Dark matter and light spin-0 mediators: self-interactions & direct detection

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[1704.02149]: Felix Kahlhoefer, Kai Schmidt-Hoberg, SW

[1707.08571]: Felix Kahlhoefer, Suchita Kulkarni, SW

Self-Interacting Dark Matter Workshop 2017
NBI Copenhagen

Recap: DM self-interactions via a light scalar mediator

Exploring light mediators with low-threshold direct detection experiments

DM self-interactions from a general spin-0 mediator (i.e. CP-violating DM interactions)

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DM self-interactions via a scalar mediator

$$\mathcal{L} = -y_\psi \bar{\psi}\psi\phi - y_{\rm SM}\sum_f \frac{m_f}{v_{\rm EW}}\bar{f}f\phi$$

$m_\psi \sim 10\,\text{MeV} - 10\,\text{TeV}$

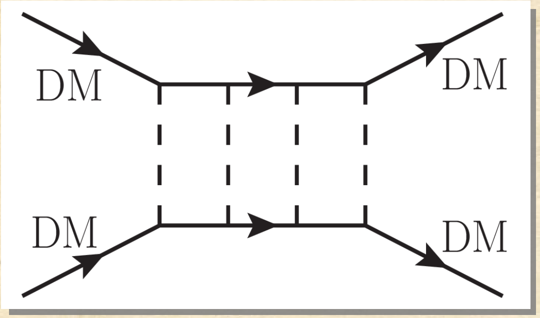
$m_\phi \sim 0.1\,\text{MeV} - 100\,\text{MeV}$

Buckley/Fox [0911.3898]
Feng/Kaplinghat/Tu/Yu [0905.3039]
Tulin/Yu/Zurek [1302.3898]

- $y_{\rm SM} \lesssim 10^{-4}$ (constrained e.g. by rare Kaon decays)
  - → freeze-out via $\psi\bar{\psi} \to \phi\phi$ ("hidden sector freeze-out")
  - → $y_\psi \sim 0.01 - 1$

- CP-conserving interaction
  - → $\psi\bar{\psi} \to \phi\phi$ (and also $\psi\bar{\psi} \to \phi\phi\phi$) are p-wave suppressed
  - → no relevant constraints from CMB

DM self-interactions via a scalar mediator



$$\longrightarrow \quad V(r) \propto \frac{e^{-m_\phi r}}{r}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{PP}} = |f(\theta) \pm f(\pi - \theta)|^2$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{PA}} = |f(\theta)|^2$$

$$\sigma_{\widetilde{\mathrm{T}}}^{\mathrm{PP,PA}} \equiv \int \mathrm{d}\Omega \, (1 - |\cos\theta|) \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{PP,PA}}$$

$$\sigma_{\widetilde{\mathrm{T}}} = \frac{1}{2}\left(\sigma_{\widetilde{\mathrm{T}}}^{\mathrm{PP}} + \sigma_{\widetilde{\mathrm{T}}}^{\mathrm{PA}}\right)$$

→ automatically leads to the desired velocity-dependence of $\sigma_{\widetilde{\mathrm{T}}}$!

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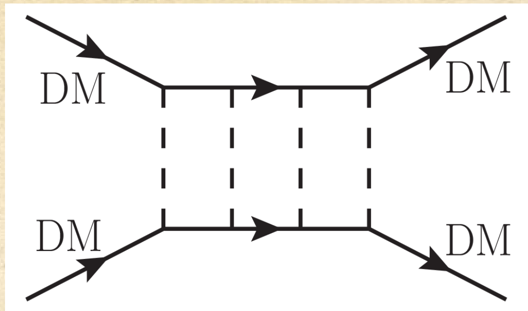

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Kahlhoefer/Schmidt-Hoberg/SW [1704.02149]
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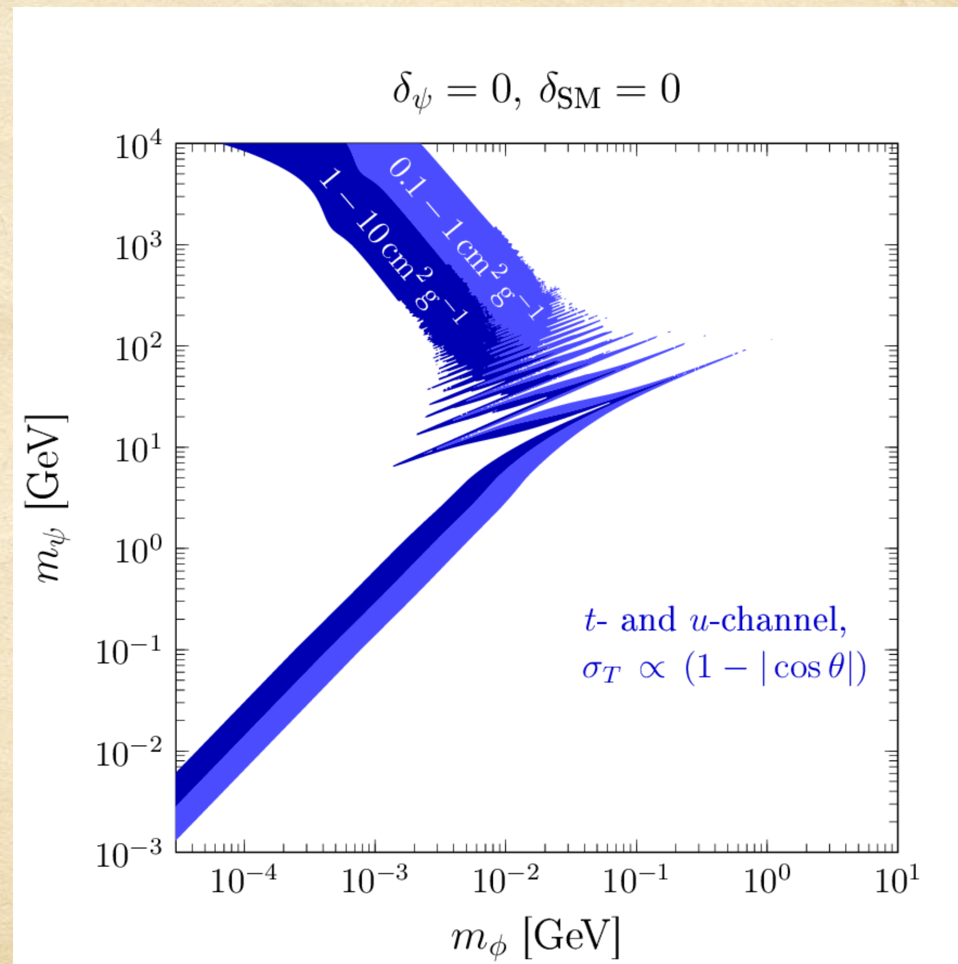

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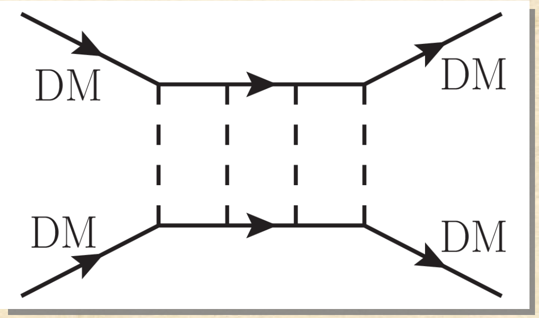

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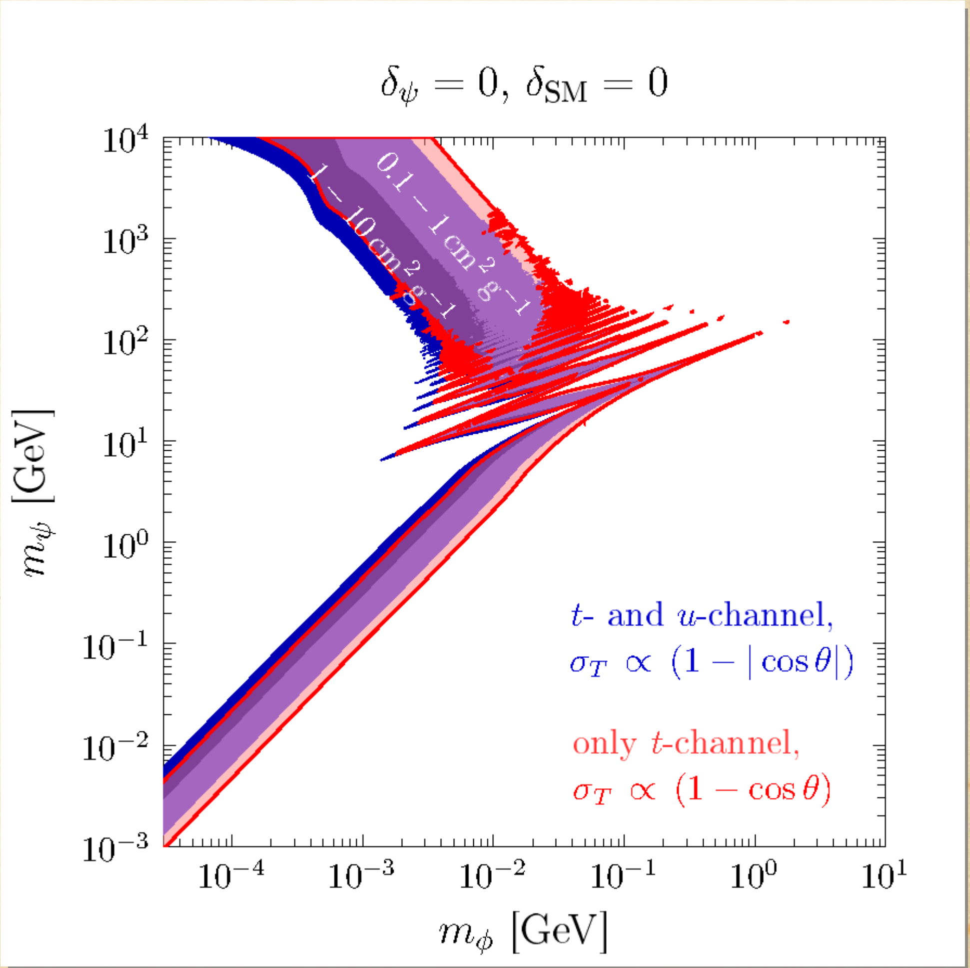
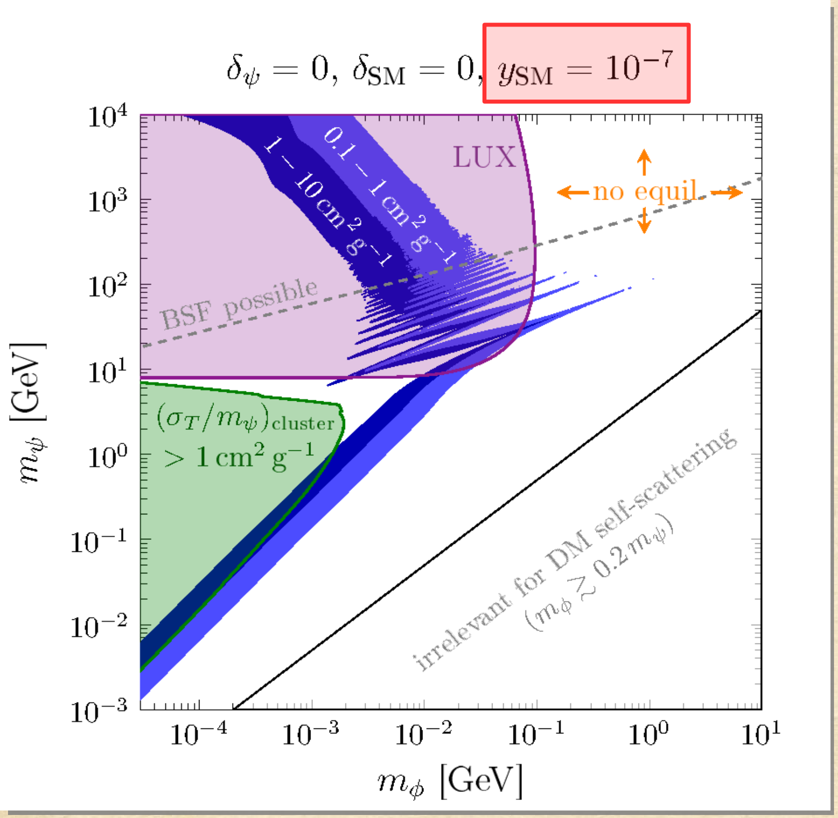
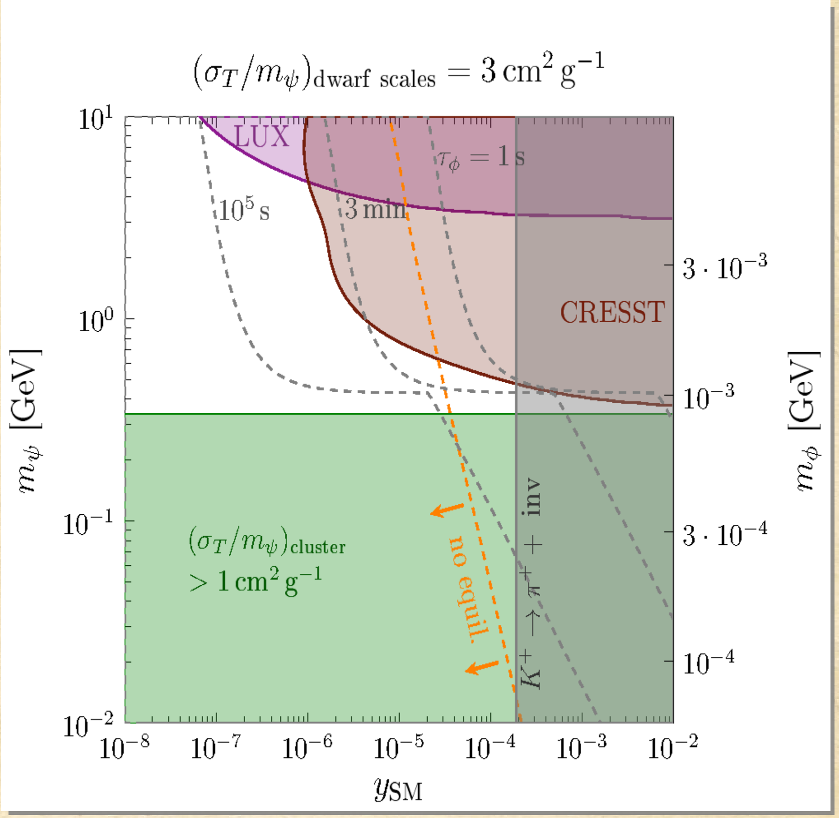


