = -A_{[\rho}{}^{i}(L) \star \Phi_{ii'} \star F_{\mu\nu]}{}^{i'}(R) = -T^{-}_{\mu\nu\rho}$$

- One can show that (to linearised order) both the gravitational Bianchi identities

$$DT = R \wedge e \tag{2}$$

$$DR = 0 \tag{3}$$

follow from those of Yang-Mills

$$D_{[\mu}(L) F_{\nu\rho]}{}^{I}(L) = 0 = D_{[\mu}(R) F_{\nu\rho]}{}^{I'}(R)$$

To do

- Convoluting the off-shell Yang-Mills multiplets $(4+4, \mathcal{N}_L = 1)$ and $(3+0, \mathcal{N}_R = 0)$ yields the $12+12$ new-minimal off-shell $\mathcal{N} = 1$ supergravity.
- Clearly two important improvements would be to generalise our results to the full non-linear transformation rules and to address the issue of dynamics as well as symmetries.
- But on shell $(2+2, \mathcal{N}_L = 1)$ and $(2+0, \mathcal{N}_R = 0)$ yields the $4+4$ on-shell $\mathcal{N} = 1$ supergravity plus a $(2+2, \mathcal{N}_L = 1)$ chiral multiplet.

BRST: The gravity theory

First, we give the BRST Lagrangians for the graviton $h_{\mu\nu}$ and dilaton φ and the two-form, $B_{\mu\nu}$. In Einstein frame, it reads

$$
\begin{aligned}
\mathcal{L}_{h,\varphi,B} = & -\frac{1}{4}h^{\mu\nu}E_{\mu\nu} + \frac{1}{2\xi_{(h)}}\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h\right)^2 \\
& -\frac{1}{4}(\partial\varphi)^2 - \bar{c}^\mu \Box c_\mu \\
& -\frac{1}{24}H^{\mu\nu\rho}H_{\mu\nu\rho} \\
& +\frac{1}{2\xi_{(B)}}\left(\partial_\mu B^{\mu\nu} + \partial^\nu \eta\right)^2 - \bar{d}_\nu \Box d^\nu \\
& +\frac{\xi_{(d)} - m_{(d)}}{\xi_{(d)}}\bar{d}_\mu \partial_\mu \partial^\nu d_\nu + m_{(d)}\bar{d}\Box d
\end{aligned}
$$

where $E_{\mu\nu}$ is the linearised Einstein tensor and we average over de Donder gauge fixings, controlled by $\xi_{(h)}$. Note the first-level ghosts for the two-form gauge invariance, $(d_\mu, \bar{d}_\mu)$ at ghost number $(1, -1)$, as well as the second-level bosonic ghosts, $(d, \bar{d}, \eta)$ at ghost number $(2, -2, 0)$ respectively.

BRST: The gravity theory

- The equations of motion are given by

$$\Box h_{\mu\nu} - 2\xi'_{(h)}\partial^\rho\partial_{(\mu}h_{\nu)\rho} + \xi'_{(h)}\partial_\mu\partial_\nu h = j_{\mu\nu}(h)$$

$$\Box B_{\mu\nu} - \xi'_{(B)}\partial^\rho\partial_{[\mu}B_{\nu]\rho} = j_{\mu\nu}(B)\Box\varphi = j(\varphi)$$

with $\xi'_{(h)} \equiv \frac{\xi_{(h)}+2}{\xi_{(h)}}$ and $\xi'_{(B)} \equiv -2\frac{\xi_{(B)}+2}{\xi_{(B)}}$, complemented by those for the ghosts, which we omit for brevity.

BRST transformations

- The Lagrangian is invariant under the BRST transformations

$$
\begin{aligned}
Qh_{\mu\nu} &= 2\partial_{(\mu}c_{\nu)} & Qc_\mu &= 0 \\
Q\bar{c}_\mu &= \frac{\left(\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h\right)}{\xi_{(h)}} & & \\
QB_{\mu\nu} &= 2\partial_{[\mu}d_{\nu]} & Qd_\mu &= \partial_\mu d \\
Q\bar{d}_\mu &= \frac{(\partial^\nu B_{\mu\nu} + \partial_\mu \eta)}{\xi_{(B)}} & Qd &= 0 \\
Q\bar{d} &= \frac{1}{\xi_{(d)}}\partial^\mu \bar{d}_\mu & Q\eta &= \frac{m_{(d)}}{\xi_{(d)}}\partial^\mu d_\mu
\end{aligned}
$$

  where our choice of Einstein frame implies the BRST invariance of the dilaton, $Q\varphi = 0$.
- Seek a YM/gravity dictionary that yields gravitational transformation rules and field equations from those of Yang-Mills

