

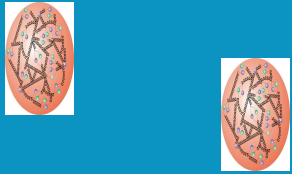


# “Long-range collectivity” in small systems

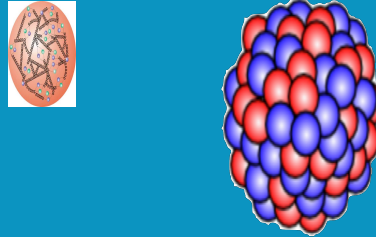
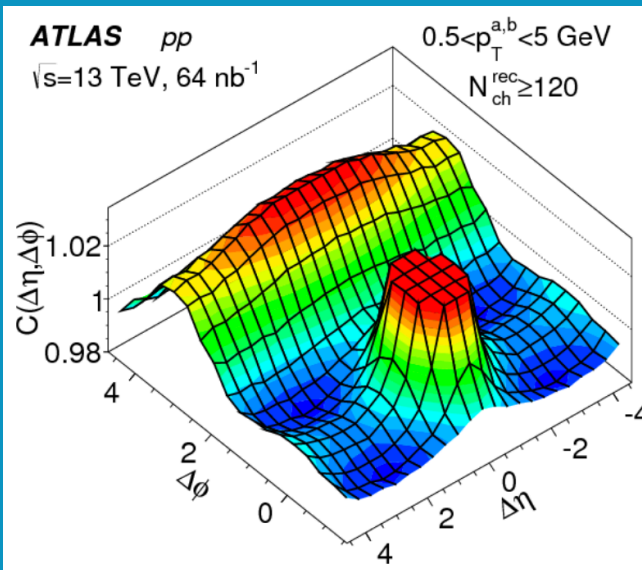
Jiangyong Jia, BNL and Stony Brook University

- What is collectivity?
- How to distinguish initial vs final state effects ?
- How are cumulants related to collectivity?

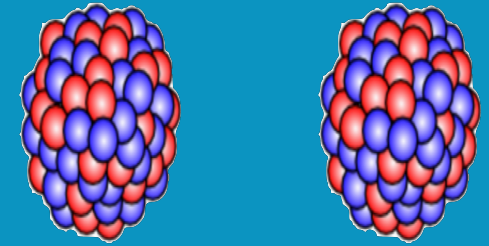
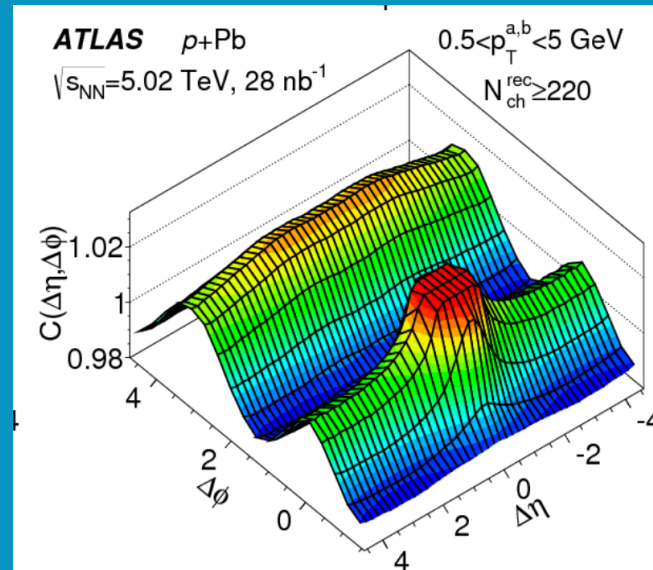
# Long-range collectivity in different systems



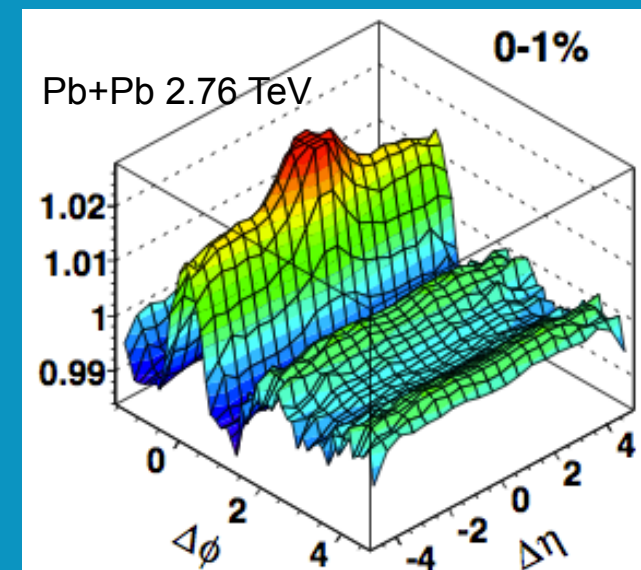
p+p



p+Pb



Pb+Pb



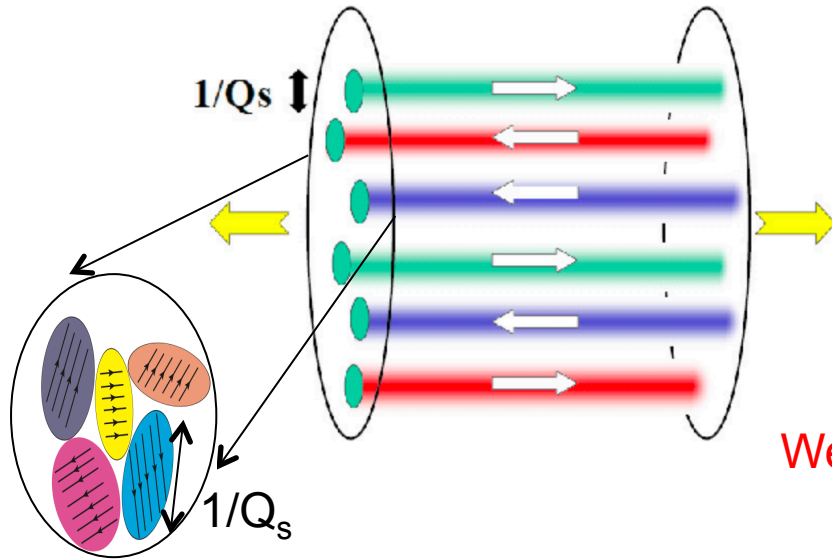
■ Long-range correlation in momentum space comes

- directly from early time  $t \sim 0$  (CGC)
- or it is a final state response to spatial fluctuation at  $t=0$  (hydro).

**What is the timescale for emergence of collectivity?**

# Examples of initial vs final state scenarios

## CGC



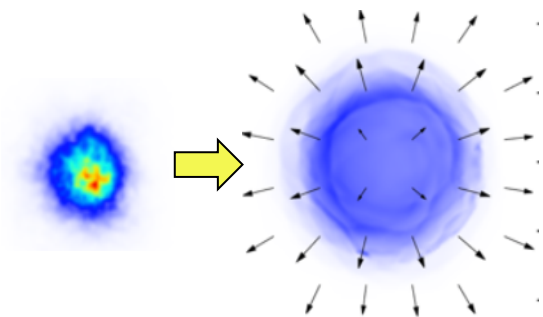
Domain of color fields of size  $1/Q_s$ , each produce multi-particles correlated across full  $\eta$ .

Uncorr. between domains, strong fluct. in  $Q_s$

More domains, smaller  $v_n$ , more  $Q_s$  fluct, stronger  $v_n$

Well motivated model framework, lack systematic treatment

## Hydro



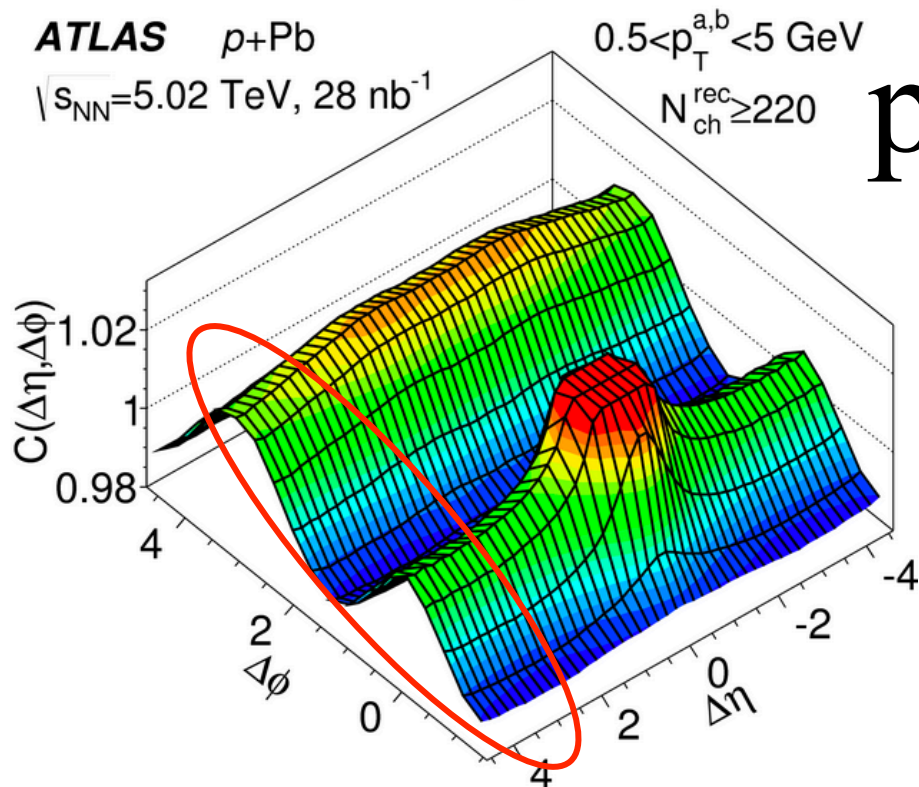
Hot spots (domains) in transverse plane e.g IP-plasma, boost-invariant geometry shape

Expansion and interaction of hot spots generate collectivity

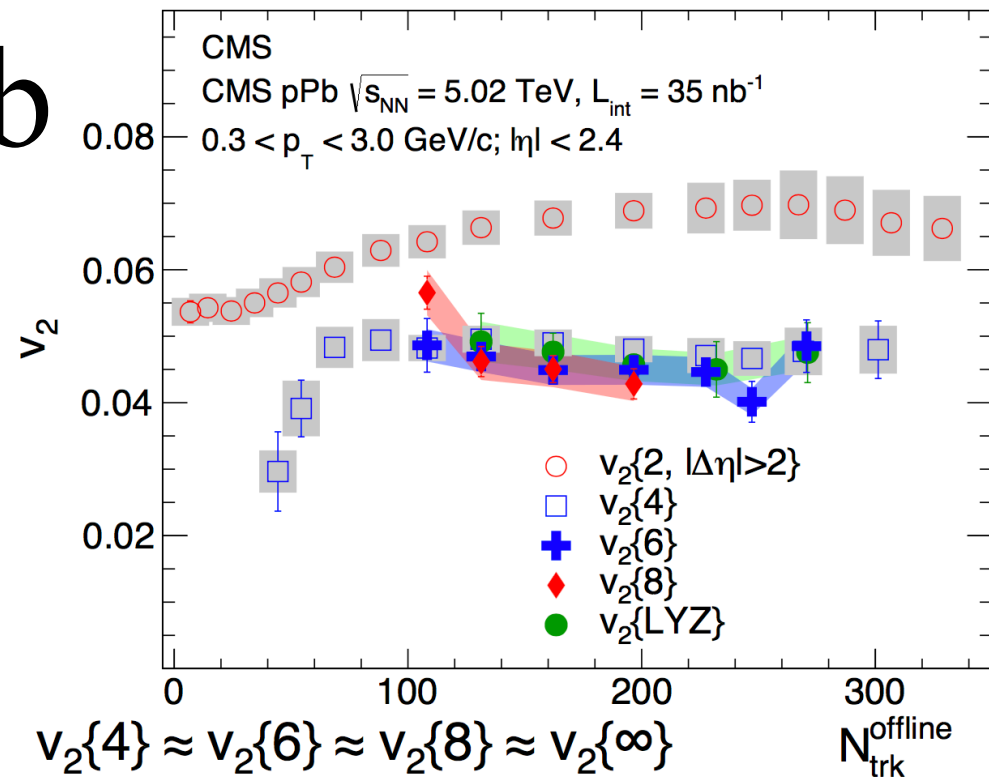
$v_n$  depends on distribution of hot spots ( $\epsilon_n$ ) and transport properties.