Status of SIDM via a light mediator





- This model faces **strong direct detection constraints**, even for very small y_{SM} see in particular DelNobile/Kaplinghat/Yu [1507.04007]
- y_{SM} can't be arbitrarily small due to BBN/CMB constraints on τ_ϕ
  - → however, precise bound is unknown so far.
  - → naive estimate $\tau_\phi \lesssim 1\,\mathrm{s}$ might be too aggressive for MeV-scale mediators

  Work in progress: Hufnagel, Schmidt-Hoberg, SW

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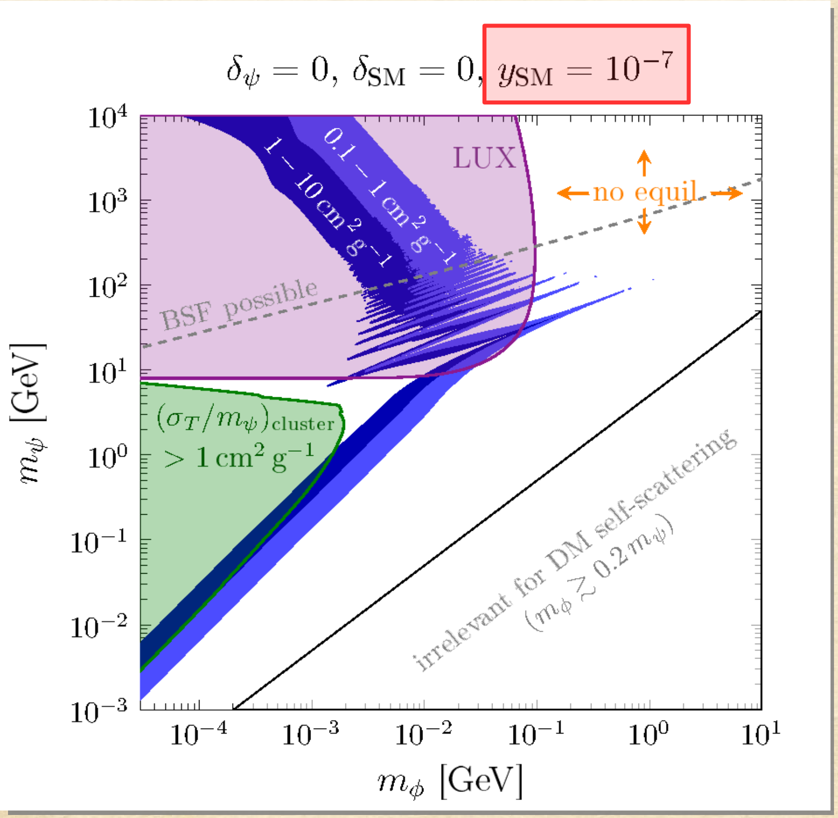

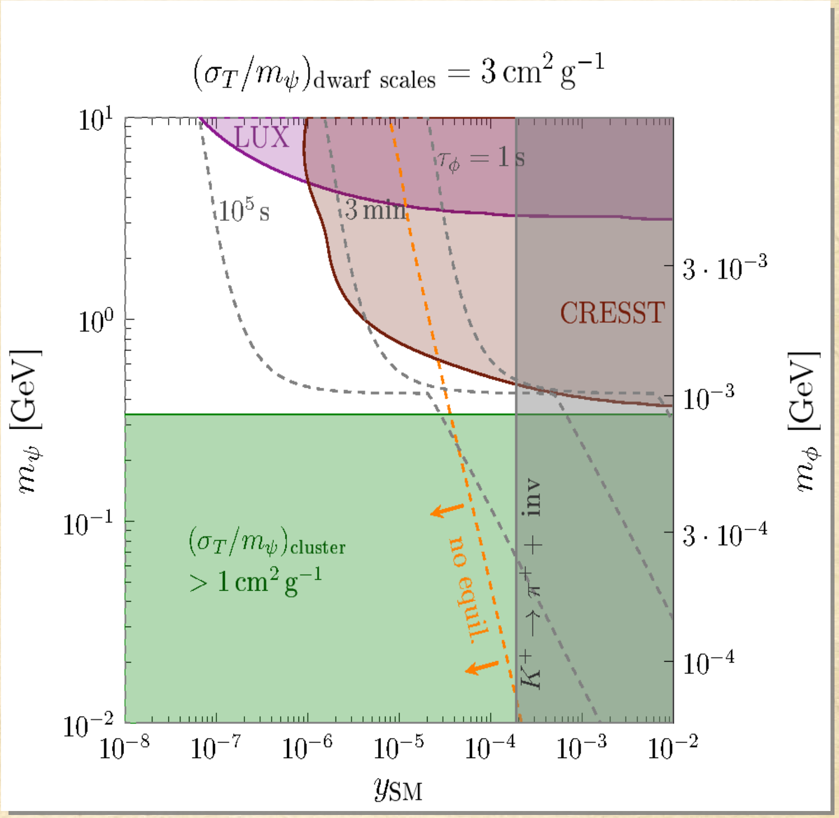

What to do with this model?

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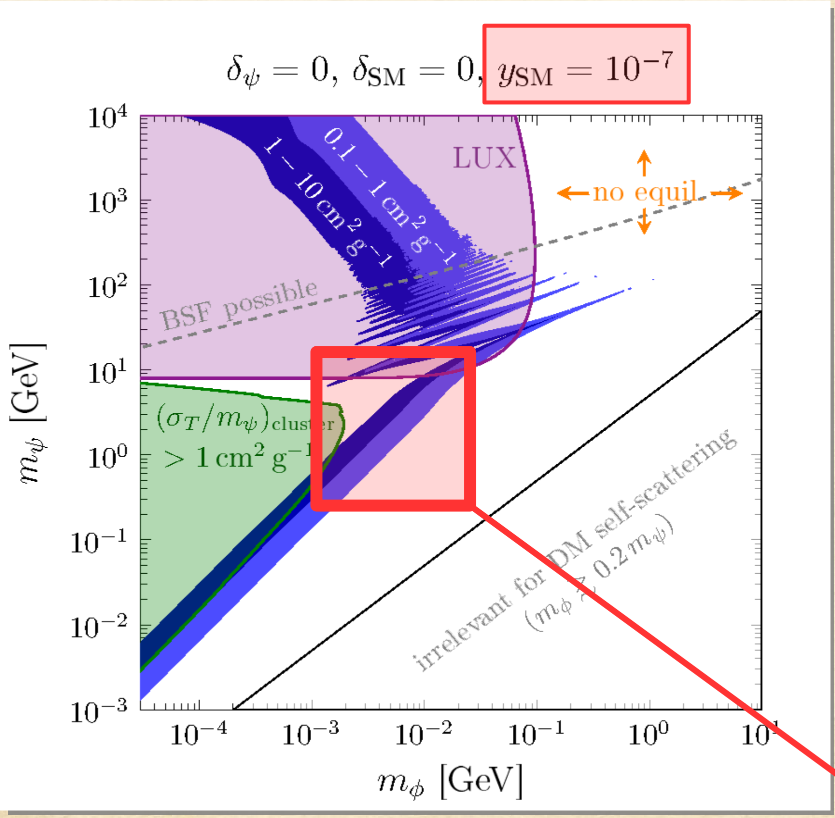

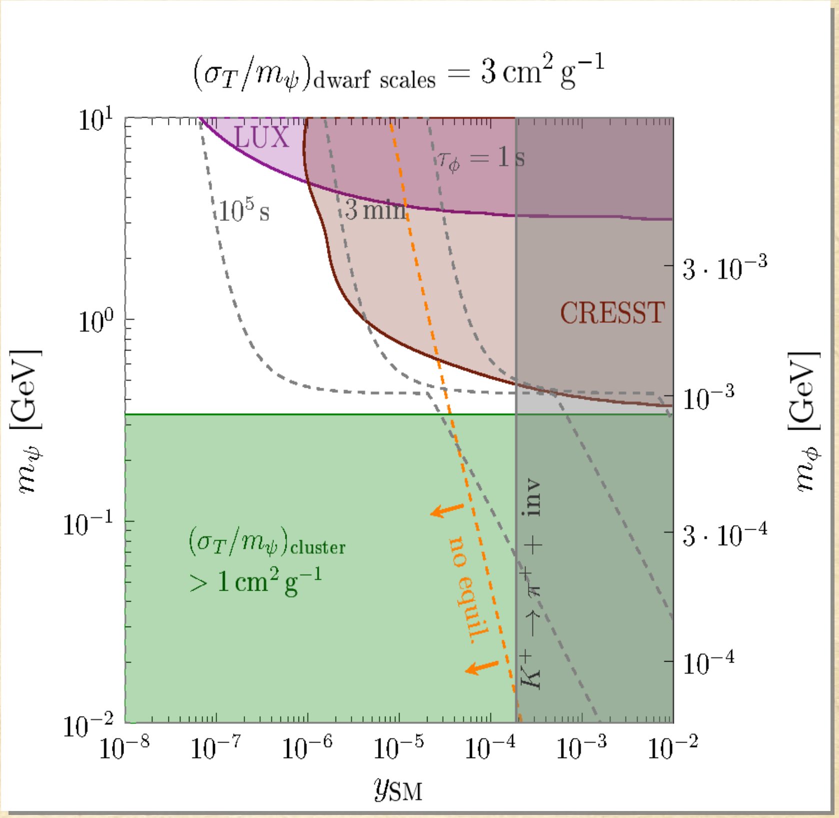

What to do with this model?

(1) Potentially remaining parameter space: $m_{\rm DM} \sim \mathrm{GeV}\,,\ m_\phi \sim 1 - 10\ \mathrm{MeV}$

→ what can we learn from **future direct detection experiments**?

