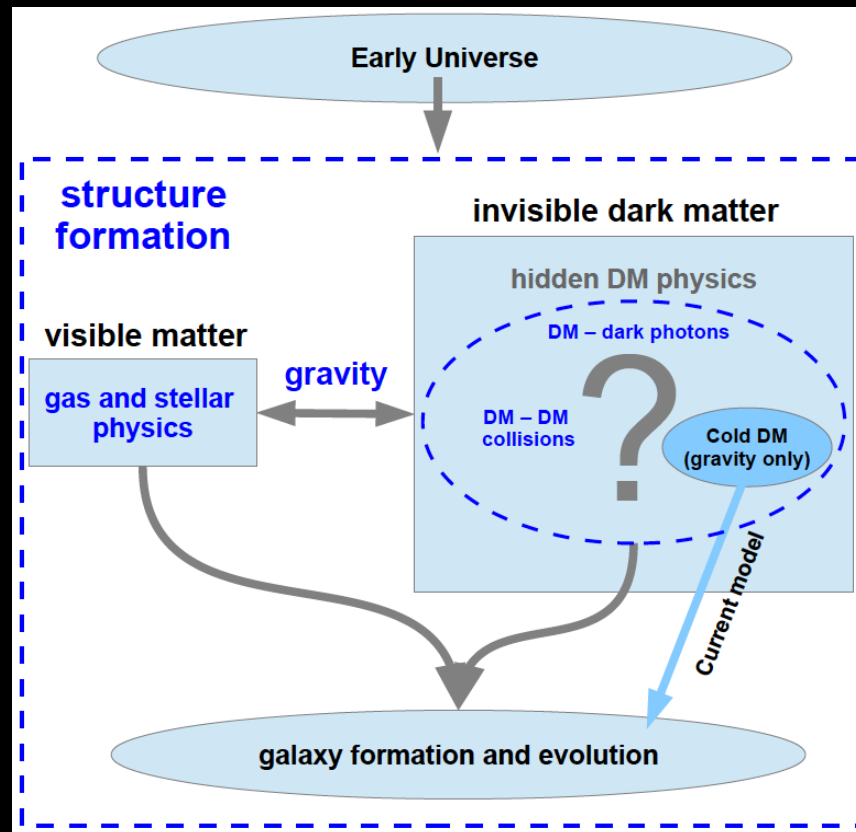


# Structure formation in a SIDM cosmology: an overview

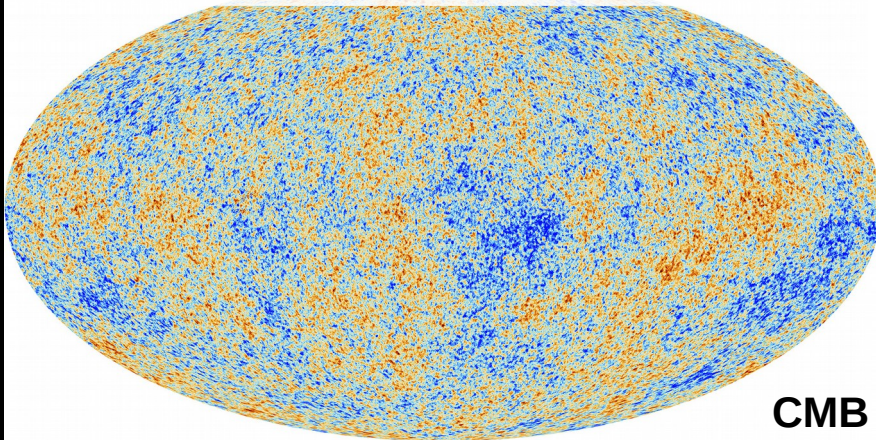
Jesús Zavala Franco

Faculty of Physical Sciences, University of Iceland



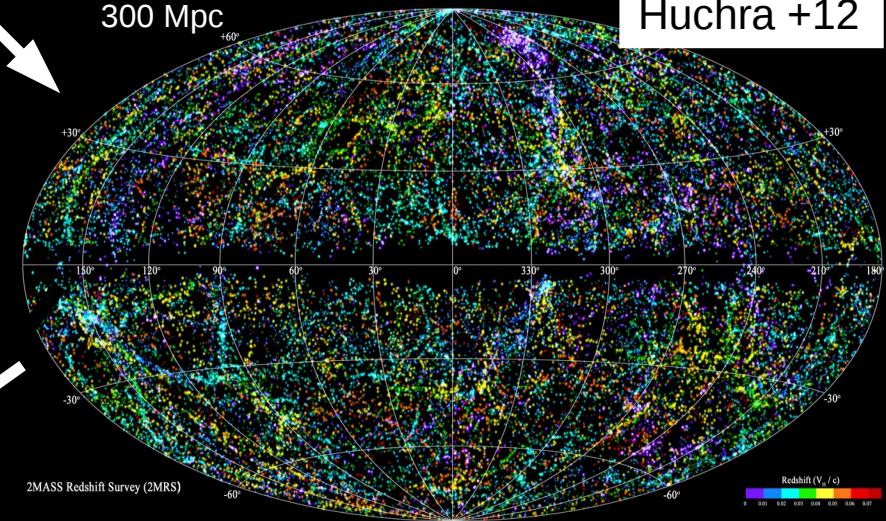
The **goal of structure formation** is to explain the growth of cosmic structures across time (DM is seemingly essential)

Early Universe (t ~ 0.4 Myrs)



$$\frac{\delta\rho_m}{\rho_m} \sim 10^{-3}$$

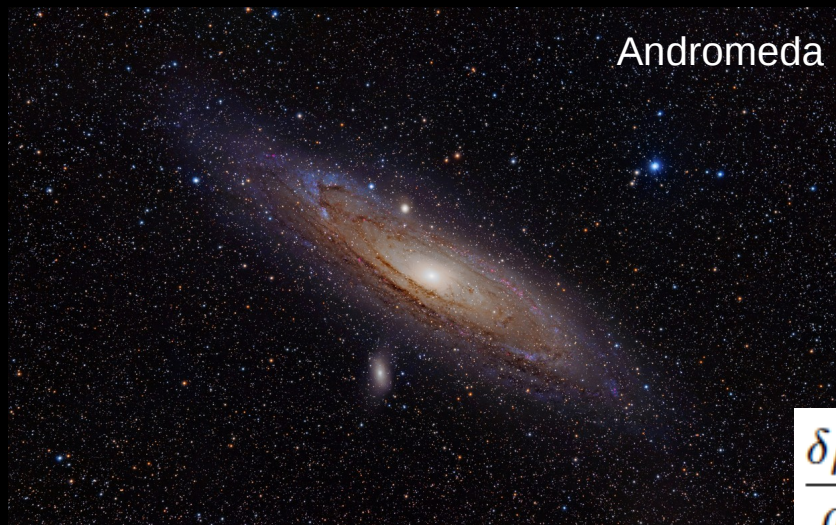
Universe today (t ~ 13.8 Gyrs)



2MRS galaxy "map", large-scale structure

$$\frac{\delta\rho_m}{\rho_m} \gtrsim 1$$

Andromeda

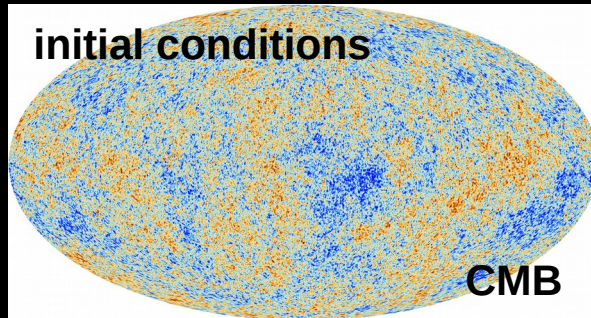


$$\frac{\delta\rho_m}{\rho_m} \gg 1$$

galactic scales

Credit: ESA and the Planck Collaboration

The **Cold Dark Matter (CDM) hypothesis** is the cornerstone of the current structure formation theory

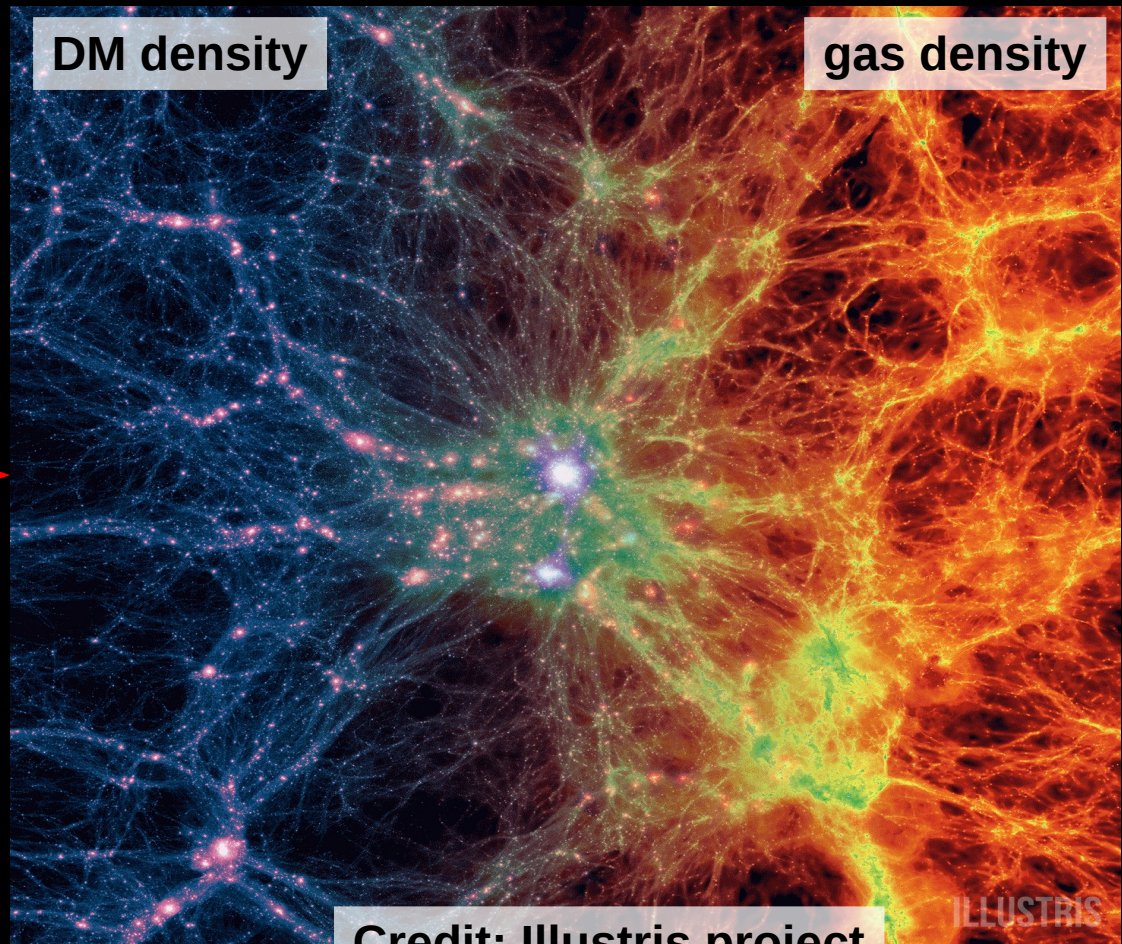


*CDM assumes that the only DM interaction that matters is gravity!!*

cosmological simulations

DM gravity only  
+  
"baryonic" physics  
(radiative cooling,  
gas hydrodynamics,  
star formation,  
supernova and AGN  
feedback,...)

A grey box containing text describing the components of cosmological simulations. It lists 'cosmological simulations' and 'DM gravity only' with a plus sign, followed by '"baryonic" physics' and a list of physical processes: radiative cooling, gas hydrodynamics, star formation, supernova and AGN feedback, etc.



**2000 CPU years!!**

Credit: Illustris project

# Opening remarks

Structure formation theory has become powerful enough to predict the phase-space distribution of dark matter across time down to galactic scales.

- The Cold Dark Matter (CDM) hypothesis has been the standard for nearly three decades and implies that DM gravity is the only relevant interaction (for galactic scales and above). It implies that structure formation within CDM has no free DM parameters. However:

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**CDM/WDM/SIDM are incomplete DM theories**

**They are effective structure formation theories that need completion from a particle physics model (all beyond SM: “exotic”)**

# Opening remarks

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- Given the current situation (obs. constraints, complexity of baryonic physics), it is timely to consider additional free DM parameters, which might play a key role in the physics of galaxies. The window is relatively narrow and within reach of upcoming observations:

SIDM transfer cross section

$$0.1 \text{ cm}^2 / \text{gr} \lesssim \sigma / m \lesssim 2 \text{ cm}^2 / \text{gr}$$



below this value, the behaviour is the same as CDM



above this value constraints are strong (at cluster scales)

'cutoff' halo mass at z=0

$$10^{9.5} M_{Sun} \lesssim M_{cut} \lesssim 10^{10.5} M_{Sun}$$



below this value galaxy formation is highly suppressed (reionisation)



above this value DM clustering must be as in CDM

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# Structure formation within SIDM: could DM particles collide with themselves?

case in this talk:  
rare interactions,  
large momentum transfer

opposite case: afternoon session

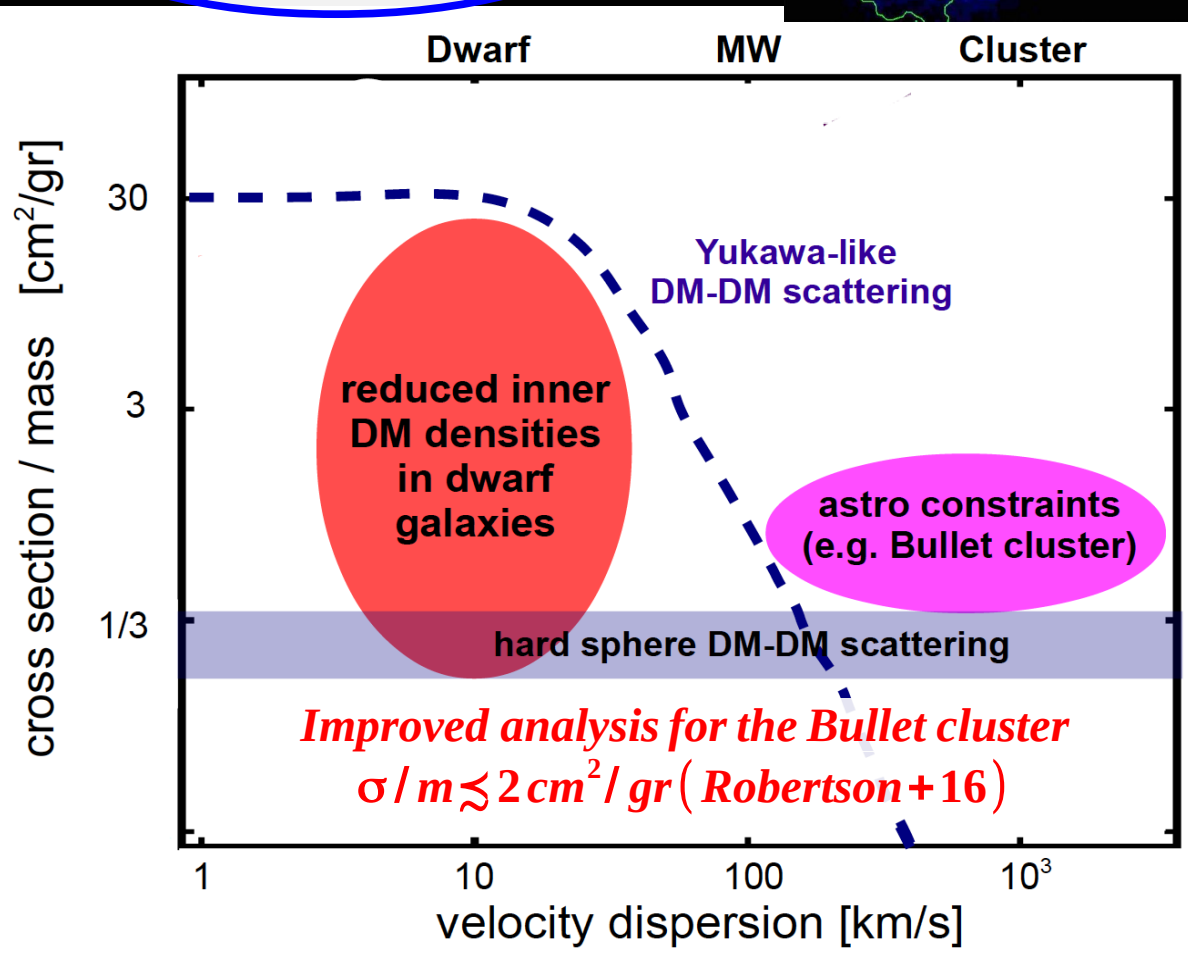
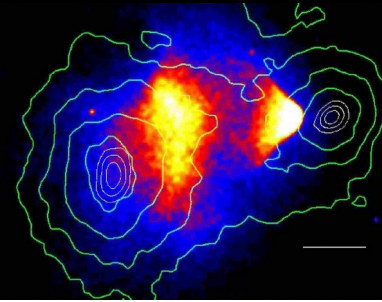
average scattering rate per particle:

$$\frac{\bar{R}_{sc}}{\Delta t} = \left( \frac{\sigma_{sc}}{m_\chi} \right) \bar{\rho}_{dm} \bar{v}_{typ}$$