Ongoing debate whether hydro is applicable in small systems

# Features of collectivity in HM pPb

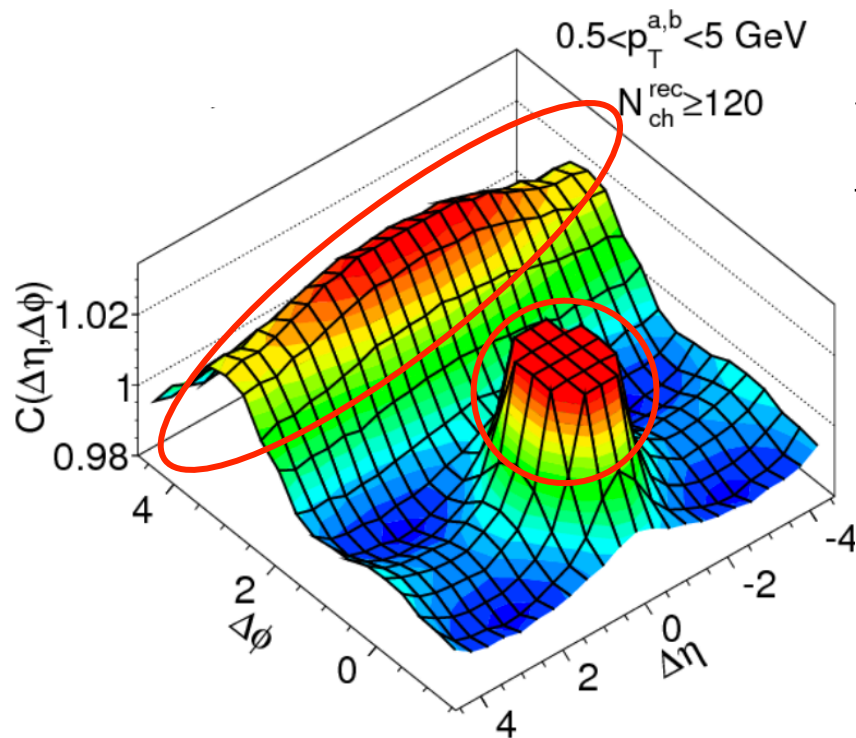


Long-range in  $\eta$

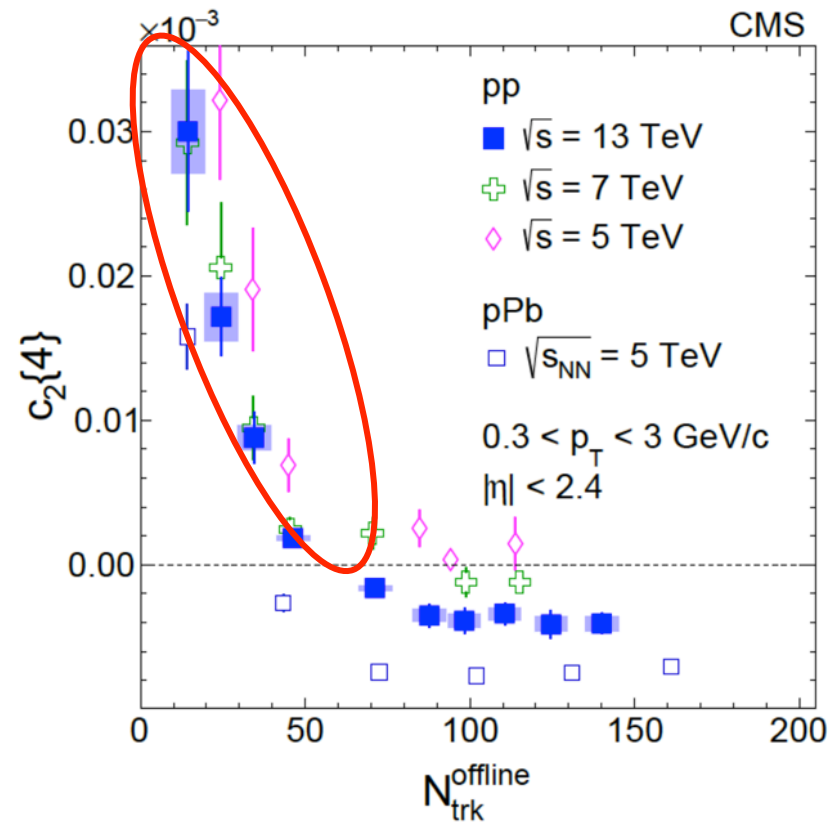


Multi-particle signals

# Features of collectivity in HM pp



pp



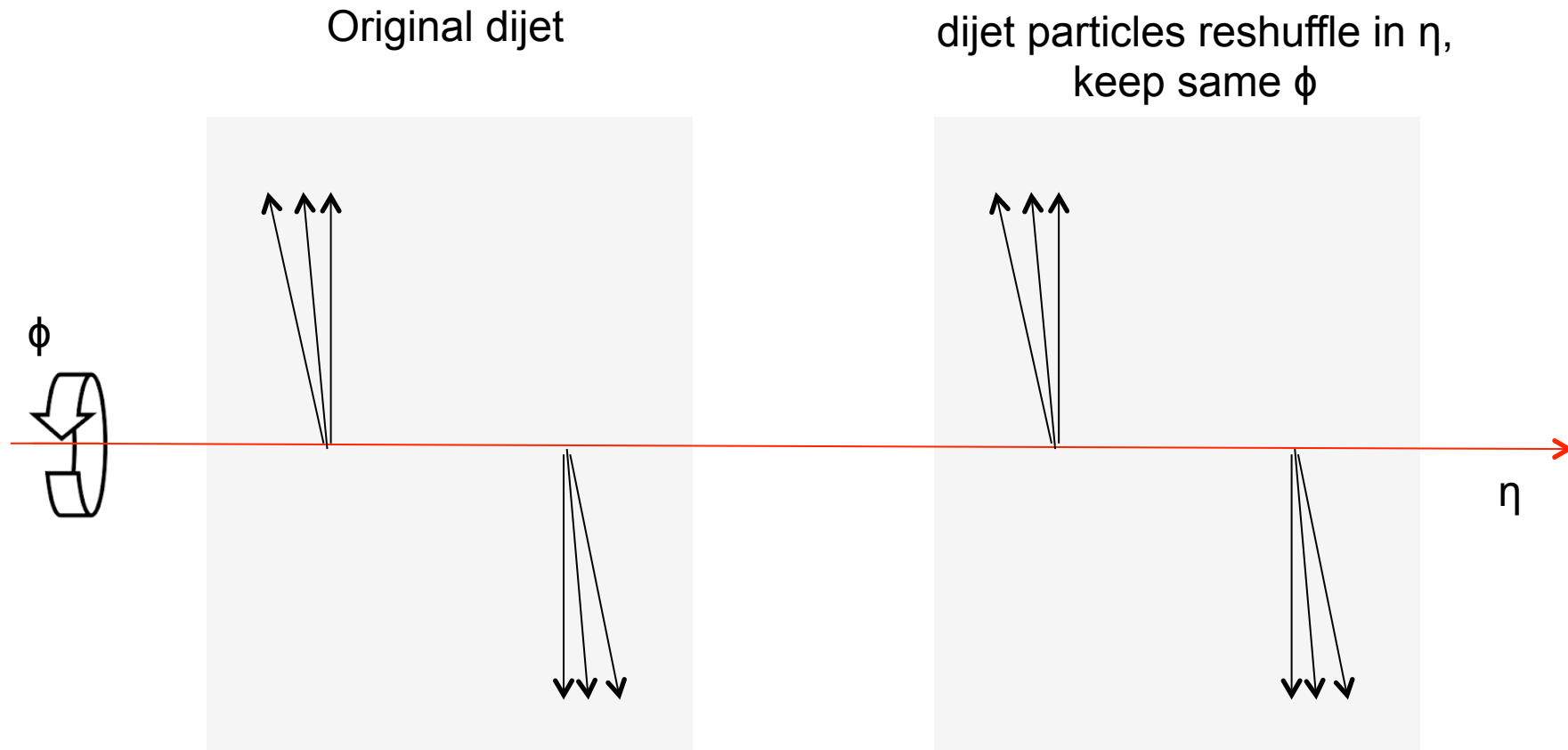
Long-range in  $\eta$

Multi-particle signals

Non-flow can generate long-range (away-jet) or multi-particle correlation (fragmentation) but not both

Collectivity must mean both

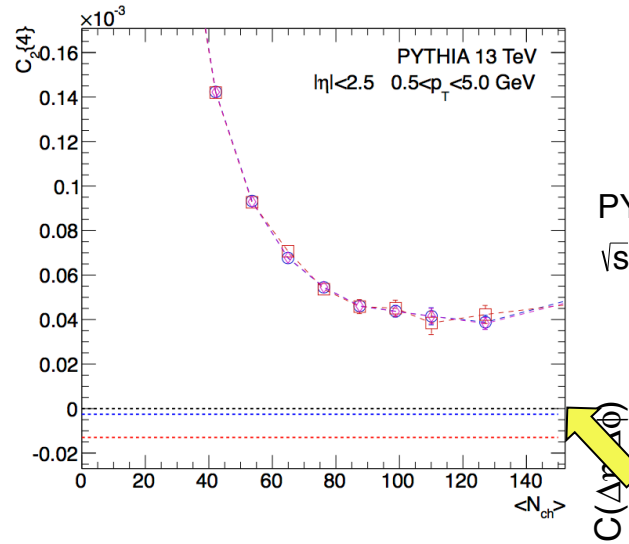
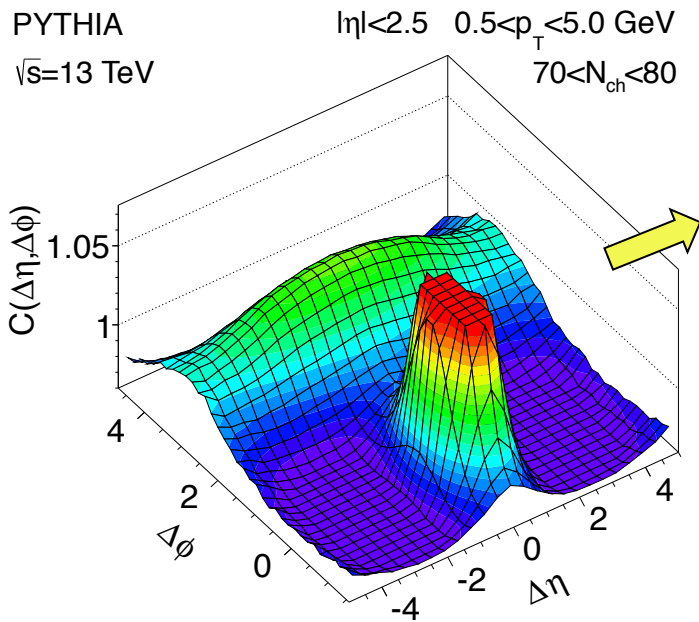
# Azimuthal correlation from collectivity



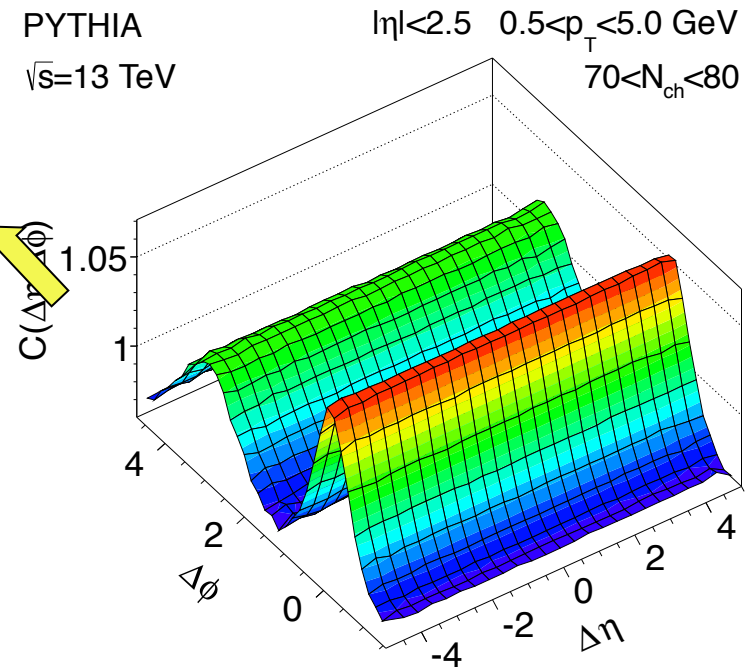
They give the same flow coefficient  $c_n\{4\}$  and  $v_n\{4\}$ , although clearly the first case is **non-flow** and the second case would be classified as **flow**

# Azimuthal correlation from collectivity

original



$\eta$  reshuffled



By mingliang Zhou

They give the same flow coefficient  $c_n\{4\}$  and  $v_n\{4\}$ , although clearly the first case is **non-flow** and the second case would be classified as **flow**

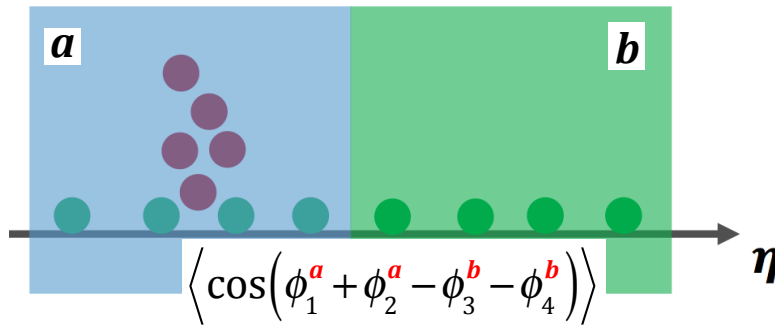
**Azimuthal corr. alone can't distinguish flow & non-flow.**