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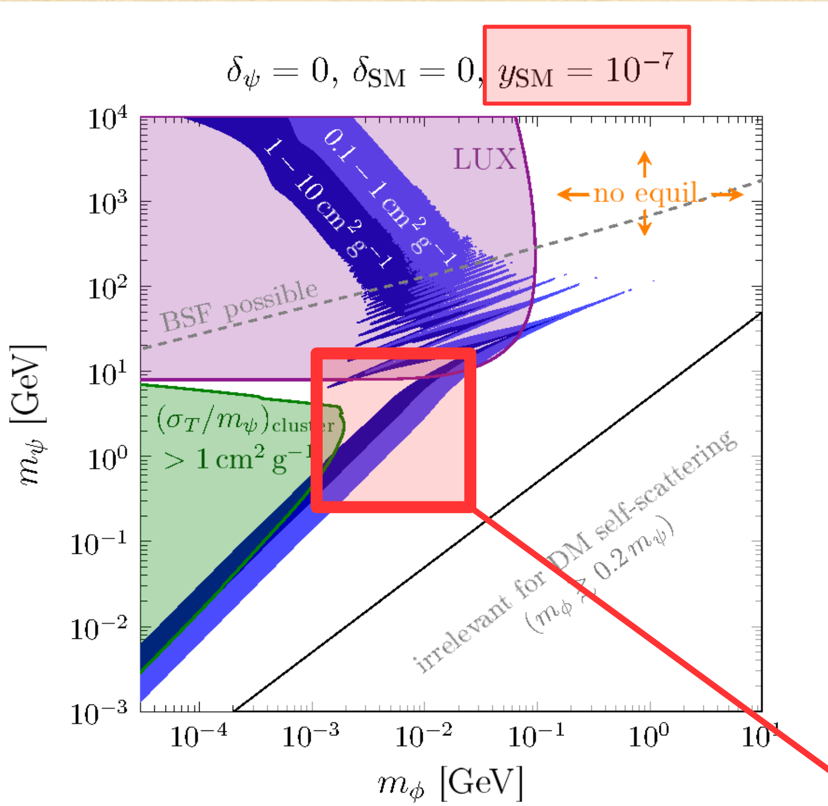

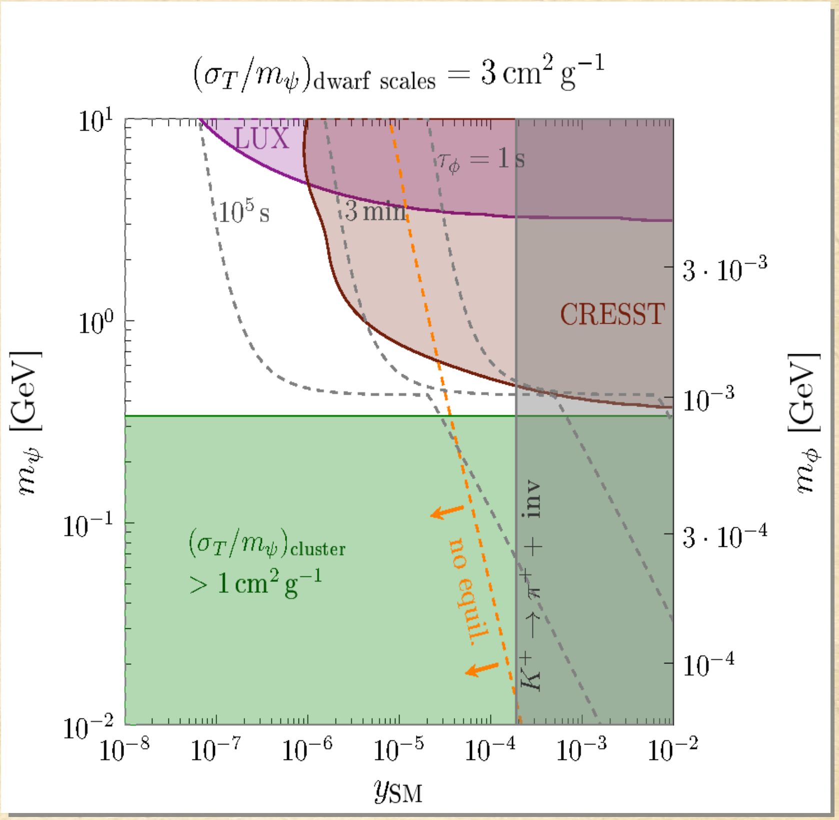

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→ what can we learn from **future direct detection experiments**?

(2) Extend the model: **CP-violating interactions** of ϕ

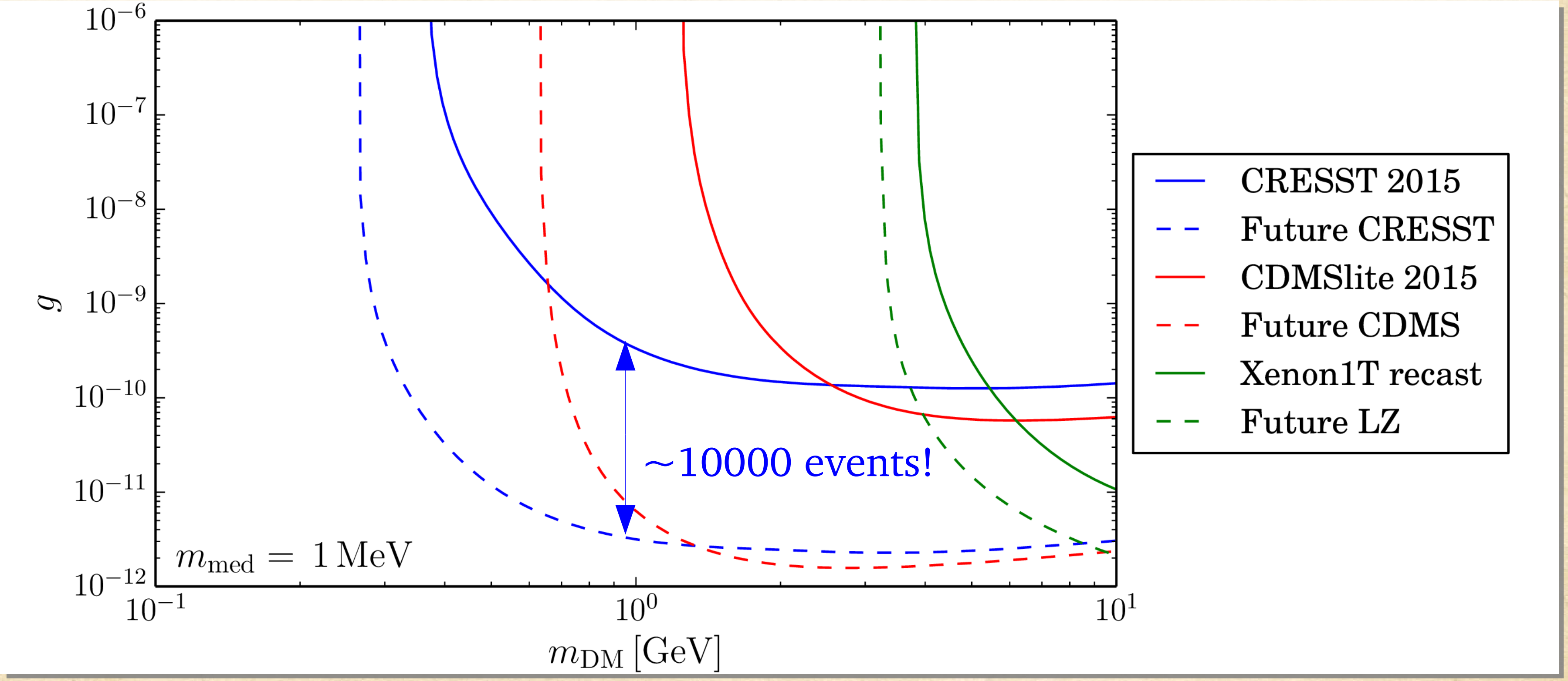
Recap: DM self-interactions via a light scalar mediator

Exploring light mediators with low-threshold direct detection experiments

Kahlhoefer/Kulkarni/SW [1707.08571]

DM self-interactions from a general spin-0 mediator (i.e. CP-violating DM interactions)

Sensitivity of DD to light mediators



- Low threshold is even more important for light mediators (steeply falling spectrum)

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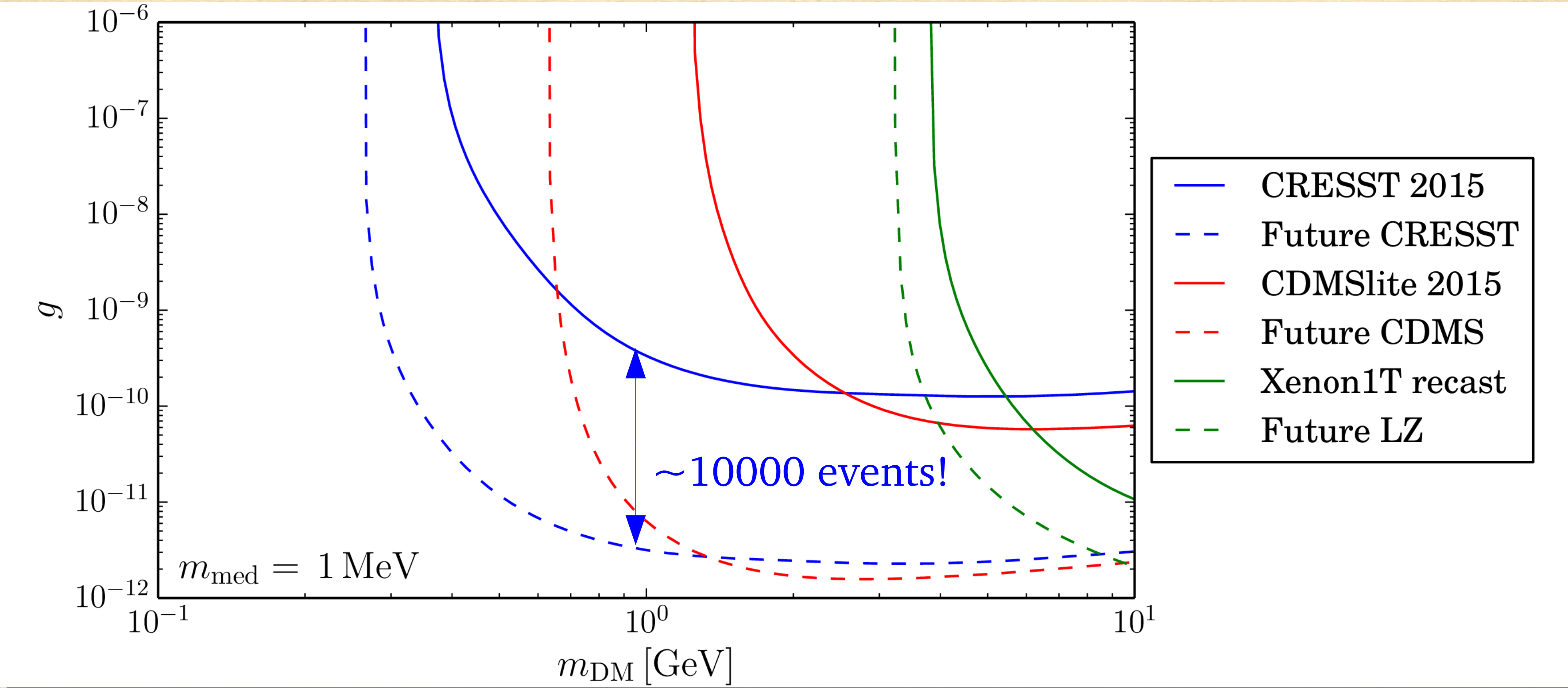

- Low threshold is even more important for light mediators (steeply falling spectrum)
- Rate $\propto \left(q^2 + m_\phi^2\right)^{-2}$ → spectrum depends sensitively on m_ϕ if

$$m_\phi \sim q \equiv \sqrt{2 m_T E_T} \sim \boxed{(1-100)\,\mathrm{MeV}}$$

SIDM!

→ Measuring **mediator masses** and **self-interaction cross section** with direct detection?

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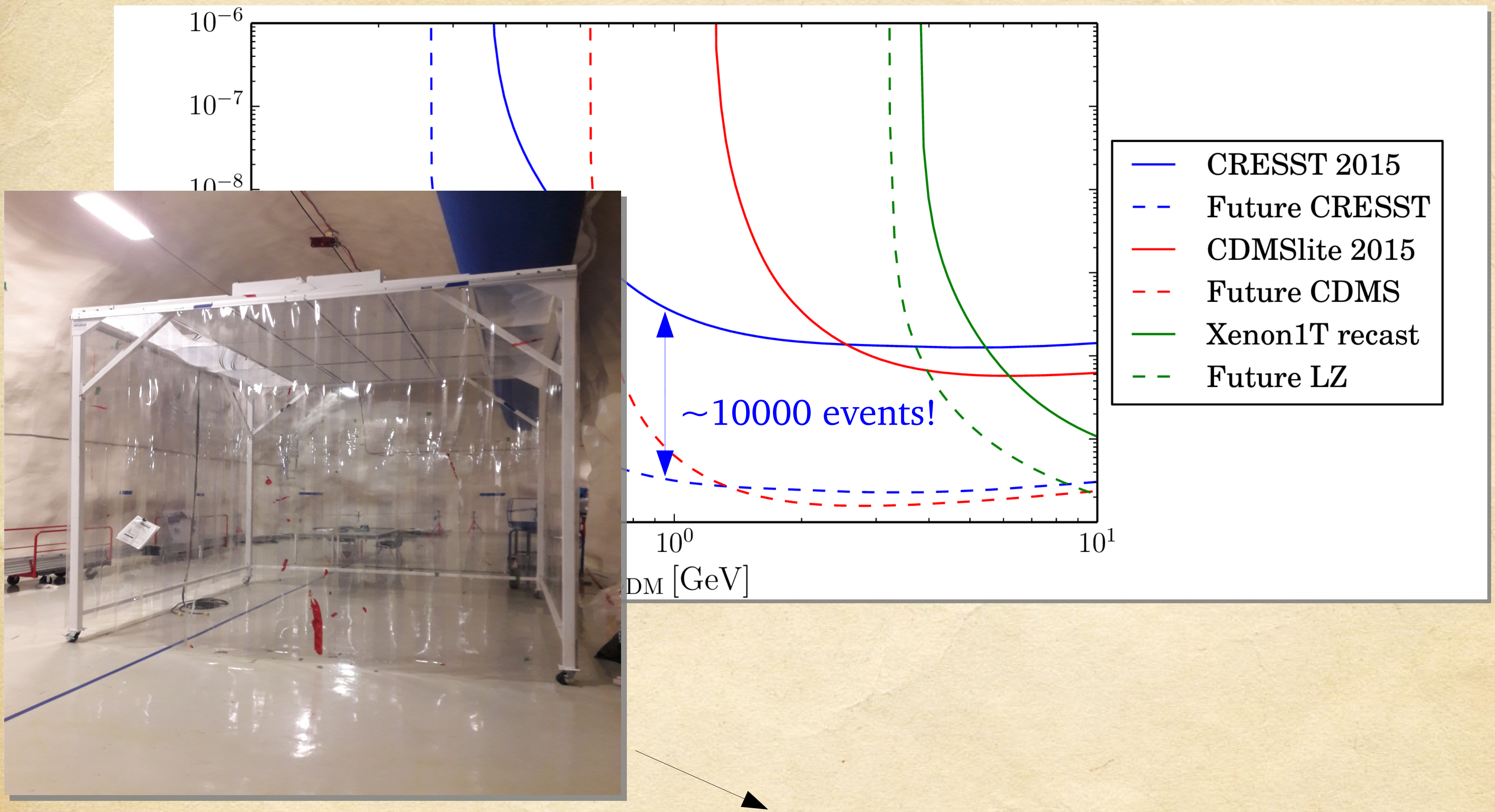


TAUP 2017 excursion last week:
Current status of SuperCDMS@SNOLAB

Parameter reconstruction: method

(1) Choose a benchmark point $(m_{\rm DM}, m_\phi, g)$

→ must be compatible with current exclusion limits

(2) Generate mock data for CRESST-III and SuperCDMS@SNOLAB

→ set of observed events N_i binned in energy (incl. energy resolution)

→ typically between 500 and 8000 events in total

(3) Which combinations of $(m_{\rm DM}, m_\phi)$ provide a good fit to the data?