Dictionary

- The graviton

$$h_{\mu\nu} = A_{(\mu} \circ \tilde{A}_{\nu)} + a_1 \frac{\partial_\mu \partial_\nu}{\Box} A \circ \tilde{A} + a_2 \frac{\partial_\mu \partial_\nu}{\Box} c^\alpha \circ \tilde{c}_\alpha + \frac{a_3}{\Box} \left(\partial A \circ \partial_{(\mu} \tilde{A}_{\nu)} + \partial_{(\mu} A_{\nu)} \circ \partial \tilde{A} \right) + \eta_{\mu\nu} \left(b_1 A \circ \tilde{A} + b_2 c^\alpha \circ \tilde{c}_\alpha + \frac{b_3}{\Box} \partial A \circ \partial \tilde{A} \right)$$

- The Kalb-Ramond two-form

$$B_{\mu\nu} = A_{[\mu} \circ \tilde{A}_{\nu]} + \frac{2\xi - 1}{\Box} \left(\partial A \circ \partial_{[\mu} \tilde{A}_{\nu]} - \partial_{[\mu} A_{\nu]} \circ \partial \tilde{A} \right)$$

- The dilaton

$$\varphi = A^\rho \circ \tilde{A}_\rho + \frac{1}{\xi} c^\alpha \circ \tilde{c}_\alpha + \left(\tfrac{1}{\xi^2} - 1 \right) \frac{1}{\Box} \partial A \circ \partial \tilde{A}$$

where

$$\begin{aligned} a_1 &= \tfrac{1}{1-\xi}, & a_2 &= \tfrac{1+\xi}{2(\xi-1)} & a_3 &= -1/2, \\ b_1 &= \tfrac{\xi}{(2-D)(\xi-1)} & b_2 &= b_1/\xi, & b_3 &= \left(\tfrac{1}{\xi^2} - 1\right) b_1. \end{aligned}$$

Global symmetries

- GLOBAL SYMMETRIES

Diversion: Super Yang-Mills and division algebras

A summary of the division algebras used in D dimensions with N supersymmetries

N	1	2	4	8
D				
10	$\mathbb{O}\mathbb{R} \sim \mathbb{O}$			
6	$\mathbb{H}\mathbb{R} \sim \mathbb{H}$	$\mathbb{H}\mathbb{C} \sim \mathbb{O}$		
4	$\mathbb{C}\mathbb{R} \sim \mathbb{C}$	$\mathbb{C}\mathbb{C} \sim \mathbb{H}$	$\mathbb{C}\mathbb{H} \sim \mathbb{O}$	
3	$\mathbb{R}\mathbb{R} \sim \mathbb{R}$	$\mathbb{R}\mathbb{C} \sim \mathbb{C}$	$\mathbb{R}\mathbb{H} \sim \mathbb{H}$	$\mathbb{R}\mathbb{O} \sim \mathbb{O}$

Master Lagrangian

$$S\left(A_n, A_{nN}\right) = \int d^{n+2}x \left(-\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} - \frac{1}{2} D_\mu \phi^{A*} D^\mu \phi^A - \mathrm{Re}(i\Psi^{\dagger A}\bar{\sigma}^\mu D_\mu \Psi^A) \right.$$
$$- g f_{BC}{}^A \mathrm{Re}\left(i\Psi^{\dagger A}\varepsilon \phi^B \Psi^C\right)$$
$$\left. -\frac{1}{16} g^2 f_{BC}{}^A f_{DE}{}^A (\phi^{B*}\phi^D + \phi^{D*}\phi^B)(\phi^{C*}\phi^E + \phi^{E*}\phi^C) \right),$$

The supersymmetry transformations are

$$\delta \bar{A}^A = i(\Psi^A \epsilon^\dagger - \epsilon \Psi^{\dagger A})$$
$$\delta \phi^A = -\frac{i}{2} \mathrm{tr}\left(\epsilon(\Psi^A \epsilon^\dagger - \epsilon \Psi^{\dagger A})\right),$$
$$\delta \Psi^A = \frac{1}{2} \hat{F}^A \epsilon + \frac{1}{2} \sigma^\mu \epsilon (D_\mu \phi^A \epsilon) + \frac{1}{4} f_{BC}{}^A \phi^C (\phi^B \epsilon)$$

Failure of maximal supersymmetry algebras to close OFF-SHELL may be attributed to the non-associativity of the octonions.

