Exploring collectivity with multi-particle correlations

status, new developments and limitations

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Workshop on collectivity in small systems – NBI – Copenhagen

09/05/17

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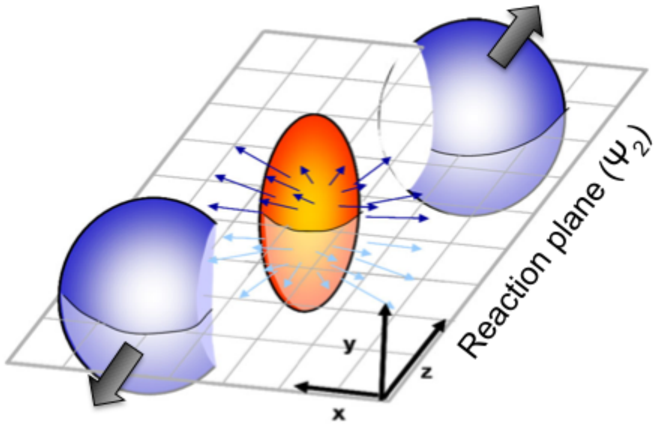
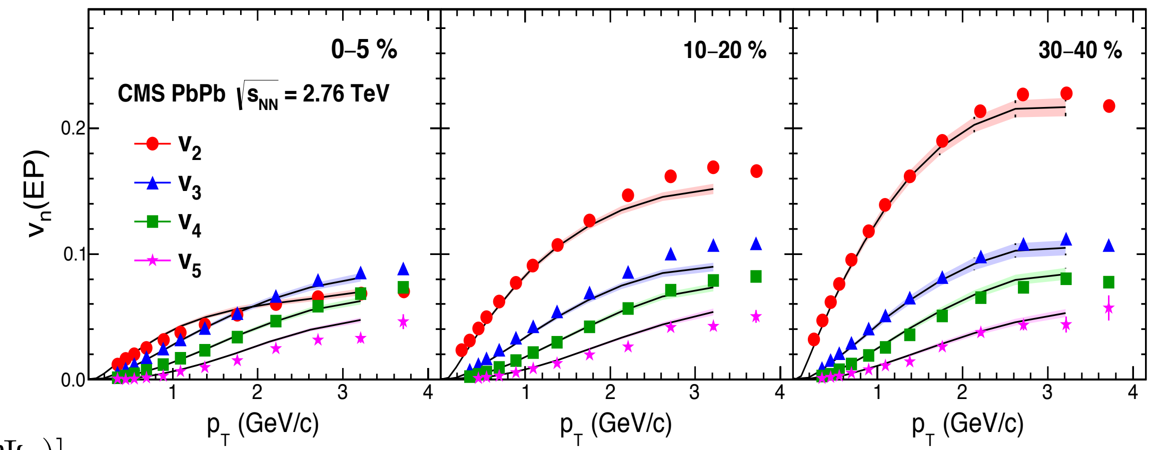
What do we learn in AA?



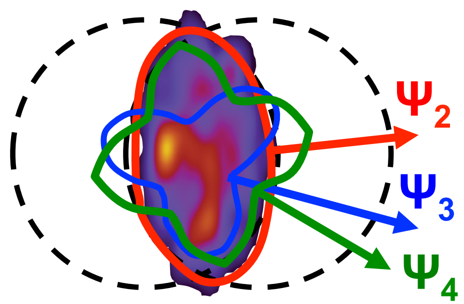


$$f(p_T,\phi,\eta) \sim 1 + 2 \cdot \sum_n v_n(p_T,\eta) \cdot \cos[\mathrm{n}(\phi - \Psi_\mathrm{n})]$$

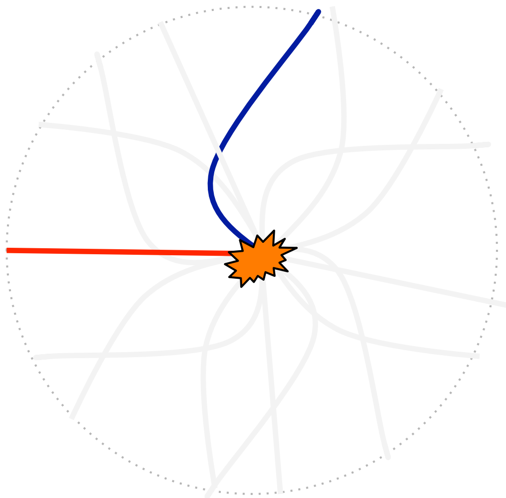
Phys. Rev. C 89, 0449076 **CMS results**

Phys. Rev. Lett. 110, 012302 **IP glasma + MUSIC**

- v_n depends on:
  - Initial state geometry and its fluctuations
  - Medium transport coefficients (η/s, …)
- v_n well understood in A-A collisions with hydro.
  - **Initial state ε_s** at τ = 0 + pressure gradient (**final state** interactions)