~ 1 scatter / particle / Hubble time

Neither a fluid nor a  
collisionless system:  
~ rarefied gas  
(Knudsen number =  $\lambda_{mean}/L > \sim 1$ )

constraints allow  
collisional DM that is  
astrophysically significant  
in the center of galaxies



**A. Robertson's talk**



# **structure formation theory (linear regime)**

# Standard structure formation theory

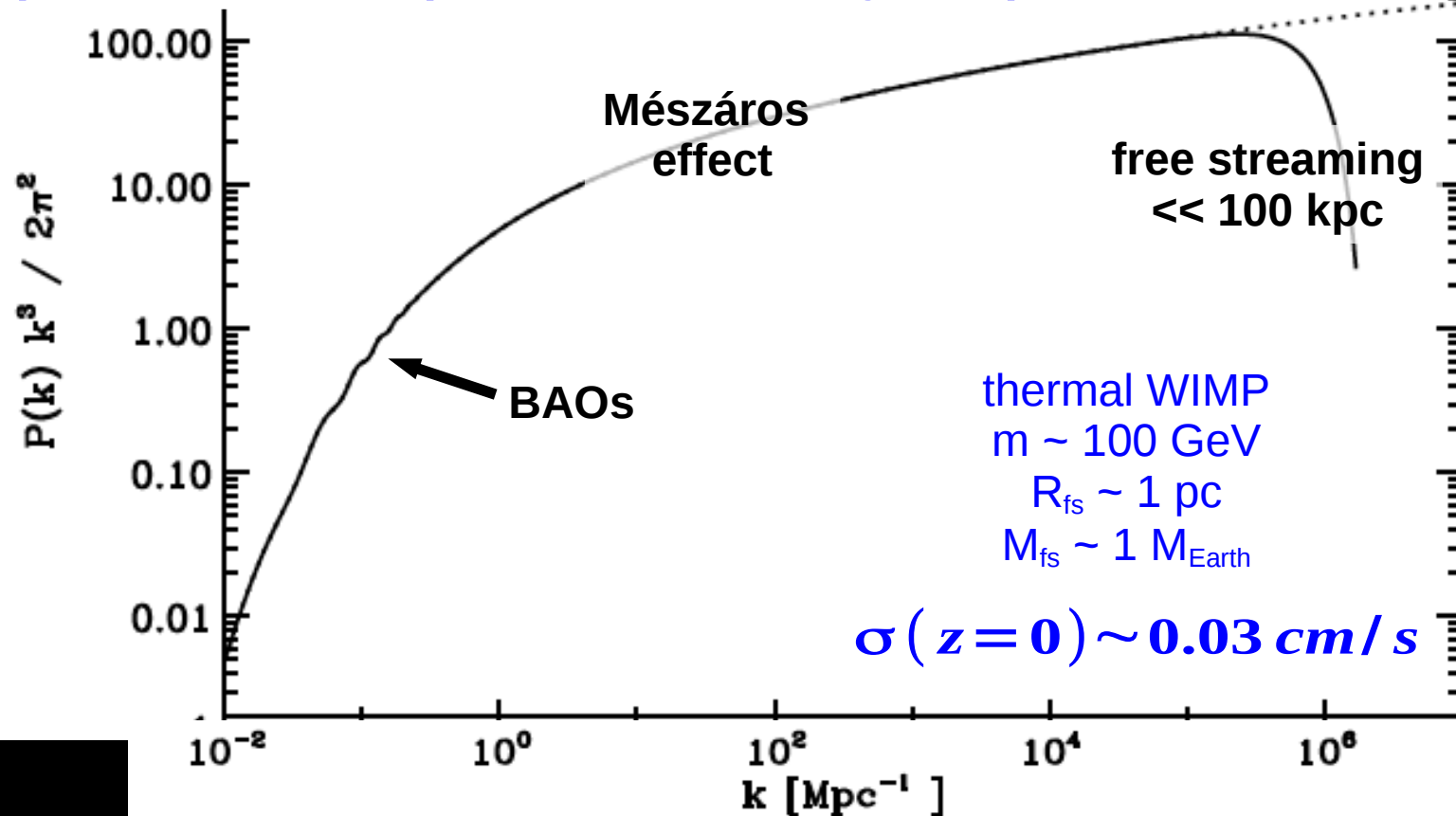
## LINEAR REGIME (cosmological perturbation theory)

$$\delta(x, t) = \frac{\rho(x, t) - \rho_B(t)}{\rho_B(t)} \ll 1$$

**Standard hypotheses:**  
DM is cold and collisionless  
(Cold Dark Matter model)

linear power spectrum  
(statistical description of the density field)

amplitude of DM clustering

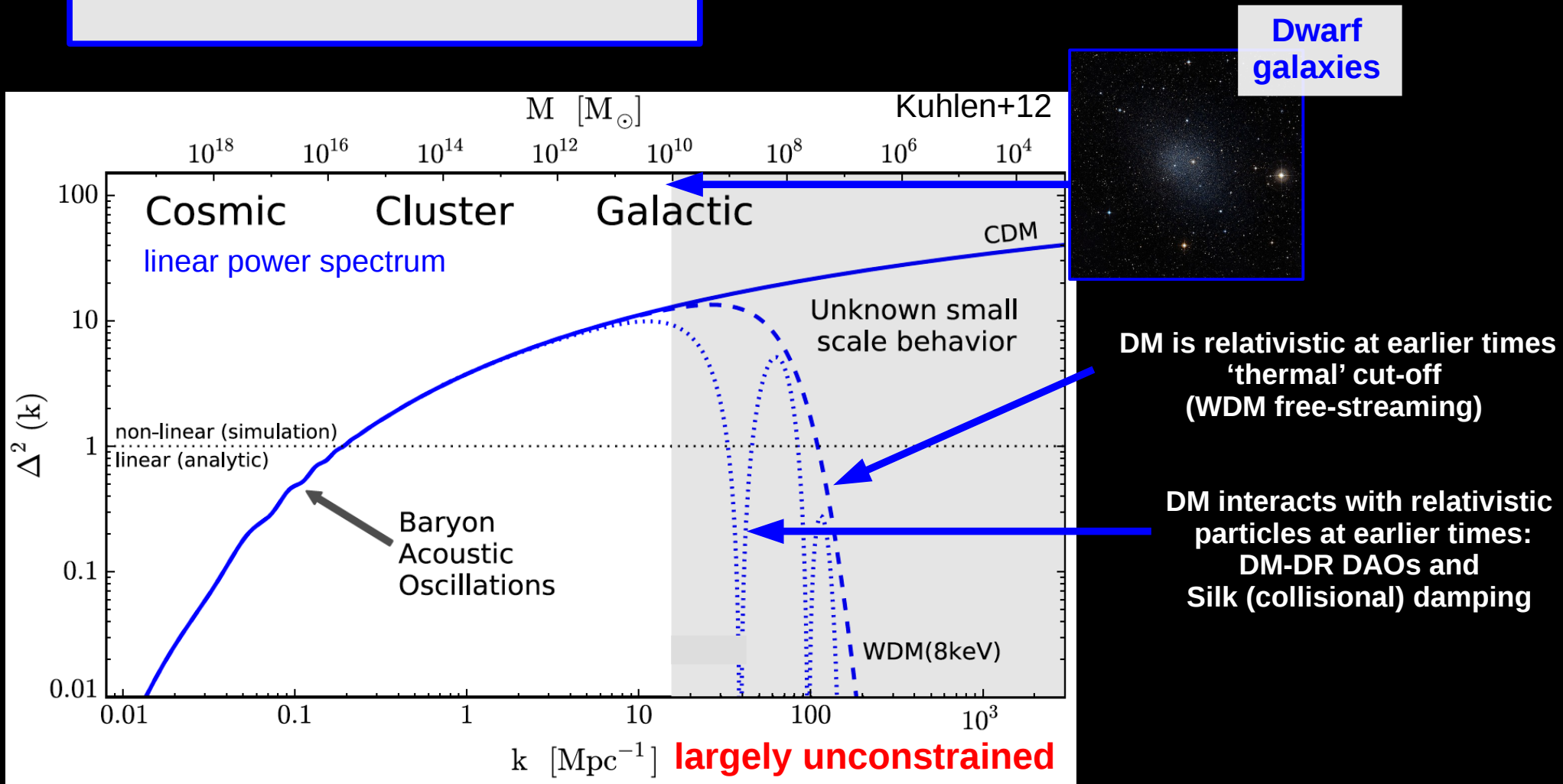


Angulo & White, 2010

# LINEAR REGIME: a 'relevant' P(k) cutoff ?

**Unsolved question:**  
 is the minimum mass scale for galaxy formation set by the DM nature or by gas physics (or by both)?

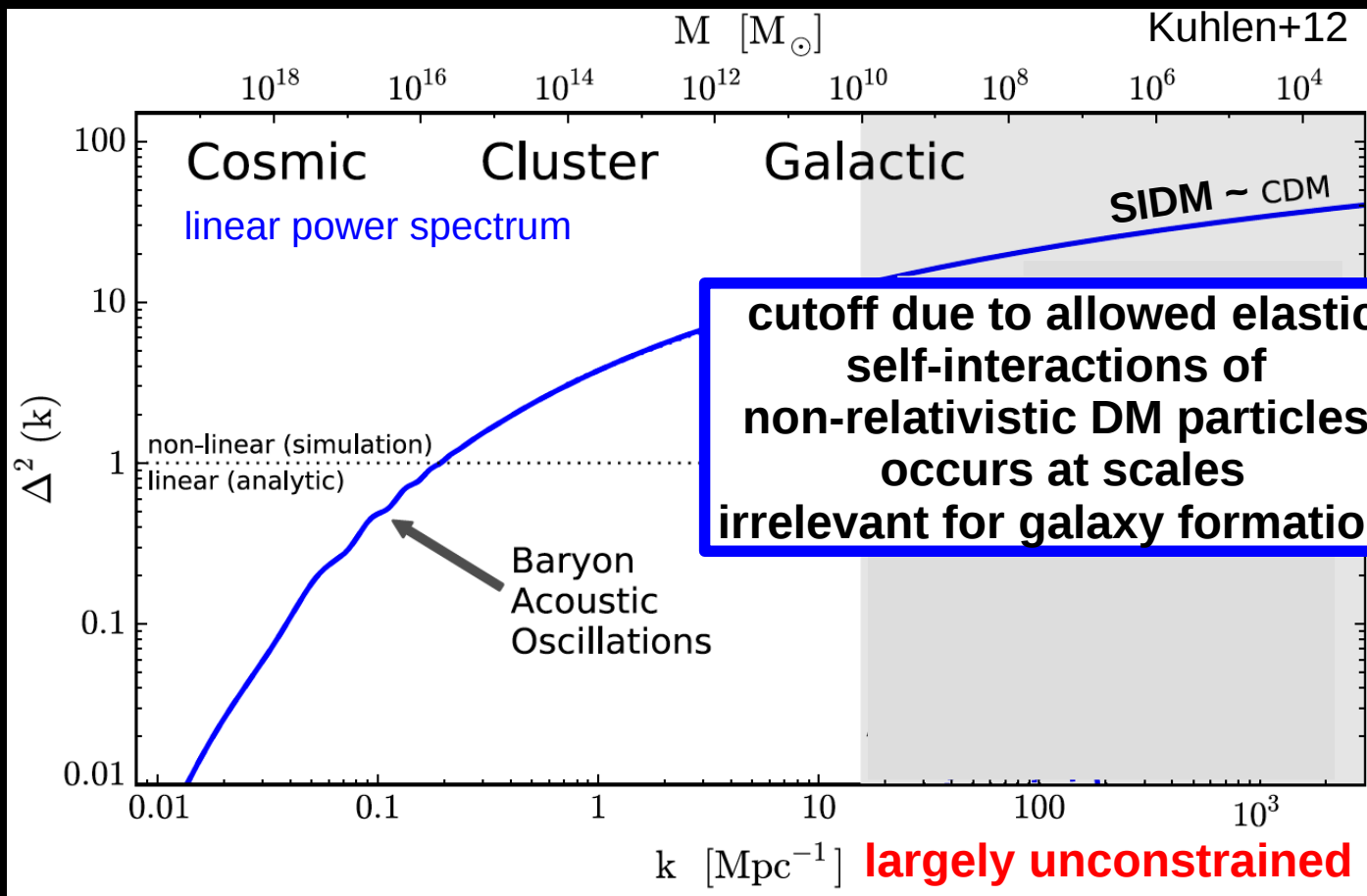
Observations have yet to measure the clustering of dark matter at the scale of the smallest galaxies



# LINEAR REGIME: a 'relevant' P(k) cutoff ?

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 is the minimum mass scale for galaxy formation set by the DM nature or by gas physics (or by both)?

