Gerard 't Hooft

The quantization of black holes

and its impact on particle physics and general relativity

starting with confirmed physics to obtain new insights.

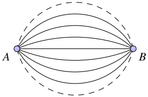
Raymond and Beverly Sackler Distinguished Lecture 2018,
Niels Bohr Institute.

Centre for Extreme Matter and Emergent Phenomena,
Science Faculty, Utrecht University,
POBox 80.089, 3508 TB, Utrecht

What hasn't worked in quantum gravity? and why?

Many attempts *nearly* worked:

$1^{\rm st}$ attempts	Perturbation expansion, renormalizable? rearrange Feynman diagrams: "superpropagators"
next: It should be done "non-perturbatively",	Wheeler-DeWitt equations, Ashtekar variables
next: space-time variables should be discrete	Dynamical triangulation Loop quantum gravity
next: a new, smart idea:	(super)string theory
Still smarter:	*M*-theory



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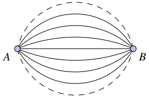

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Still smarter: — M-theory

What went wrong?

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1) The correct theory should explain *the hierarchy problem*:
Where do large dimensionless constants of Nature come from?

and 2) The correct theory should *not be quantum mechanical.*
How else can we restore the logic of "quantum" cosmology?

The correct path towards the truth will be a narrow and winding one.
The only way to achieve our goals will be by making

small steps at the time.

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Are there such spots?

Yes!

Whenever a **black hole** forms,
and we want to apply QM !

Schwarzschild, Reissner-Nordström, Kerr, Kerr-Newman BH:

What does the text book say?

- Hawking 1975: BH emits particles! However:
  - BH consists of interior part and exterior part.
  - Particles entering the interior part cannot come out.
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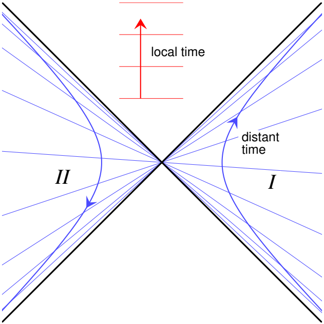

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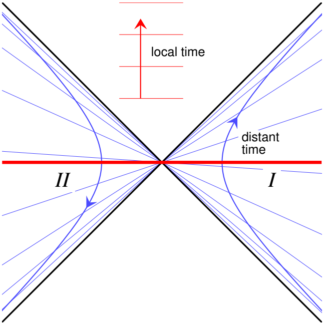

Hartle-Hawking vacuum:

$$|\,HH\,\rangle \;=\; C\sum_{E,n} e^{-\frac{1}{2}\beta E}\,|E,n\rangle_I\,|E,n\rangle_{II}$$

I = outside
II = inside [?]

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...and this cannot be right ... It should be possible to describe the physics of a horizon in terms of pure quantum states

so, people start guessing again

...What does superstring theory say?

They find something that looks like a black hole,
and it is described in terms of pure quantum states !
"microstates". But it is not understood ...

How are these microstates related to vacuum fluctuations?

The string theory "text book" is very unclear about these issues.
It claims that Hawking radiation must be in a pure state. But then:

Difficulties regarding entanglement and no-cloning:

Almheiri, Marolf, Polchinski, Sully (AMPS): *If we start in a pure state, and consequently Hawking radiation is in a pure state, then that pure state must be entangled also with earlier radiation. Therefore it can't be in the state originally used by Hawking. This produces a curtain of infinitely energetic particles along the future event horizon: a firewall*

This cannot be right.
As we discovered, you can do better.

And yes, there will be new physics.
But here it can be derived precisely. . .

We begin with getting the maximum out of *standard Einstein General Relativity* and *standard Quantum Mechanics*

Then, one has to discover 3 important things:

(*i*) Particles going into a black hole, interact *gravitationally* with particles going out. If you want to describe these as pure quantum states, you cannot ignore that (only if they are in mixed states, you may) because this grav. force is **strong**

(*ii*) This force generates an algebra that is *linear* in the coordinates & momenta of the in- and out-particles,
and therefore, you can superimpose solutions!
$\longrightarrow$ Make an expansion in spherical harmonics.

(*iii*) We had always been wrong in thinking that the entangled partners of the Hawking particles, going into the BH, were lost.
They appear at the other side!

If we ignore any of these 3 points, we fail
to understand what happens

What makes **black holes** stand out in physics, is that they are very delicate and unconventional solutions of Einstein's theory of General Relativity, which require special attention when we try to subject them to the laws of quantum mechanics.