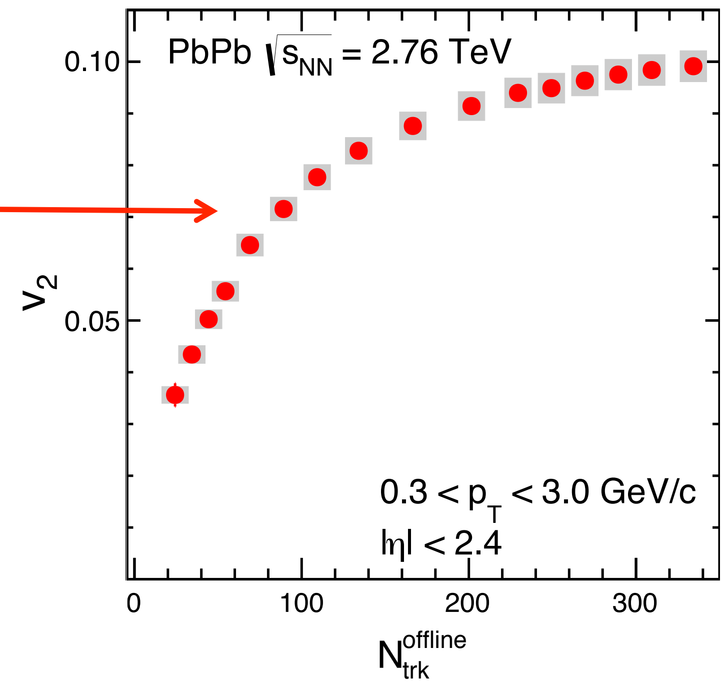


Collectivity in AA



2 particle correlations

$v_2\{2\}$



PhysLettB.2016.12, 009

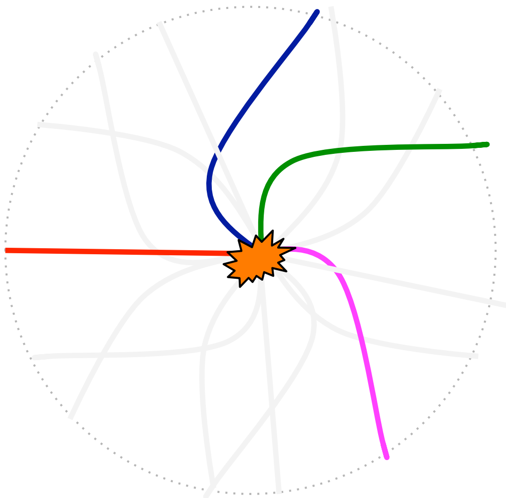
- **High-Multiplicity regime**

$$\langle 2 \rangle = \frac{1}{P_{M,2}} \sum_{i,j} e^{in(\phi_i - \phi_j)},$$

$$P_{M,2} = \frac{M!}{2!(M-2)!}$$

$$c_n\{2\} = \langle\langle 2 \rangle\rangle \;, \qquad v_n\{2\} = \sqrt{c_n\{2\}}$$

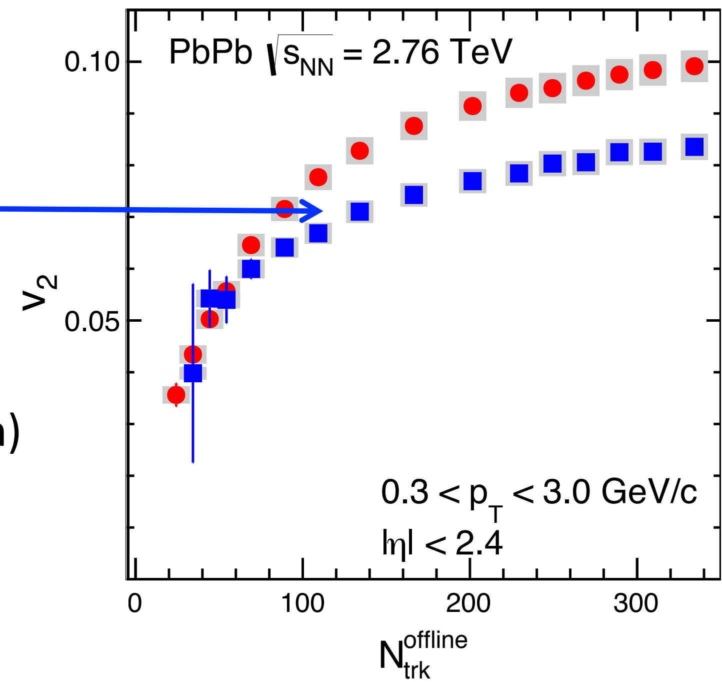
Collectivity in AA



4 particle correlations

$v_2\{4\}$

- Less sensitive to non-flow (i.e. jet induced correlation)
- Needs larger sample of events



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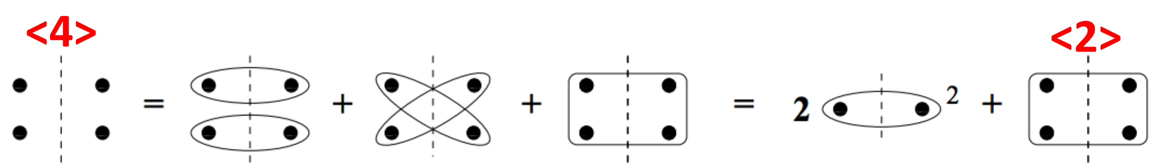
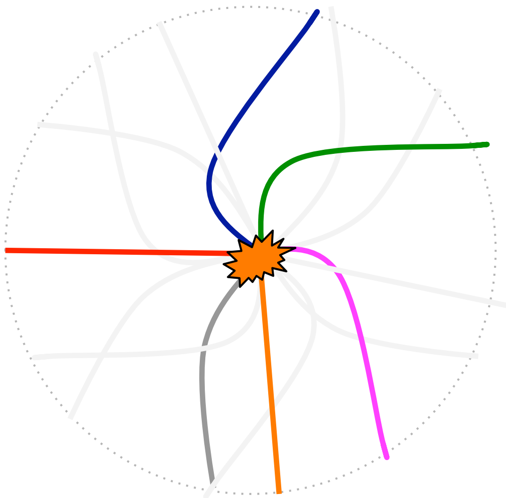


$$\langle 4\rangle = \frac{1}{P_{M,4}} \sum_{i,j,k,l} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)},$$

$$P_{M,4} = \frac{M!}{4!(M-4)!}$$

$$c_n\{4\} = \langle\langle 4\rangle\rangle - 2\langle\langle 2\rangle\rangle^2, \qquad v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

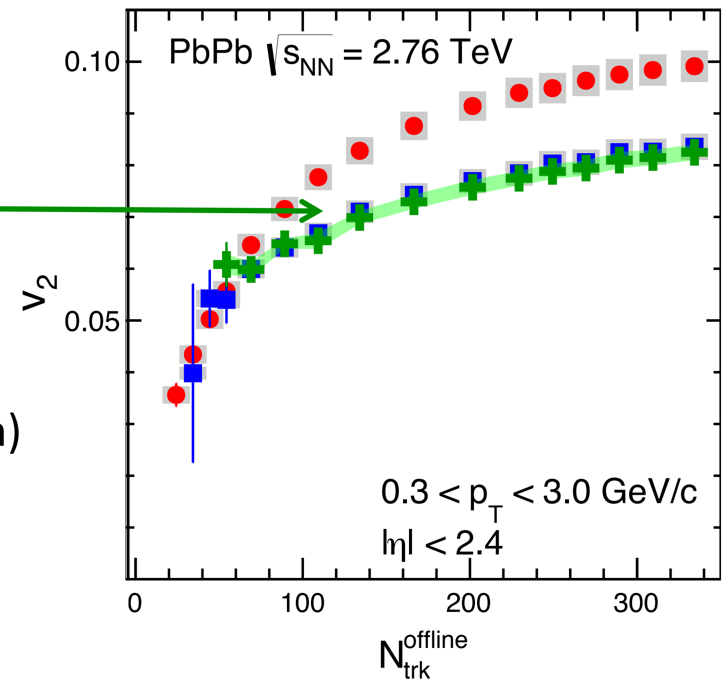
Collectivity in AA



6 particle correlations

$v_2\{6\}$

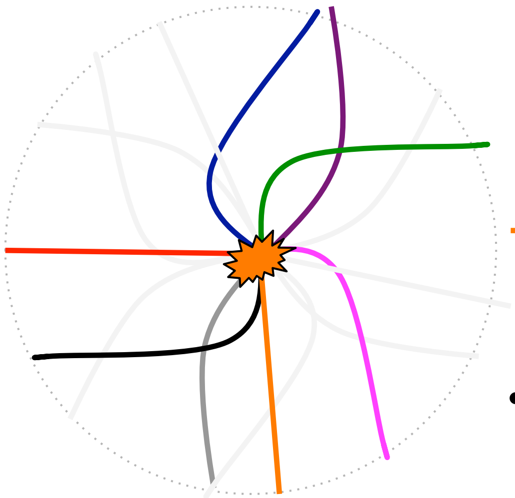
- Less sensitive to non-flow (i.e. jet induced correlation)
- Needs larger sample of events