Observations have yet to measure the clustering of dark matter at the scale of the smallest galaxies



~ 1 scatter / particle / t<sub>age</sub>  
 (scatt. rate ~ exp. rate)



self-decoupling temp. and Jeans mass

e.g.  $M_J \sim 10^{-11} M_{\text{Sun}}$

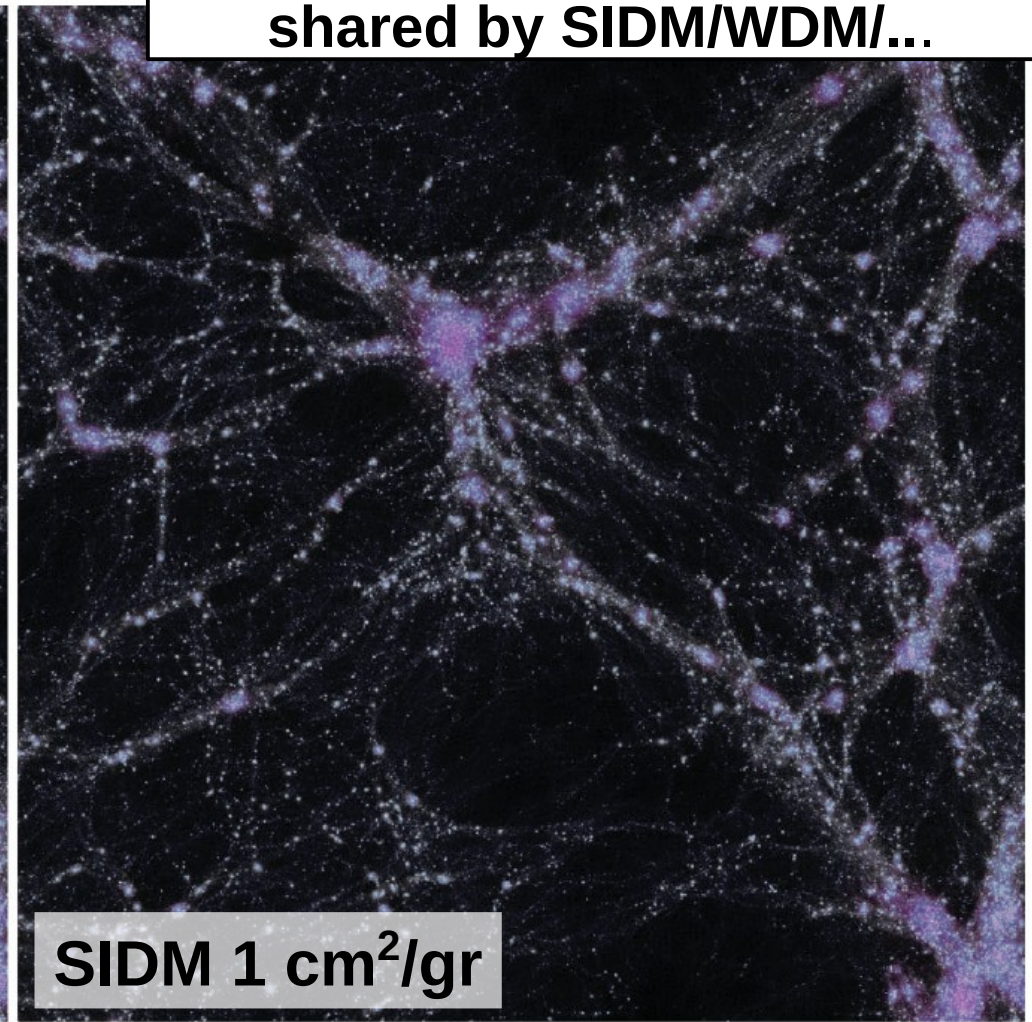
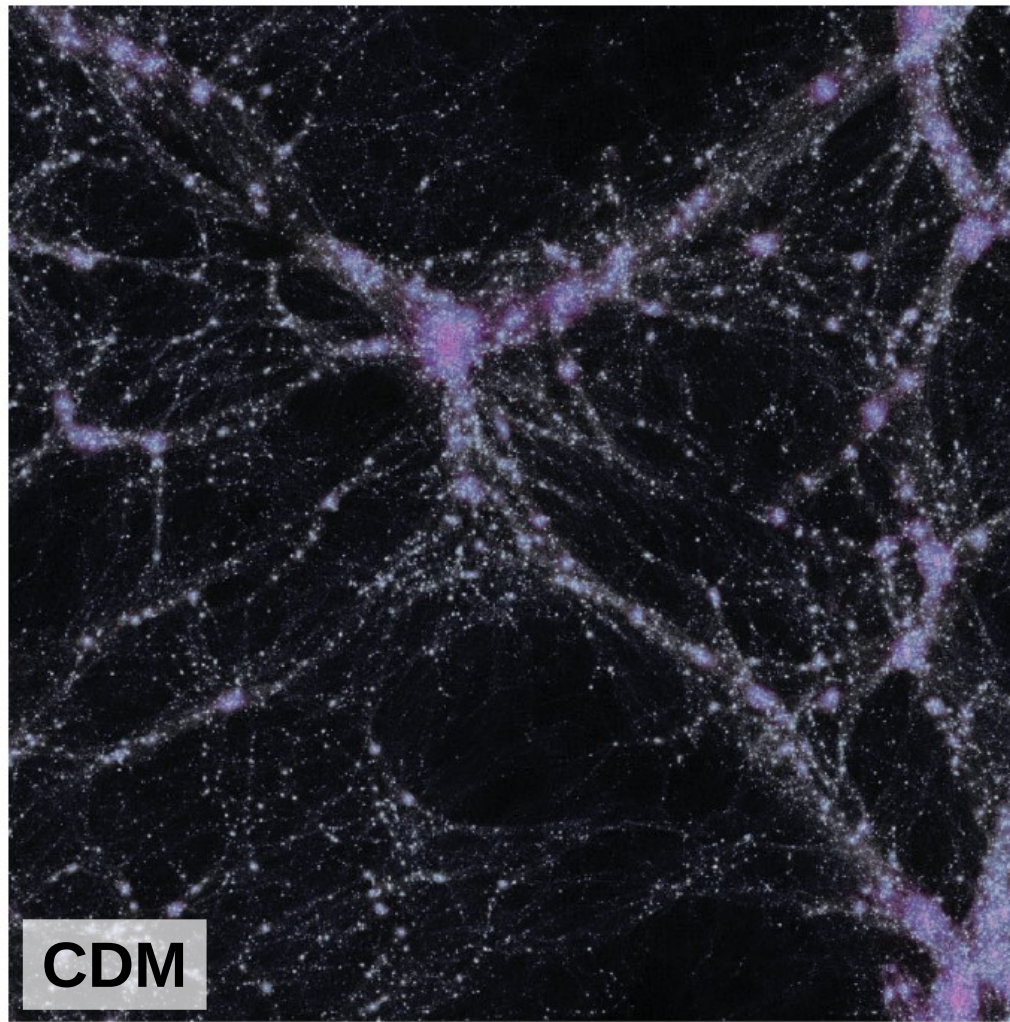
$\sigma/m \sim 0.1 \text{ cm}^2/\text{gr}$

$m \sim 100 \text{ GeV}$

# Linear evolution in SIDM is the same as in CDM (if DM is cold, and there are no additional interactions)

Rocha et al. 2013

CDM large scale successes are  
shared by SIDM/WDM/...



# **structure formation theory (non-linear regime)**

# Standard structure formation theory

## NON-LINEAR REGIME

*If  $\delta(\mathbf{x}, t) \gtrsim 1$  perturbation theory breaks down*

**Standard hypotheses:  
DM is cold and collisionless  
(Cold Dark Matter model)**

**the only DM interaction  
that matters is gravity!!**

In principle: solve Collisionless Boltzmann Equation (coupled with the Poisson equation) with the initial conditions given by linear perturbation theory

$$\frac{df}{dt} = 0$$

$$\nabla^2 \phi = 4\pi G \rho$$

i.e., find the local DM distribution in phase space at all points and at all times:

$$f(\vec{x}, \vec{v}, t) d^3 \vec{x} d^3 \vec{v}$$



$$\rho(\vec{x}, t) = \int f(\vec{x}, \vec{v}, t) d^3 \vec{v}$$

In practice however, we can only compute, measure, the DM distribution averaged over a certain macroscopic scale (coarse-grained distribution)

# Standard structure formation theory

## NON-LINEAR REGIME (N-body simulations)

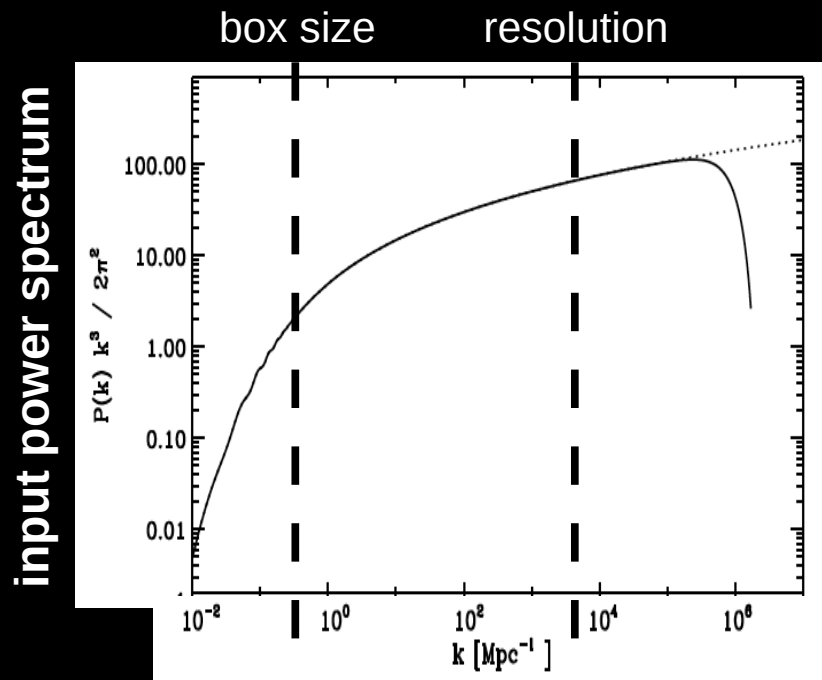
N-body sim: the coarse-grained distribution is given by a discrete representation of N particles:

$$\hat{f}(\mathbf{x}, \mathbf{v}, t) = \sum_i (M_i/m) W(|\mathbf{x} - \mathbf{x}_i|; h_i) \delta^3(\mathbf{v} - \mathbf{v}_i)$$

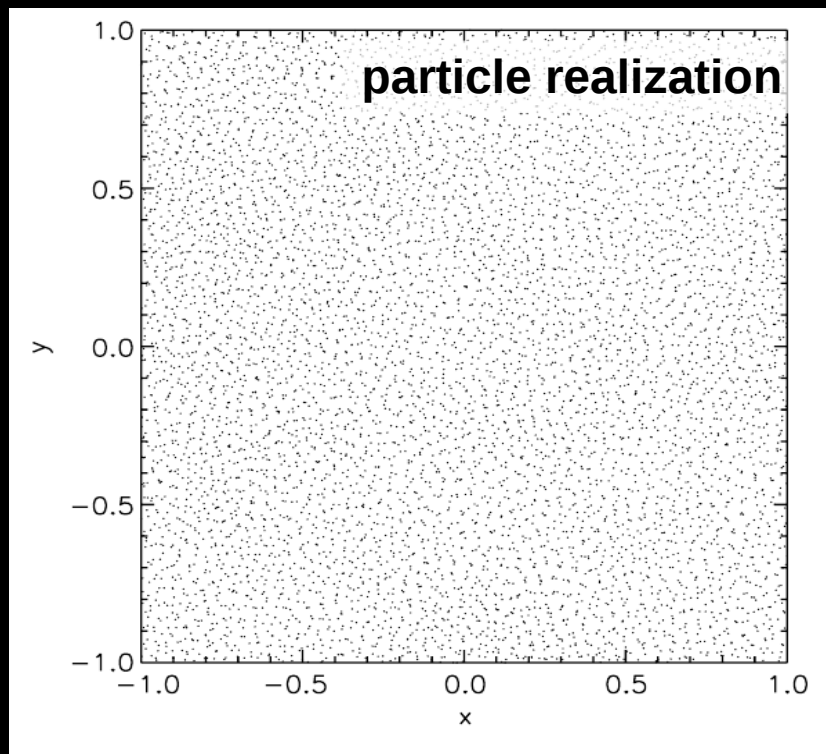
macro-to-micro-particle mass ratio

each particle is smoothed in space to give a smooth local density

each macro-particle travels at one speed

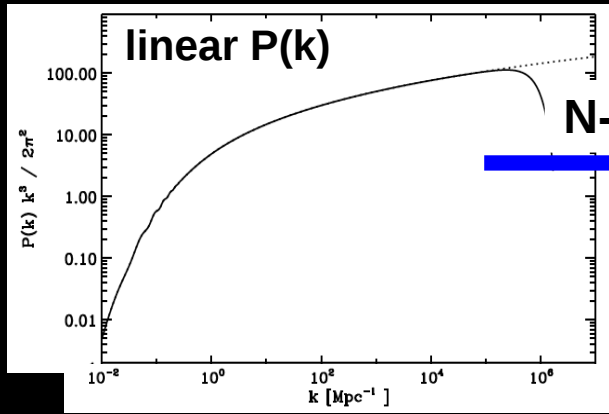


mapping

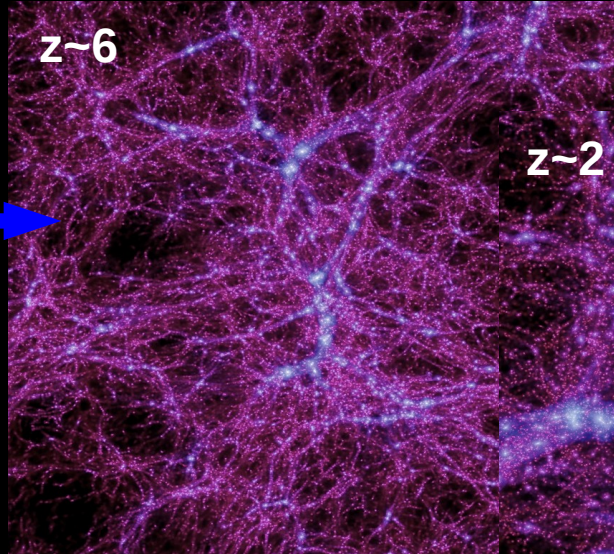




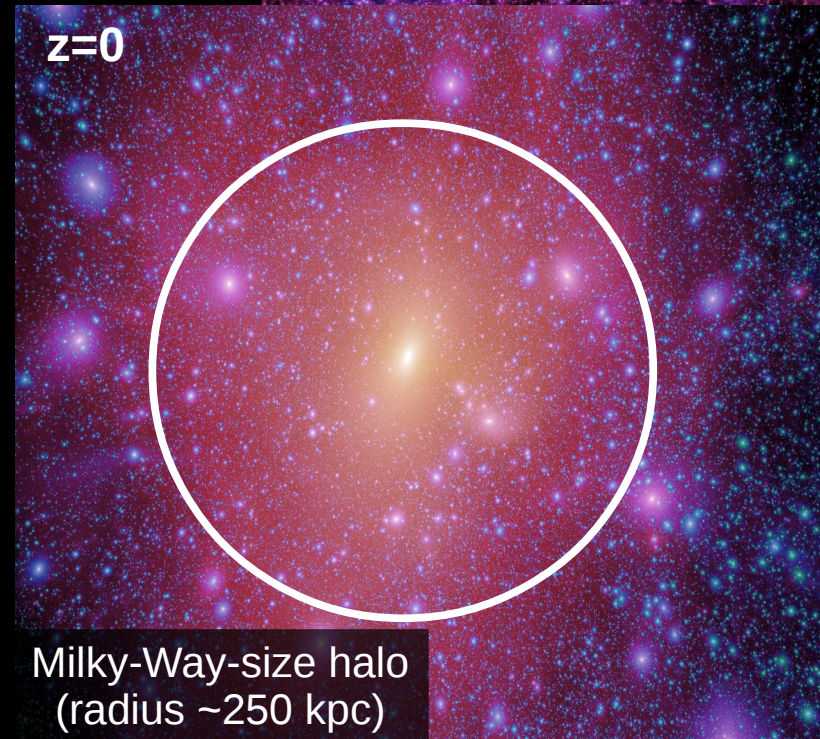
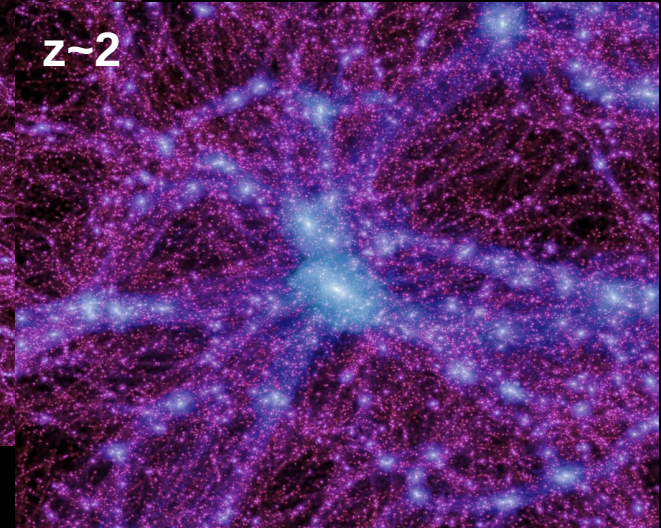
# Self-gravitating DM structures: haloes