In Einstein's theory, black holes are just "ordinary" objects, like solitons, which behave like heavy particles, accept that they capture things falling in without leaving a trace.

Naturally, one would expect them to be like that also when quantum mechanics is applied. They now emit radiation. There should be a Schrödinger equation for black holes. Since this equation should have microscopic accuracy, the quantum states for a black hole are called "microstates".

We claim that *microstates* can be understood very well, but unexpected refinements in General Relativity are needed.

In particular, we emphasise that one can use any set of ordinary coordinates in the visible parts of space-time, while at the (past as well as future) *event horizons* something else happens.

Some researchers assume that 'chaos' occurs. But, if you do things correctly, the only "chaos" one will find is comparable to other phenomena in ordinary physics.

We shall have all we need to characterise the microstates.

A beautiful discovery was that we can use expansions in spherical harmonics, just as in the hydrogen atom. *These spherical waves factorise.*

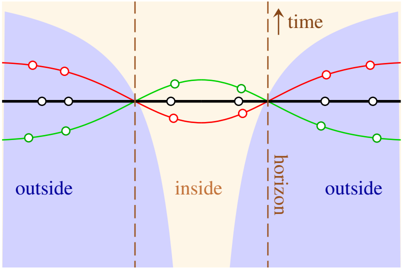
The situation in a black hole is then much like non-interacting particles in a rectangular box. Every single particle is described by a simple, one dimensional (quantum) equation of motion, easy to solve by an undergraduate student.

There are cases where interactions do occur, and there are still questions waiting for an answer.

The gravitational back reaction also factorises. This is a fortunate accident, enabling us now to easily solve the mathematical equations and see what happens.

It has been argued that the quantum black hole, as a "theoretical laboratory", replaces *experimental tests* that are urgently needed to understand quantum gravity.

At first sight, it will seem that our refinements of the textbook on quantum gravity are plainly wrong, and they were criticised as such in email exchanges with colleagues. But in spherical harmonics, the equations are so simple that one can now investigate all alternatives.



It is essential to have Cauchy surfaces, and examine how the data on these surfaces evolve with time. What is the time coordinate? How can we ensure that the Cauchy data evolve through unitary equations?

The Cauchy data should not get lost between the crevices of the horizons, which can easily happen while you think you are doing things correctly.

We have to use the same time coordinates as the external observer.

So, the quantum black hole does here what *particle experiments* did when physicists constructed the Standard Model:

It tells us how to improve our physics text books.

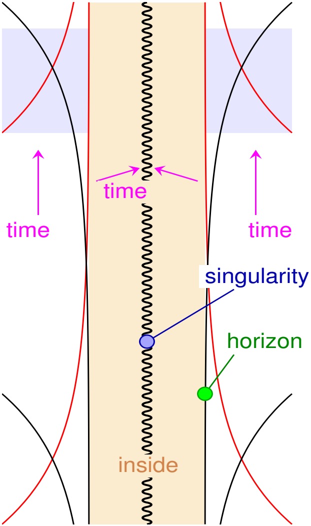
We have no experiments for black holes,
and therefore, we have to be especially careful and use devices such as spherical wave expansions or whatever else we can put our hands on, to get a clearer picture and to simplify our subject.

Our prototype is the Schwarzschild black hole, with $M_{\rm BH} \gg M_{\rm Planck}$

No serious complications when generalised to Kerr-Newman or such. The *extreme* black hole would not serve our purposes, because it is a limiting case, and its horizon is fundamentally different from the more generic black holes.

Schwarzschild metric: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$,

$$\mathrm{d}s^2 = \frac{1}{1-\frac{2GM}{r}}\mathrm{d}r^2 - \left(1-\tfrac{2GM}{r}\right)\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2 \ ; \quad \begin{cases} \Omega & \equiv & (\theta,\,\varphi)\ , \\ \mathrm{d}\Omega & \equiv & (\mathrm{d}\theta,\,\sin\theta\,\mathrm{d}\varphi)\ . \end{cases}$$



light-cone coordinates
(light-like geodesics)
near horizon

The tortoise coordinates

spanned by the light-like, radial geodesics:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\left(1 - \tfrac{2M}{r}\right) \quad , \quad (r,t) \to (u^+, u^-)$$

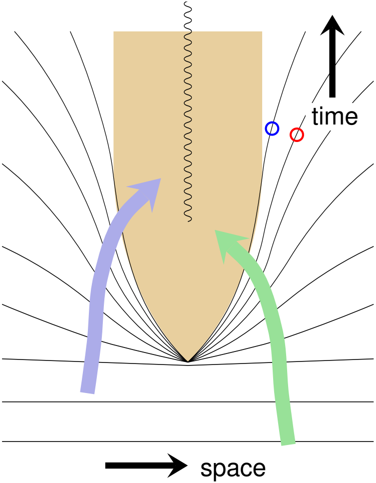
At the *black hole horizon* , time slows down to a stop.