PhysLettB.2016.12, 009

$$v_n\{6\} = \sqrt[4]{\frac{1}{4}c_n\{6\}}$$

Collectivity in AA



8 particle correlations

$v_2\{8\}$

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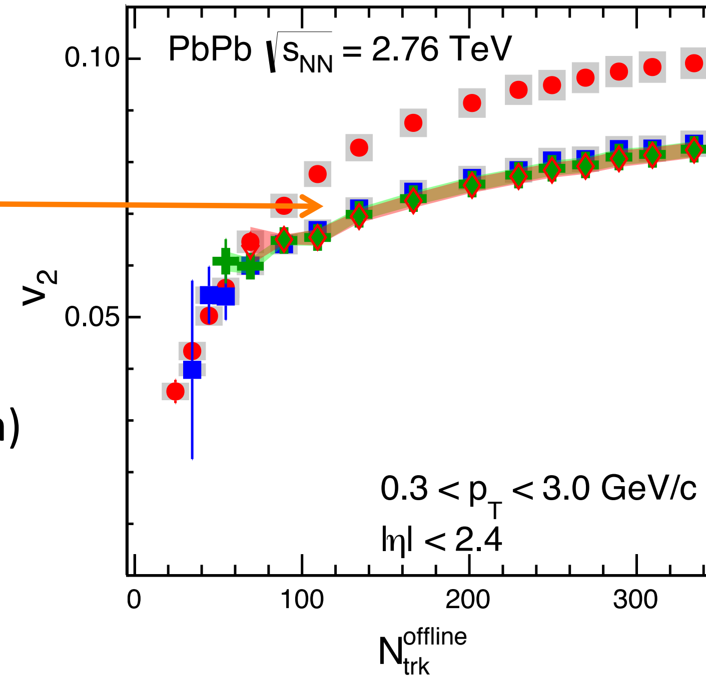
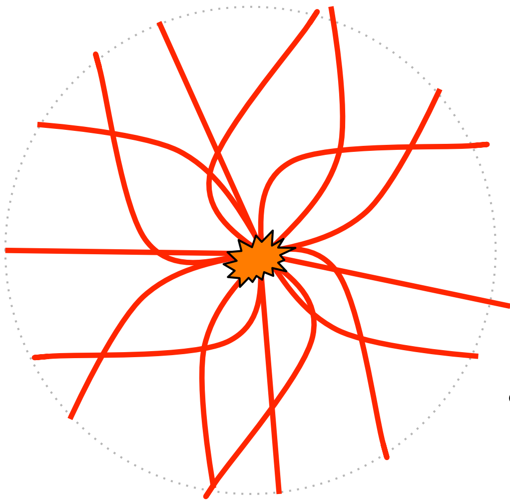


PhysLettB.2016.12, 009

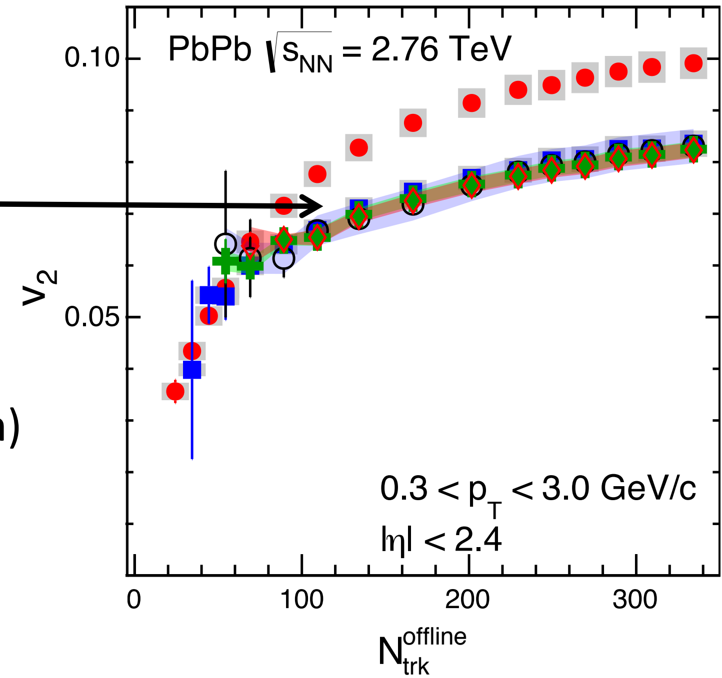
$$v_n\{8\} = \sqrt[4]{-\frac{1}{33}c_n\{8\}},$$

Collectivity in AA



All particles

$v_2\{LYZ\}$



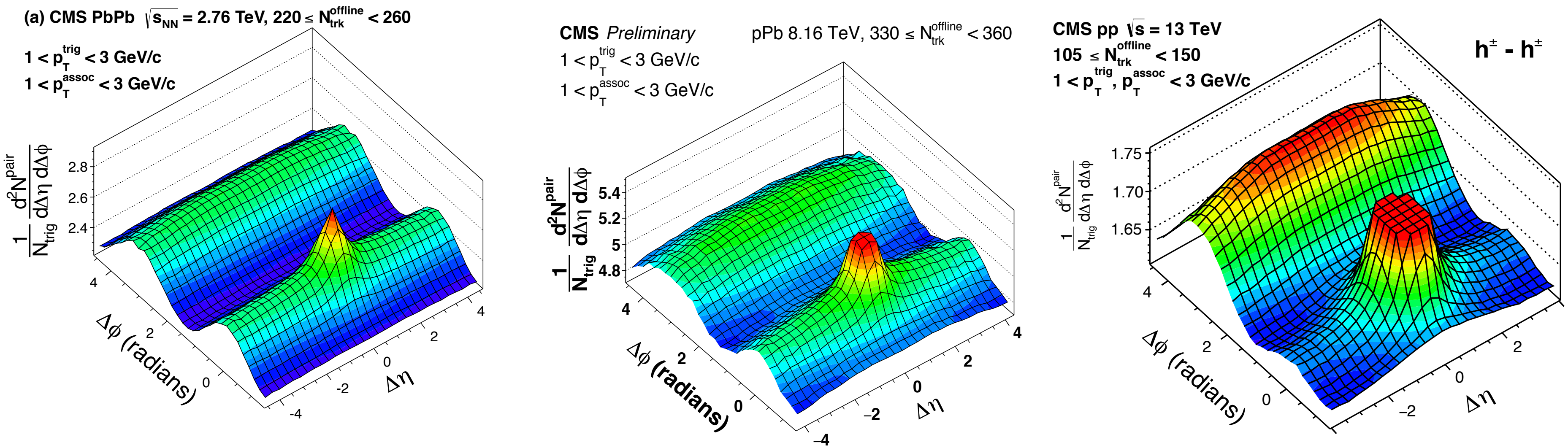
PhysLettB.2016.12, 009

- Less sensitive to non-flow (i.e. jet induced correlation)
- Needs larger sample of events