N-body

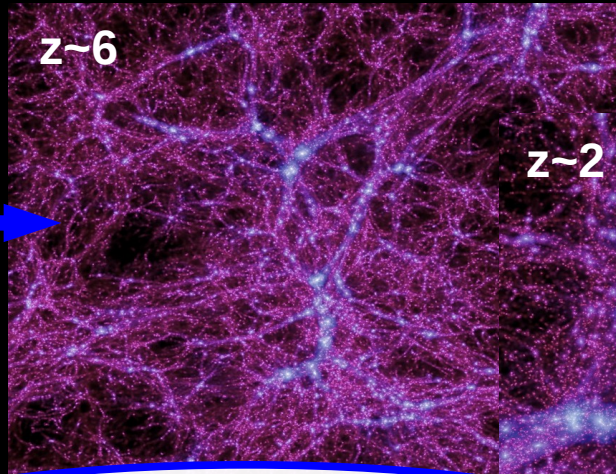
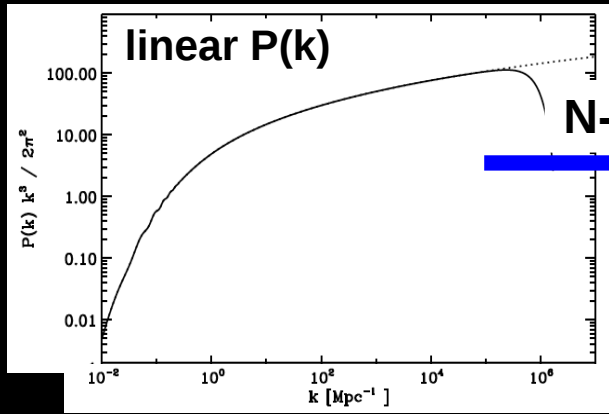


Boylan-Kolchin+2009

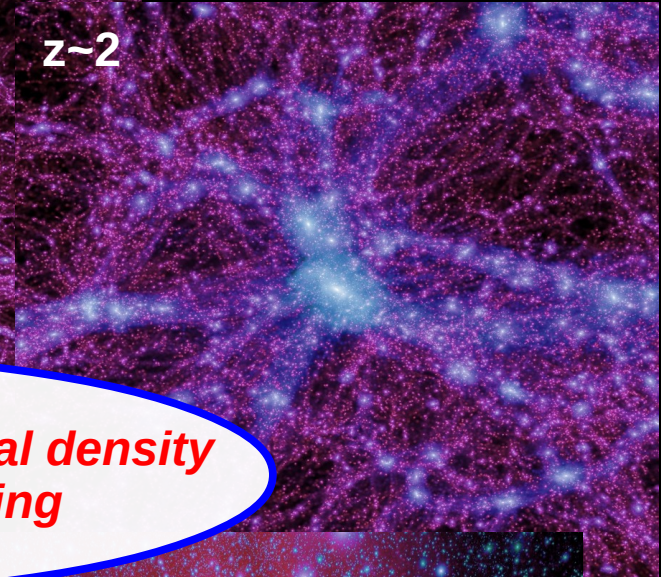


Aquarius project Springel+08

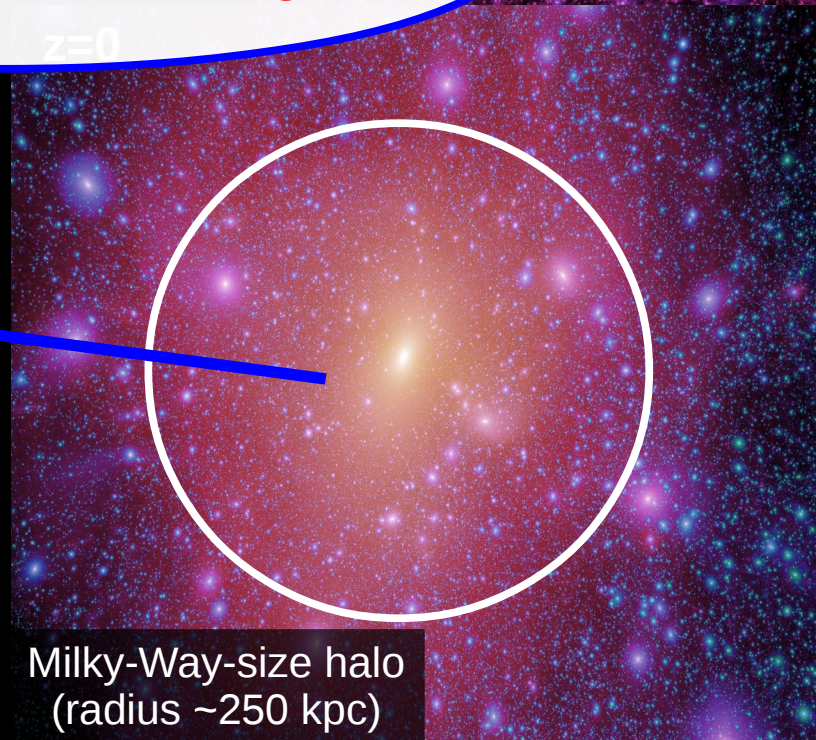
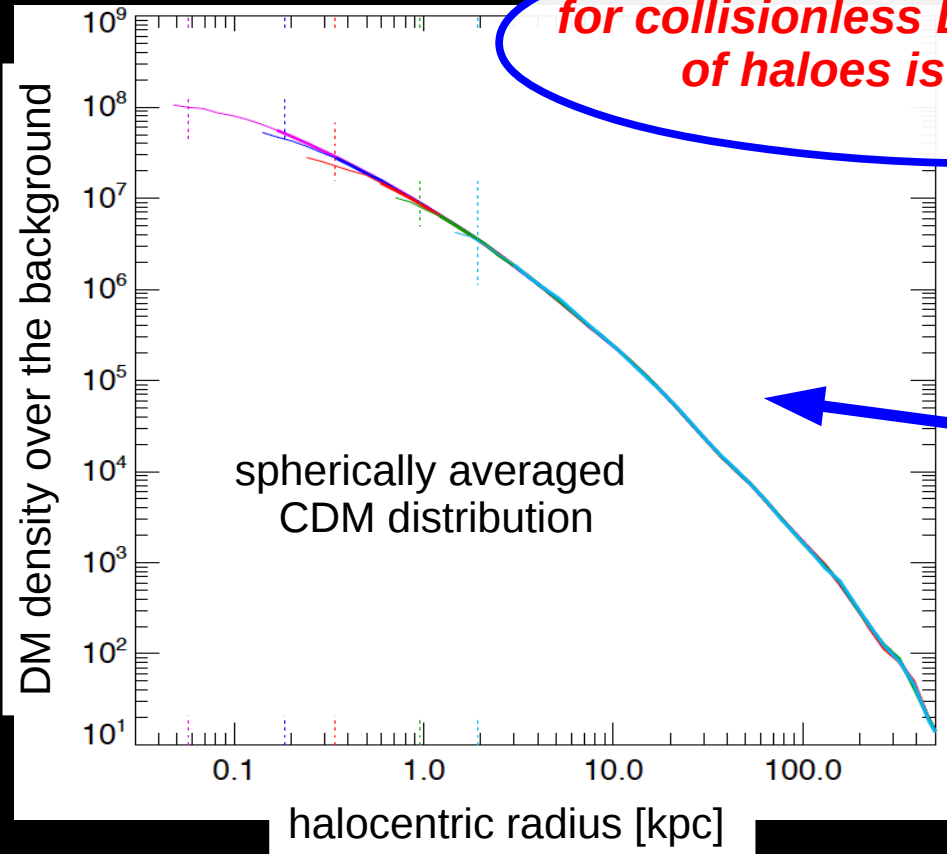
# Self-gravitating DM structures: haloes



Boylan-Kolchin+2009



*for collisionless DM, the central density of haloes is ever increasing*



Aquarius project Springel+08

# **structure formation theory with DM self-interactions**

# DM self-collisions in N-body simulations (probabilistic approach)

Far from the fluid and  
collisionless regimes  
(Knudsen number =  $\lambda_{\text{mean}}/L \gtrsim 1$ )



Collisional  
Boltzmann equation  
(elastic)

$$\frac{Df(\mathbf{x}, \mathbf{v}, t)}{Dt} = \Gamma[f, \sigma]$$

$$= \int d^3\mathbf{v}_1 \int d\Omega \frac{d\sigma}{d\Omega} |\mathbf{v} - \mathbf{v}_1| [f(\mathbf{x}, \mathbf{v}', t) f(\mathbf{x}, \mathbf{v}'_1, t) - f(\mathbf{x}, \mathbf{v}, t) f(\mathbf{x}, \mathbf{v}_1, t)]$$

Differential  
cross section

Rate of scattered particles  
into phase-space patch

Rate of scattered particles  
out of phase-space patch

$$|\vec{v}_{\text{rel}}| = |\vec{v}_1 - \vec{v}| = |\vec{v}'_1 - \vec{v}'|$$

Ansatz for N-body simulation: same solution for “coarse-grained” distribution function

$$\frac{D\hat{f}}{Dt} = \int d^3\mathbf{v}_1 \int d\Omega \frac{d\sigma}{d\Omega} |\mathbf{v} - \mathbf{v}_1| [\hat{f}(\mathbf{x}, \mathbf{v}', t) \hat{f}(\mathbf{x}, \mathbf{v}'_1, t) - \hat{f}(\mathbf{x}, \mathbf{v}, t) \hat{f}(\mathbf{x}, \mathbf{v}_1, t)]$$

# DM self-collisions in N-body simulations (probabilistic approach)

The coarse-grained distribution is given by a discrete representation of N particles:

$$\hat{f}(\mathbf{x}, \mathbf{v}, t) = \sum_i (M_i/m) W(|\mathbf{x} - \mathbf{x}_i|; h_i) \delta^3(\mathbf{v} - \mathbf{v}_i)$$

## Algorithm: Gravity + Probabilistic method for elastic scattering

Consider a neighbourhood around each particle:

in pairs:

$$P_{ij} = \frac{m_i}{m_\chi} W(r_{ij}, h_i) \sigma_T(v_{ij}) v_{ij} \Delta t_i$$

total for a particle:

$$P_i = \sum_j P_{ij} / 2$$

**discrete version of the collisional operator**

A collision happens if:  $x \leq P_i$ , where x is a random number between 0 and 1

sort neighbours by distance and pick the one with:

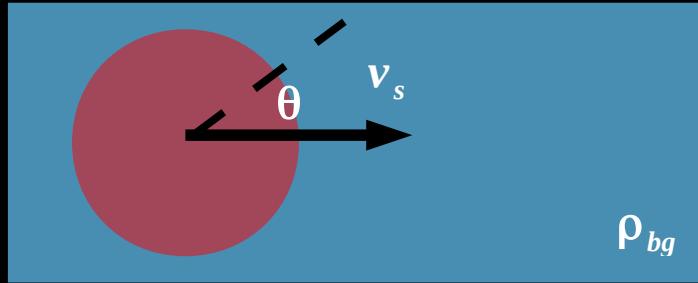
$$x \leq \sum_i^l P_{ij}$$

*Isotropic Elastic collision:*

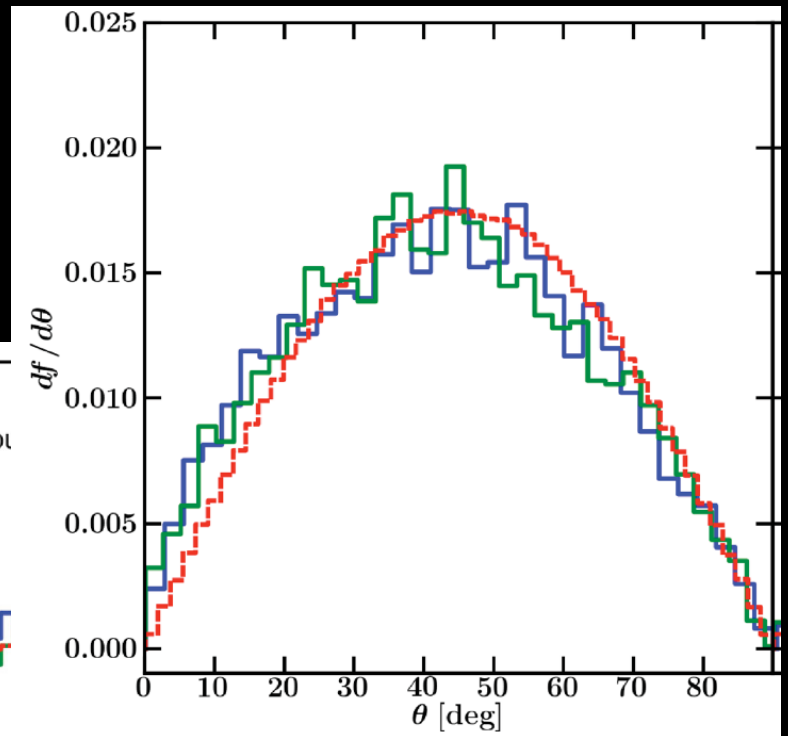
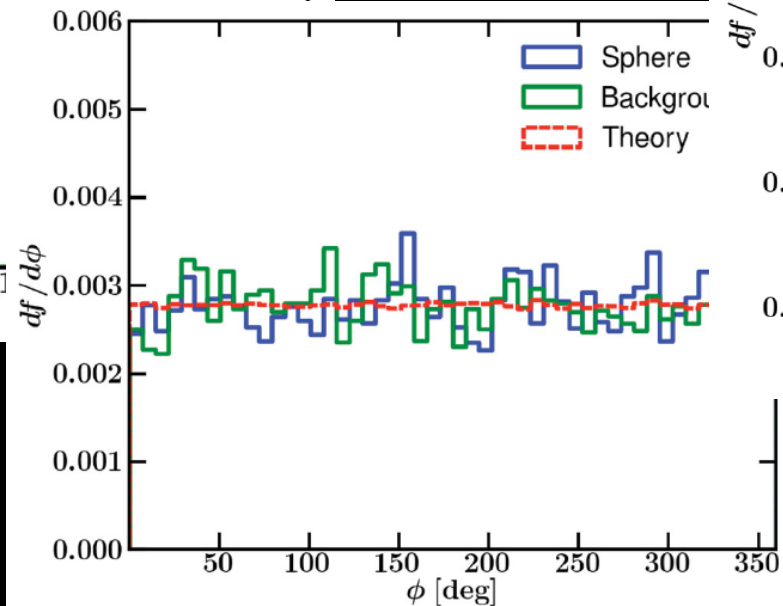
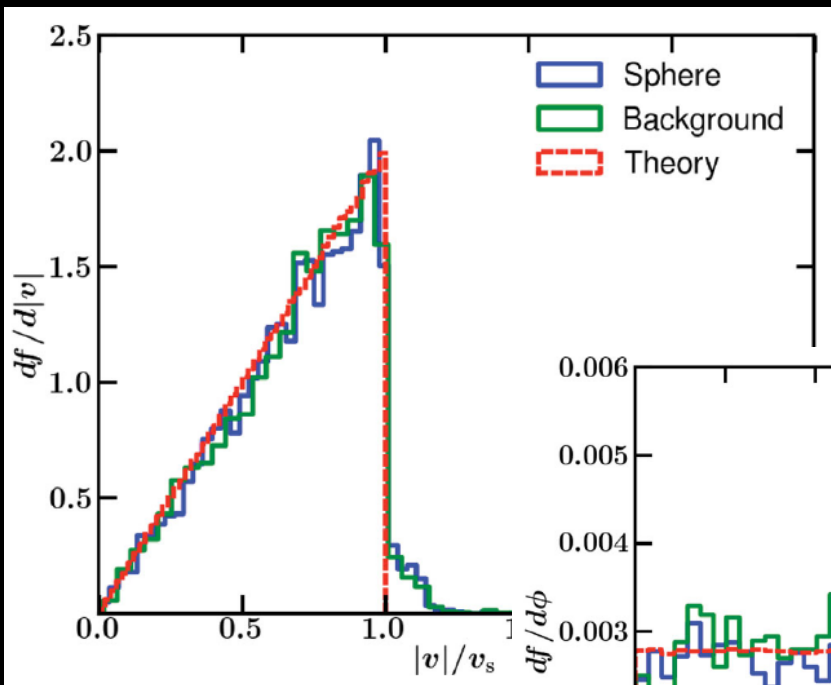
$$\begin{aligned} \vec{v}_i &= \vec{v}_{cm} + (\vec{v}_{ij}/2) \hat{e} \\ \vec{v}_j &= \vec{v}_{cm} - (\vec{v}_{ij}/2) \hat{e} \end{aligned}$$