# Long-range collectivity via subevent cumulants

arXiv:1701.03830

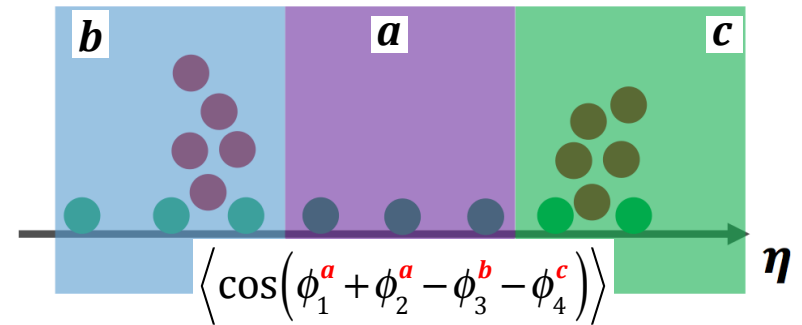
Event with jet

Event with dijet



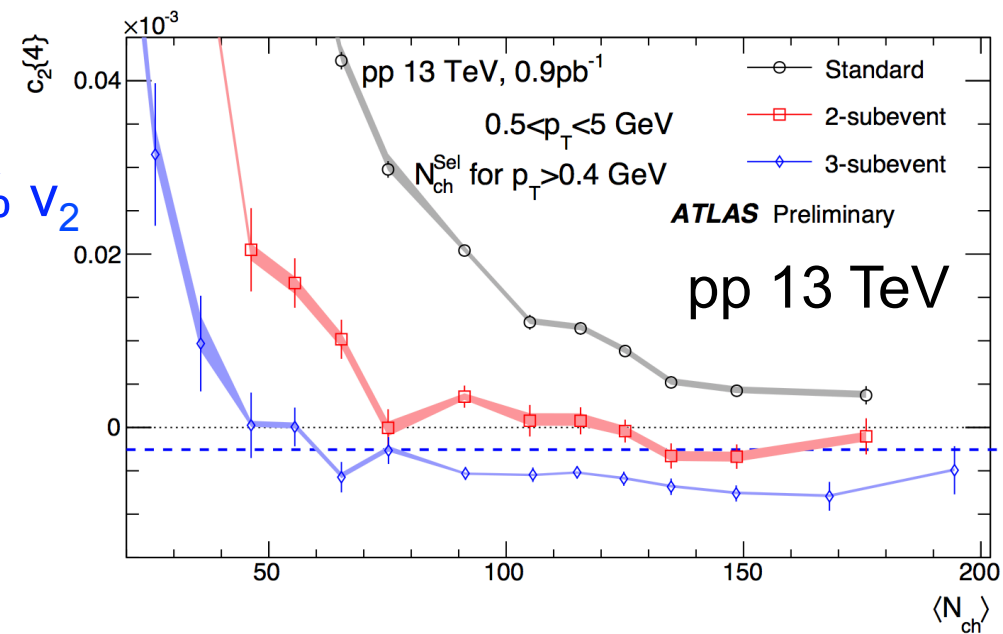
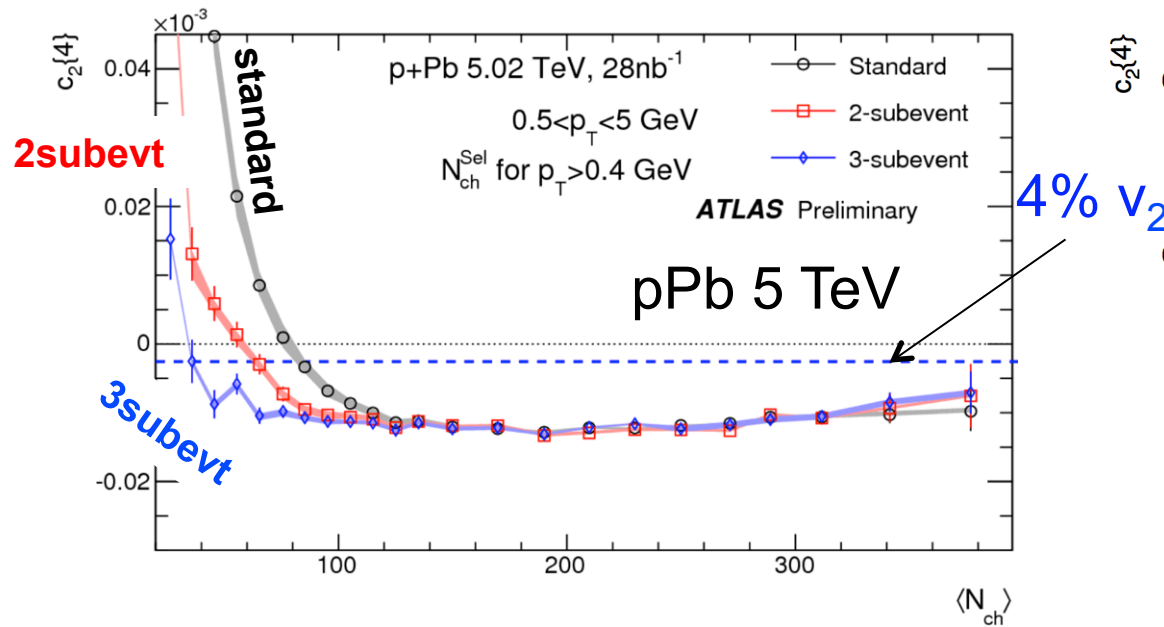
2 sub-event

removes intra-jet correlations



3 sub-event

removes inter-jet correlations



pPb: methods consistent for  $N_{ch} > 100$ , but split below that

pp: Only subevent method gives reliable negative  $c_2\{4\}$  in broad range of  $N_{ch}$



# Sign-change of $c_2\{4\}$

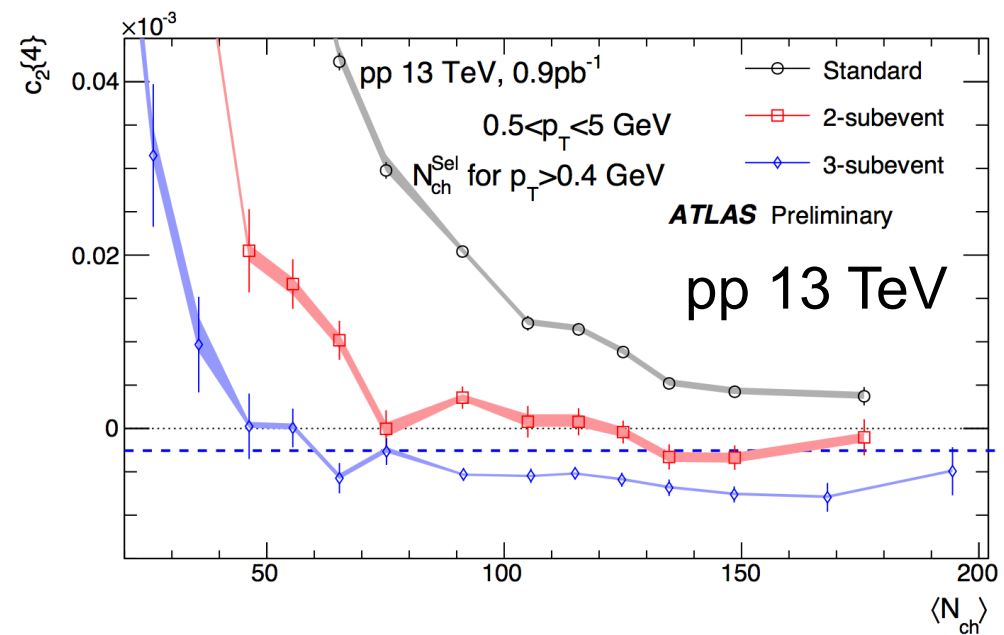
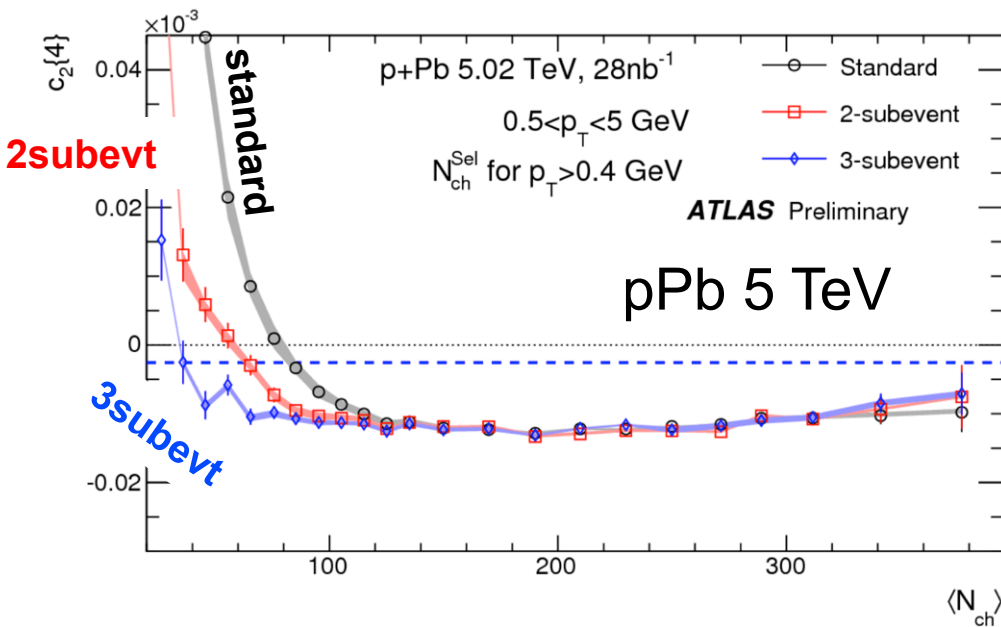
- Most positive  $c_2\{4\}$  in standard cumulants are jets and dijets.
  - Remaining positive  $c_2\{4\}$  in 3-subevent due to residual dijets.

- CGC expect sign-change at low  $N_{ch}$ 

$$c_2\{4\} = \frac{1}{N_D^3} \left( \frac{1}{4(N_c^2 - 1)^3} - A^4 \right)$$

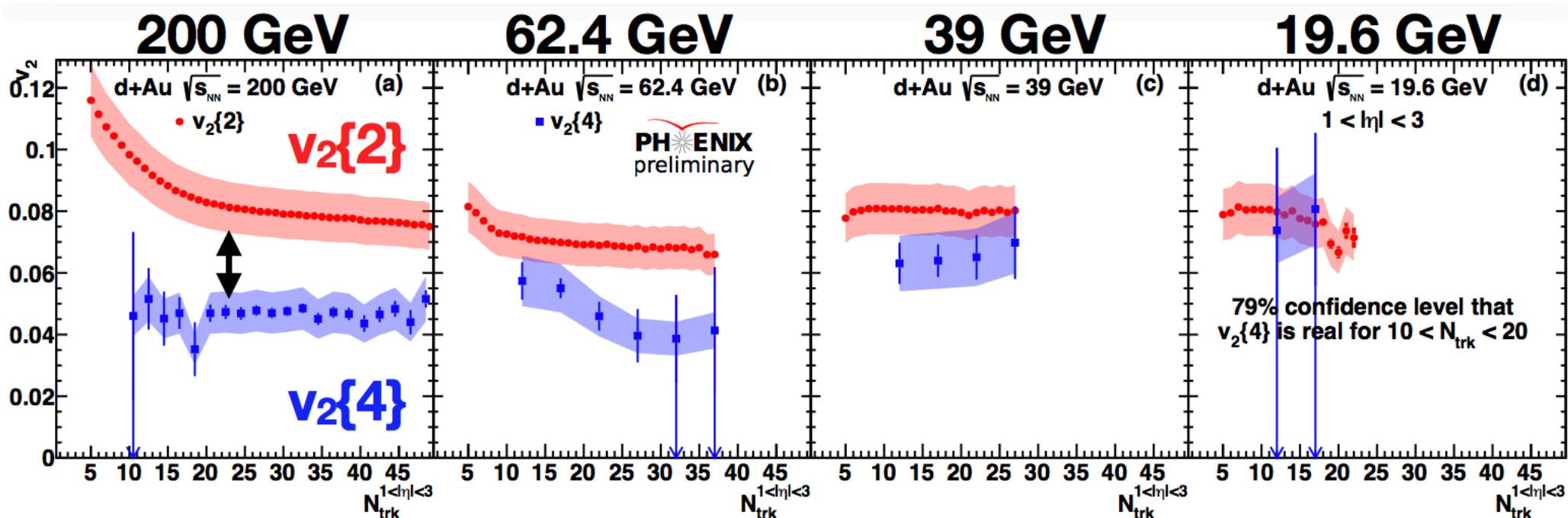
Dumitru, McLerran, Skokov

Glasma diagram
non-linear/non-Gaussian effects



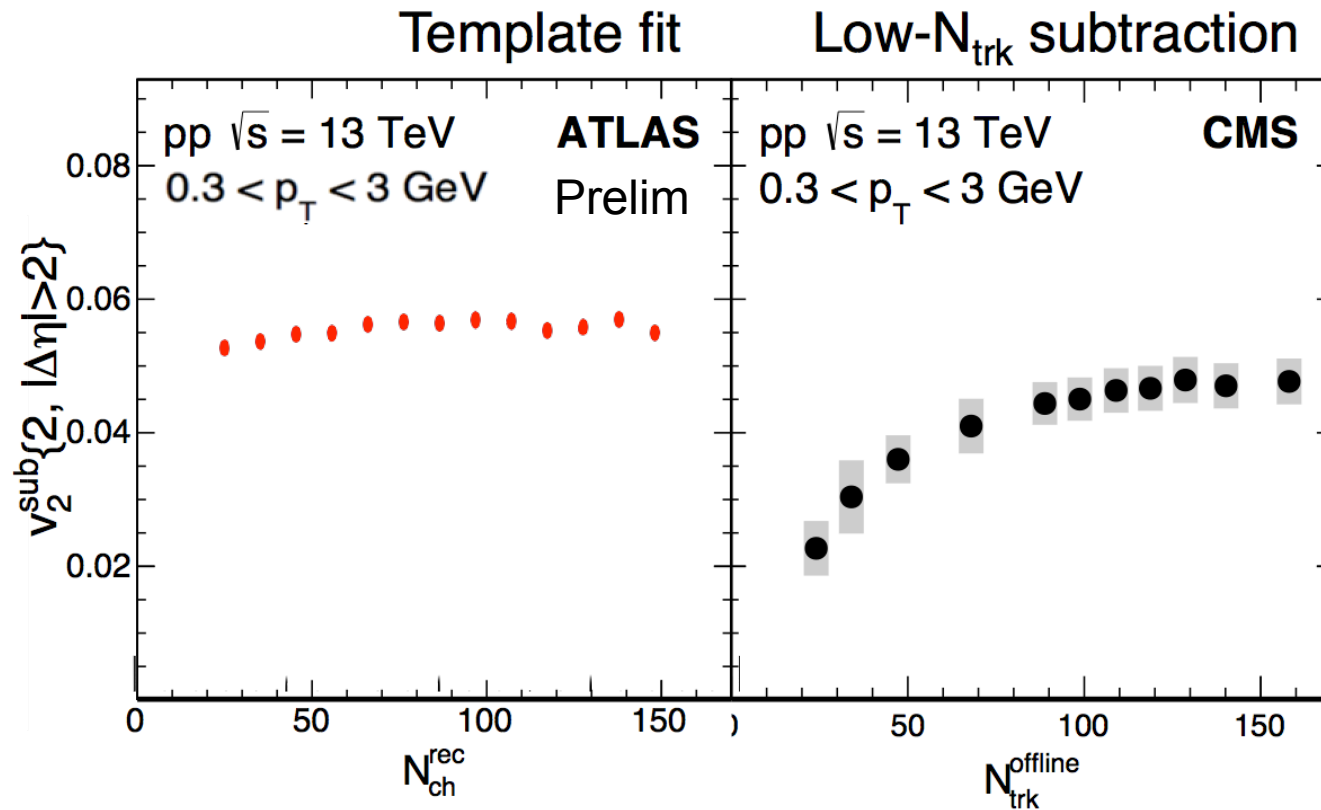
Glasma diagram contribution is small?