➢ Collectivity: $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \approx v_2\{LYZ\}$

➢ Well describe by **hydrodynamic** at low p_T (< 3 GeV/c)

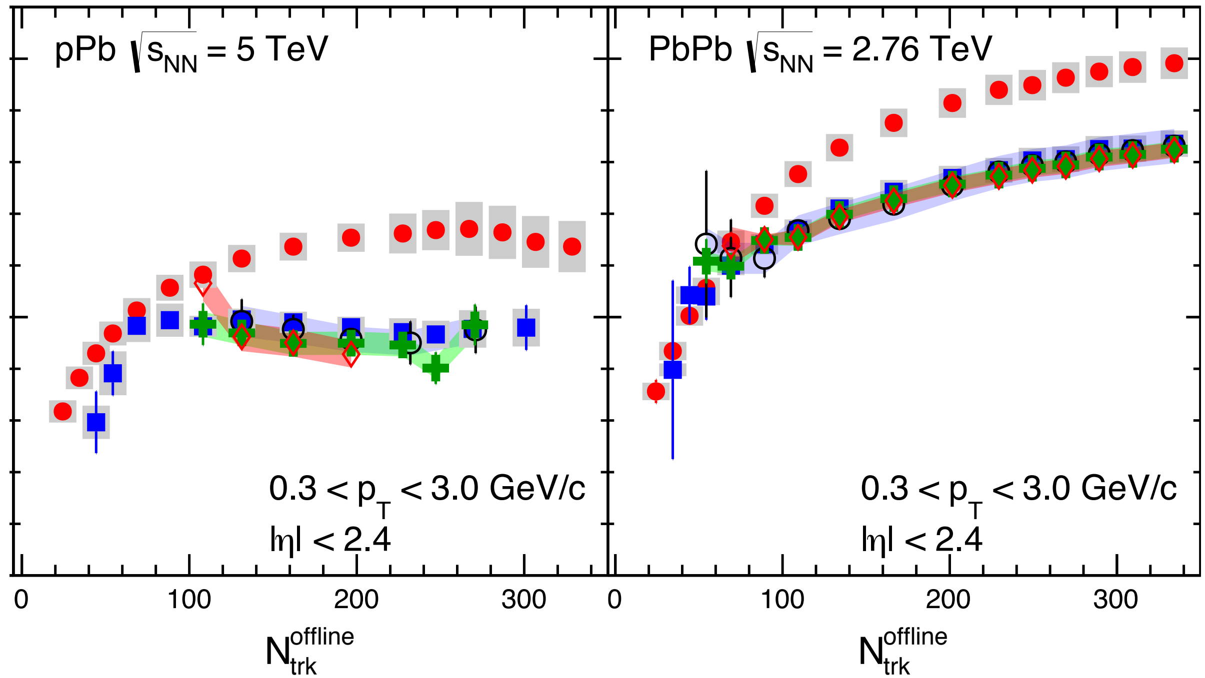
“Surprising” long range correlation in small systems



PhysLettB.2013.06, 028, PhysLettB.2016.12, 009

- **“Collective effect”**
  - Two articles with very different η are “connected”
  - Also observed in small colliding systems for **high multiplicity events**
- **Is it a sign of QGP formation in small system?**

Evidence of collectivity in small systems



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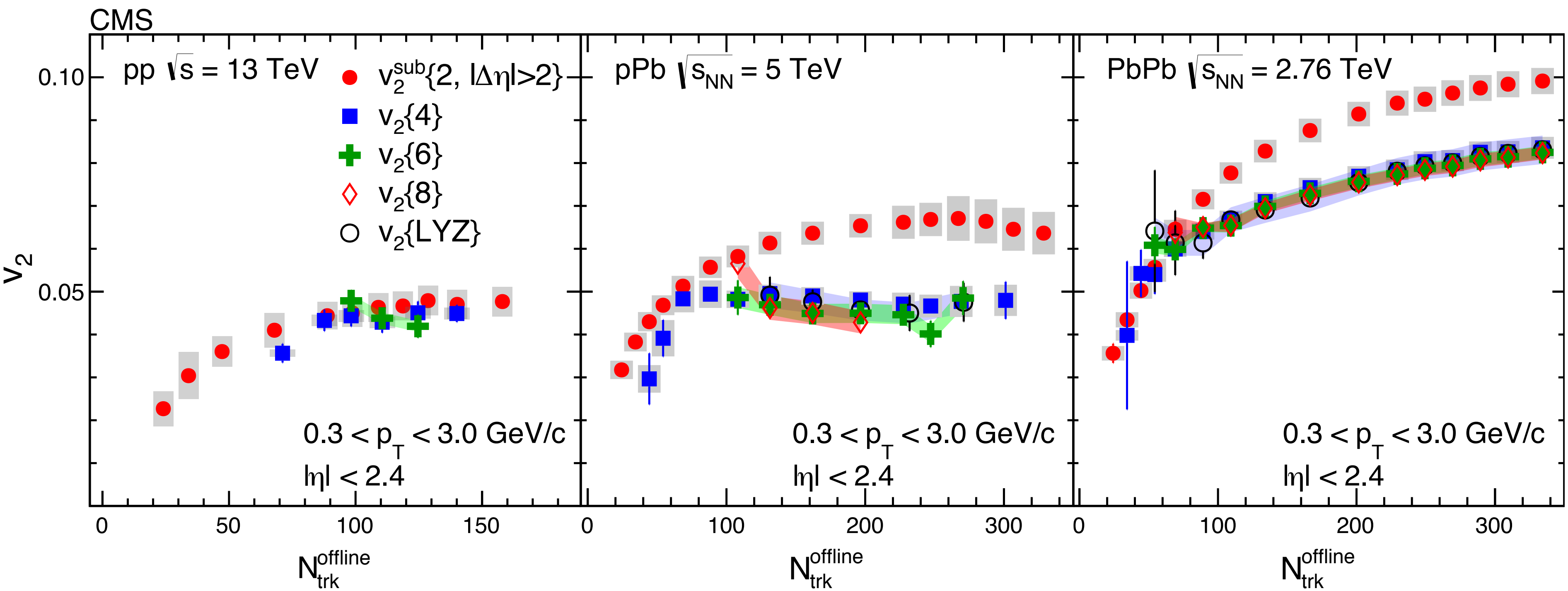
- $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ ⟶ **Collectivity!**