Triality Algebra

- Second, the triality algebra $\mathfrak{tri}(\mathbb{A})$

$$\mathfrak{tri}(\mathbb{A}) \equiv \{(A, B, C)|A(xy) = B(x)y + xC(y)\}, \quad A, B, C \in \mathfrak{so}(n), \quad x, y \in \mathbb{A}.$$

$$\begin{aligned} \mathfrak{tri}(\mathbb{R}) &= 0 \\ \mathfrak{tri}(\mathbb{C}) &= \mathfrak{so}(2) + \mathfrak{so}(2) \\ \mathfrak{tri}(\mathbb{H}) &= \mathfrak{so}(3) + \mathfrak{so}(3) + \mathfrak{so}(3) \\ \mathfrak{tri}(\mathbb{O}) &= \mathfrak{so}(8) \end{aligned}$$

[Barton and Sudbery:2003]:

Global symmetries of supergravity in D=3

- MATHEMATICS: Division algebras: R, C, H, O

  $(DIVISION\ ALGEBRAS)^2 = MAGIC\ SQUARE\ OF\ LIE\ ALGEBRAS$

- PHYSICS: $N = 1, 2, 4, 8\ D = 3\ Yang - Mills$

  $(YANG - MILLS)^2 = MAGIC\ SQUARE\ OF\ SUPERGRAVITIES$

- CONNECTION: $N = 1, 2, 4, 8 \sim R, C, H, O$

  $MATHEMATICS\ MAGIC\ SQUARE = PHYSICS\ MAGIC\ SQUARE$

- The $D = 3\ G/H\ grav$ symmetries are given by ym symmetries

  $$G(grav) = \mathfrak{tri}\ ym(L) + \mathfrak{tri}\ ym(R) + 3[ym(L) \times ym(R)].$$

  eg

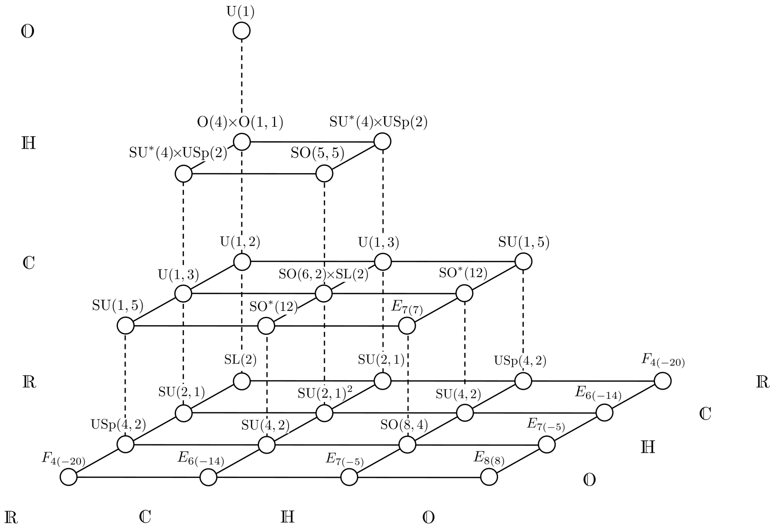
  $$E_{8(8)} = SO(8) + SO(8) + 3(\mathbb{O} \times \mathbb{O})$$
  $$248 = 28 + 28 + (8_v, 8_v) + (8_s, 8_s) + (8_c, 8_c)$$