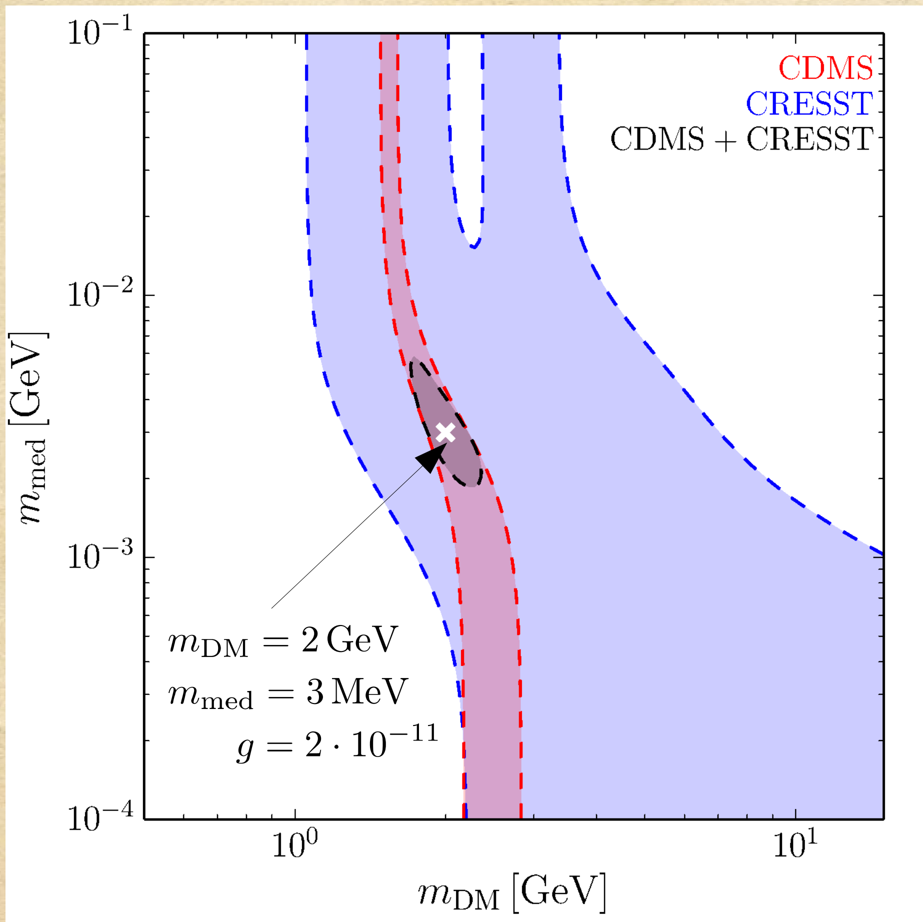
→ we always profile out the coupling strength g

→ binned profile likelihood, allowing us to consider **nuisance parameters**:

- background normalization
- coupling ratio f_n/f_p
- astrophysical uncertainties (varying v_0, $v_{\rm local}, v_{\rm esc}$)

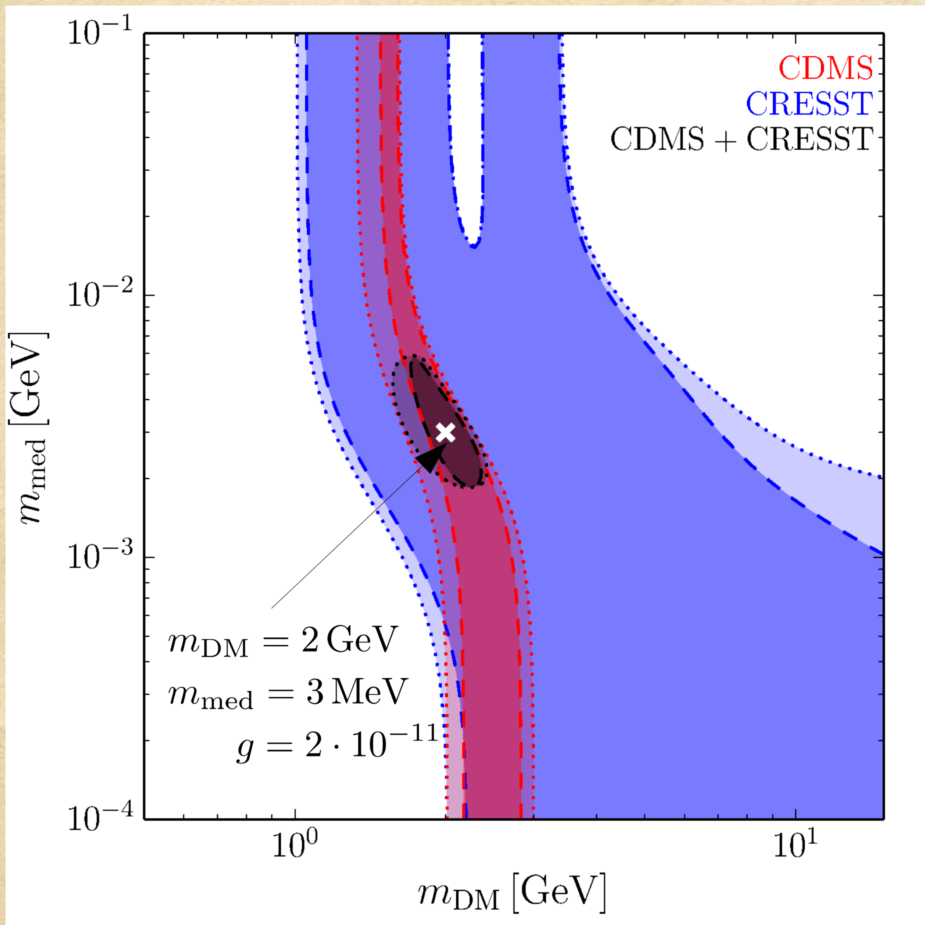
→ Realistic assessment of the potential of future low-threshold experiments!

Parameter reconstruction: an example



- For this benchmark point:
  $N_{\rm CDMS} \simeq 700, \; N_{\rm CRESST} \simeq 200$
- In this plot:
  no additional nuisance parameters
- Two branches for CRESST:
  scattering of Ca/O vs. W
- Smaller $m_{\rm DM}$ can be (partially) compensated by larger $m_{\rm med}$
- **Strong complementarity** of CDMS and CRESST!

Parameter reconstruction: an example



- Treating **background normalizations** as free parameters enlarges the allowed region in parameter space

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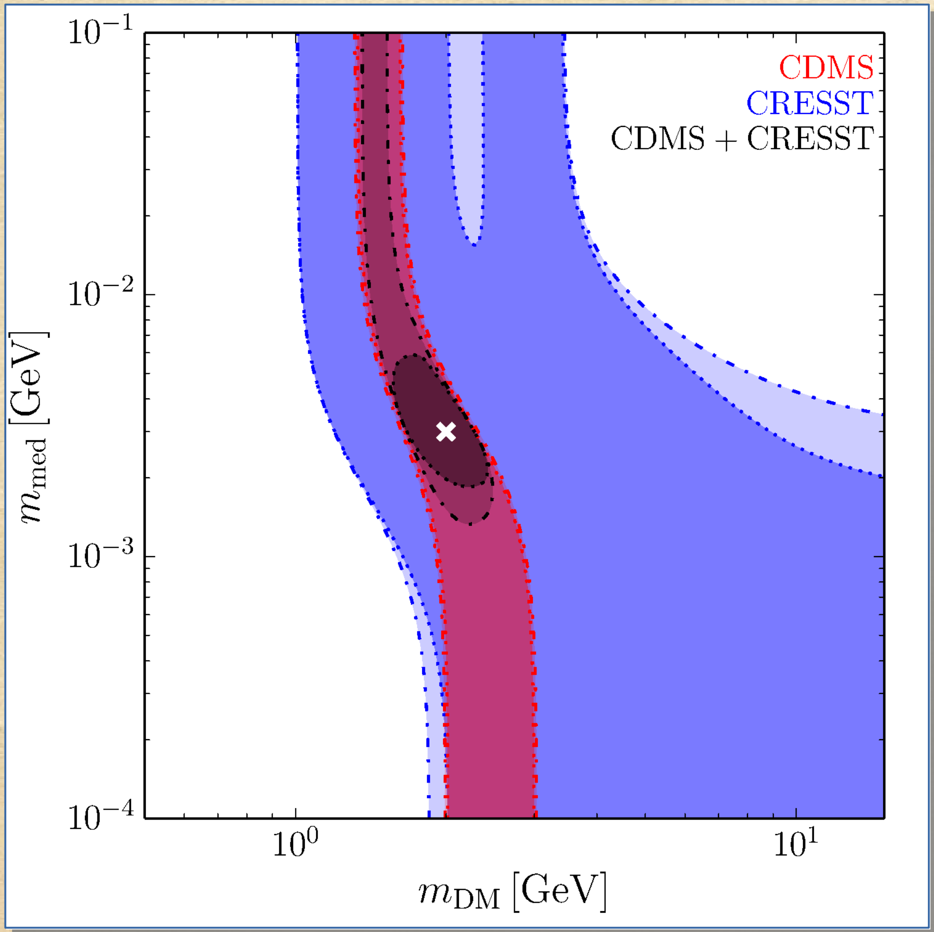