Evidence of collectivity in small systems



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- $v_2\{2\} \approx v_2\{4\} \approx v_2\{6\}$ ⟶ **Collectivity!**
- $v_2\{2\} \approx v_2\{4\}$: From hydro. picture
  - **Less fluctuating sources in the IS**

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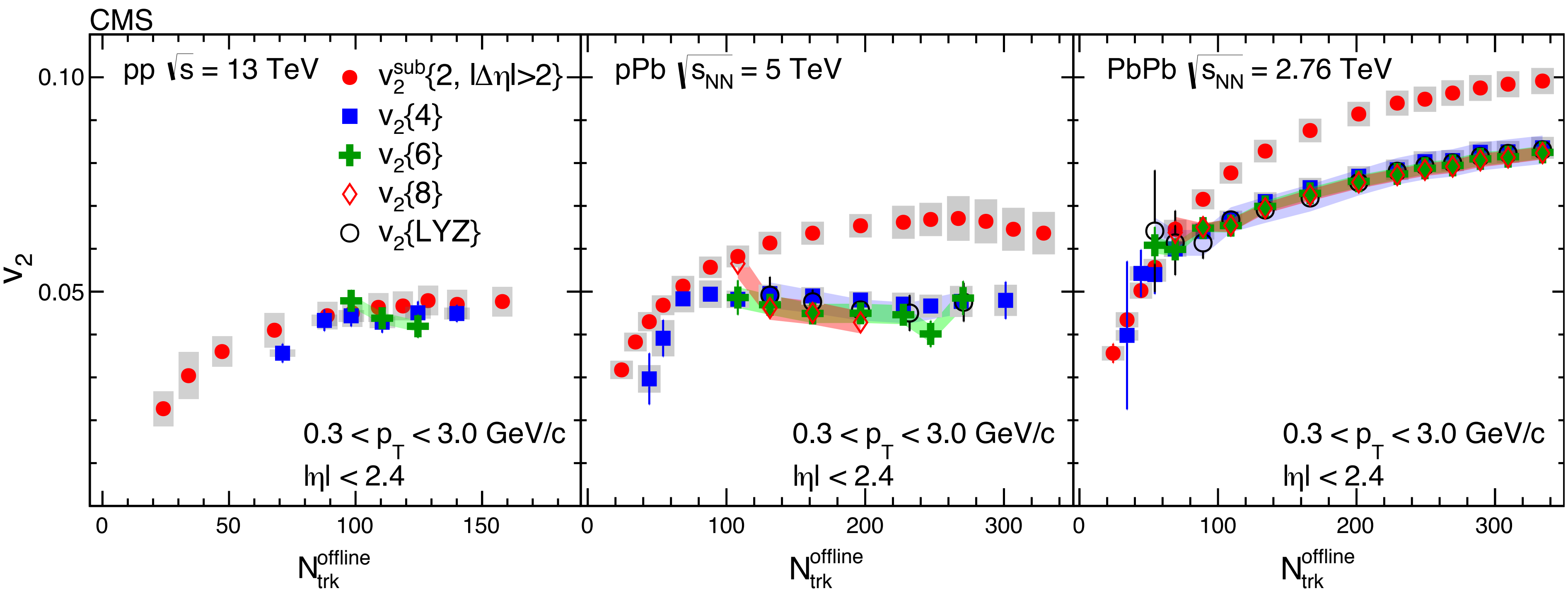

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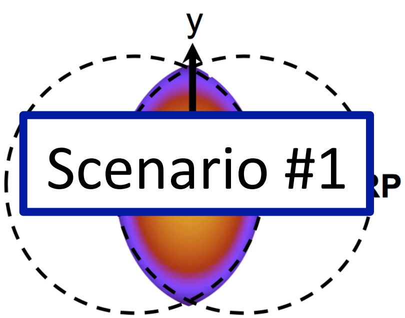
- $v_2\{2\} \approx v_2\{4\}$: From hydro. picture
  - Less fluctuating sources in the IS

Still under debate because of the subtraction

How to interpret the data?