To understand what goes on, consider a black hole during a time interval $OO(M_{\mathrm{BH}})$. Since the *lifetime* of the hole is $\mathcal{O}(M_{\mathrm{BH}})^3$, it makes sense here to consider the spacetime of an *eternal* black hole.

The *generic black hole state is now described as the metric for an eternal black hole,* with only particles in it with energies $< M_{\mathrm{Planck}}$.

Claim: one may *ignore* the far past of its history, such as the implosion that gave birth to the black hole..

This effect of gravitational force between hard and soft particles cannot be ignored:



We see that this effect has drastic consequences for the out-going particles.

The effect increases exponentially with time.

The in-particles leave their ‘footprints’ in the out-particles.

This changes everything!

But the effect can be precisely calculated.

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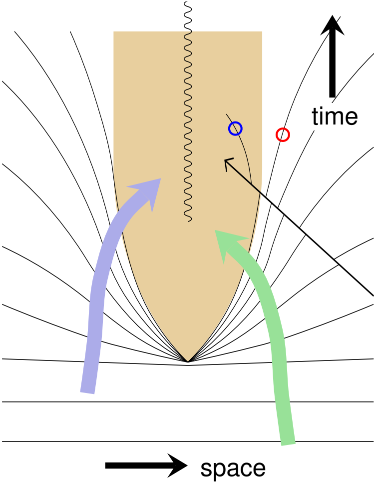

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Hard and soft particles

defined differently in other publications (!)

Particles near the horizon(s). Mass shell: $2p^+p^- + \tilde{p}^2 + \mu^2 = 0$.
$\tilde{p}$ is the transverse part of the momentum, $|\tilde{p}| \approx L/R$, $\mu =$ mass;
$p^-(\tau) = p^-(0)e^\tau$, $p^+(\tau) = p^+(0)e^{-\tau}$. $\tilde{p}$ and μ are constant.

Define *soft* particles: $|\vec{p}|,\ \mu \ll M_{\rm Planck}$ Negligible effect on space-time.
Define hard particles as particles that do cause space-time curvature.

As $\tau \to \infty$, $p^+ \to 0$, $p^- \to \infty$: all in-particles will become *hard;*
As $\tau \to -\infty$, $p^- \to 0$, $p^+ \to \infty$: all out-particles originally were *hard.*

To understand what happens with the evolution at longer time intervals, we have to understand what the hard in- and out- particles do.

As $\tau \gg 1$, the in-particles become hard. Their interactions with other in-particles are negligible (they basically move in parallel orbits), but they do interact with the out-particles. The interaction through QFT forces stay weak, but the *gravitational forces* make that (early) in-particles interact strongly with (late) out-particles.

The new rules for the text books of the future must be these:

- The complete set of microstates for a black hole must be written as a *fixed* background metric, populated exclusively by *soft* particles and gravitons (to represent sub Planckian deformations of this space-time) going in or out.
- The out-particles are *not independent* of the early in-particles, and this is why we are not allowed to add any details of the early imploding matter, or the very late Hawking particles.
- As time τ proceeds, in-particles become hard, and as soon as that happens, we have to replace them by their *gravitational footprints*, which are now soft out-particles.
- This procedure replaces momenta with positions, which is a *unitary* transformation., however:
- this is a Fourier transformation, which is only unitary if we consider the particles in region I *and* in region II.
- Region II, therefore, must also represent a physically visible part of the black hole. The only possibility for having this is to identify the space-time points of region II as the *antipodes* of region I. We call this the antipodal identification.

Note that the in-particles will never get the opportunity to become truly hard particles.

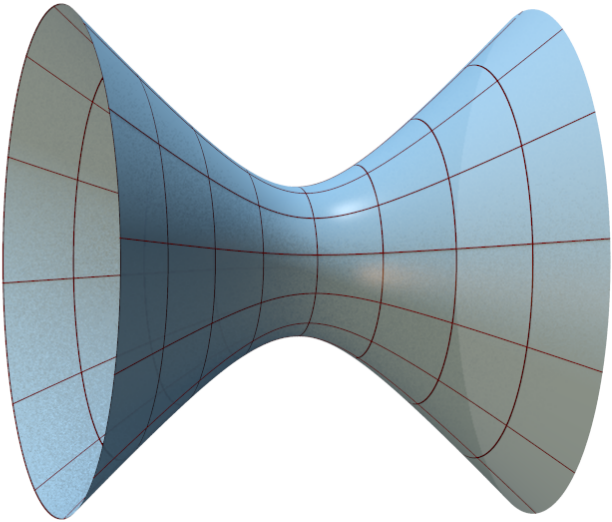
Wave functions of soft particles going in are reflected as wave functions going out. These again emerge as soft particles.