# $\sqrt{s}$ dependence of $v_2\{4\}$ at RHIC



- Surprising features:  $v_2\{4\}$  larger at lower  $\sqrt{s}$ , reaching  $v_2\{2\}$ .
- Difficult to describe in both CGC and hydro
- Important to understand non-flow in standard cumulant method

# Does collectivity turn off at low $N_{\text{ch}}$ ?

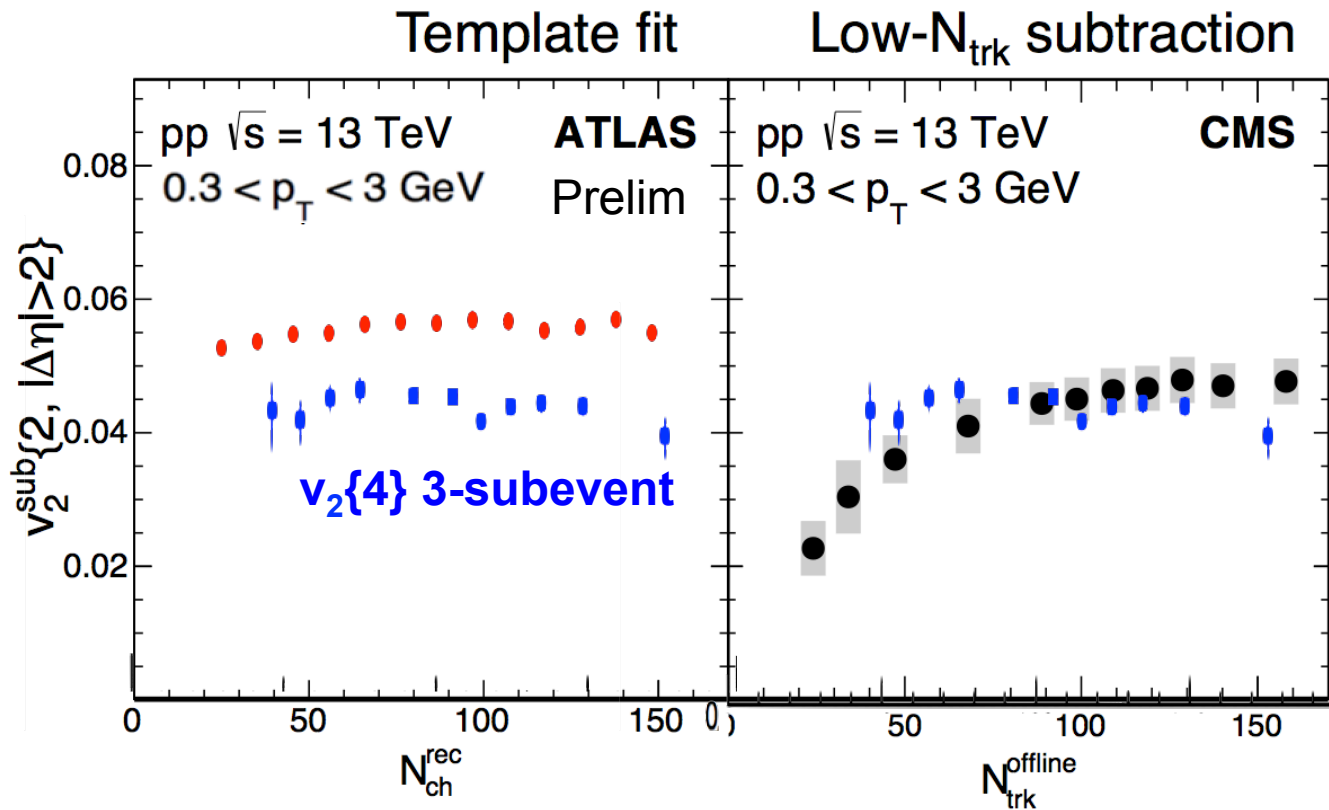


peripheral subtraction including peripheral pedestal (assuming the peripheral also has flow)  
 → so called template fit

peripheral subtraction **not** including peripheral pedestal (assuming the peripheral has **no** flow)  
 → so call peripheral sub.

Mingliang Zhou's talk for more detail

# Does collectivity turn off at low $N_{ch}$ ?



- $v_2\{4\}$  from 3-subevent show no dependence on  $N_{ch}$ .
- Why  $v_2\{2\}_{\text{peri. sub}} \approx v_2\{4\}$  in pp? surprising because:

$$v_n\{2\}^4 - v_n\{4\}^4 = \langle v_n^4 \rangle - \langle v_n^2 \rangle^2 = \langle (v_n^2 - \langle v_n^2 \rangle)^2 \rangle \geq 0$$

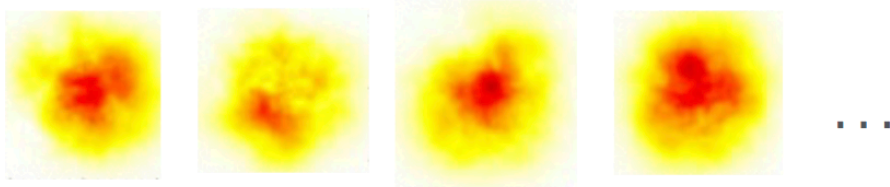
$v_2\{4\}$  also show No hint of collectivity turning-off at low  $N_{ch}$ !

Challenge both CGC and standard hydro?

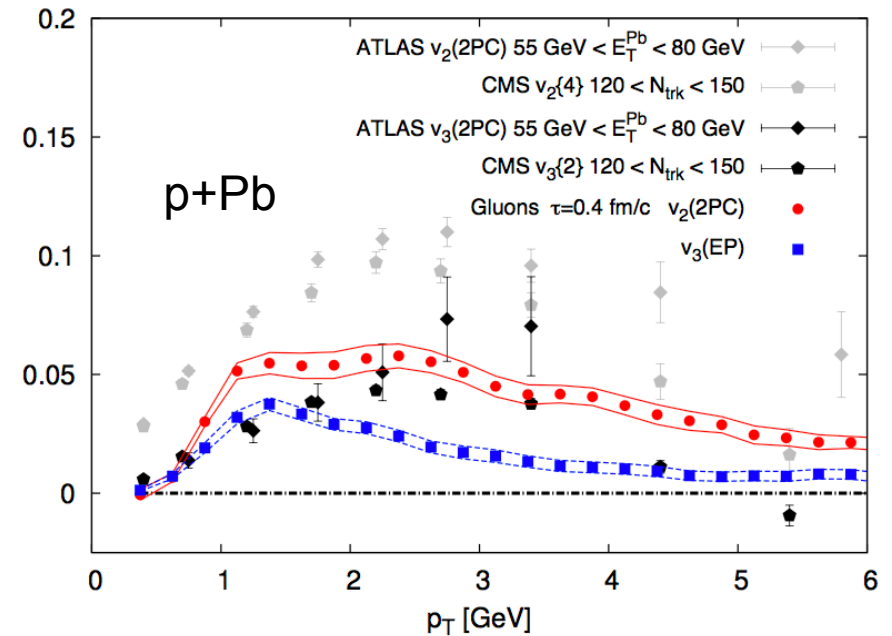
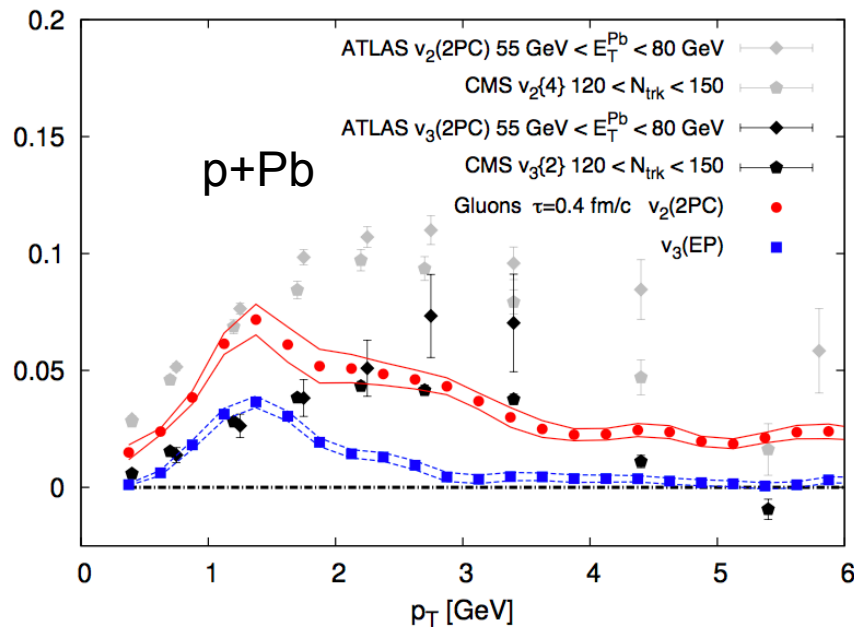
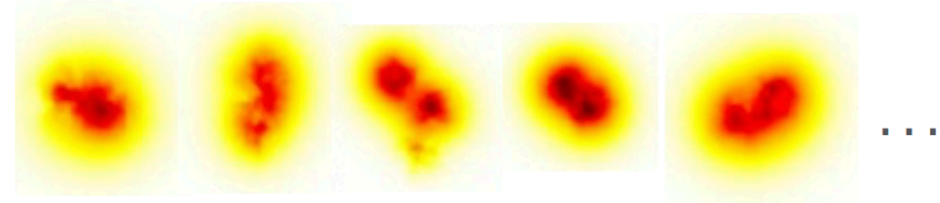
# Role of initial geometry is very different

From Schenke, Schlichting, Venugopalan,

'Spherical' proton



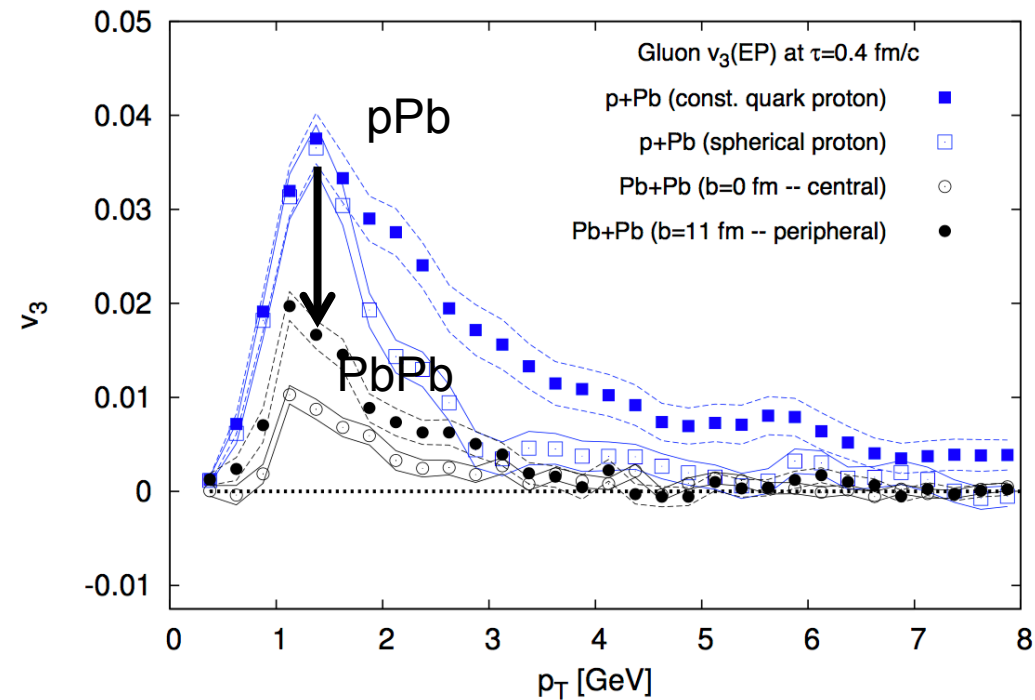
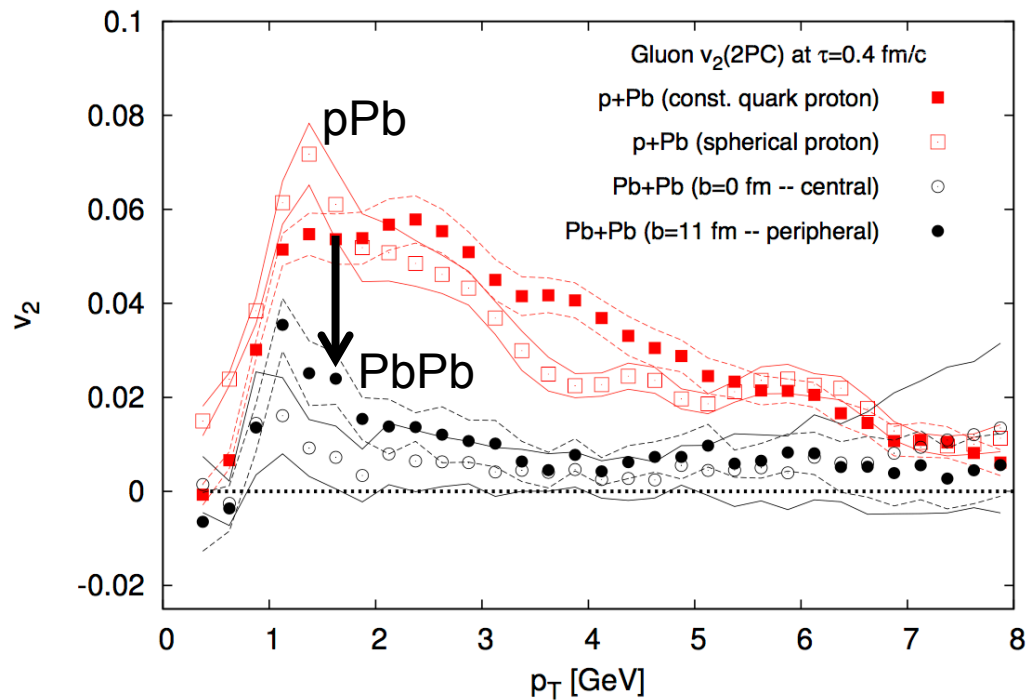
'Eccentric' proton



The orientation of collectivity is unrelated to initial eccentricity  
 → Very different from hydrodynamics