Observed long-range correlations in η is rooted in **the early stage**



Scenario #1

Initial spatial ε_s **+ final** interactions

Hydrodynamics
Parton transport, escape

0 τ

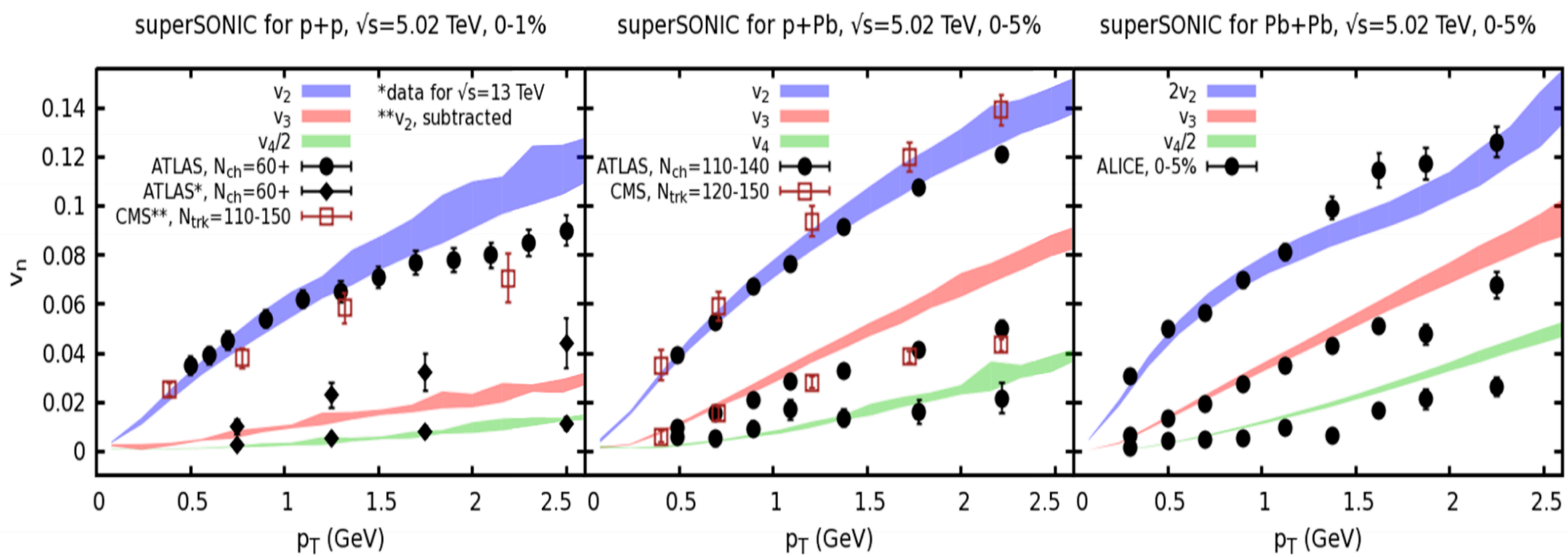
Scenario #2

Initial spatial ε_p by initial interactions

CGC Glasma
Color-field domains, etc.

- **Challenge:** How to disentangle them experimentally?
- Can we find a universal description for all hadronic colliding systems?

One fluid to rule them all?



First attempt of unification…

arXiV:1701.071459

- Reproduce the data quite well
- **Emergence of a unified paradigm?**
  - Not yet… Applicability of hydro? Experimental method? Cumulant?

Other contestant strike back!

Very interesting results from CGC like scenario

- Reproduce the now famous: $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$

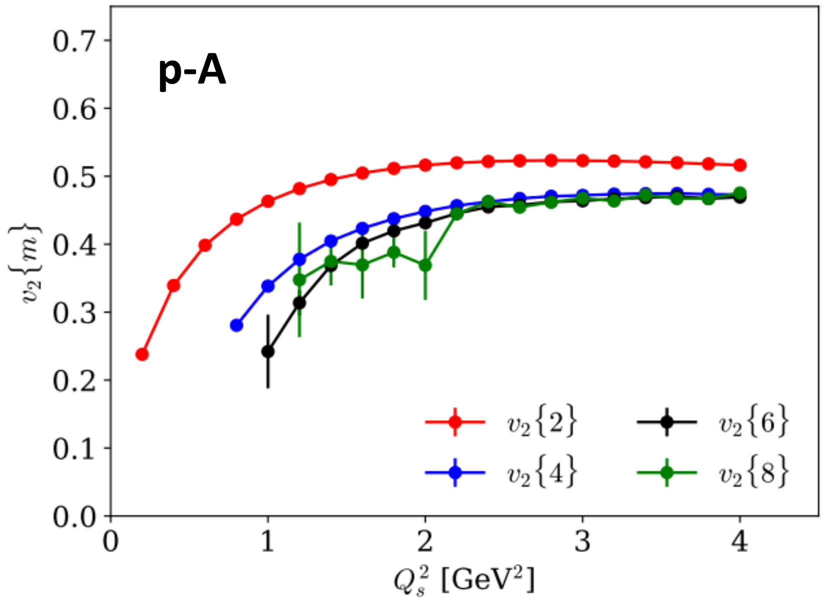


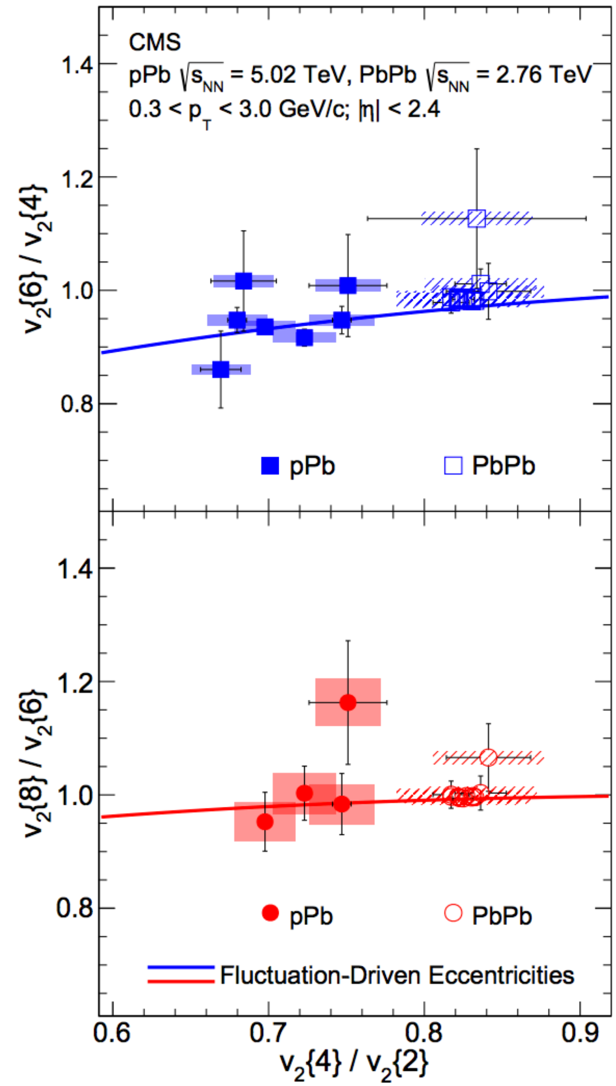
FIG. 6. Two, four, six and eight particle Fourier harmonics for coherent multiple scattering off Abelian fields plotted as a function of $Q^2_{s,T}$.

arXiV:1705.00745

$v_n\{m\}$ measurement might not be sufficient to disentangle two scenarii

Universal behavior of initial geometry fluctuations?

- Fine splitting between $v_2\{4\}$, $v_2\{6\}$ and $v_2\{8\}$ observed
- Universal behavior for PbPb and pPb



Phys.Rev.Lett. 115, 012301 (2015)