*randomly scattered*

# DM self-collisions in N-body simulations (probabilistic approach: simple kinematic test)



$$N_{\text{exp}}(t) = \sum_{i \in S, j \in \text{bg}} P_{ij} = N_s(\sigma/m)\rho_{\text{bg}}v_s t$$



# Structure of SIDM haloes

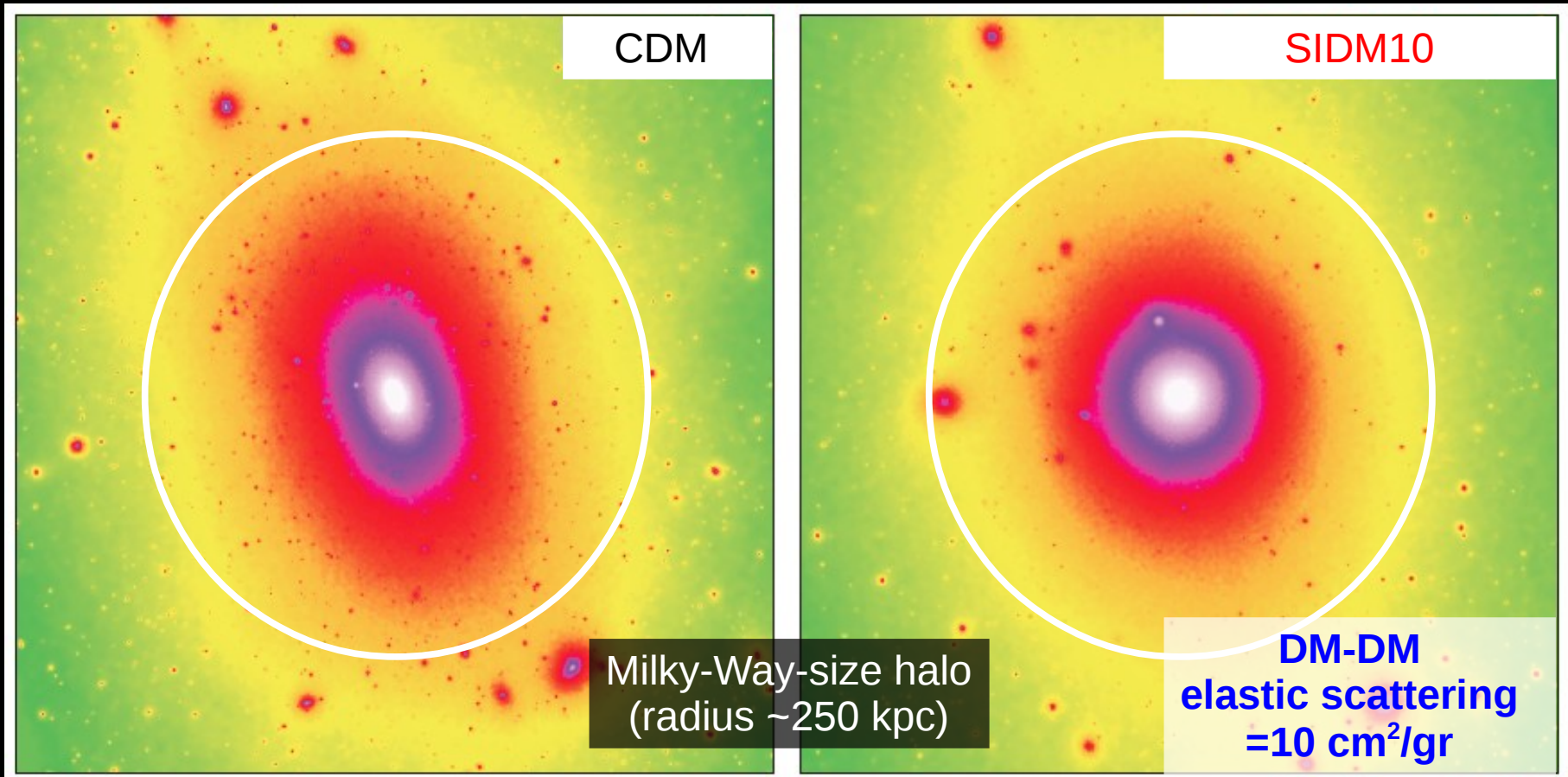
# Structure of SIDM haloes

If gravity is the only relevant DM interaction, the central density of haloes is ever increasing

With strong self-interactions ( $\sigma/m \gtrsim 0.5 \text{ cm}^2/\text{gr}$ ) DM haloes develop nearly spherical “isothermal” cores

M. Vogelsberger’s talk

Vogelsberger, Zavala & Loeb 2012



DM-only simulations

(Carlson+92, Spergel & Steinhardt 00, Yoshida+00, Davé+01, Colín+02, Rocha+13, Peter+13....)

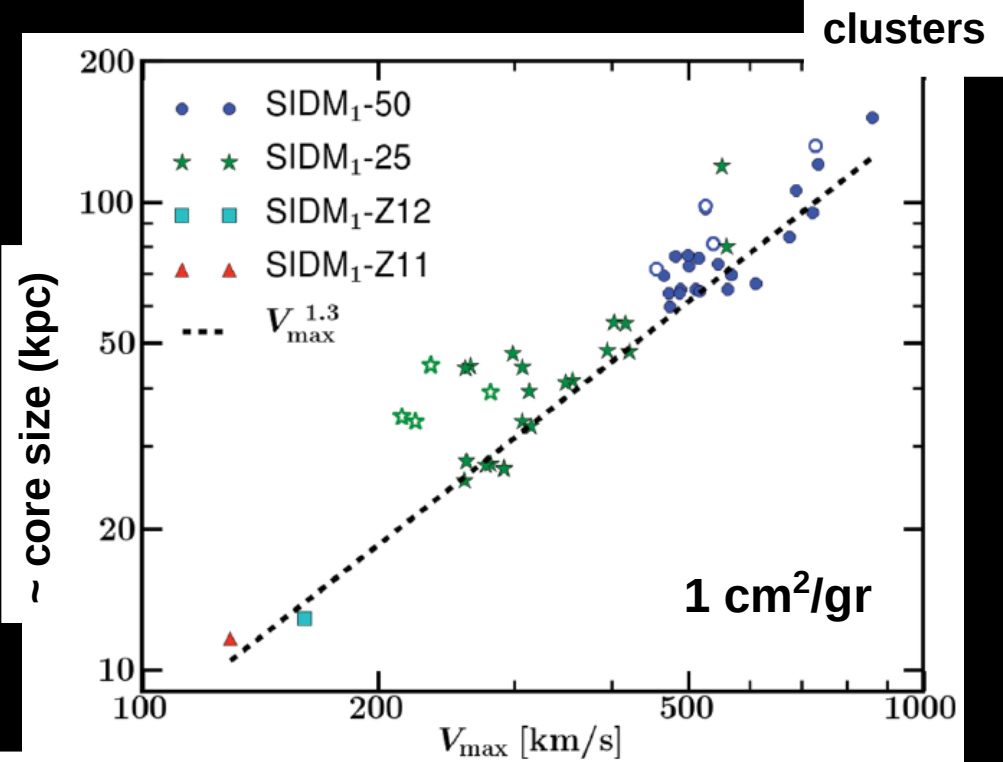
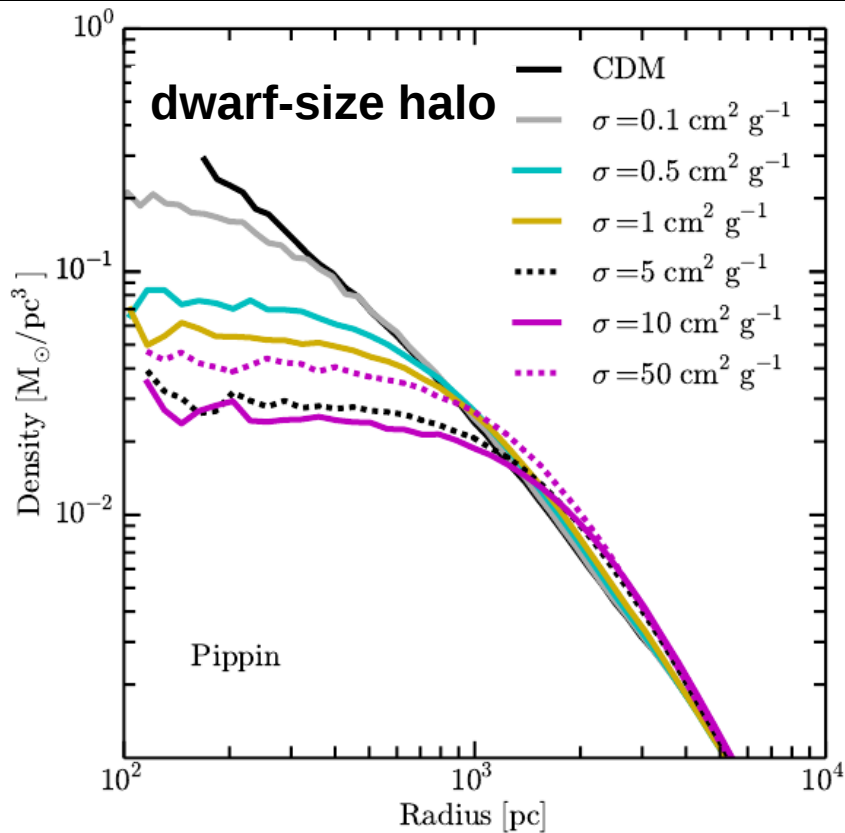


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spherically averaged  
DM distribution



dwarfs

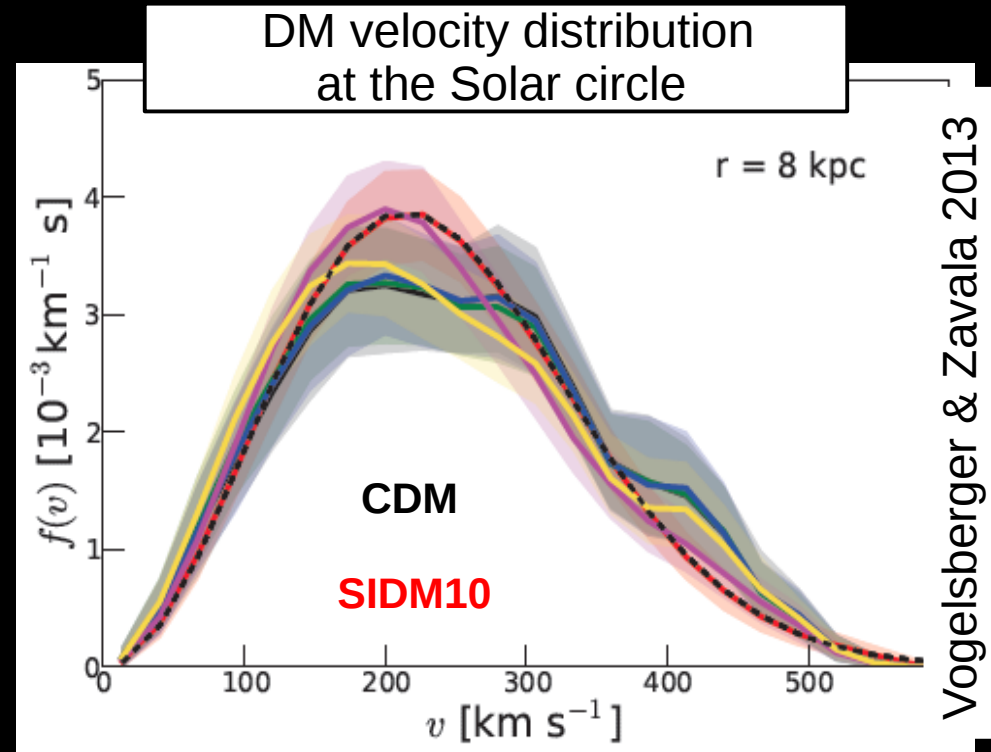
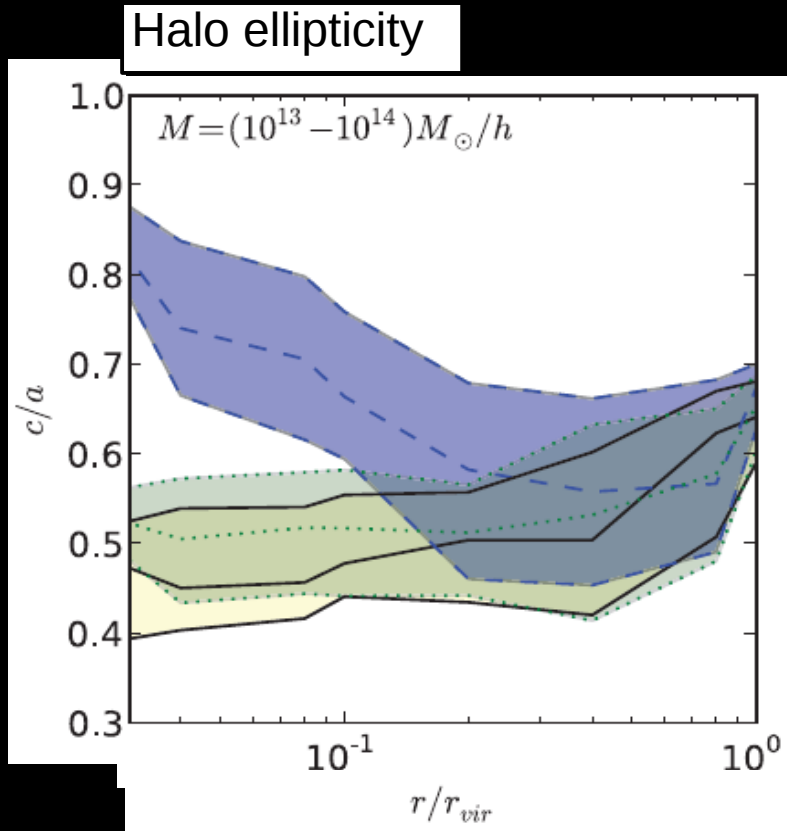
Rocha et al. 2013