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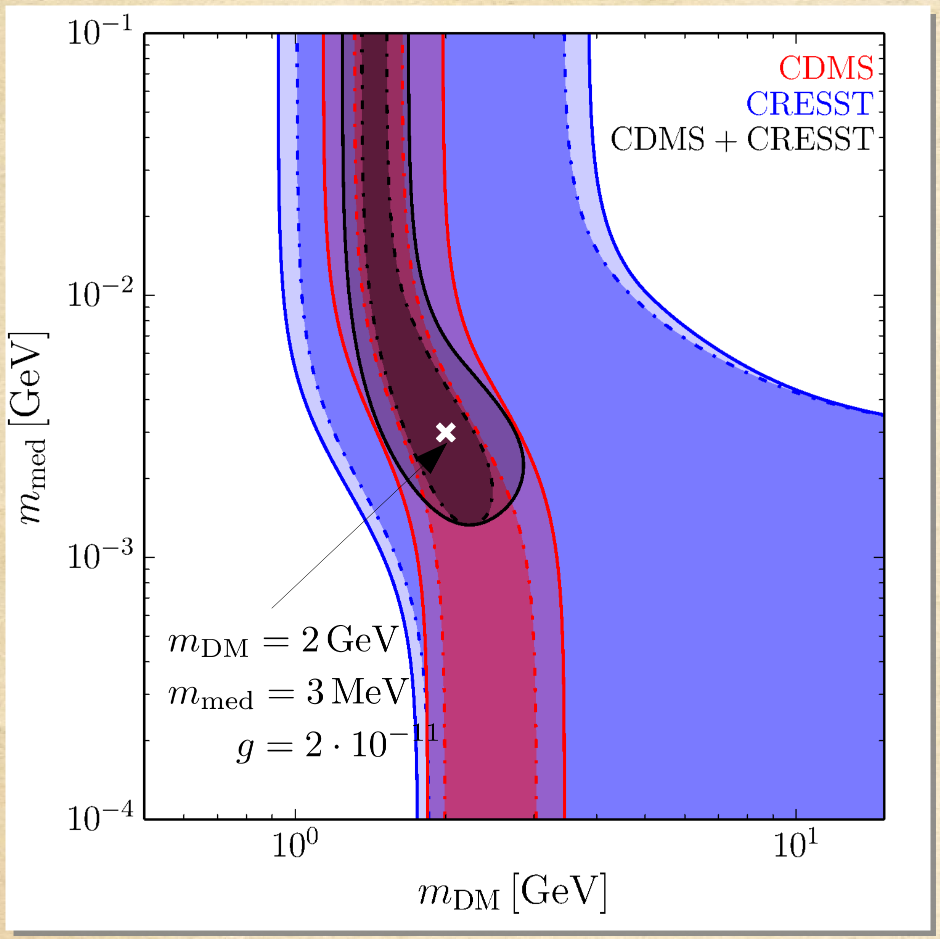
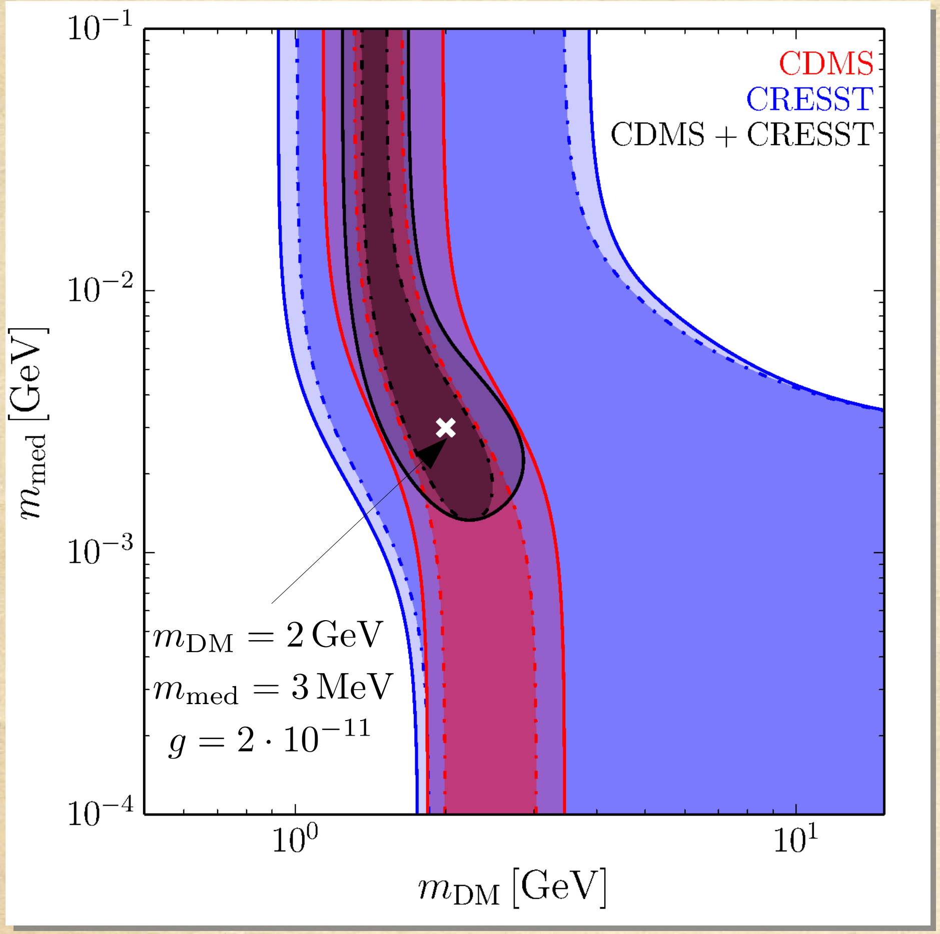
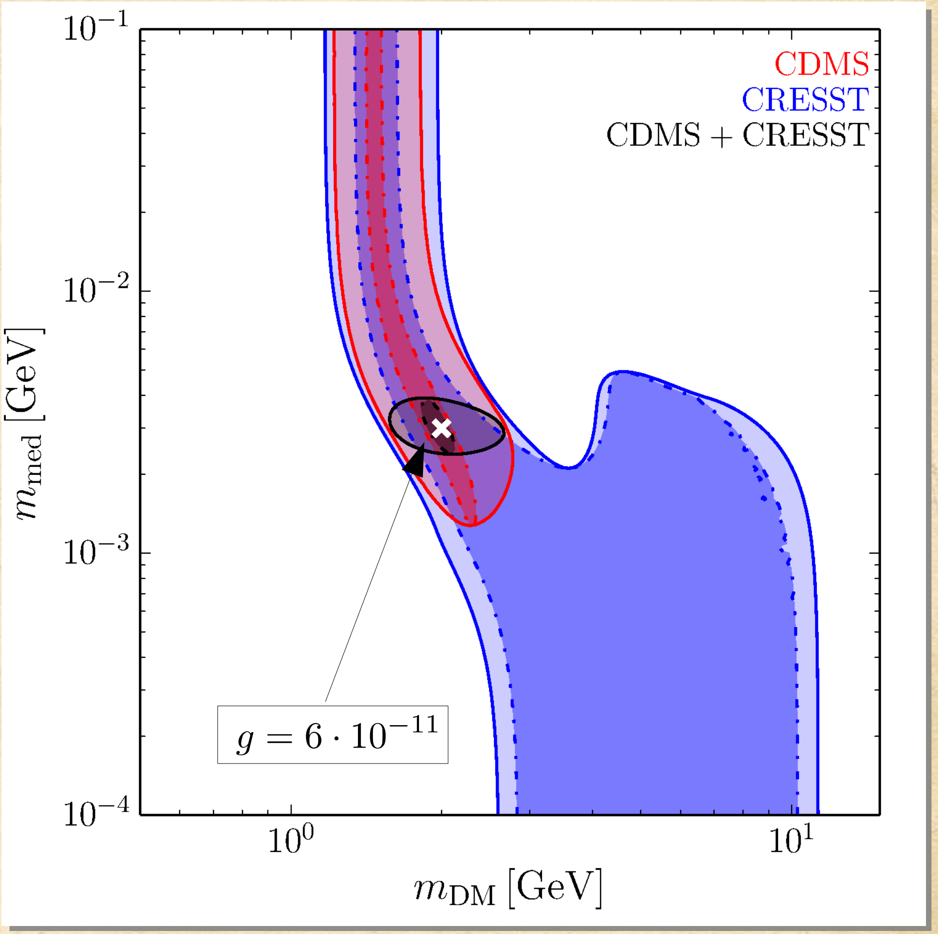


- Treating **background normalizations** as free parameters enlarges the allowed region in parameter space
- In this example, allowing f_n/f_p to float freely opens the combined CDMS+CRESST contour to large $m_{\rm med}$
- We take into account **astrophysical uncertainties** by varying v_0, $v_{\rm local}$, $v_{\rm esc}$ within their 95% C.L. allowed ranges
  → this essentially corresponds to a smearing of $m_{\rm DM}$

Parameter reconstruction: an example





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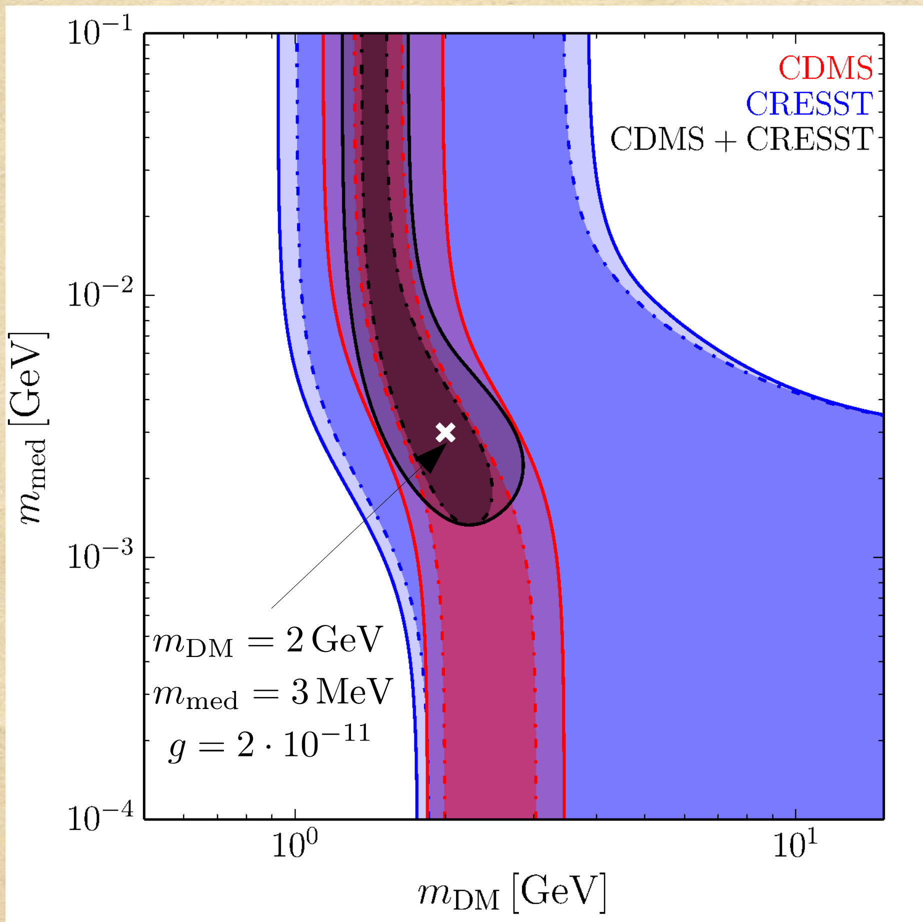
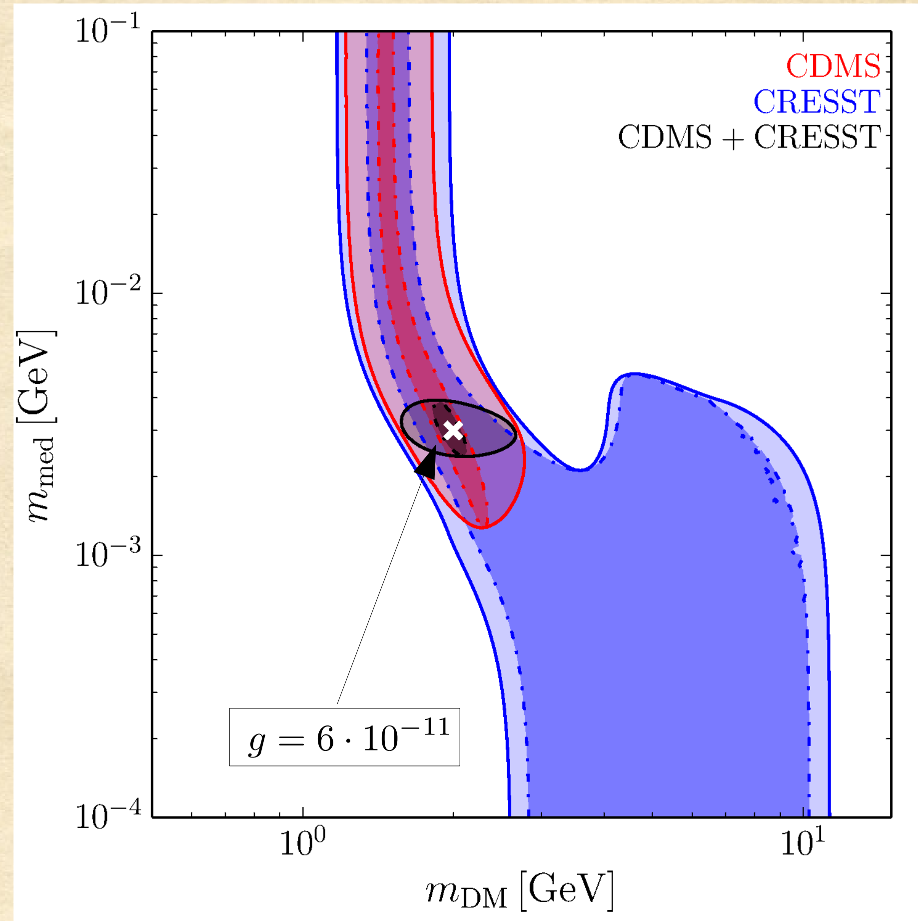