Final result

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$\mathcal{N}=2, f=4$ $G=\mathrm{SL}(2,\mathbb{R})$, dim 3 $H=\mathrm{SO}(2)$, dim 1	$\mathcal{N}=3, f=8$ $G=\mathrm{SU}(2,1)$, dim 8 $H=\mathrm{SU}(2)\times\mathrm{SO}(2)$, dim 4	$\mathcal{N}=5, f=16$ $G=\mathrm{USp}(4,2)$, dim 21 $H=\mathrm{USp}(4)\times\mathrm{USp}(2)$, dim 13	$\mathcal{N}=9, f=32$ $G=F_{4(-20)}$, dim 52 $H=\mathrm{SO}(9)$, dim 36
$\mathbb{C}$	$\mathcal{N}=3, f=8$ $G=\mathrm{SU}(2,1)$, dim 8 $H=\mathrm{SU}(2)\times\mathrm{SO}(2)$, dim 4	$\mathcal{N}=4, f=16$ $G=\mathrm{SU}(2,1)^2$, dim 16 $H=\mathrm{SU}(2)^2\times\mathrm{SO}(2)^2$, dim 8	$\mathcal{N}=6, f=32$ $G=\mathrm{SU}(4,2)$, dim 35 $H=\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{SO}(2)$, dim 19	$\mathcal{N}=10, f=64$ $G=E_{6(-14)}$, dim 78 $H=\mathrm{SO}(10)\times\mathrm{SO}(2)$, dim 46
$\mathbb{H}$	$\mathcal{N}=5, f=16$ $G=\mathrm{USp}(4,2)$, dim 21 $H=\mathrm{USp}(4)\times\mathrm{USp}(2)$, dim 13	$\mathcal{N}=6, f=32$ $G=\mathrm{SU}(4,2)$, dim 35 $H=\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{SO}(2)$, dim 19	$\mathcal{N}=8, f=64$ $G=\mathrm{SO}(8,4)$, dim 66 $H=\mathrm{SO}(8)\times\mathrm{SO}(4)$, dim 34	$\mathcal{N}=12, f=128$ $G=E_{7(-5)}$, dim 133 $H=\mathrm{SO}(12)\times\mathrm{SO}(3)$, dim 69
$\mathbb{O}$	$\mathcal{N}=9, f=32$ $G=F_{4(-20)}$, dim 52 $H=\mathrm{SO}(9)$, dim 36	$\mathcal{N}=10, f=64$ $G=E_{6(-14)}$, dim 78 $H=\mathrm{SO}(10)\times\mathrm{SO}(2)$, dim 46	$\mathcal{N}=12, f=128$ $G=E_{7(-5)}$, dim 133 $H=\mathrm{SO}(12)\times\mathrm{SO}(3)$, dim 69	$\mathcal{N}=16, f=256$ $G=E_{8(8)}$, dim 248 $H=\mathrm{SO}(16)$, dim 120

- The $\mathcal{N}>8$ supergravities in $D=3$ are unique, all fields belonging to the gravity multiplet, while those with $\mathcal{N}\leq 8$ may be coupled to k additional matter multiplets [Marcus and Schwarz:1983; deWit, Tollsten and Nicolai:1992]. The real miracle is that tensoring left and right YM multiplets yields the field content of $\mathcal{N}=2,3,4,5,6,8$ supergravity with $k=1,1,2,1,2,4$: just the right matter content to produce the U-duality groups appearing in the magic square.

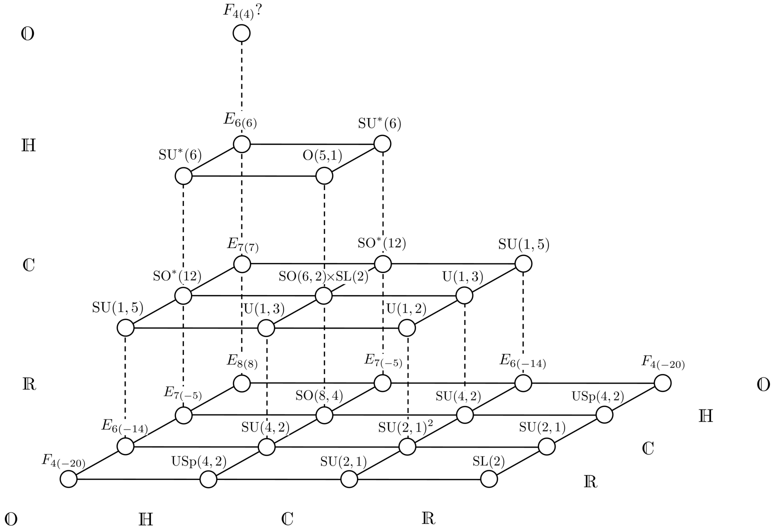
Magic Pyramid: G symmetries



Summary Gravity: Conformal Magic Pyramid

- We also construct a *conformal* magic pyramid by tensoring conformal supermultiplets in D = 3, 4, 6.
- Is the missing entry in D = 10 is suggestive of an exotic theory with G/H duality structure $F_{4(4)}/Sp(3) \times Sp(1)$?