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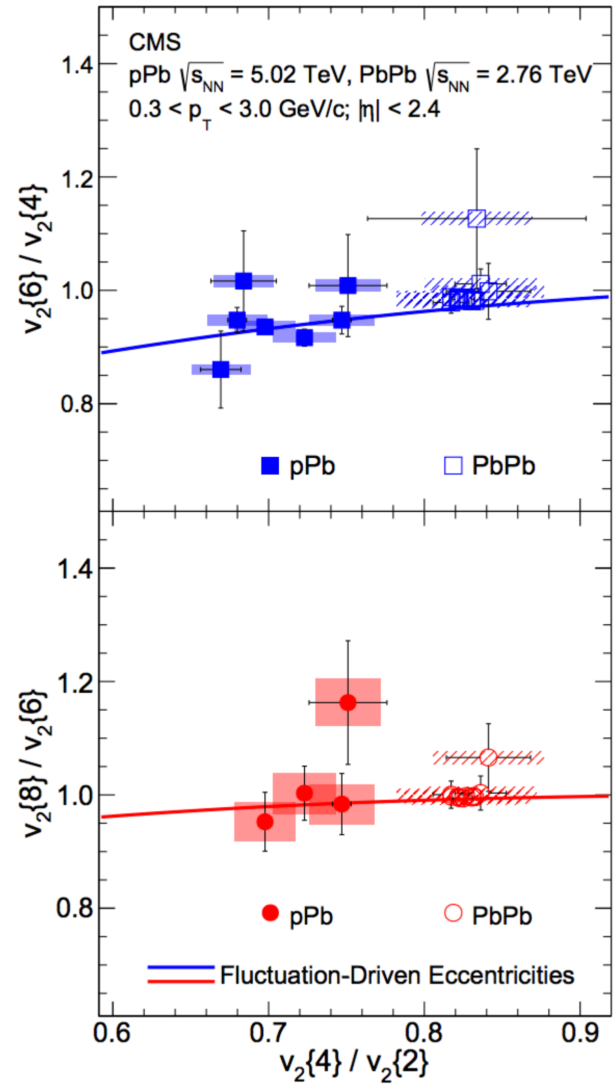


- **According to predictions**:
  - Universal behavior **does not** depend on the harmonic order *n*
  - $v_3\{m\}$ is a very good test for universal eccentricity-driven behavior

Phys.Rev.Lett. 115, 012301 (2015)

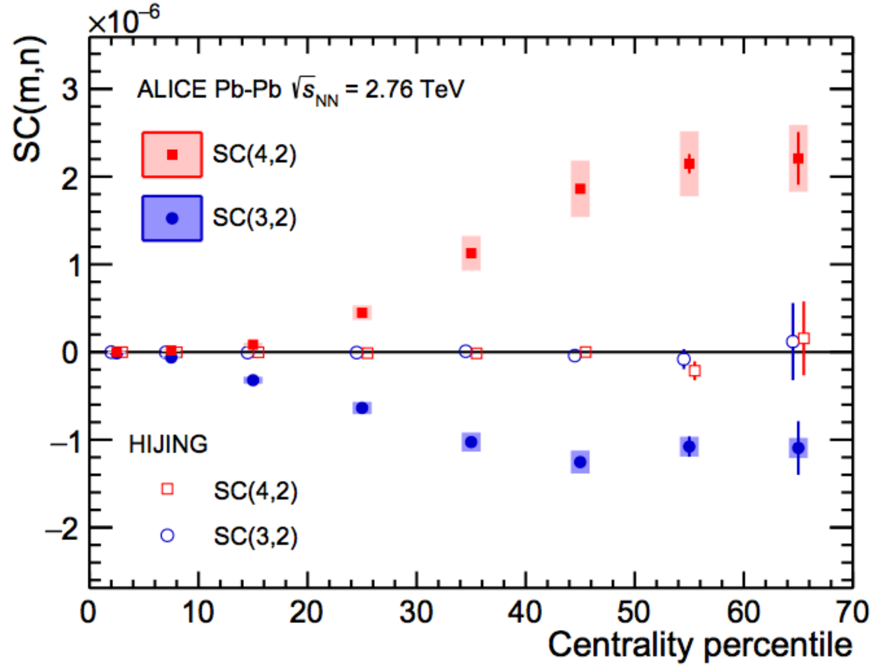
Looking into the details!

New insights to IS fluctuation of pp and pPb

- Correlation between harmonics:

$$SC(n,m) = \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$$

- Symmetric Cumulant (SC) developed by ALICE
  - New observable
  - **Base on 4-particle cumulant technique**
  - Non-flow free at first order



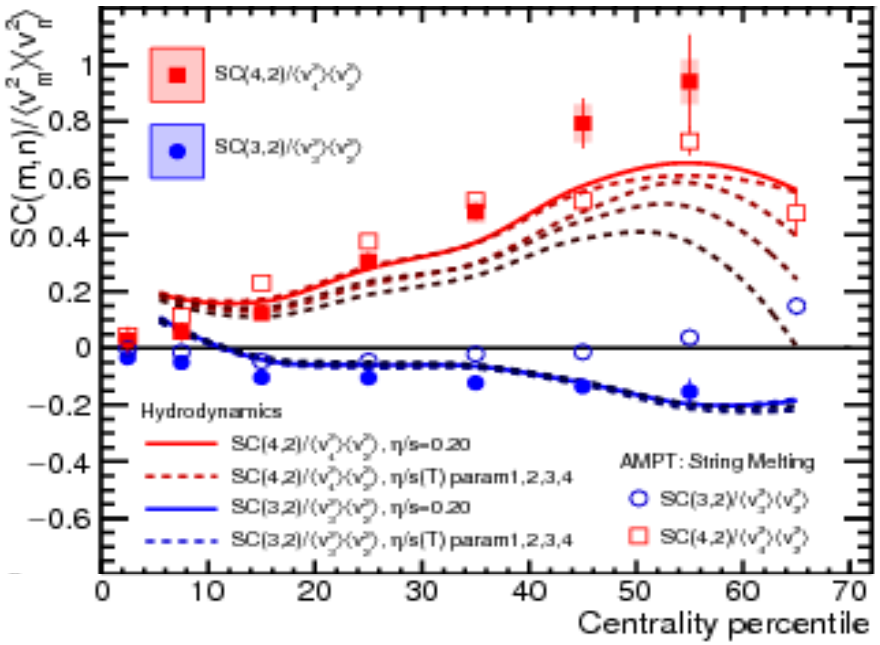
PhysRevLett.117, 182301 **ALICE results**

SC(2,3) < 0 ➔ v_2 and v_3 are **anti-correlated**

SC(2,4) > 0 ➔ v_2 and v_4 are **correlated**

Normalized SC in A-A (NSC)

- SC normalized by $<\varepsilon_n^2>.<\varepsilon_m^2>$
  - Only $<v_n^2>.<v_m^2>$ accessible experimentally
  - Normalized by v_n magnitude



PhysRevLett.117, 182301 **ALICE results**

- ✓ **(2,3) correlation: IS fluctuation**
- ✓ **(2,4) correlation: medium response + IS fluctuation**

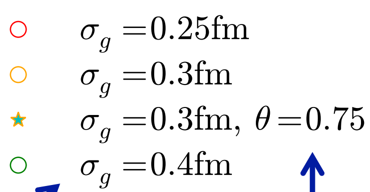
  - Apples-to-apples comparison across systems (p-p, p-Pb and Pb-Pb)
  - **Stringent constraints on models!**
    - Giacalone et al. **arXiv 1605.08303**
    - Gardim et al. **arXiv 1608.02982**
    - Norhona-Holster et al. **arXiv 1609.05171**
    - Welsh et al. **arXiv 1605.09418**

What about small colliding systems?