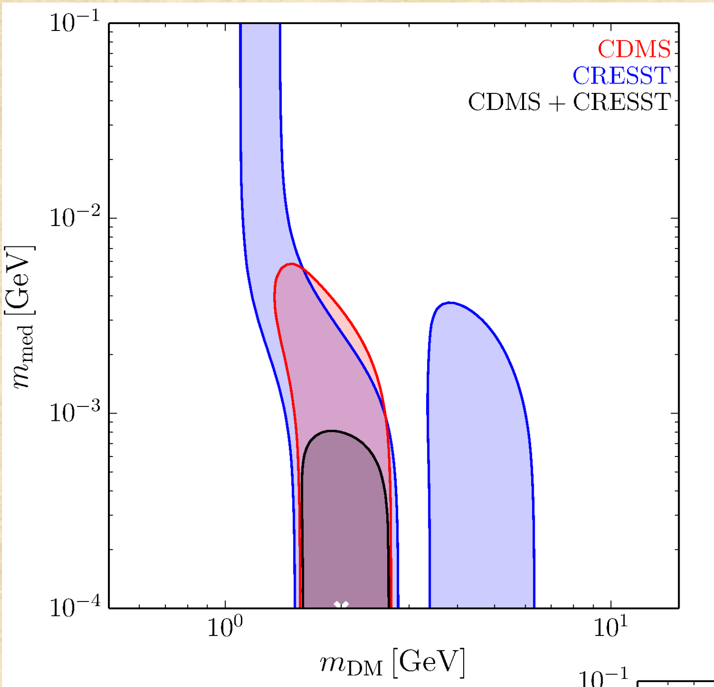


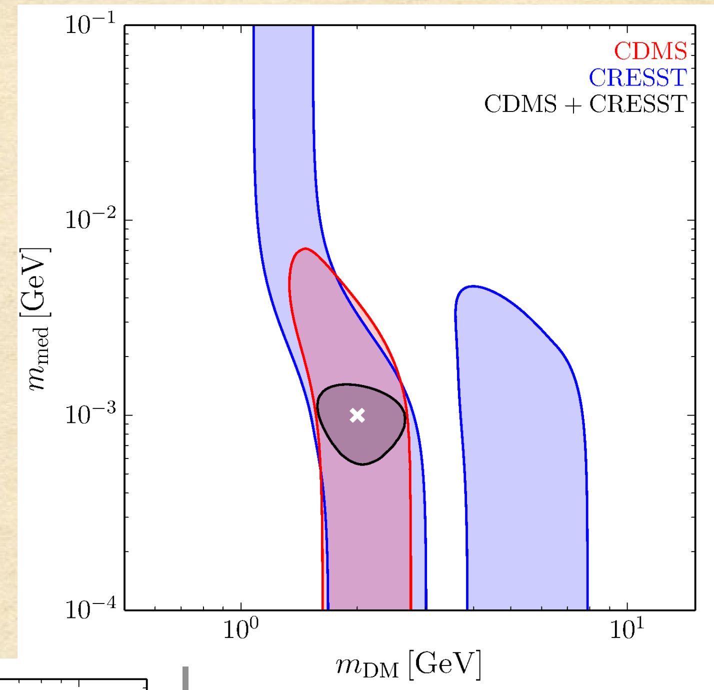
Using the combined information from two experiments, it is feasible to **measure simultaneously** $m_{\rm DM}$ and $m_{\rm med}$!

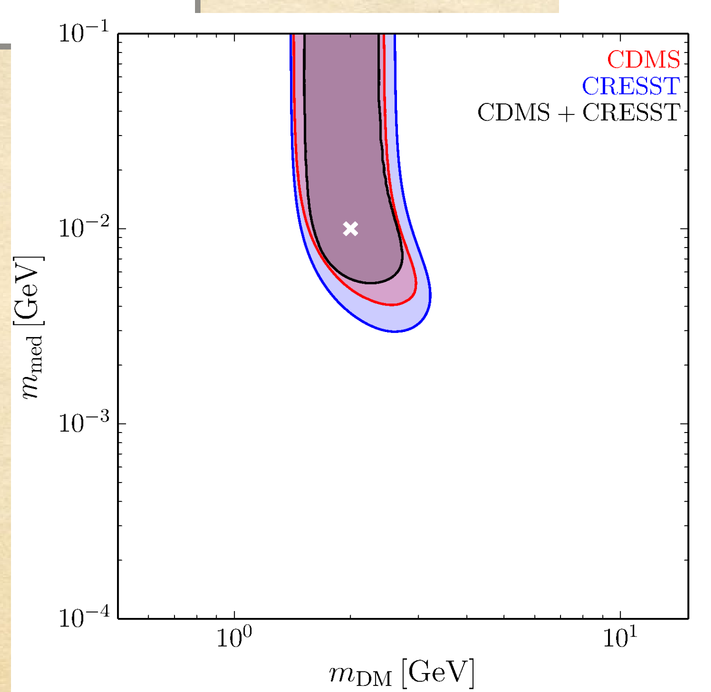
→ for realistic exposures and not-yet-excluded scenarios

→ even after taking into account various nuisance parameters

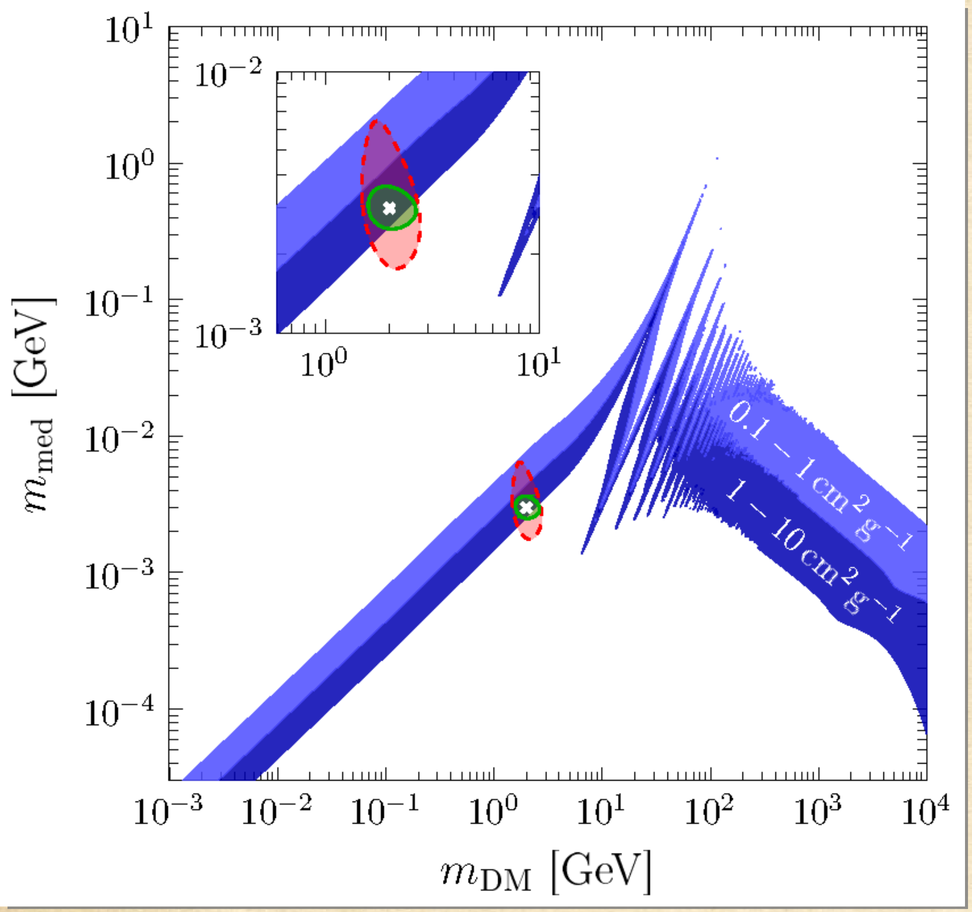
Some more examples

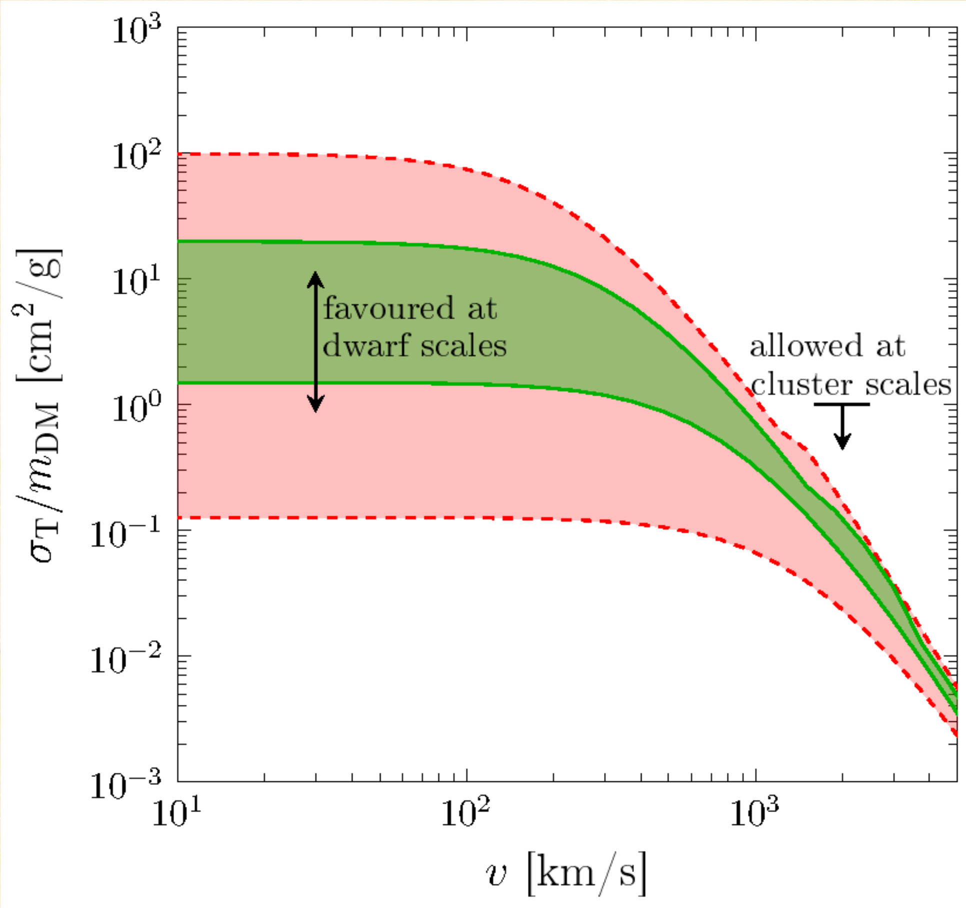






Interplay: SIDM ←→ direct detection





Future low-threshold DD experiments can potentially probe the behaviour of DM on astrophysical scales!

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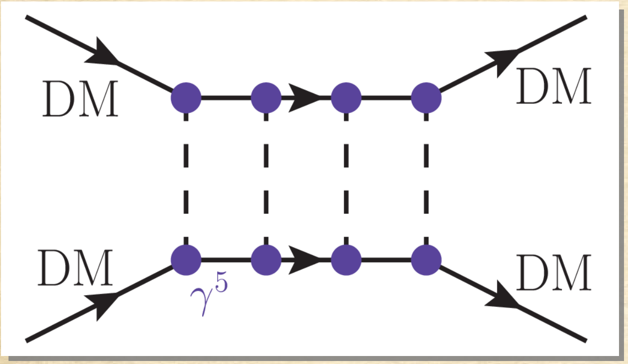
SIDM via pseudoscalar exchange?

$$\mathcal{L} = -iy_\psi \bar{\psi}\gamma^5 \psi \phi - iy_{\rm SM} \sum_f \frac{m_f}{v_{\rm EW}} \bar{f}\gamma^5 f \phi$$

→ DD constraints are easily evaded ($\sigma \propto q^4$)



$$V_{\rm PS}(r) = \frac{y_\psi^2}{48\pi} \frac{m_\phi^2}{m_\psi^2} \left(\frac{e^{-m_\phi r}}{r} - \frac{4\pi}{m_\phi^2} \delta^{(3)}(\vec{r}) \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \frac{y_\psi^2}{48\pi} \frac{m_\phi^2}{m_\psi^2} \left(1 + \frac{3}{m_\phi r} + \frac{3}{m_\phi^2 r^2} \right) \frac{e^{-m_\phi r}}{r} S_{12}(\vec{r})$$

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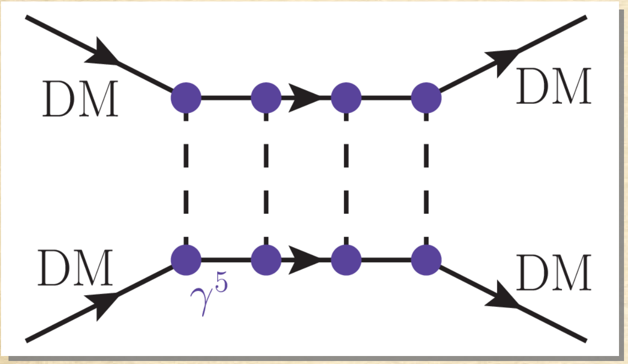

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- Yukawa term $\propto e^{-m_\phi r}/r$ is suppressed by m_ϕ^2/m_ψ^2 Bellazzini+ [1307.1129]
- $1/r^3$ term leads to singular solutions of Schrödinger equation
  - → renormalization techniques do not give predictive results
  - → notice: same problem (in principle) appears already for standard vector/scalar exchange: $V_{LS} \propto L\cdot S/r^3$
- We have repeated the full one-loop calculation of the box diagram
  - → no “effective scalar” interaction claimed in Fujita+ [1209.3067]