# Role of initial geometry is very different

From Schenke, Schlichting, Venugopalan,

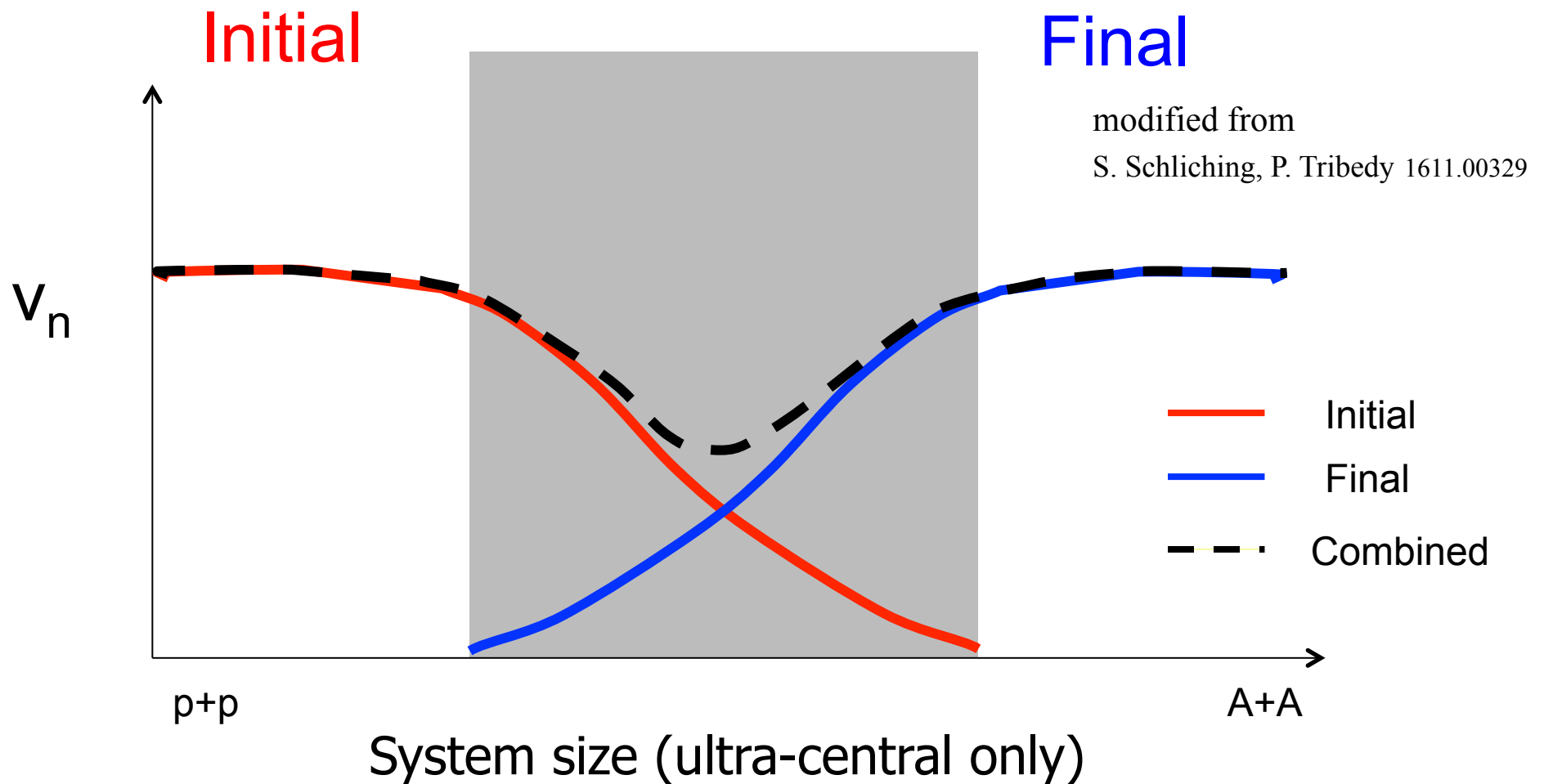


$$c_2\{4\} = \frac{1}{N_D^3} \left( \frac{1}{4(N_c^2 - 1)^3} - A^4 \right)$$

The orientation of collectivity is unrelated to initial eccentricity

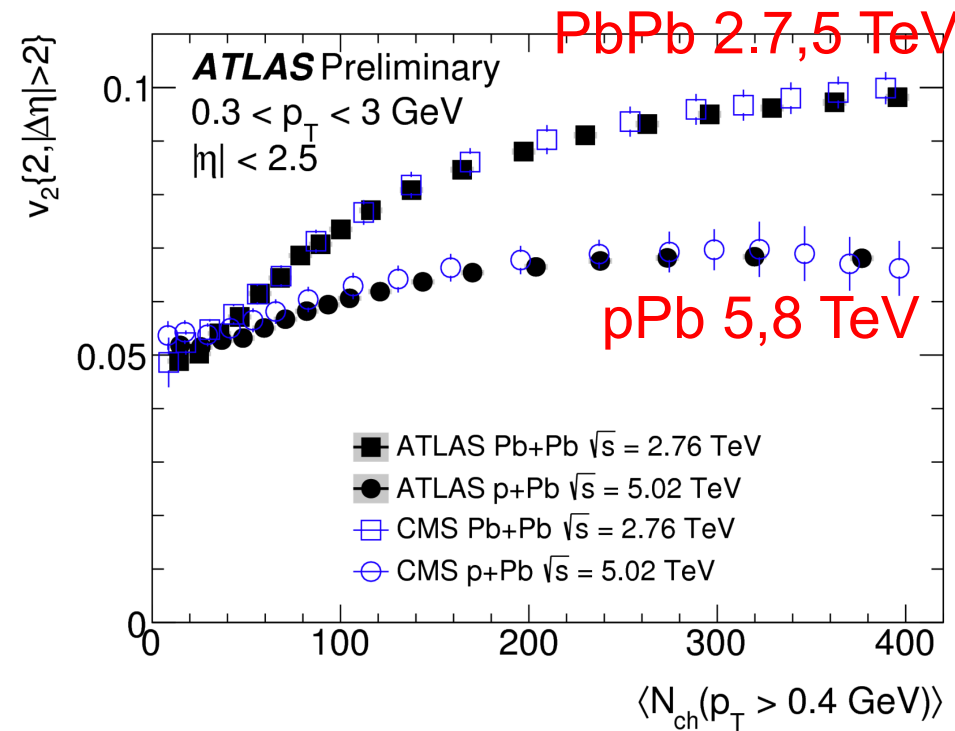
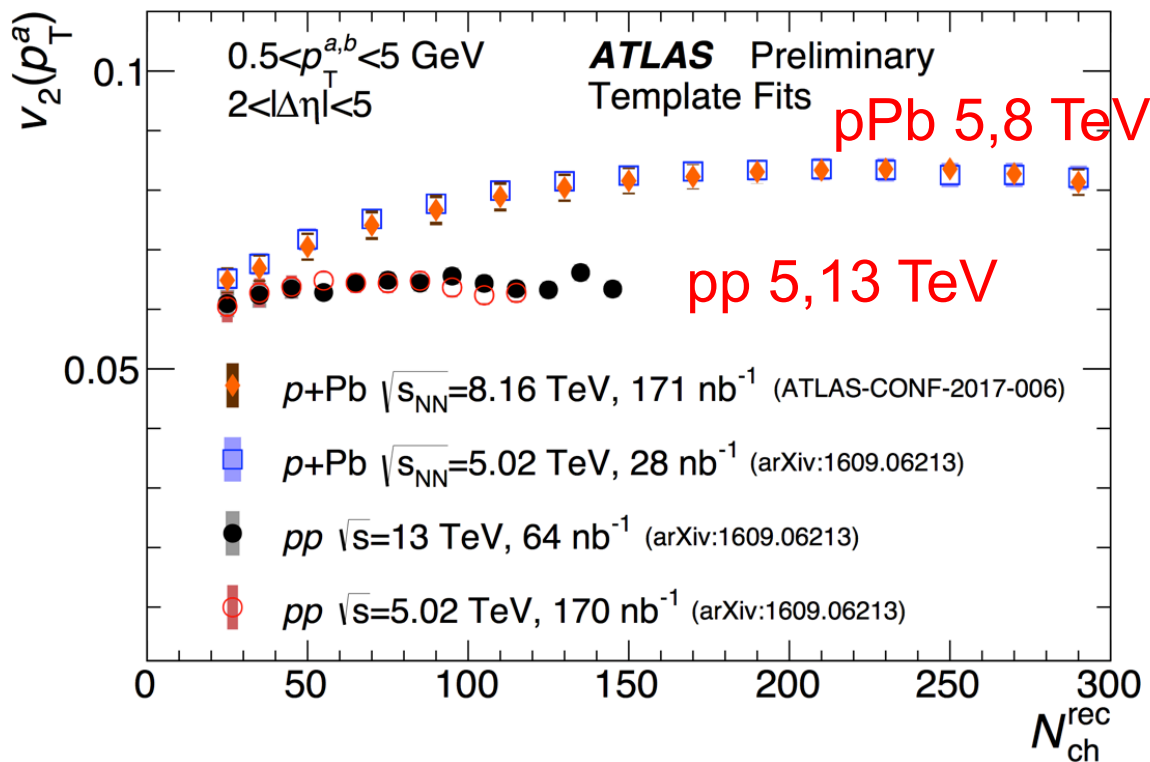
→ Very different from hydrodynamics

Expect contribution diminish as system size is increased



Phases of collectivity from CGC and hydro are unrelated  
→ a minimum of total  $v_n$  at certain system size?

# System size dependence



Clear dependence on collision systems but  $\sim$ no dependence on  $\sqrt{s}$

$$v_2^{pp}(\text{high-mul}) < v_2^{pPb}(\text{low-mul})!$$

CGC

Unclear if the pp/pPb hierarchy is expected.

Hydro

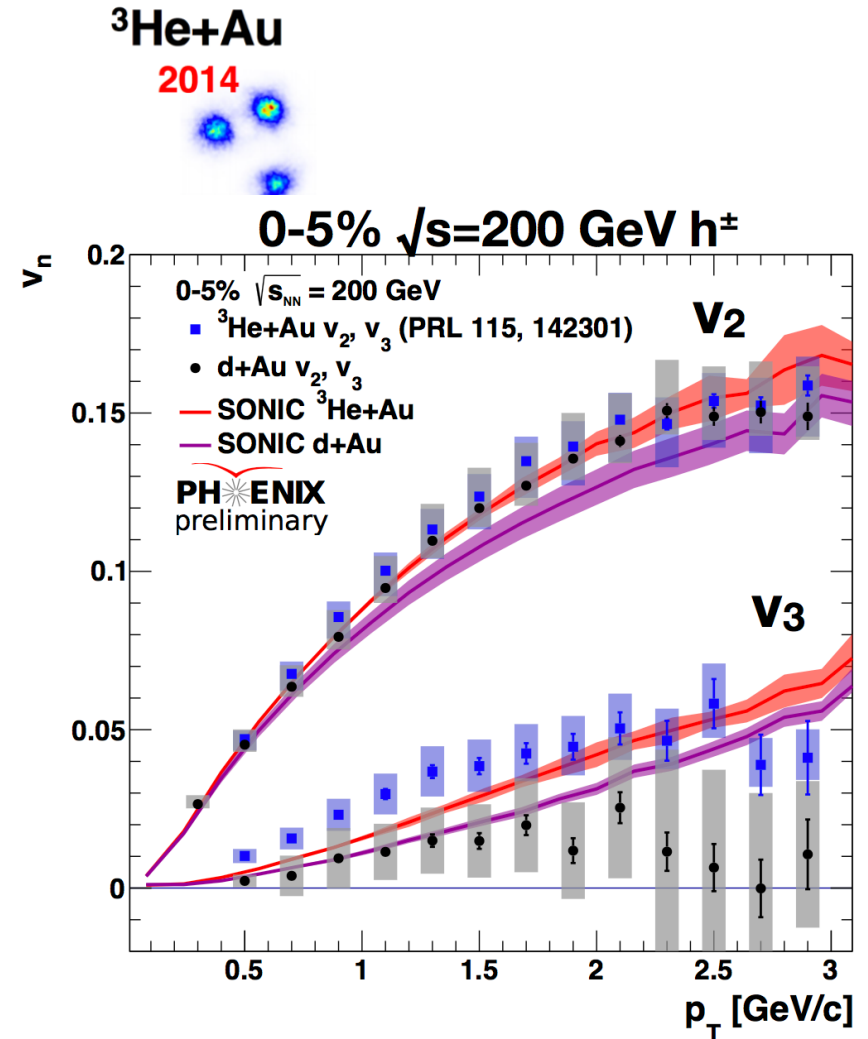
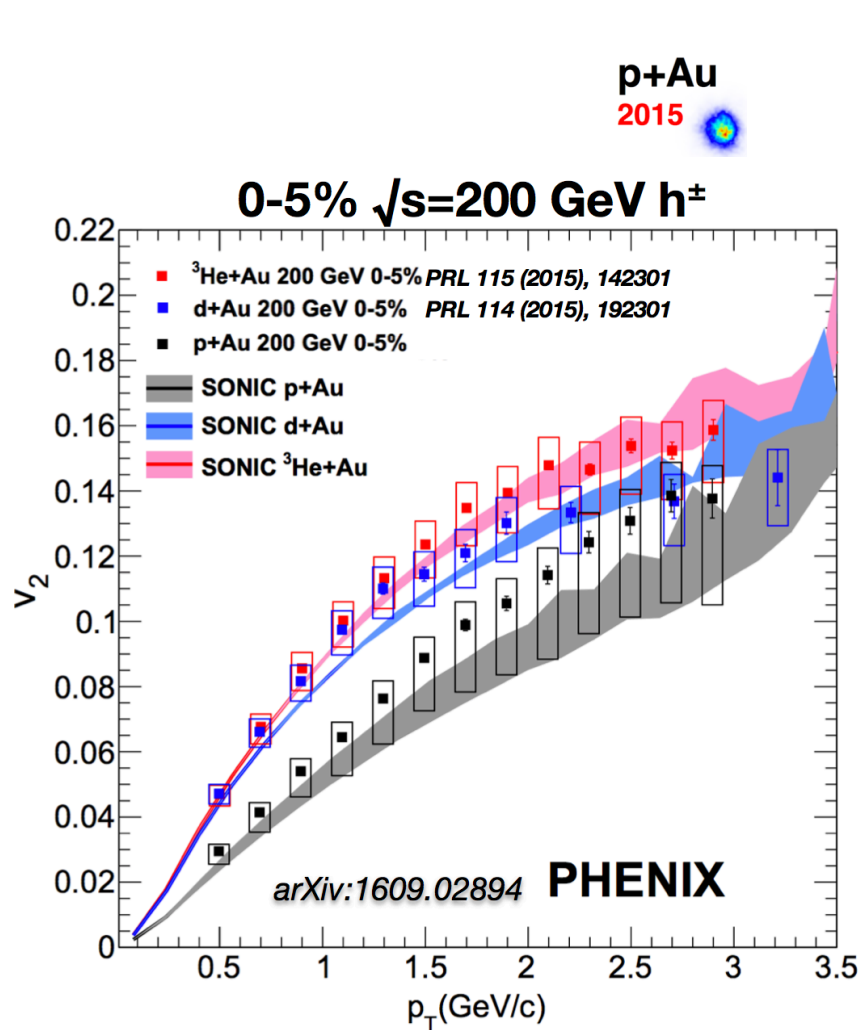
Interplay between viscous damping and initial  $\epsilon_n$

pPb: may see an average geometry effect

pp: geometry maybe poorly correlated with  $N_{ch}$ .