Conformal Magic Pyramid: G symmetries



- TWIN SUPERGRAVITIES

. Twins?

- We consider so-called 'twin supergravities' - pairs of supergravities with $\mathcal{N}_+$ and $\mathcal{N}_-$ supersymmetries, $\mathcal{N}_+ > \mathcal{N}_-$, with identical bosonic sectors - in the context of tensoring super Yang-Mills multiplets.
  [Gunaydin, Sierra and Townsend Dolivet, Julia and Kounnas Bianchi and Ferrara]
- Classified in [Roest and Samtleben Duff and Ferrara]
- Related work in [Chiodaroli, Gunaydin, Johansson, Roiban, 2015]

Example: $\mathcal{N}_+ = 6$ and $\mathcal{N}_- = 2$ twin supergravities

- The $D = 4, \mathcal{N} = 6$ supergravity theory is unique and determined by supersymmetry. The multiplet consists of

$$\mathbf{G}_6 = \{g_{\mu\nu}, 16A_\mu, 30\phi; 6\Psi_\mu, 26\chi\}$$

- Its twin theory is the magic $\mathcal{N} = 2$ supergravity coupled to 15 vector multiplets based on the Jordan algebra of 3×3 Hermitian quaternionic matrices $\mathfrak{J}_3(\mathbb{H})$. The multiplet consists of

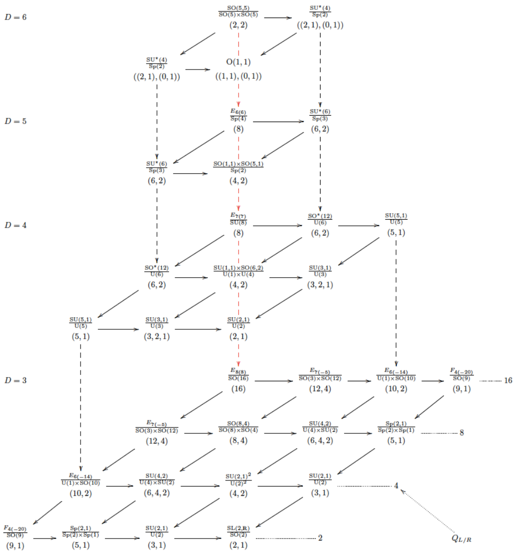
$$\mathbf{G}_2 \oplus 15\mathbf{V}_2 = \{g_{\mu\nu}, 2\Psi_\mu, A_\mu\} \oplus 15\{A_\mu, 2\chi, 2\phi\}$$

- In both cases the 30 scalars parametrise the coset manifold

$$\frac{\mathrm{SO}^\star(12)}{\mathrm{U}(6)}$$

and the 16 Maxwell field strengths and their duals transform as the **32** of $\mathrm{SO}^\star(12)$ where $\mathrm{SO}^\star(2n) = O(n, H)$

Pyramid of twins



Yang-Mills origin of twin supergravities

- Key idea: reduce the degree of supersymmetry by using 'fundamental' matter multiplets

$$\chi^{\mathsf{adj}} \longrightarrow \chi^{\mathsf{fund}}$$

- Twin supergravities are systematically related through this process

- Generates new from old (supergravities that previously did not have a Yang-Mills origin)

Yang-Mills origin of $(6, 2)$ twin supergravities

$\mathcal{N} = 6$

- The $\mathcal{N} = 6$ multiplet is the product of $\mathcal{N} = 2$ and $\mathcal{N} = 4$ vector multiplets,

$$[\mathbf{V}_2 \oplus \mathbf{C}_2^{\rho}] \otimes \tilde{\mathbf{V}}_4 = \mathbf{G}_6,$$