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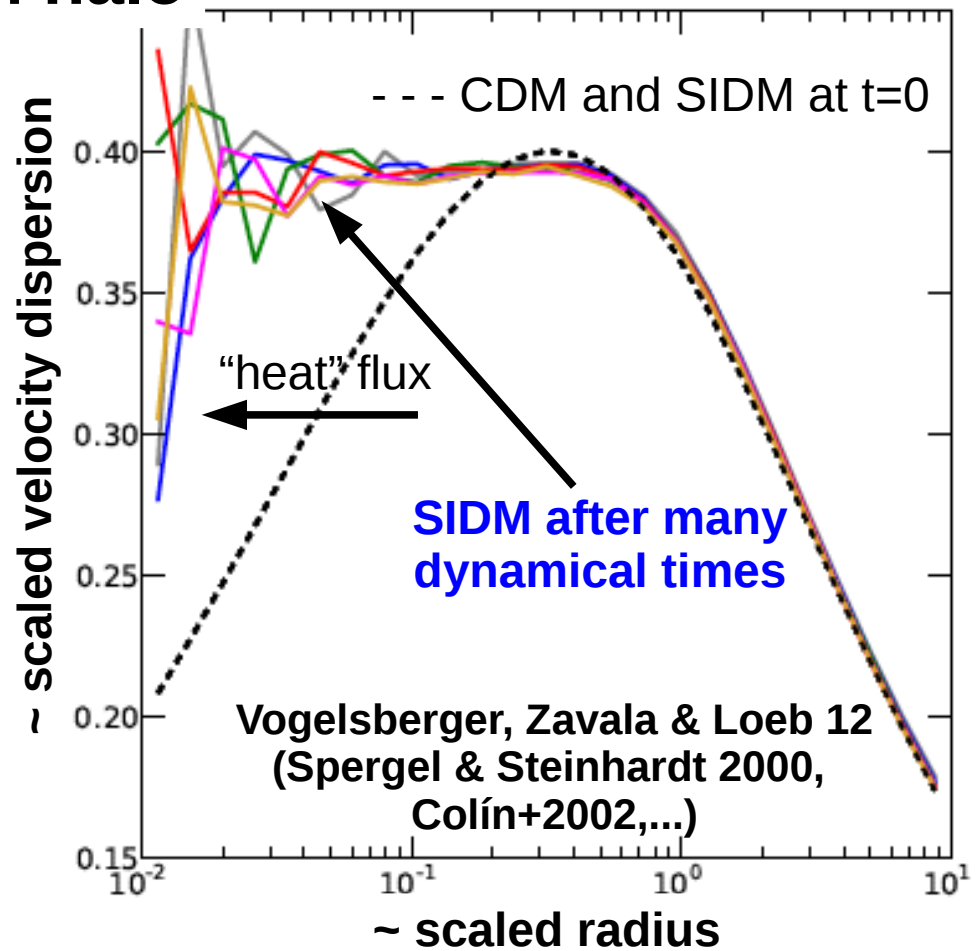
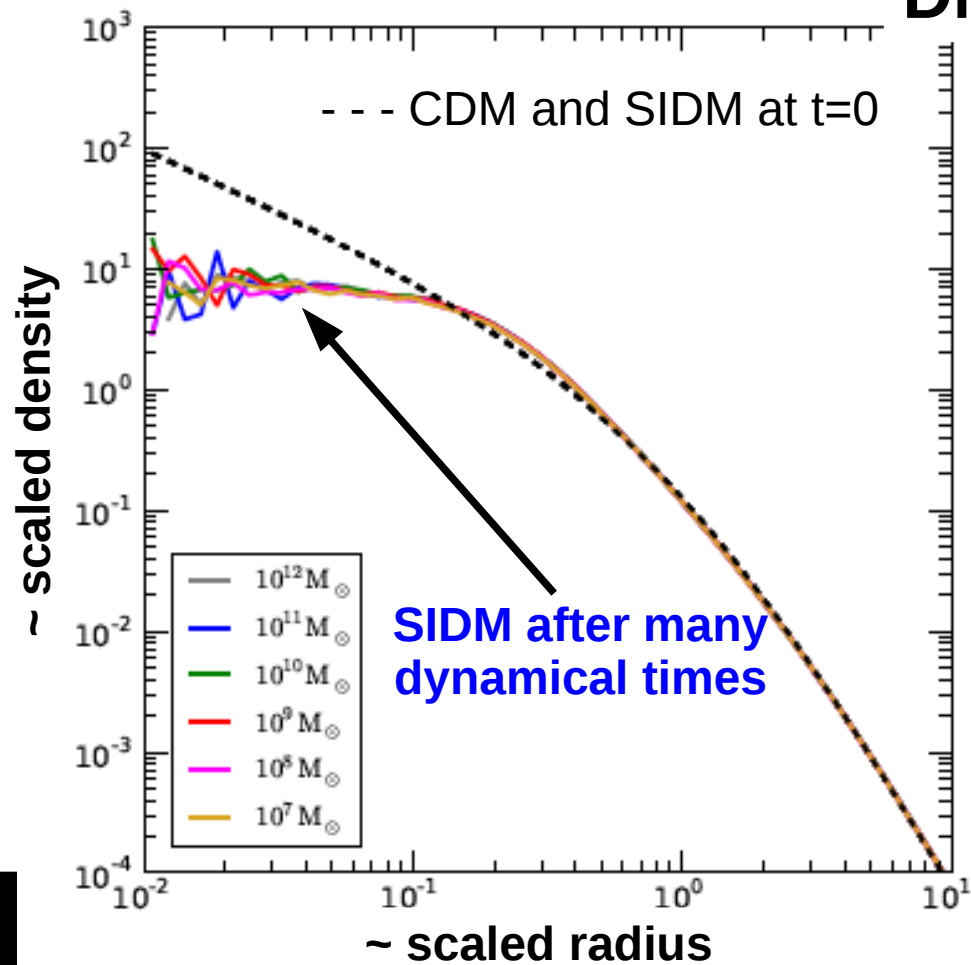
Peter et al. 2013



Vogelsberger & Zavala 2013

T. Brinckmann's talk

# Core formation with DM self-collisions



# DM self-collisions (gravothermal fluid approximation)

*spherically symmetric ideal gas  
in hydrostatic equilibrium*  
Lynden-Bell & Eggleton 1980

since  $Kn \sim 1$  conductivity is found as an  
empirical interpolation between fluid  
and collisionless regimes

$$\frac{\partial(\rho v^2)}{\partial r} = -\frac{GM\rho}{r^2}$$

isotropic  
Jeans equation

$$\lambda \rightarrow l_{mean} = 1/(\rho \sigma) \quad Kn \ll 1$$

$$\lambda \rightarrow \lambda_J^2 = v^2 / (4\pi G \rho) \quad Kn \gg 1 \quad (LBE)$$

heat flux

$$\frac{L}{4\pi r^2} = -\kappa \frac{\partial T}{\partial r}$$

conductivity

requires calibration from N-body sims

$$\frac{\partial L}{\partial r} = -4\pi \rho r^2 v^2 \left( \frac{\partial}{\partial t} \right)_M \ln \frac{v^3}{\rho},$$

1<sup>st</sup> law

mass shell

$$\kappa \sim (3k/2m) \rho \lambda^2 / \tau$$

$\tau \equiv$  relaxation time

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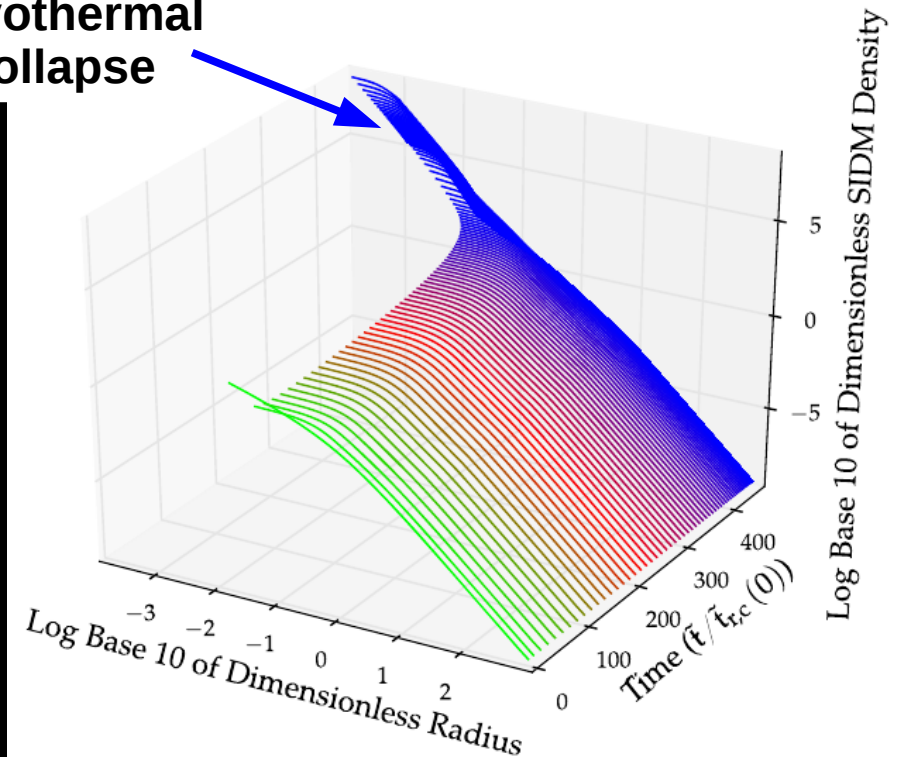
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$$\lambda \rightarrow \lambda_J^2 = v^2/(4\pi G\rho) \quad Kn \gg 1 \quad (LBE)$$

gravothermal  
collapse



Pollack, Spergel & Steinhardt 2015

# DM self-collisions (isothermal solution to 'relaxed' SIDM haloes)

isotropic  
Jeans equation

$$\frac{\partial(\rho v^2)}{\partial r} = -\frac{GM\rho}{r^2}$$

+

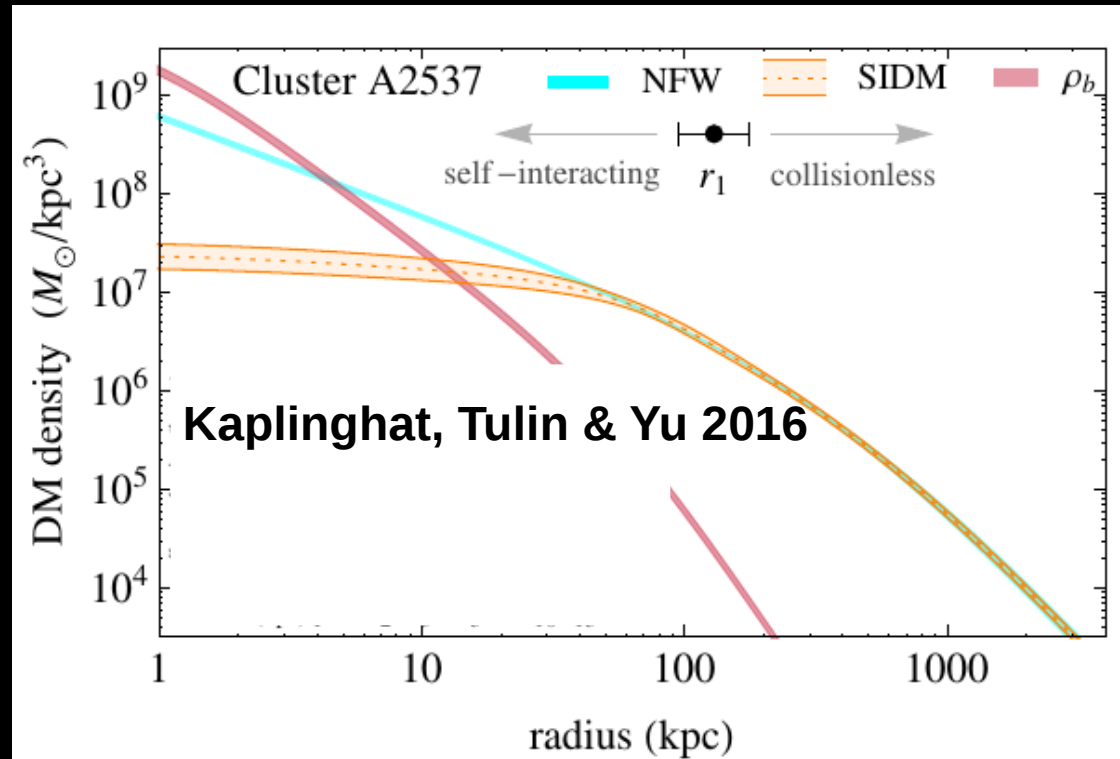
isothermal core

+

DM is eff. collisionless beyond  $r_1$ , which is given by the condition:

**~ 1 scatter / particle / Hubble time**

$$\rho_{\text{dm}}(r) = \begin{cases} \rho_{\text{iso}}(r), & r < r_1 \\ \rho_{\text{NFW}}(r), & r > r_1 \end{cases}$$



O(10-15%) level agreement with N-body  
straightforward to add a baryonic component

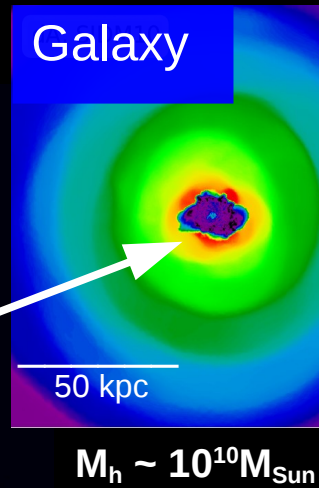
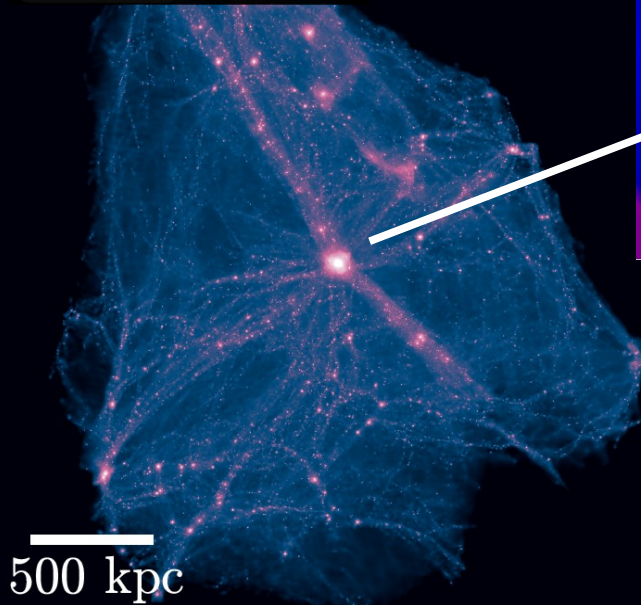
# **SIDM structure formation theory: beyond DM physics**