# Geometry scan at RHIC



$$v_2^{pAu} < v_2^{dAu} \leq v_2^{HeAu}$$

$$v_3^{dAu} < v_3^{HeAu}$$

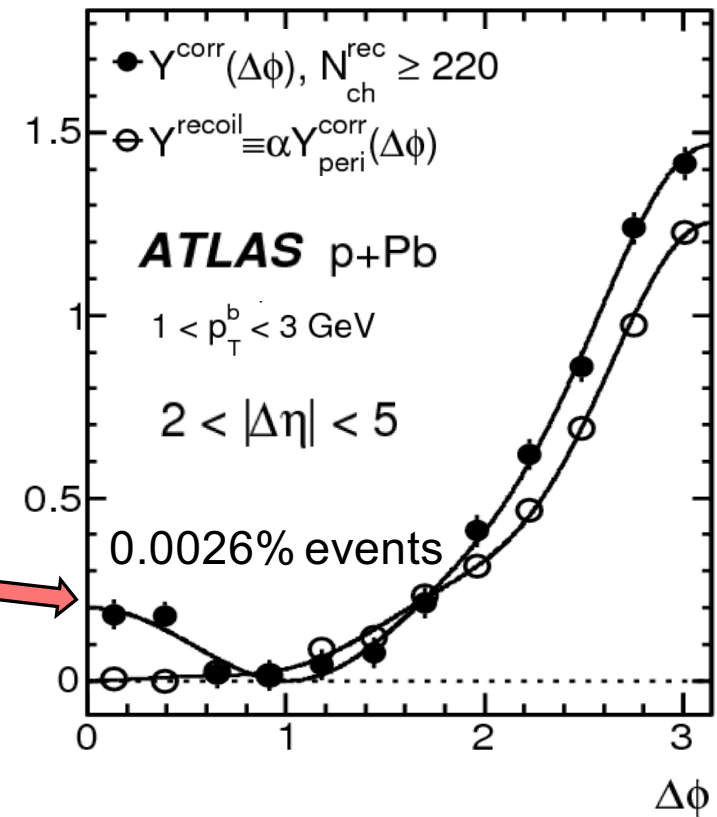
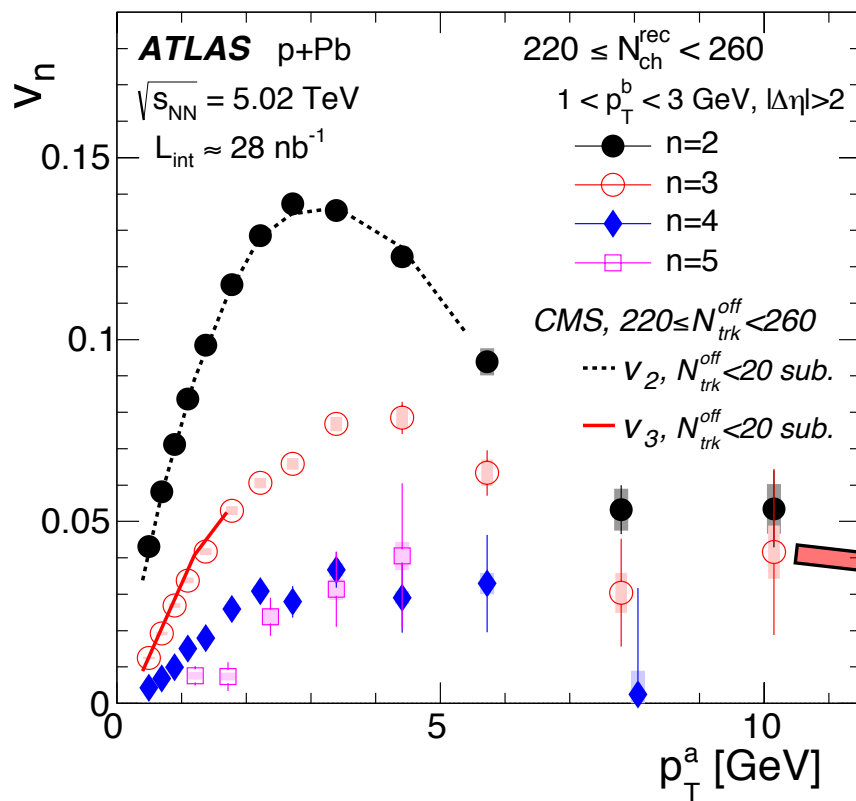
Hierarchy compatible with initial geometry + final state effects  
 Look forward to the CGC predictions

# Original of high- $p_T$ $v_2$ ?

## ■ Ridge seen directly at 10 GeV or 5% $v_2$ in pPb

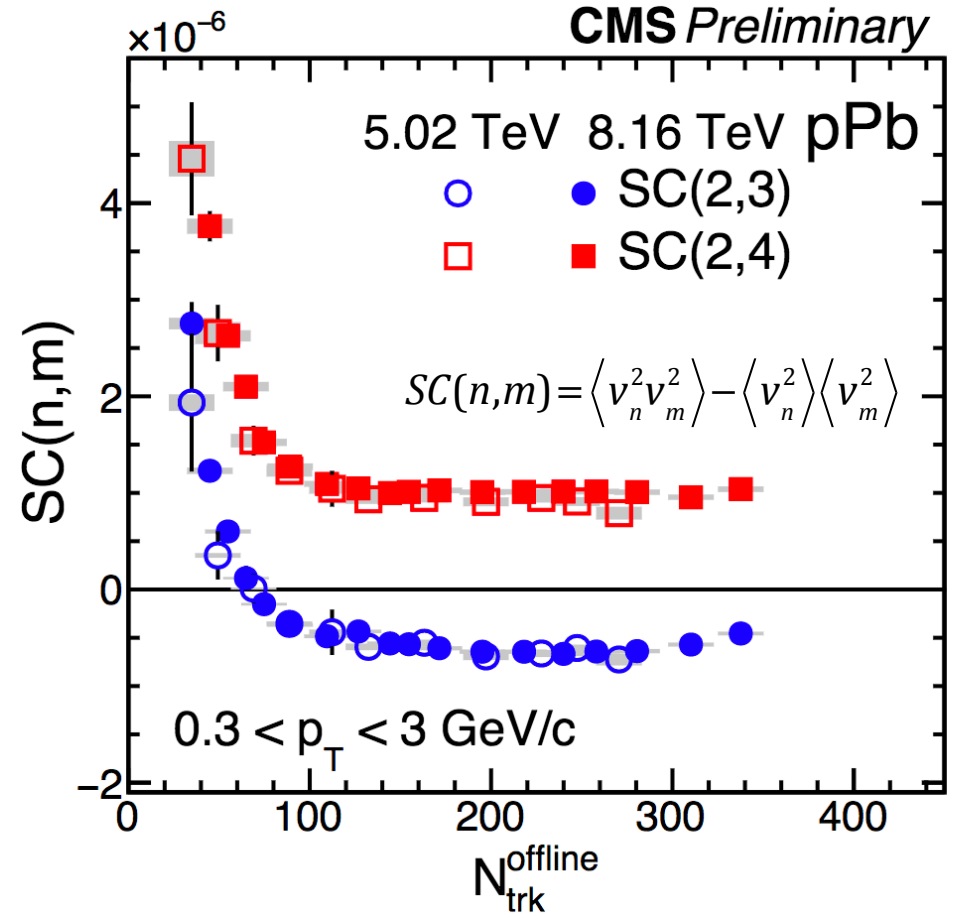
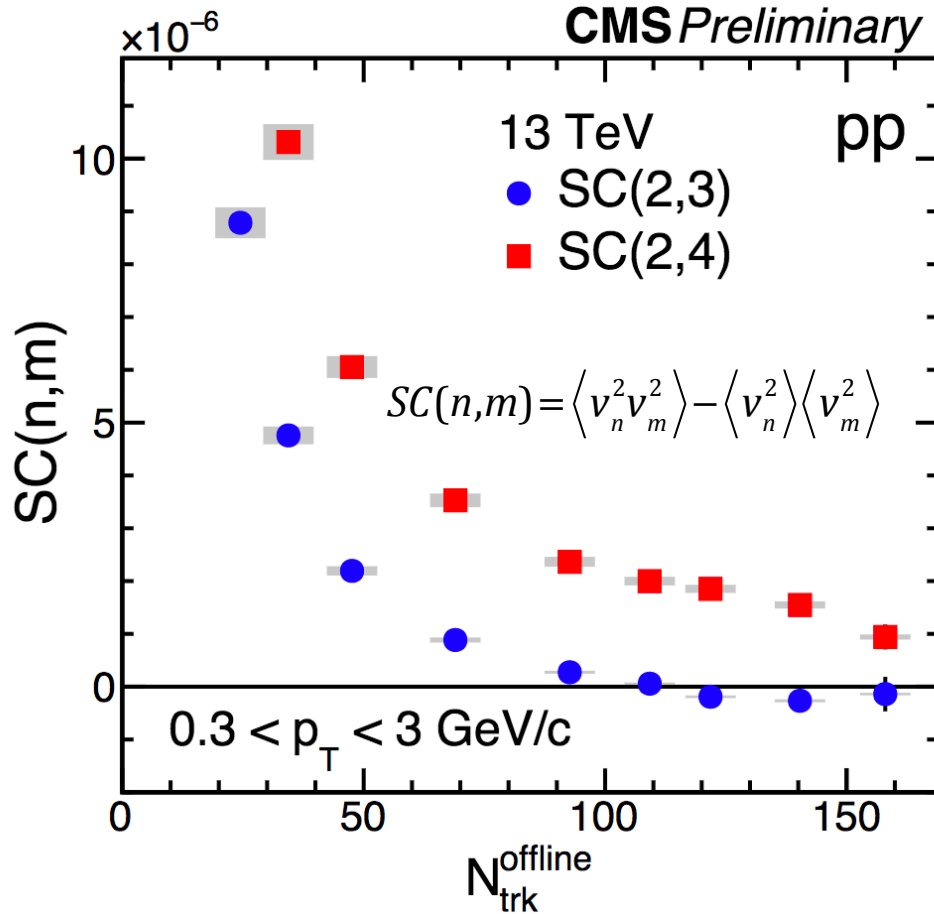
→ final state effects, e.g. jet quenching (better observable than  $R_{AA}$ )?

→ initial state effects, rare  $Q_s$  fluctuation?



Outlook: more precision and higher  $p_T$  with 8 TeV pPb data

# Symmetric cumulants



- Influence of non-flow need to be taken out, but see anti-correlation between  $v_2 v_3$  and correlation between  $v_2 v_4$ .
- Naturally understood in hydrodynamics
  - $v_2 v_3$  reflects  $\varepsilon_2 \varepsilon_3$  correlation,  $v_2 v_4$  correlation reflects mode-mixing effects
- In principle, some processes in CGC can also produce this [1705.00745](#)

# Summary of collectivity in small system

- Collectivity associated with ridge must involve many particles in multiple  $\eta$  ranges  $\rightarrow$  subevent methods

Challenge for both initial & final state scenarios?

- LHC  $v_2$  associated with ridge does not turn off at low  $N_{ch}$ .
- RHIC  $v_2\{4\}$  increases and approaches  $v_2\{2\}$  at lower  $\sqrt{s}$

Challenge for initial state only scenarios?

- LHC  $v_2^{pp} < v_2^{pPb}$  in all  $N_{ch}$  and all  $\sqrt{s}$ .
- LHC  $c_2\{4\} < 0$  down to very low  $N_{ch}$  and more negative at higher  $p_T$ .
- RHIC geometry scan suggest ordering of  $v_n$  follows that of  $\epsilon_n$ .
- LHC 5%  $v_2$  at  $p_T \sim 10$  GeV.

...

Coexistence of initial state & final state scenarios?

**Key issue: How to constrain timescales for emergence of collectivity?  
the role of CGC, preflow and hydro?**

# How are cumulants related to collectivity?

— the role of flow, non-flow and multiplicity fluctuation

1701.03830, 1412.4759

# Role of flow & nonflow in multi-particle cumulant<sup>23</sup>

- Flow vector for event with M particles:  $\mathbf{q}_n \equiv \frac{\sum_i e^{in\phi_i}}{M} = q_n e^{in\Psi_n}$

- Contains contribution from flow and non-flow  $\mathbf{q}_n = \mathbf{v}_n + \mathbf{s}_n$

- Cumulant is **additive** for convolution: (v and s are independent)

$$p(\mathbf{q}_n) = p(\mathbf{v}_n) \otimes p(\mathbf{s}_n) \implies c_n\{2k\} = c_n\{2k, \text{flow}\} + c_n\{2k, \text{non-flow}\}$$

- i.e. for four-particle cumulants:

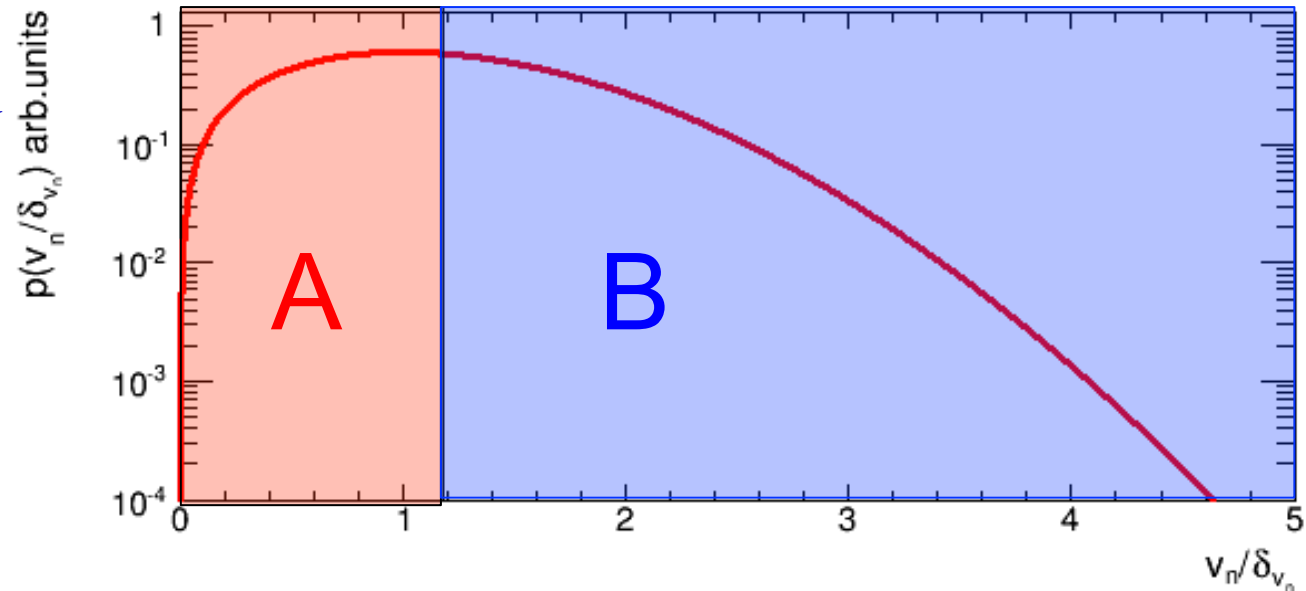
$$c_n\{4\} = \frac{\langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2}{c_n\{4, \text{flow}\}} + \frac{\langle s_n^4 \rangle - 2 \langle s_n^2 \rangle^2}{c_n\{4, \text{non-flow}\}}$$

The sign of  $c_n\{2k\}$  depends on the nature of the p(flow) and p(non-flow)

# Properties of flow cumulants

- Gaussian fluctuation without average geometry

$$p(v_n) = \frac{v_n}{\delta_{v_n}^2} e^{-v_n^2 / (2\delta_{v_n}^2)}$$



- Divide to 2 equal parts, and calculate cumulants separately.