- $\mathbf{G}_{\mathcal{N}}, \mathbf{V}_{\mathcal{N}}$ and $\mathbf{C}_{\mathcal{N}}$ denote the $\mathcal{N}$-extended gravity, vector, and spinor multiplets

- The hypermultiplet $\mathbf{C}_2^{\rho}$ carries a non-adjoint representation ρ of G

- $\mathbf{C}_2^{\rho}$ does not ‘talk’ to the right adjoint valued multiplet $\tilde{\mathbf{V}}_4$

Yang-Mills origin of $(6,2)$ twin supergravities

To generate the twin $\mathcal{N}_- = 2$ theory:

- Replace the right $\mathcal{N}=4$ Yang-Mills by an $\mathcal{N}=0$ multiplet

$$[\mathbf{V}_2 \oplus \mathbf{C}_2^\rho] \otimes \tilde{\mathbf{V}}_4 \quad \longrightarrow \quad [\mathbf{V}_2 \oplus \mathbf{C}_2^\rho] \otimes \left[\tilde{A} \oplus \tilde{\chi}^{\rho\alpha} \oplus \tilde{\phi}^{[\alpha\beta]}\right]$$

- Here $\tilde{\chi}^\alpha$ in the adjoint of $\tilde{G}$ and **4** of $SU(4)$ is replaced by $\tilde{\chi}^{\rho\alpha}$ in a non-adjoint representation of $\tilde{G}$

- $\tilde{\chi}^{\rho\alpha}$ does not 'talk' to the right adjoint valued multiplet $\mathbf{V}_2$, but does with $\mathbf{C}_2^\rho$

- Gives a "sum of squares"

$$[\mathbf{V}_2 \oplus \mathbf{C}_2^\rho]\otimes\left[\tilde{A} \oplus \tilde{\chi}^{\rho\alpha} \oplus \tilde{\phi}^{[\alpha\beta]}\right] = \mathbf{V}_2\otimes\left[\tilde{A} \oplus \tilde{\phi}^{[\alpha\beta]}\right]\oplus[\mathbf{C}_2^\rho \otimes \tilde{\chi}^{\rho\alpha}] = \mathbf{G}_2\oplus 15\mathbf{V}_2$$

Sum of squares

Introduce bi-fundamental scalar $\Phi^{a\tilde{a}}$ to obtain sum of squares off-shell:

- Block-diagonal spectator field Φ with bi-adjoint and bi-fundamental sectors
$$\Phi = \begin{pmatrix} \Phi^{i\tilde{i}} & 0 \\ 0 & \Phi^{a\tilde{a}} \end{pmatrix}.$$

- The off-shell dictionary correctly captures the sum-of-squares rule:
$$[\mathbf{V}_{\mathcal{N}_L} \oplus \mathbf{C}^{\rho}_{\mathcal{N}_L}] \circ \Phi \circ [\tilde{\mathbf{V}}_{\mathcal{N}_R} \oplus \tilde{\mathbf{C}}^{\tilde{\rho}}_{\mathcal{N}_R}] = \mathbf{V}^{i}_{\mathcal{N}_L} \circ \Phi_{i\tilde{i}} \circ \tilde{\mathbf{V}}^{\tilde{i}}_{\mathcal{N}_R} \oplus \mathbf{C}^{a}_{\mathcal{N}_L} \circ \Phi_{a\tilde{a}} \circ \tilde{\mathbf{C}}^{\tilde{a}}_{\mathcal{N}_R}$$

- Crucially, the gravitational symmetries are correctly generated by those of the Yang-Mills-matter factors via $\star$ and Φ.

Universal rule

- This construction generalises: all pairs of twin supergravity theories in the pyramid are related in this way

Super Yang-Mills factors

$$\begin{array}{ccc}
\left[\mathbf{V}_{\mathcal{N}_L} \oplus \mathbf{C}_{\mathcal{N}_L}\right] \otimes \tilde{\mathbf{V}}_{\mathcal{N}_R} & \longrightarrow & \left[\mathbf{V}_{\mathcal{N}_L} \oplus \mathbf{C}_{\mathcal{N}_L}\right] \otimes \left[\tilde{A} \oplus \tilde{\chi} \oplus \tilde{\phi}\right] \\
\downarrow & & \downarrow \\
\mathbf{G}_{\mathcal{N}_+} + \mathit{matter} & \underset{\text{twin}}{\longrightarrow} & \mathbf{G}_{\mathcal{N}_-} + \mathit{matter}
\end{array}$$

Twin supergravities

. Remarks

- Twin relations relations gives new from old
- Raises the question: what class of gravitational theories are double-copy constructible?
- What about supergravity coupled to the MSSM: is it a double-copy?