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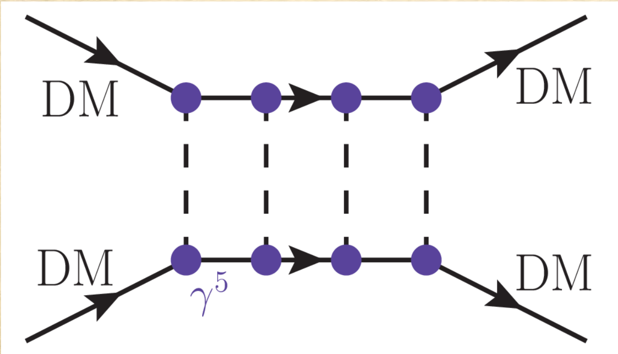

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→ No significant self-interactions expected from pseudoscalar exchange

Maximally CP-violating DM interactions

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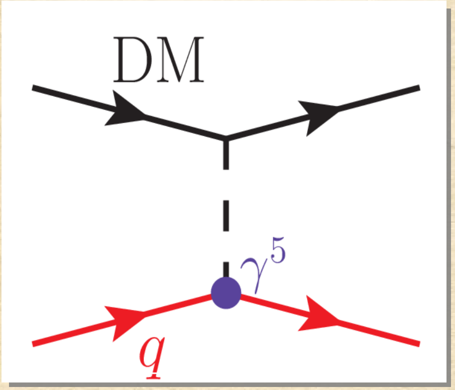
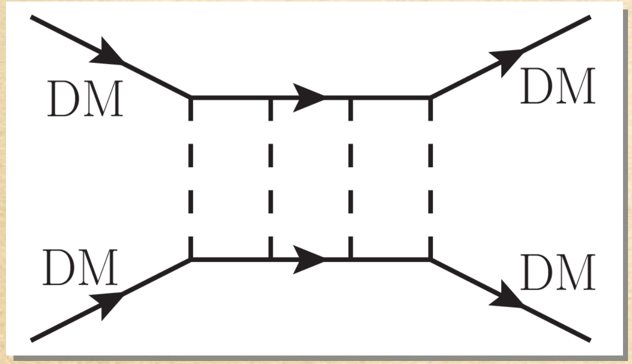
Direct detection:



$\propto q^2$ for a mixed scalar/pseudoscalar mediator
→ direct detection constraints are still evaded

Self-interactions:



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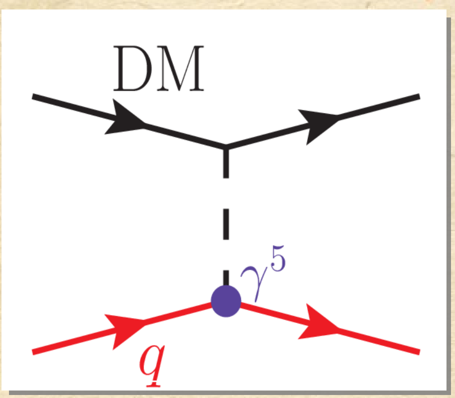

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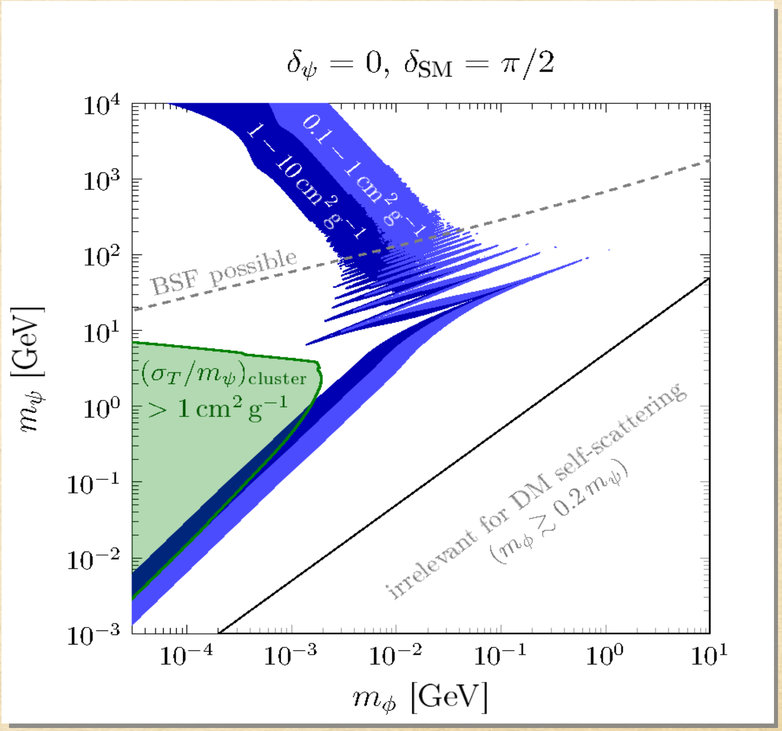

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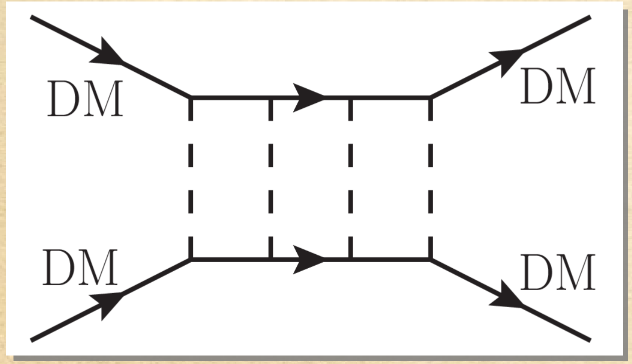

$$V(r) \propto \frac{e^{-m_\phi r}}{r}$$

Phenomenologically viable also for large $m_{\rm DM}$!

Maximally CP-violating DM interactions

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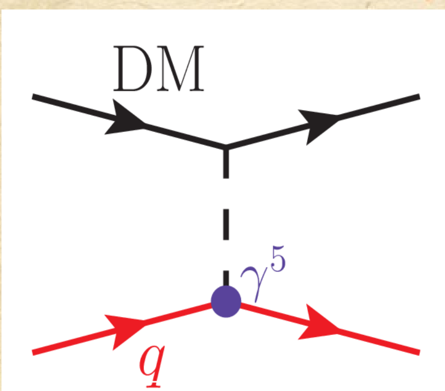
Unmotivated/tuned

Direct detection:



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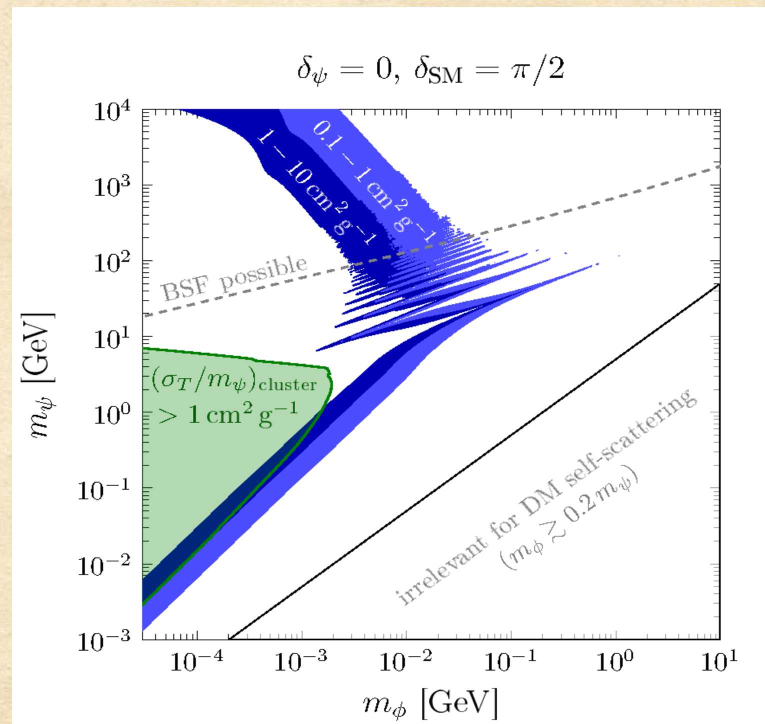
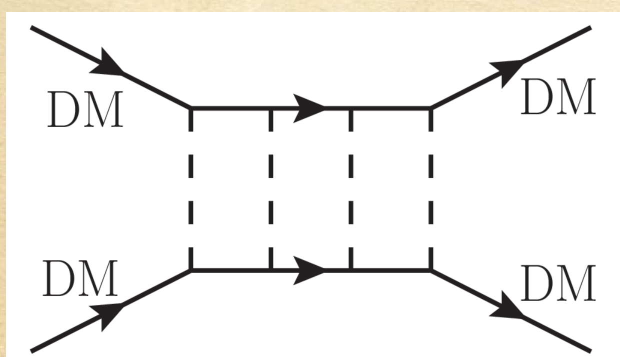


Self-interactions:



$$V(r) \propto \frac{e^{-m_\phi r}}{r}$$

Phenomenologically viable also for large m_{DM}!

CP violating SIDM: general case

$$\mathcal{L} = -y_\psi \cos \delta_\psi \bar{\psi}\psi\phi - i y_\psi \sin \delta_\psi \bar{\psi}\gamma^5\psi\phi$$
$$-y_{\rm SM} \sum_f \left[\frac{m_f}{v_{\rm EW}} \cos \delta_{\rm SM} \bar{f} f \phi + i \frac{m_f}{v_{\rm EW}} \sin \delta_{\rm SM} \bar{f}\gamma^5 f \phi \right]$$

- No a priori reason for CP conservation in the dark sector
  - → in fact, such a general setup can arise from **spontaneous CP violation**, starting from a CP conserving coupling to a pseudoscalar
  - → we treat δ_ψ and $\delta_{\rm SM}$ as free parameters

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- s-wave contribution to lowest order DM annihilation process:

$$(\sigma v)_{\psi\bar{\psi}\to\phi\phi} \simeq \frac{y_\psi^4 \sin^2(2\delta_\psi)}{32\pi m_\psi^2} \neq 0 \quad \text{for } \delta_\psi \neq 0, \pi/2$$

  → unsuppressed annihilation $\psi\bar{\psi} \to \phi\phi$ for CP-violating couplings!
  → for $m_\phi \ll m_\psi$ this annihilation channel is strongly **Sommerfeld enhanced** during CMB

Similar as in Bringmann/Kahlhoefer/Schmidt-Hoberg/Walia [1612.00845] for a vector mediator

→ Strong CMB constraints on CP-violating phase δ_ψ

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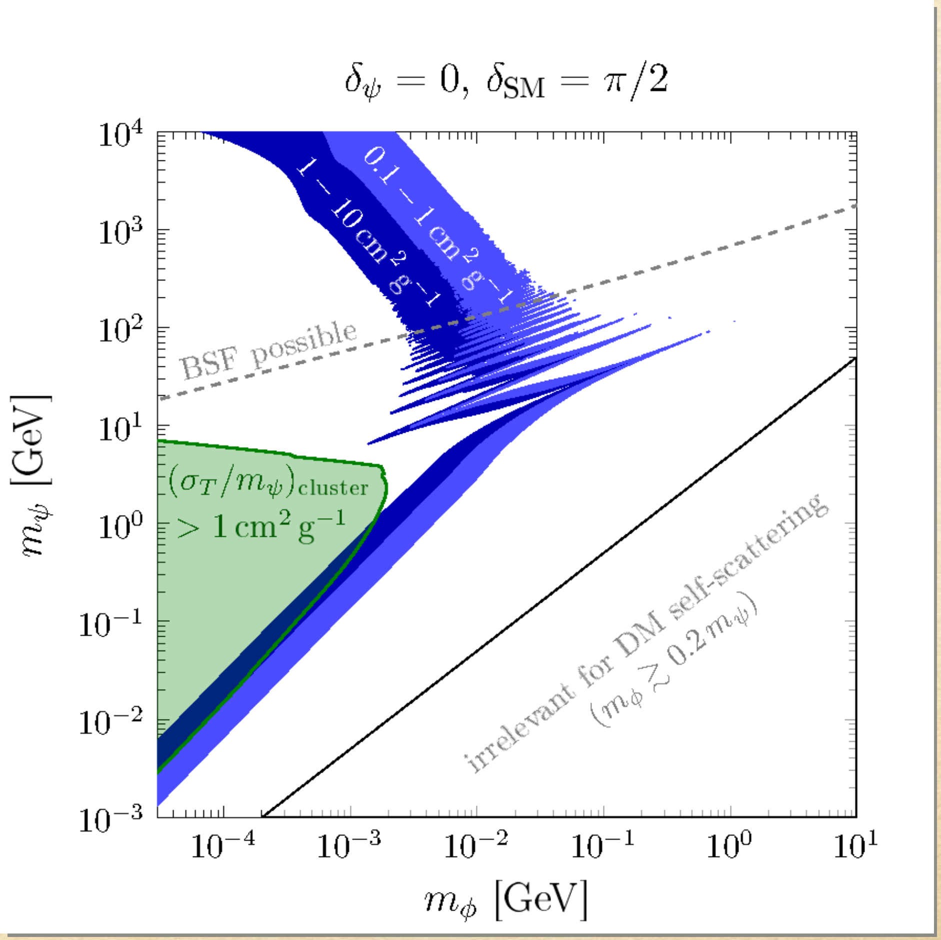