	$v_n\{2\}$	$v_n\{4\}$	$v_n\{6\}$	$v_n\{8\}$	in units of $\delta$
all	1.414	0	0	0	
A	0.783	0.685	0.671	0.667	
B	1.840	1.653	1.680	1.681	

- Not additive due to non-linear term:  $c_n\{4\}_{ev1+ev2} \neq \frac{c_n\{4\}_{ev1} + c_n\{4\}_{ev2}}{2}$

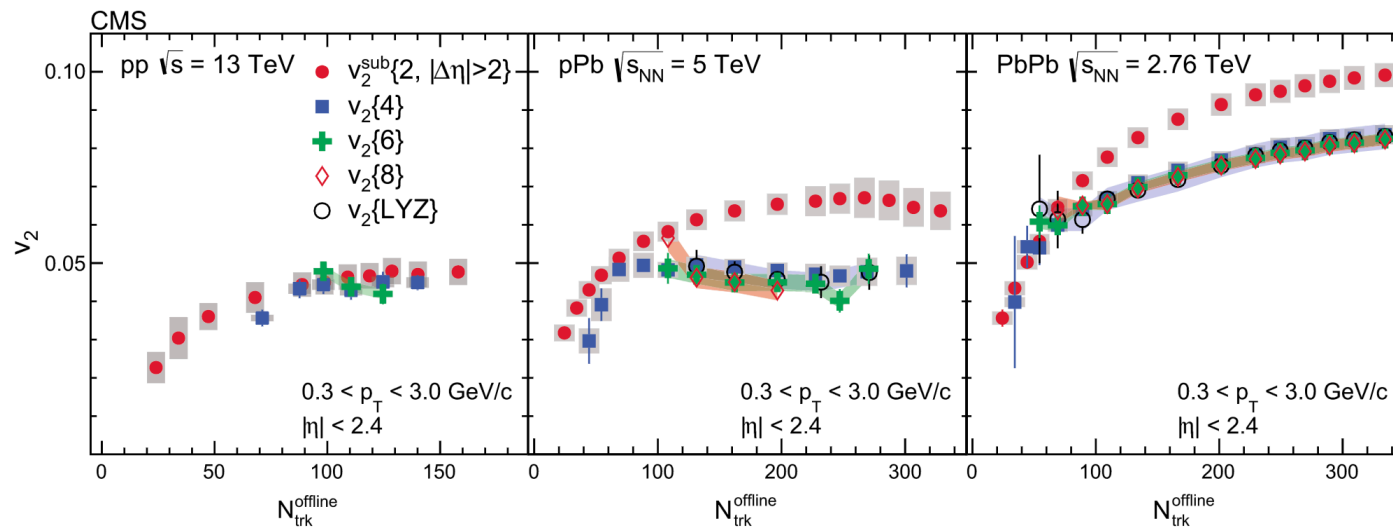
$$c_n\{4\} = \langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2$$

- Cumulants not very sensitive to  $p(v_n)$  shape beyond 4<sup>th</sup>-order!

# Nature of collectivity fluctuations?

- Arguments based on initial eccentricity fluctuations
  - A+A system: Bessel-Gaussian, confirmed by  $p(v_n)$  obtained from unfolding ✓
  - Small system: power distribution based on independent source model ?

$$p(v_n) = 2 \frac{\alpha}{\kappa} \frac{v_n}{\kappa} \left(1 - \left(\frac{v_n}{\kappa}\right)^2\right)^{\alpha-1} \quad 2\alpha = N_s - 1 \quad \text{PRL112,082301(2014)}$$



We can't know  $p(v_n)$  given current precision of  $v_2\{2k\}$ !

Important to directly measure  $p(v_n)$



# “wrong” sign of $c_n\{2k\}$ ?

- In principle,  $c_2\{2k\}$  with “correct” sign &  $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \approx v_2\{\infty\}$  neither necessary nor sufficient condition for collectivity.

- The sign & hierarchy controlled by shape of  $p(v_n)$ . More examples given  
in 1412.4759
  - Mixing 1/3 events with flow and 2/3 with zero flow

$$p(v_n) = \frac{1}{3}\delta(v_n - v_0) + \frac{2}{3}\delta(v_n)$$

- Cumulants have “wrong” sign, despite only flow is present

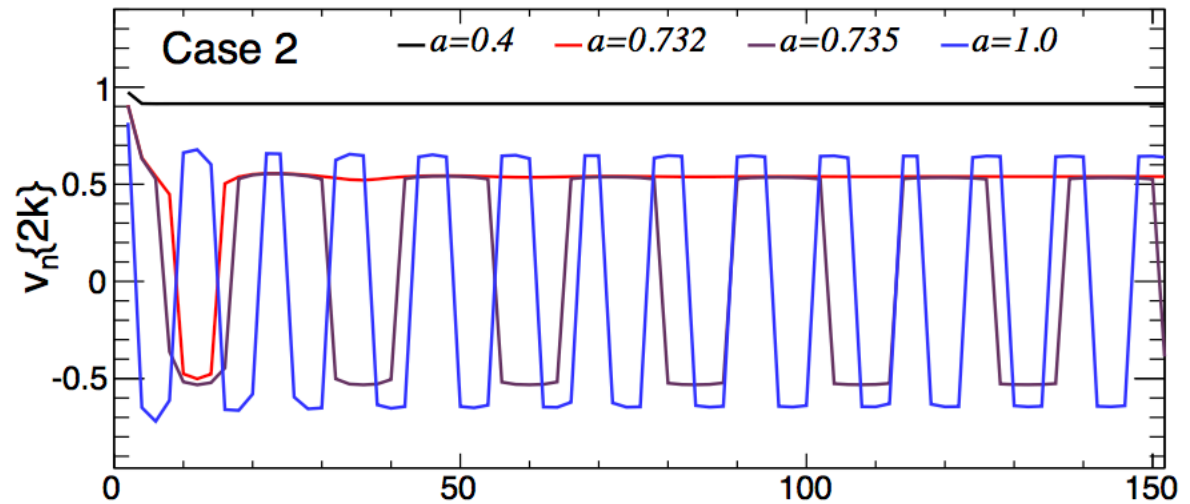
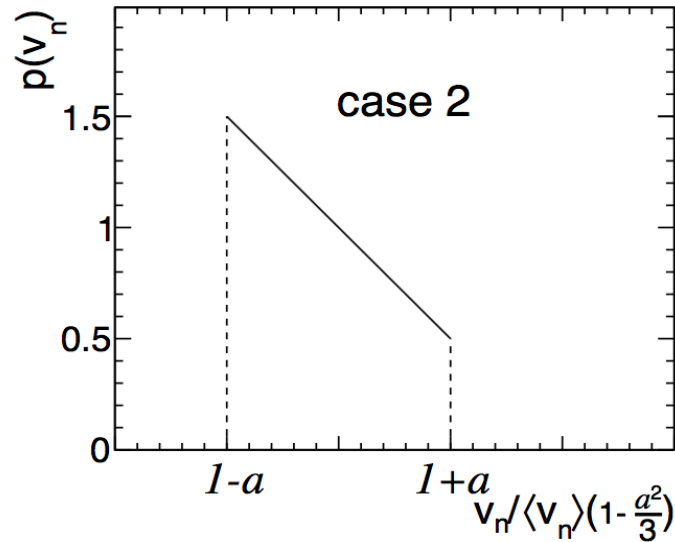
$$c_n\{4\} = \frac{1}{9}v_0^4, \quad c_n\{6\} = -\frac{2}{9}v_0^6, \quad c_n\{8\} = \frac{71}{9}v_0^8$$

- Same discussion applies for non-flow as well: the sign of  $c_n\{4, \text{non-flow}\}$  depends on EbyE fluctuation of non-flow  $p(s_n)$

$$c_n\{4\} = \langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2 + \boxed{\langle s_n^4 \rangle - 2 \langle s_n^2 \rangle^2}$$

Understanding EbyE **non-flow fluctuation** is important for  
understanding flow fluctuation in small system

# How cumulants depends on $p(v)$ ?



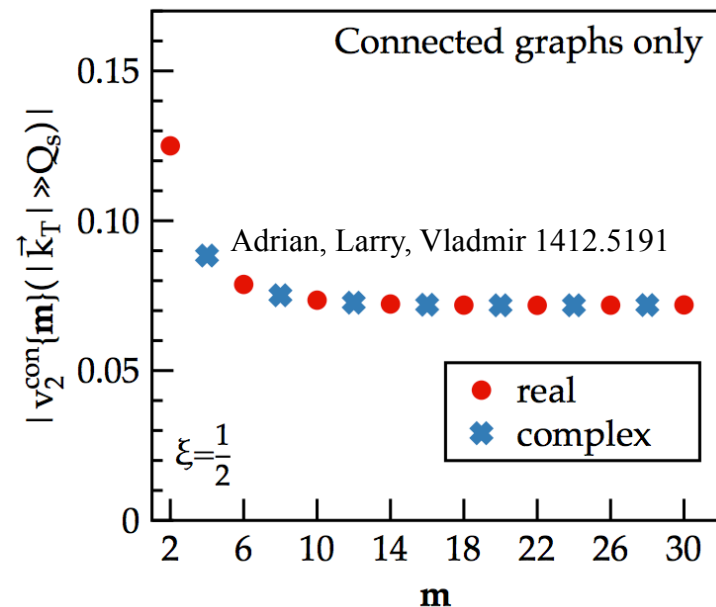
- Convergence of  $\lim_{k \rightarrow \infty} v\{2k\}$  requires  $\langle J_0(2vz) \rangle = \int J_0(2vz)p(v)dv$  has 0 in complex plane, i.e. LYZ method.
- $p(v)$  from initial state color fluc is strongly non-Gaussian

$$c_2\{2\} = \frac{1}{N_D} \left( \mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right)$$

$$c_2\{4\} = -\frac{1}{N_D^3} \left( \mathcal{A}^4 - \frac{1}{4(N_c^2 - 1)^3} \right)$$

non-linear/non-Gaussian effects

Glasma diagram



# Non-flow and multiplicity fluctuations

$$c_n\{4\} = \frac{\langle \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle \rangle_{\text{evt}}}{\langle \langle e^{in(\phi_1 - \phi_2)} \rangle \rangle_{\text{evt}}^2} - 2$$

calculated for **selected particles** average over an **event class**

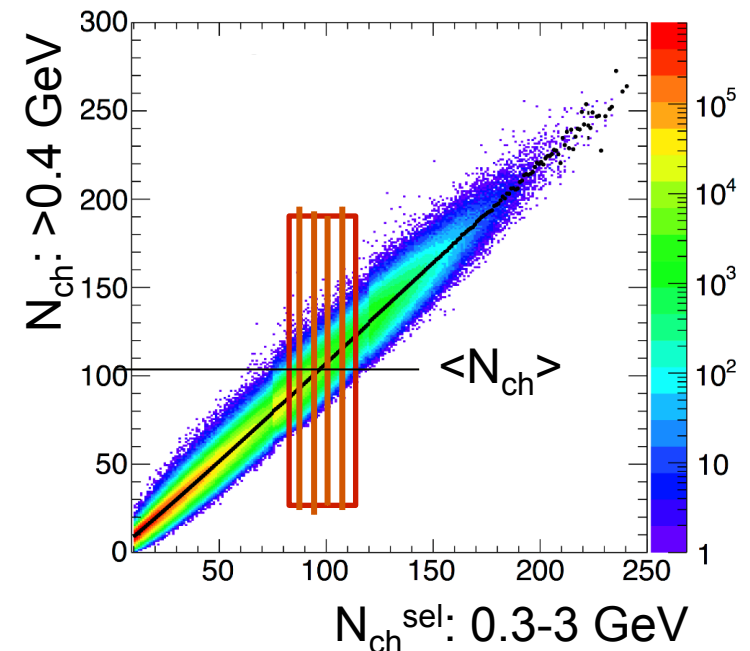
e.g. 0.5-5 GeV

$N_{\text{ch}}^{\text{Sel}}$  : number of charge particles with  
e.g. 0.3-3, >0.2, >0.4, >0.6 GeV...