# Full structure formation theory in SIDM (self-scattering DM + baryonic physics)

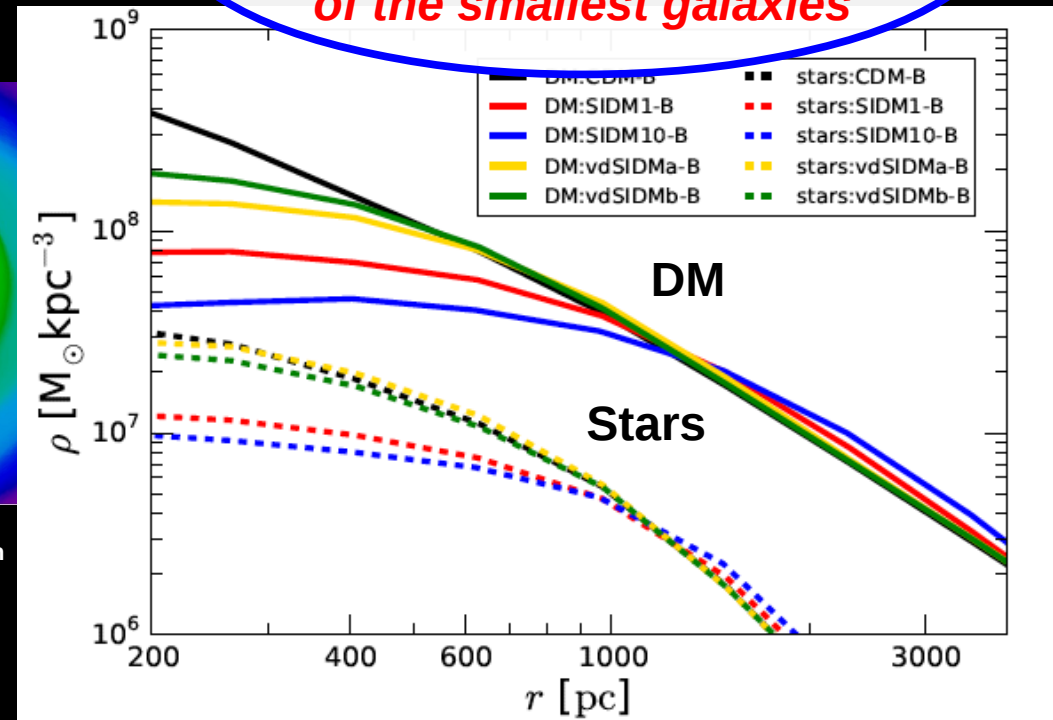
“baryonic physics”: hydrodynamics, radiative cooling of gas, stellar population modelling, SNe feedback (**non-bursty**)

simulation of a galaxy in  
Self-Interacting DM  
(Vogelsberger, Zavala +14)

dark matter



The signature of DM collisions could be imprinted in the stellar distribution of the smallest galaxies



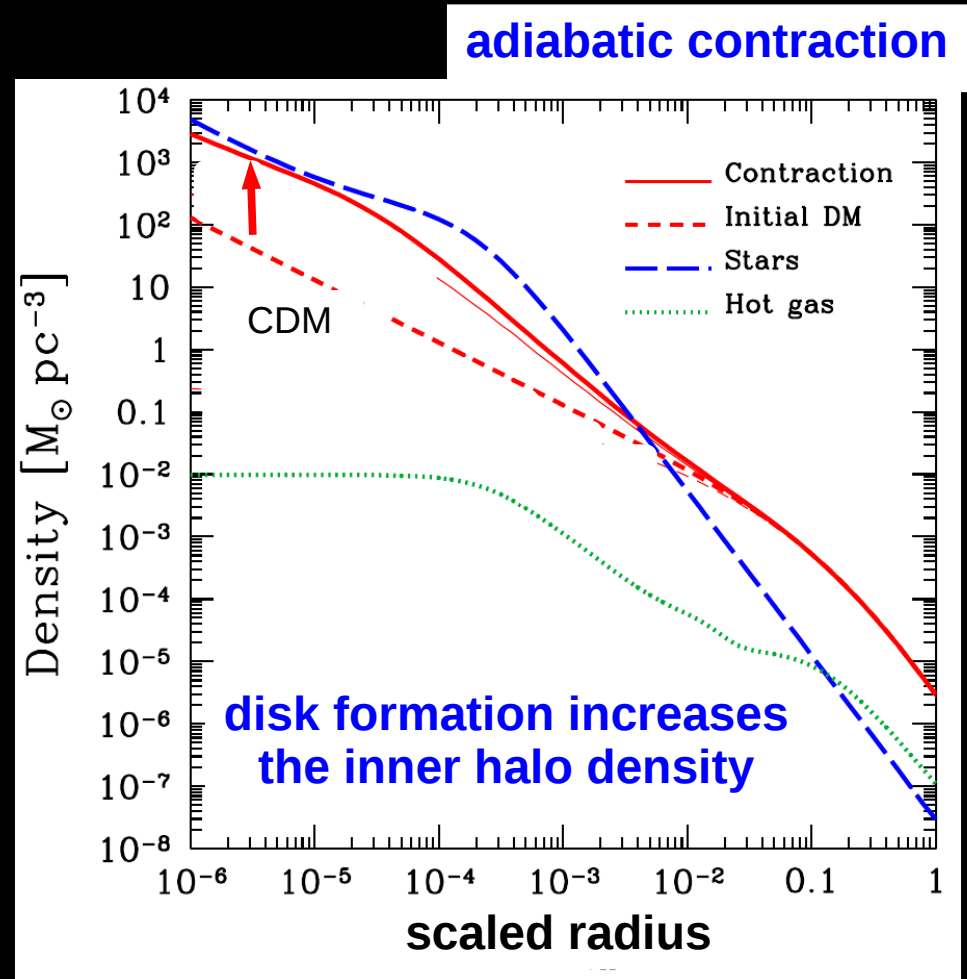
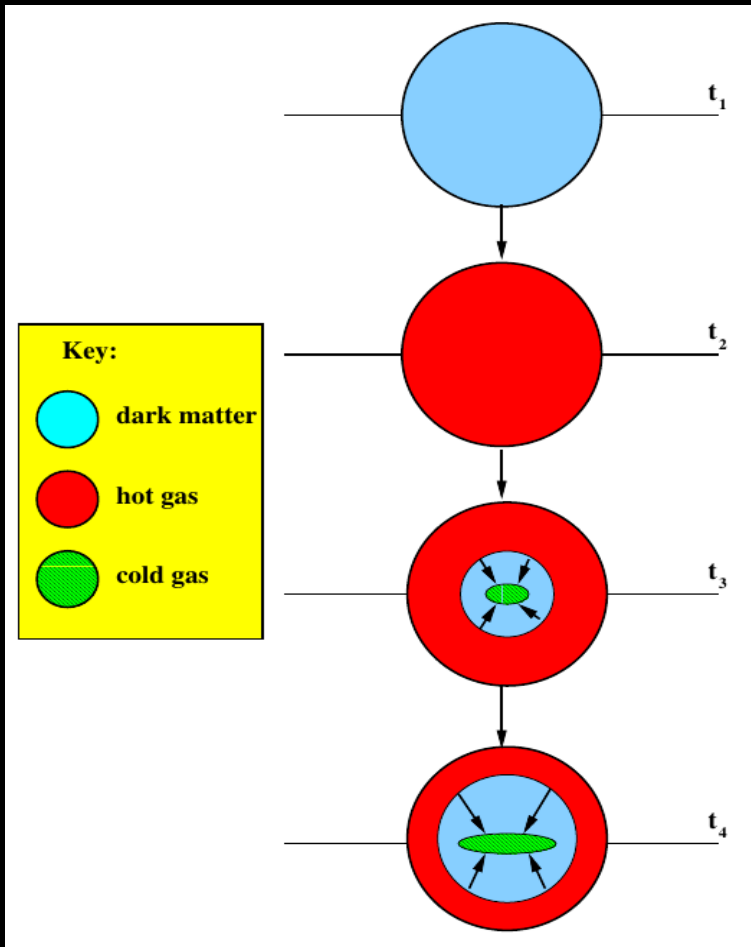
$$\sigma/m = 1 \text{ cm}^2/\text{gr}$$

$$\sigma/m = 10 \text{ cm}^2/\text{gr}$$



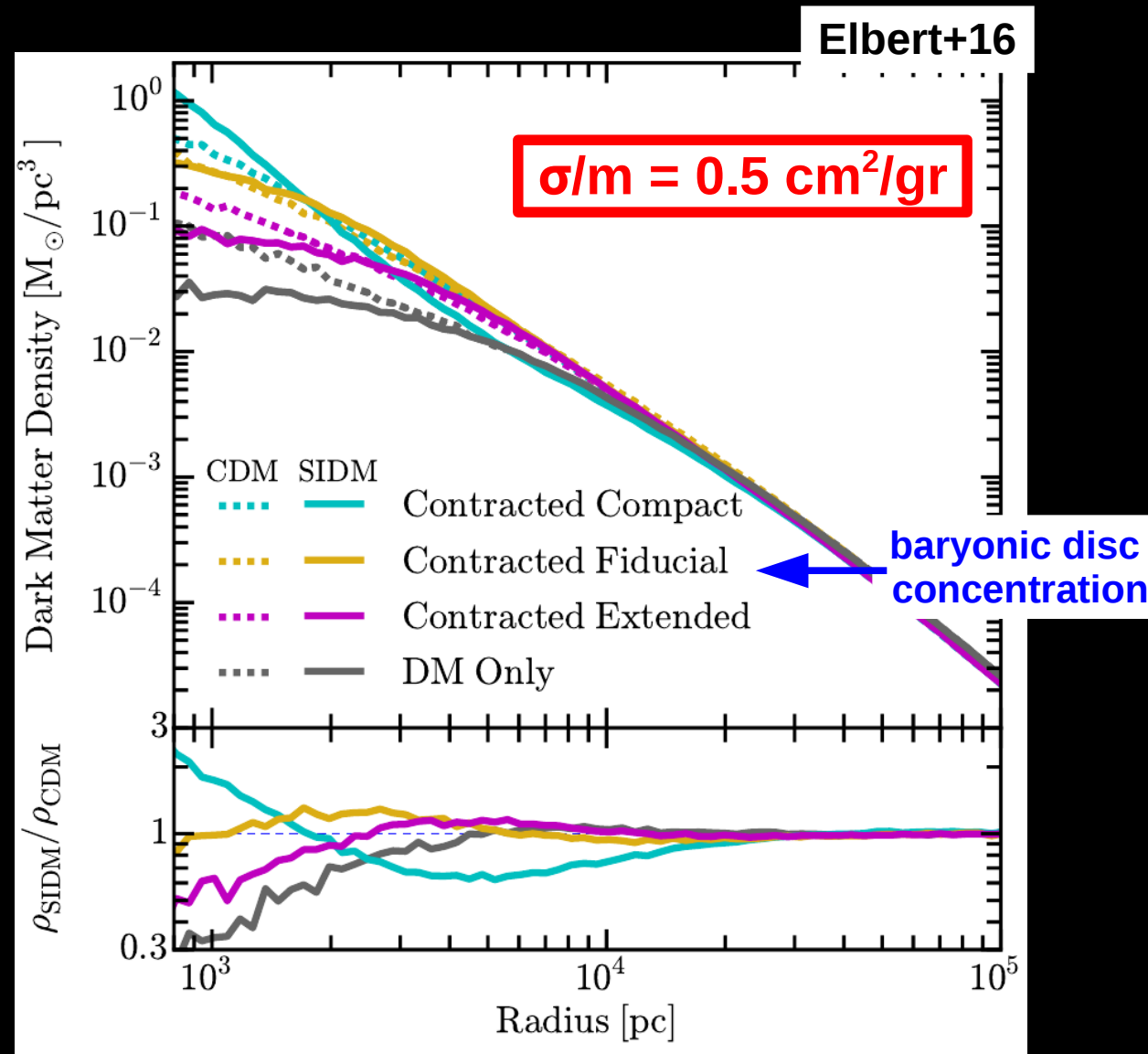
# The challenging interplay between DM/baryonic physics

Baugh 2006



# The challenging interplay between DM/baryonic physics

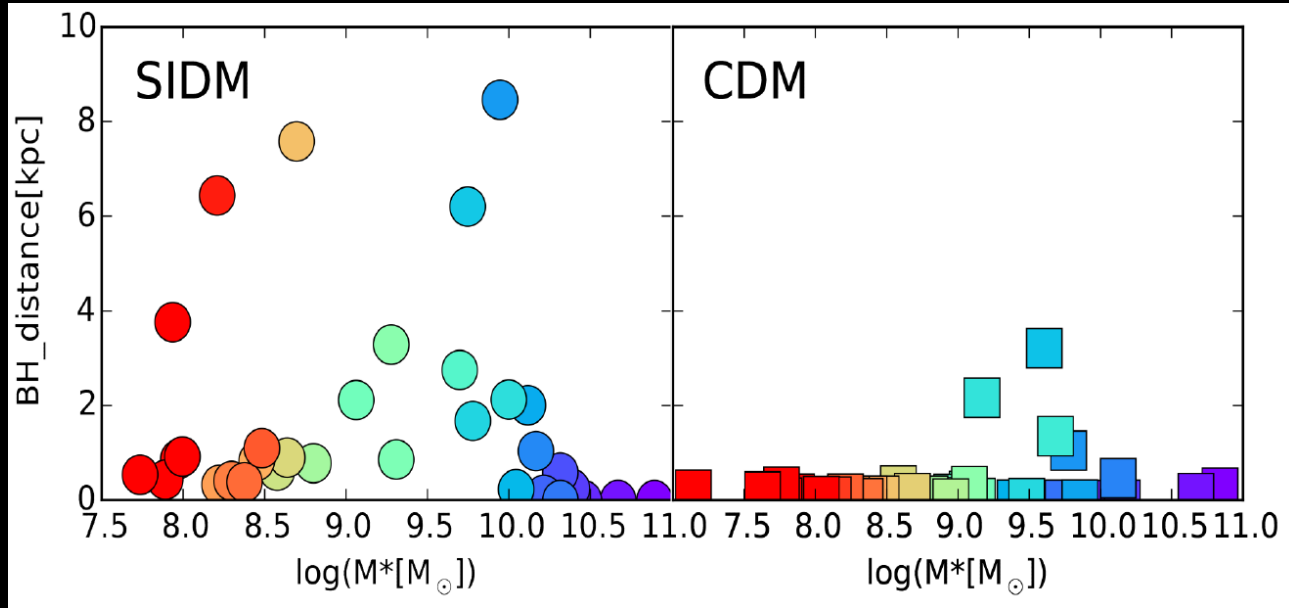
*If DM self-collisions are important, a strong contraction due to a compact massive disc, can lead to core collapse*



Milky-Way-size simulation: DM and stars (by hand)