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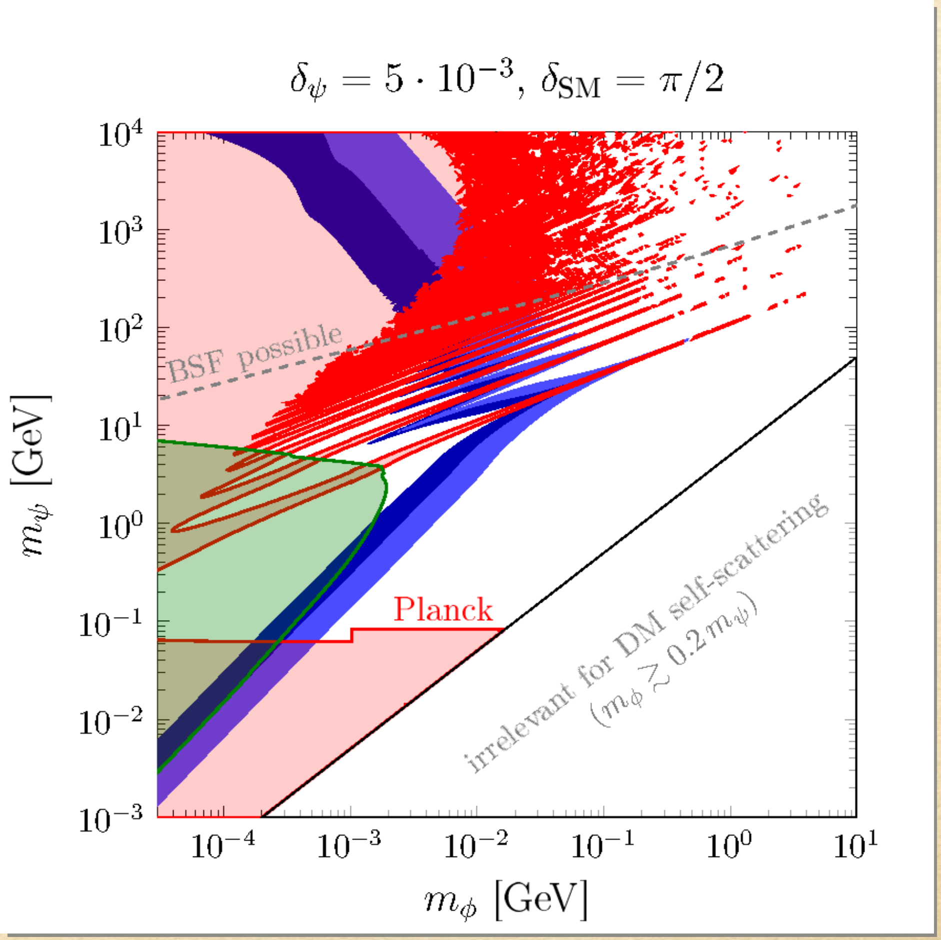

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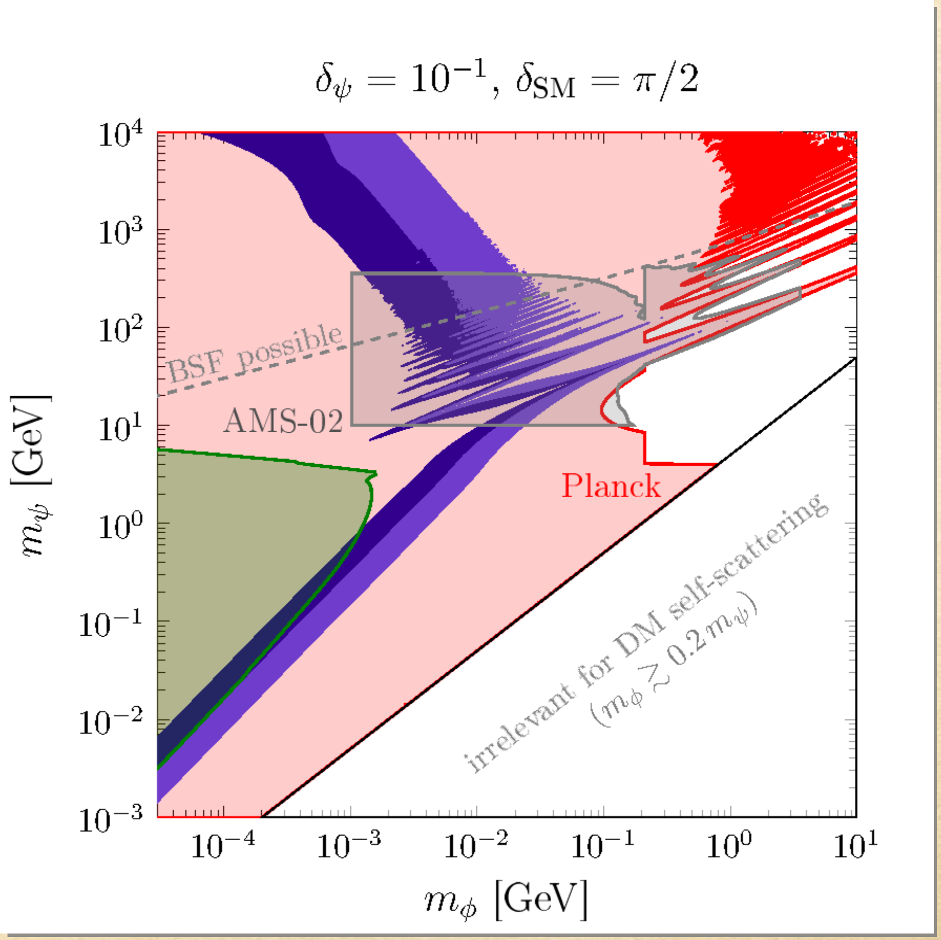
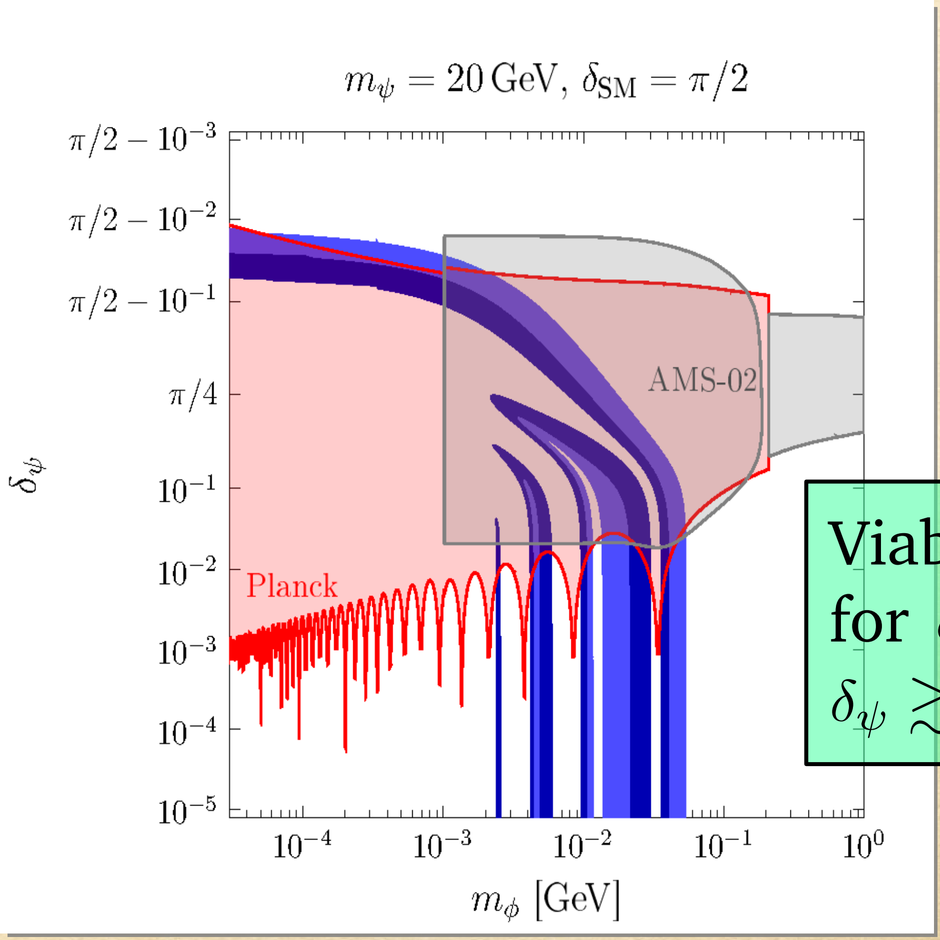


CP violating SIDM: general case

$$\mathcal{L} = -y_\psi \cos\delta_\psi \bar{\psi}\psi\phi - i y_\psi \sin\delta_\psi \bar{\psi}\gamma^5\psi\phi - y_{\rm SM}\sum_f \left[\frac{m_f}{v_{\rm EW}}\cos\delta_{\rm SM}\bar{f}f\phi + i\frac{m_f}{v_{\rm EW}}\sin\delta_{\rm SM}\bar{f}\gamma^5 f\phi\right]$$



Viable SIDM scenario for $\delta_\psi \lesssim 10^{-2}$ or $\delta_\psi \gtrsim \pi/2 - 0.1$

Conclusions

SIDM via **light mediators** is under pressure
→ vector mediator: largely excluded by CMB
→ scalar mediator: tension BBN vs. DD, in particular for $m_{\mathrm{DM}} \gtrsim 5\,\mathrm{GeV}$

(1) Test remaining parameter space at $m_{\mathrm{DM}} \lesssim 5\,\mathrm{GeV}$ with direct detection

- Interesting coincidence: $m_{\mathrm{med}} \sim q \sim 1 - 10\,\mathrm{MeV}$
- Even when taking into account exp. & astrophysical uncertainties, future low-threshold DD can **simultaneously probe** m_{DM} and m_{med}

(2) CP-violating DM: mixed scalar/pseudoscalar couplings

- Dominantly scalar DM-mediator couplings
  → large SIDM cross sections
- Dominantly pseudoscalar SM-mediator interactions
  → direct detection is suppressed
- CMB strongly constrains the CP-violating phases
  → viable scenario only for specific choices of δ_ψ and δ_{SM}

Backup material

Spontaneous CP violation

- Starting point: CP-conserving scenario involving a pseudoscalar P:

$$\mathcal{L}_{\rm DM} = \bar\psi(i\partial\!\!\!/ - m_0)\psi - iy_\psi P\bar\psi\gamma^5\psi - V(P)$$

- P acquires a vev: $P = v_P + \phi$

$$\Rightarrow\ \ \mathcal{L}_{\rm DM} = \bar\psi\left[i\partial\!\!\!/ - (m_0 + iy_\psi v_P\gamma^5)\right]\psi - iy_\psi\phi\bar\psi\gamma^5\psi - V(\phi)$$

- Chiral rotation $\psi \to \exp(i\gamma^5\alpha/2)\psi$ with $\tan\alpha = y_\psi v_P/m_0$ removes the complex mass term:

$$\mathcal{L}_{\rm DM} = \bar\psi(i\partial\!\!\!/ - m_\psi)\psi - y_\psi\phi\bar\psi(\cos\delta_\psi + i\sin\delta_\psi\gamma^5)\psi \qquad (\delta_\psi = \pi/2 - \alpha)$$

→
- Mixed scalar/pseudoscalar interactions of DM
- $\delta_\psi \ll 1$ corresponds to $y_\psi v_P \gg m_0$

Spontaneous CP violation

- Similar story for the mediator-SM coupling:

$$\mathcal{L}_{\rm mass} \supset -\sum_f \bar{f}\frac{y_f}{\sqrt{2}}(v_{\rm EW} + i\sin\theta\, v_P\gamma^5)f$$

$$\Rightarrow\ \ \mathcal{L}_{\rm mixing} = -y_{\rm SM}\sum_f \frac{y_f}{\sqrt{2}}\phi\bar{f}(\cos\delta_{\rm SM} + i\sin\delta_{\rm SM}\gamma^5)f$$

with $y_{\rm SM} = \sin\theta$ and $\delta_{\rm SM} = \pi/2 - \alpha_{\rm SM}$

- Crucial point: $\alpha_{\rm SM} \propto \sin\theta \ll 1 \ \Rightarrow\ \delta_{\rm SM} \approx 1$

→ Small coupling to SM fermions ensures that the SM-mediator coupling is nearly pseudoscalar after spontaneous CP breaking