Map final results to a **common centrality**, e.g.  $\langle N_{\text{ch}} \rangle$  for  $p_{\text{T}} > 0.4$  GeV

Because  $c_n\{4\}$  is not additive:

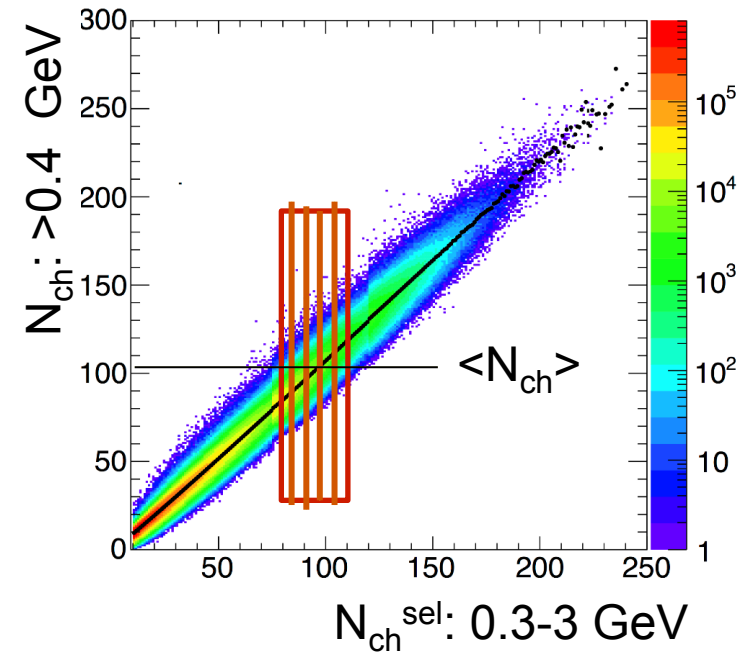
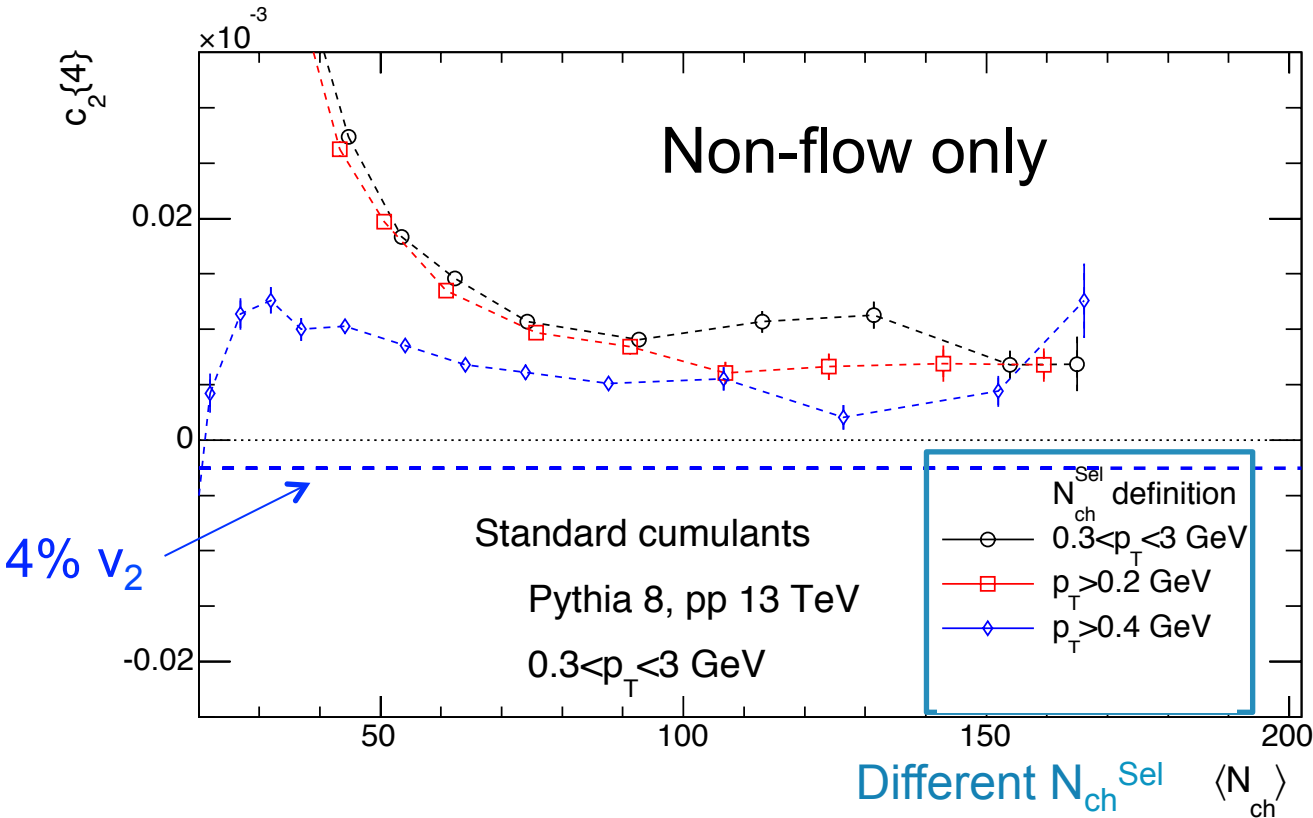
$$c_n\{4\}_{\text{ev1+ev2}} \neq \frac{c_n\{4\}_{\text{ev1}} + c_n\{4\}_{\text{ev2}}}{2}$$



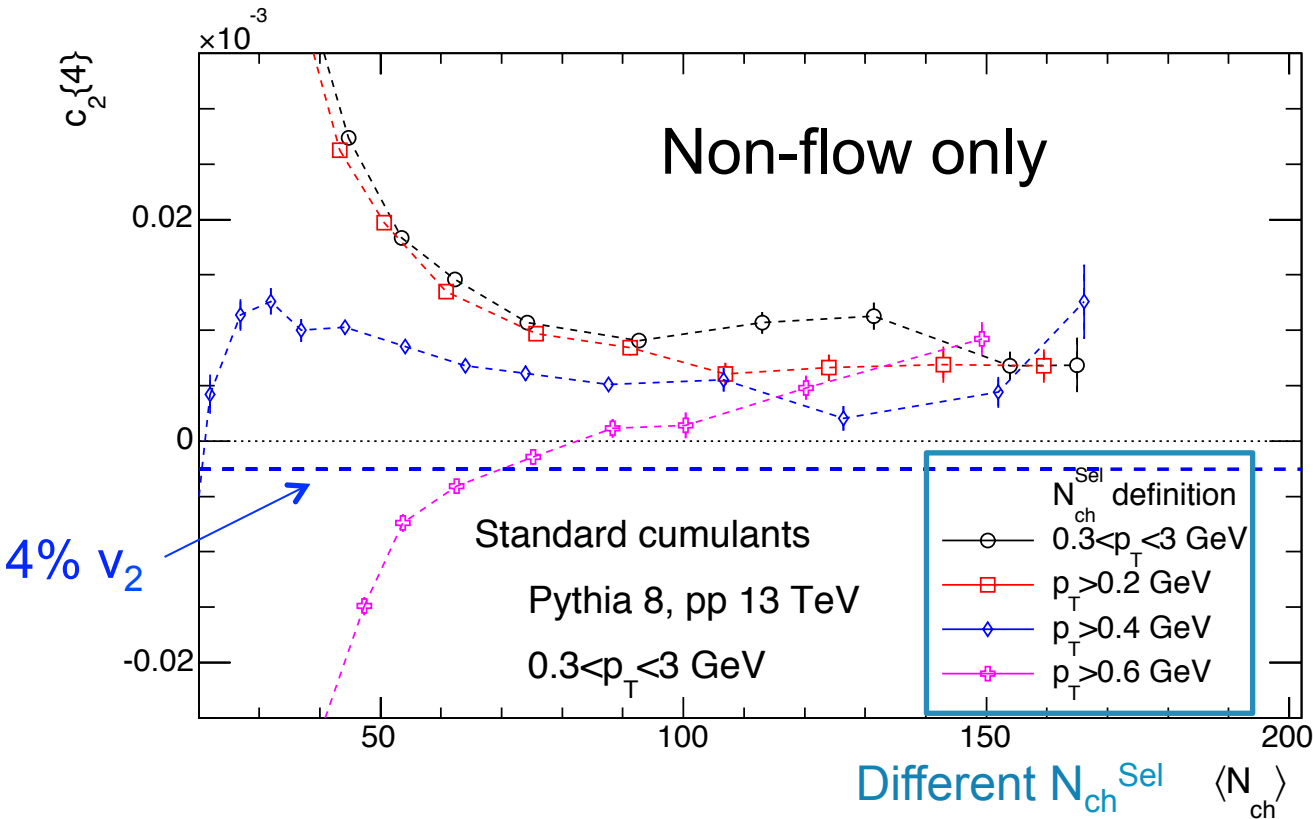
Results depend on the intermediate  $N_{\text{ch}}^{\text{Sel}}$  !

Mainly because  $p(\text{non-flow})$  has strong dependence on  $N_{\text{ch}}$

# Dependence on $N_{ch}^{Sel}$ in PYTHIA



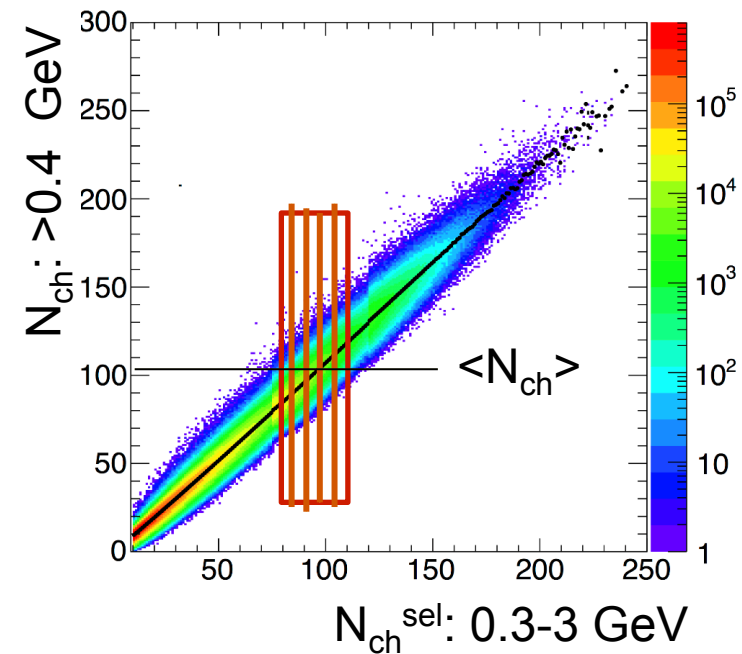
# Dependence on $N_{ch}^{Sel}$ in PYTHIA



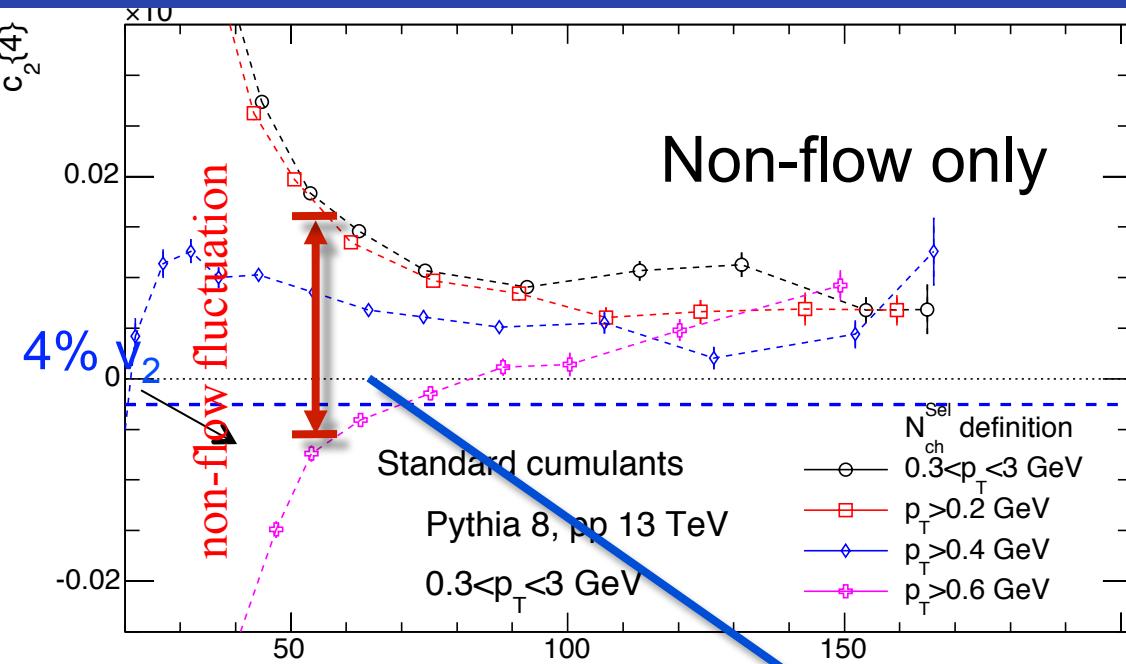
Different  $N_{ch}^{Sel}$

Different non-flow fluctuations

Different  $c_2\{4, \text{non-flow}\}$



# Standard v.s. Subevent cumulants



Different  $N_{ch}^{Sel}$

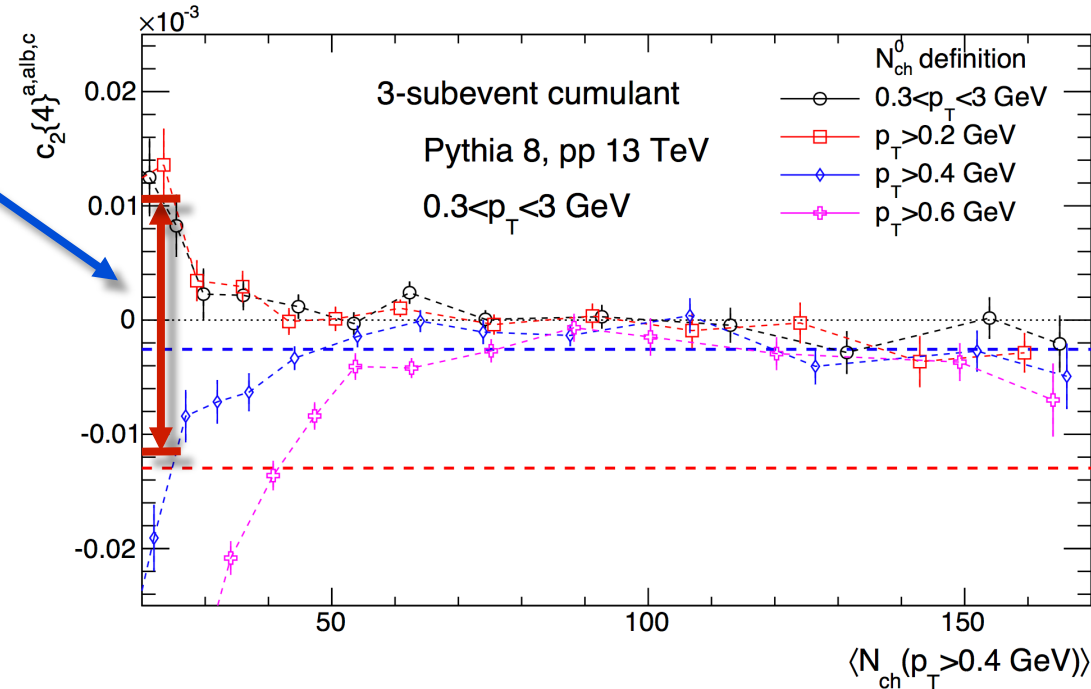
Different non-flow fluctuations

Different  $c_2\{4, \text{non-flow}\}$

Less non-flow

Less non-flow fluctuations

Less dependence on  $N_{ch}^{Sel}$



3 subevent cumulant is a more reliable method in small system

# Summary of cumulants in small system

- Current precision of  $c_n\{2k\}$  can't probe details of  $p(v_n)$  shape, other than the mean and standard deviation.
- $c_n\{2k\}$  with “correct” sign &  $v_n\{4\} \approx v_n\{6\} \approx v_n\{8\} \approx v_n\{\infty\}$  neither necessary nor sufficient condition for collectivity.
- In small systems, non-collective sources from dijet dominates the statistical properties of two- or multi-particle correlations
  - Reflected by strong sensitivity to multiplicity class definition and multiplicity bin-width.
- Cumulants based on subevents suppress such non-collective sources