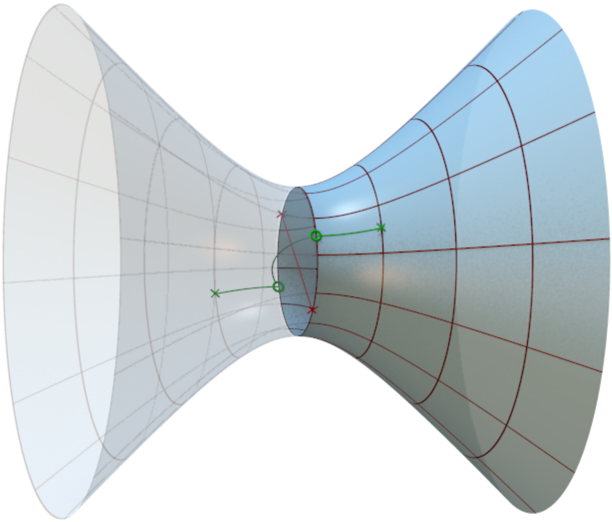
Thus, there is no firewall, ever.

The total of the in-particles in regions *I* and *II* are transformed (basically just a Fourier transform) into out-particles in the same two regions.

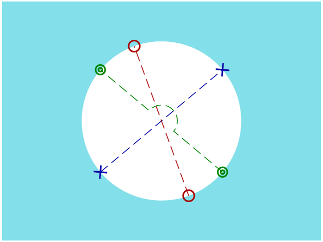
Note that the regions *III* and *IV* in the Penrose diagram never play much of a role, even if an observer falling in region *III* would want to assure us that (s)he is still alive.

Regions *III* and *IV* are best to be seen as lying somewhere on the time-line *where time t is beyond $\pm$infinity.*





Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

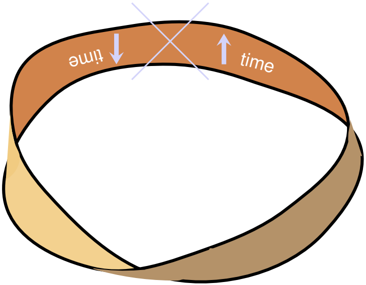


The white sphere within is *not* part of space-time. Call it a 'vacuole'. At given time t, the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time.

Space coordinates change sign at the identified points
– *and also time changes sign*
(Note: time stands still at the horizon itself).

Not that all $u^{\pm}$ and $p^{\pm}$ coordinates are odd when switching between antipodes. Therefore, only odd ℓ contribute in the spherical harmonic expansion.

A timelike Möbius strip



Draw a spacelike closed curve:
Begin on the horizon at a point
$r_0 = 2GM\ ,\ \ t_0 = 0\ ,\ \ (\theta_0, \varphi_0)\ \ .$

Move to larger r values, then
travel to the antipode:

$r_0 = 2GM\ ,\ \ t_0 = 0\ ,\ \ (\pi - \theta_0, \varphi_0 + \pi)\ .$
You arrived at the same point,
so the (space-like) curve is closed.

Now look at the environment $\{\mathrm{d}x\}$ of this curve. Continuously transport $\mathrm{d}x$ around the curve. The identification at the horizon demands

$$\mathrm{d}x \leftrightarrow -\mathrm{d}x\ , \quad \mathrm{d}t \leftrightarrow -\mathrm{d}t\ ,.$$

So this is a Möbius strip, in particular in the time direction.
Note that it makes a T inversion and a CP inversion when going around the loop. CPT is preserved. Therefore, CP must be a good symmetry!
Thus, only 'Standard Models' where CPT is at most *spontaneously* broken, are allowed.

There are numerous treatises in the literature claiming solutions to the black hole information paradox, and about as many publications that dismiss these claims.
This author dismisses all claims, from both sides, that either ignore the gravitational back reaction of quantised excitations, or ignore the antipodal identification of points on the horizon – meaning that the horizon is a *projective* 2-sphere.

Sackler Lecture

THE END