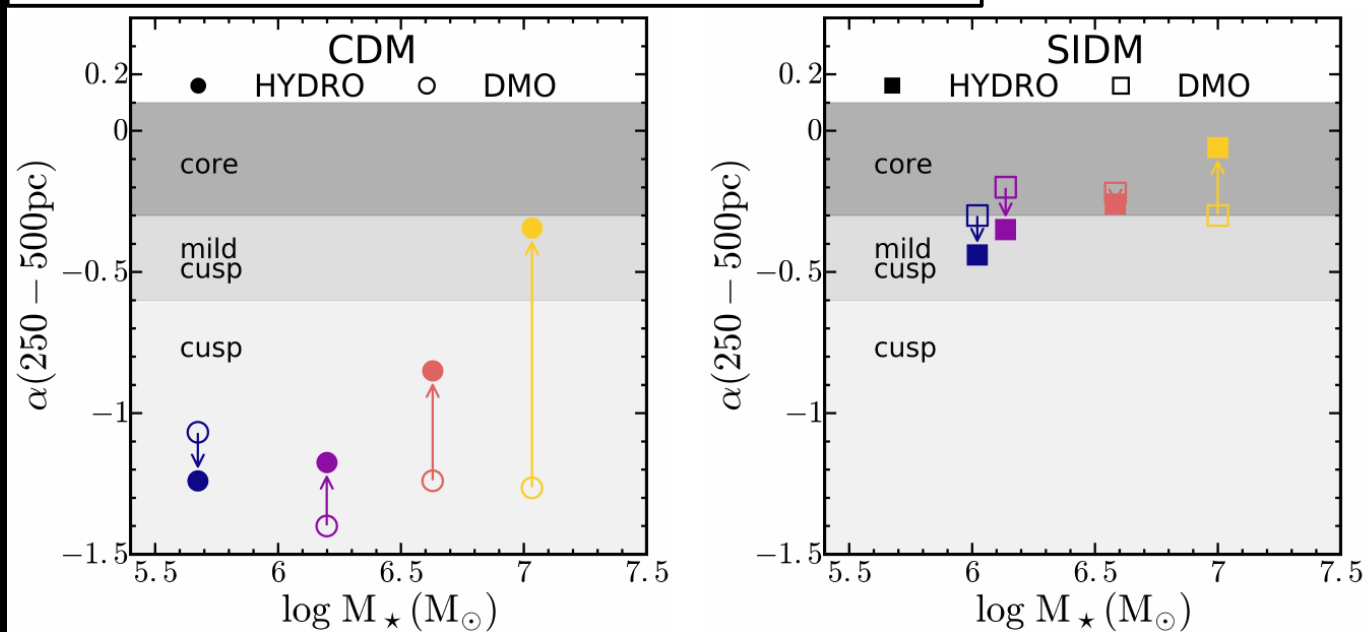
# Disentangling dark from baryonic physics

SMBH offsets



Di Cintio et al. 2017

## SNe-driven DM cores inefficient at low M\*

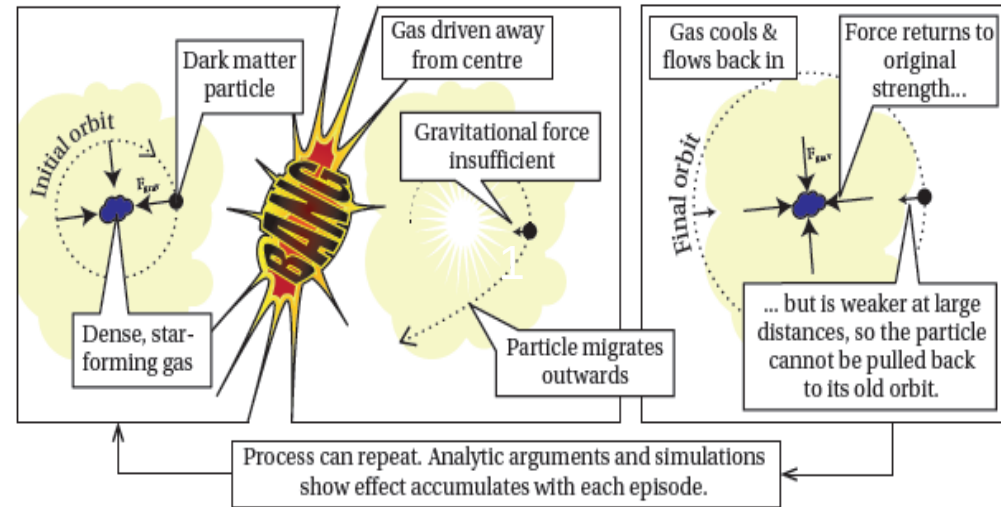
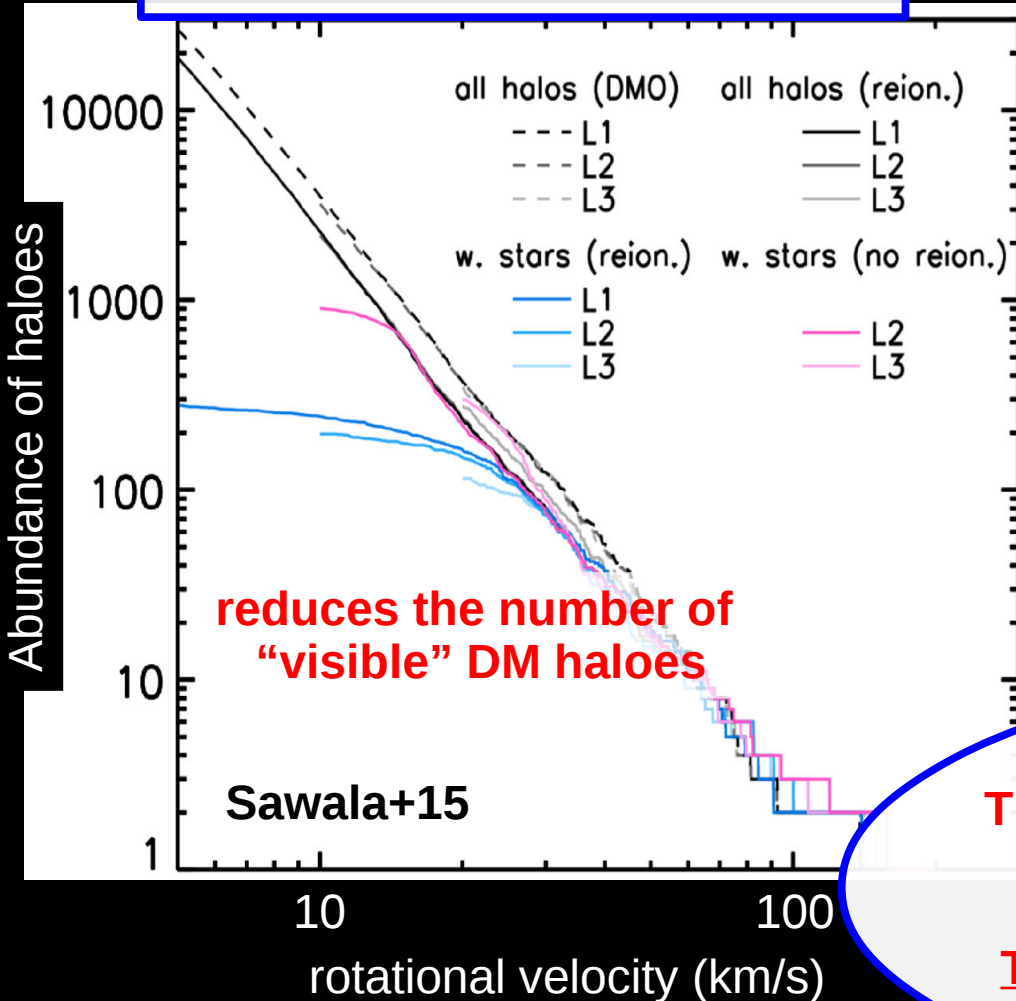


Robles et al. 2017

# The complexity of gas and stellar physics

Gas heating (UV background from first generation of stars/galaxies)

Gas and DM heating through supernovae



Credit: Pontzen & Governato 2014

reduces the inner density of DM haloes

These mechanisms are certainly there, but how efficient they are remains unclear

To some extent, they are degenerate with new DM physics

# Concluding remarks

- The window for the DM particle nature to be relevant for structure formation is narrow and within reach of upcoming observations

SIDM transfer cross section

$$0.1 \text{ cm}^2 / \text{gr} \lesssim \sigma / m \lesssim 2 \text{ cm}^2 / \text{gr}$$

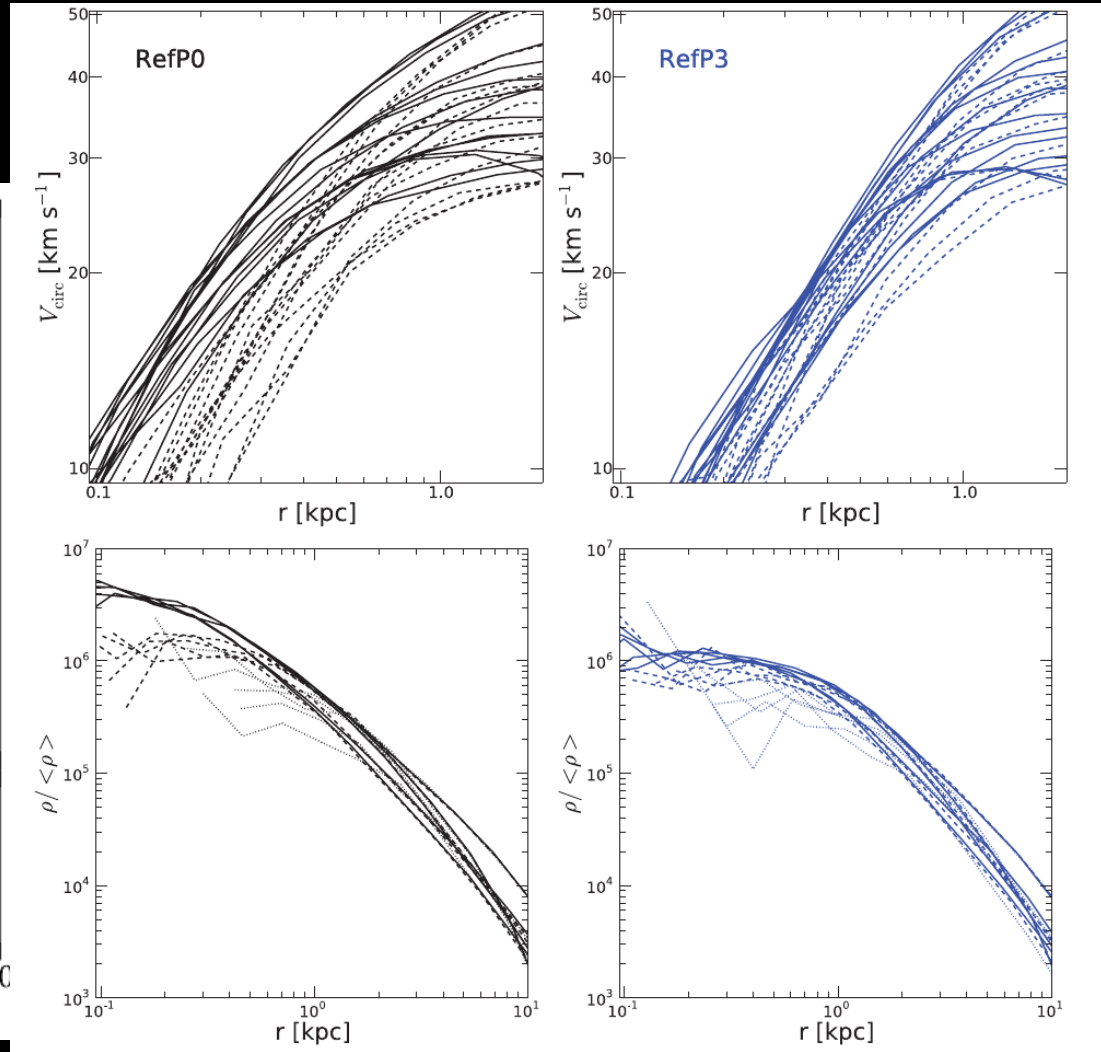
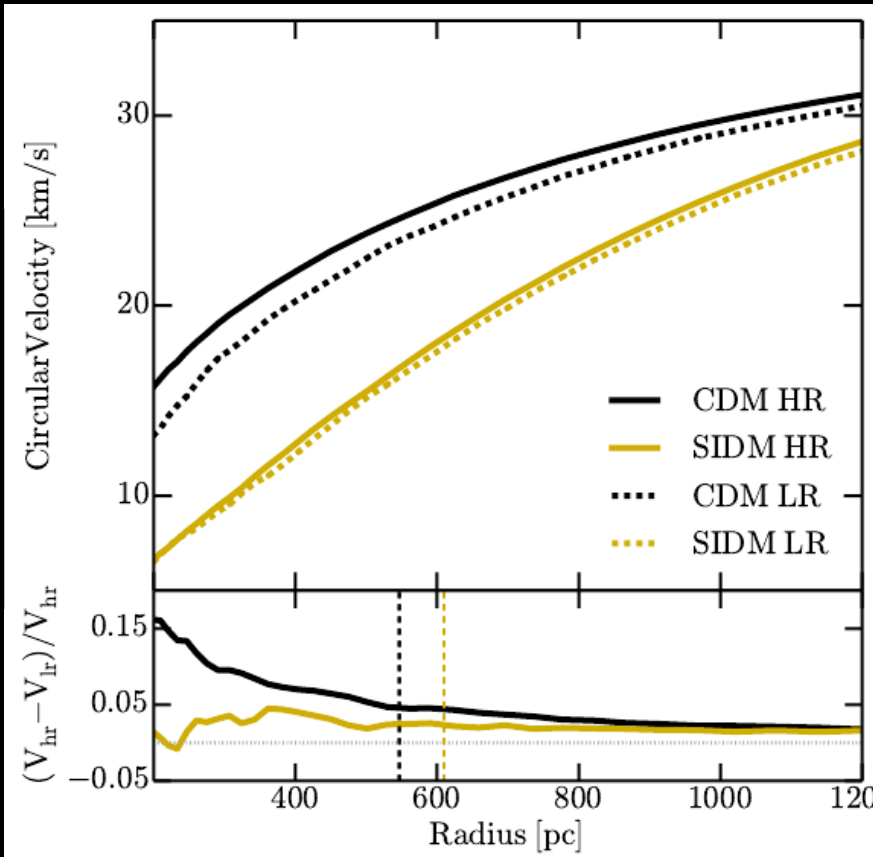
'cutoff' halo mass at  $z=0$

$$10^{9.5} M_{\text{Sun}} \lesssim M_{\text{cut}} \lesssim 10^{10.5} M_{\text{Sun}}$$

- DM self-collisions could have an impact in the non-linear evolution of haloes:
  - Spherical, Maxwellian DM cores of size  $\sim r_{\text{max}}$  are the quasi-equilibrium stage of SIDM haloes if the central scattering rate per particle is  $\sim 1/t_{\text{H}}$
  - Gravo-thermal collapse is the natural outcome for large time scales (and/or  $\sigma/m$ )
  - The DM/baryonic physics synergy remains largely unexplored: possible degeneracies in observational comparisons, albeit undesirable, reflect our current incomplete knowledge of the DM nature and galaxy formation/evolution
  - Looking for subtle changes beyond  $r_{\text{max}}$  is a promising avenue to avoid the complexities of baryonic physics (T. Brinckmann's talk)

# DM self-collisions in N-body simulations (probabilistic approach: convergence)

Elbert et al. 2015



Vogelsberger et al. 2012