Are All Supergravity Theories the Square of Yang-Mills?

- All $N \geq 2$ supergravities with arbitrary matter couplings, with scalars parametrising a symmetric manifold
  [Anastasiou, Borsten, Duff, Marrani, Nagy, Zoccali '17]

- Exceptions: $\mathcal{N} = 2$ pure sugra and the T^3 model, but see
  [Anastasiou, LB, Johansson to appear]

- Can extend to all homogenous (not necessarily symmetric) matter couplings although the BCJ compatibility remains unclear
  [Anastasiou, Borsten, Duff, Marrani, Nagy, Zoccali '17]

Twin CFTs

[Borsten, Duff, Marrani]

- Supersymmetric theories with the same bosonic content but different fermions, aka *twins*, were thought to exist only for supergravity.
- We show that pairs of super conformal field theories, for example exotic $\mathcal{N} = 3$ and $\mathcal{N} = 1$ (plus 14 vector multiplets) in $D = 4$ spacetime dimensions, can also be twin.
- We provide evidence from three different perspectives:
  (i) a twin S-fold construction,
  (ii) a double-copy argument and
  (iii) by identifying twin holographically dual gauged supergravity theories (for example $\mathcal{N} = 6$ and magic $\mathcal{N} = 2$ plus 15 vector mutiplets in D=5).
- Furthermore, twin W-supergravity theories then follow by applying the double-copy prescription to exotic super CFTs

Exotic $N = 3$

- The exotic $\mathcal{N} = 3$ SCFT is the supercurrent multipet. It corresponds to the $\mathcal{N} = 3$ super-Weyl multipet, which consists of the massive Spin(3) $\times$ Sp(3) states,

$$[3, 2] = (\mathbf{5}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{6}) \oplus (\mathbf{3}, \mathbf{14} + \mathbf{1}) \oplus (\mathbf{2}, \mathbf{14}' + \mathbf{6}) \oplus (\mathbf{1}, \mathbf{14}) \quad (4)$$

where we denote by $[\mathcal{N}, j]$ the massive $\mathcal{N}$-extended long supermultiplet with top spin j [Ferrara and Lust].

- The bosonic content is matched by

$$[1, 2] \oplus 14[1, 1]. \quad (5)$$

Interestingly there are twin $\mathcal{N} = 6$ and $\mathcal{N} = 2$ supergravities in $D = 5$, with identical bosonic sectors determined by the common scalar coset $SU^{\star}(6)/Sp(3)$, that can be gauged with respect to the same subgroup $SU(3) \times U(1) \subset Sp(3)$. The gauged $\mathcal{N} = 6$ supergravity (or more precisely, an S-duality fibration thereof) provides the bulk holographic dual of the exotic $\mathcal{N} = 3$ SCFT [Ferrara,Garcia-Etxebarria], while its $\mathcal{N} = 2$ twin provides the candidate bulk holographic dual of the proposed exotic $\mathcal{N} = 1$ twin SCFT.

Twin W supergravities

- $$exotic\ N = 4\ CFT \times\ exotic\ N = 3\ CFT \rightarrow N = 7\ W - supergravity$$

  [Ferrara,Lust]

- $$exotic\ N = 4\ CFT \times exotic\ N = 1\ twin\ CFT \rightarrow N = 5\ twin W - supergravity$$

  [Borsten, Duff, Marrani]

Supergravity@1,@25,@40

- “Supergravity is very compelling but it has yet to prove its worth by experiment”

  MJD ”What’s up with gravity?”
  New Scientist 1977

- “...a remark still unfortunately true at Supergravity@25. Let us hope that by the Supergravity@50 conference we can say something different.”

  MJD ”M-theory on manifolds of G_2 holonomy”
  Supergravity@25 2001

- “I’m glad I said 50 and not 40”
  MJD “Twin supergravities from Yang-Mills squared”
  Supergravity@40 2016