SC in small systems

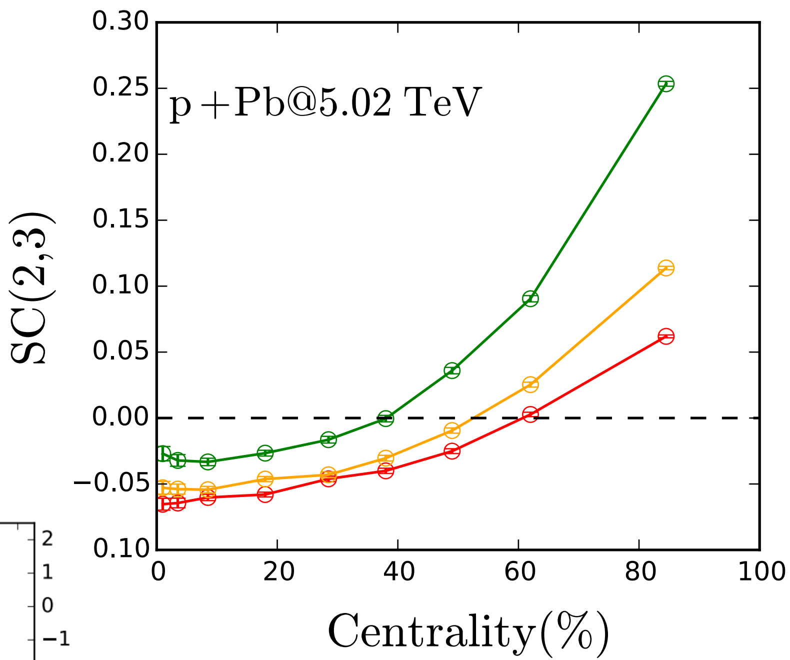
- SC can be measured in small system (p-p and p-Pb)
  - Little knowledge about the expected sign at first

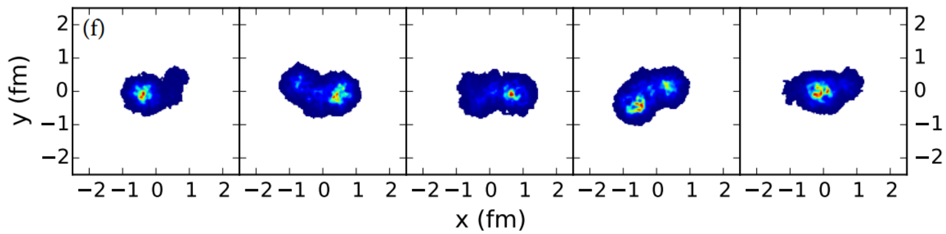
J. Quian, U. Heinz (eccentricity correlation only) arXiv 1605.09418



quark width

multiplicity fluctuations





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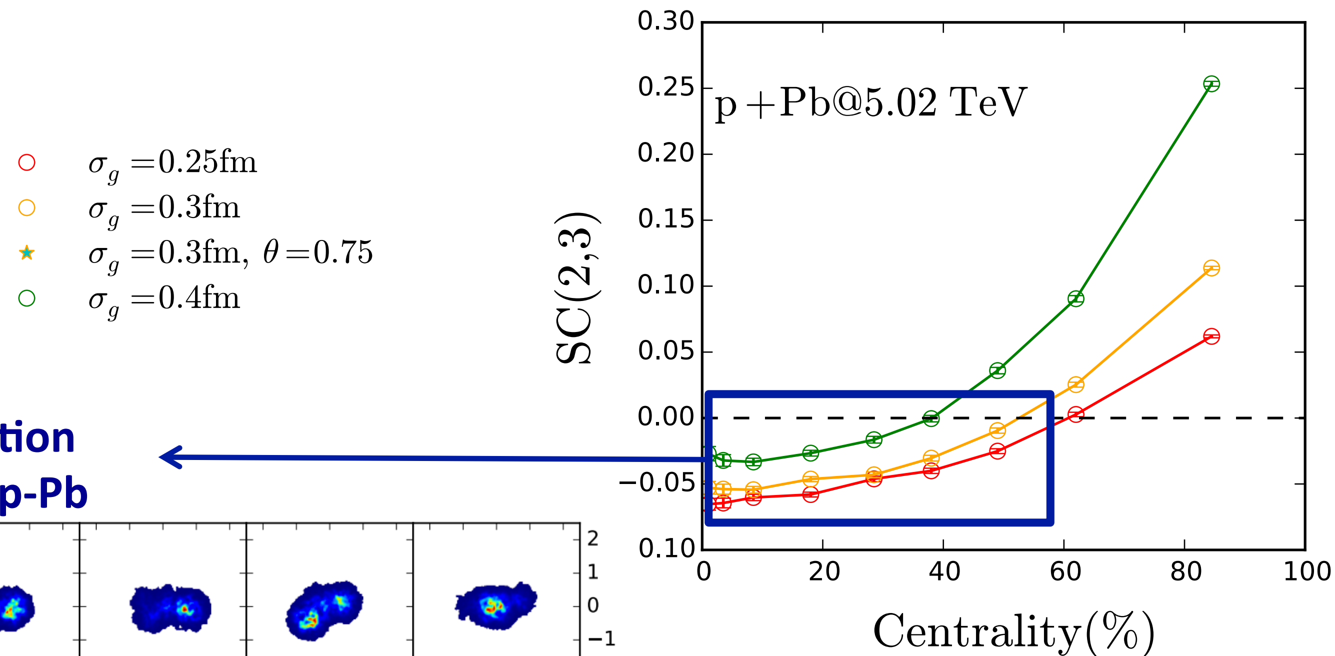
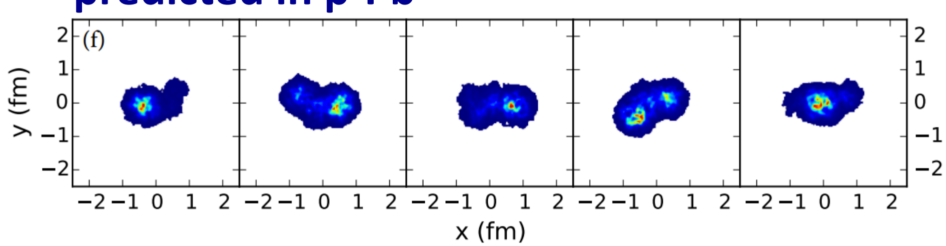


Anti-correlation predicted in p-Pb



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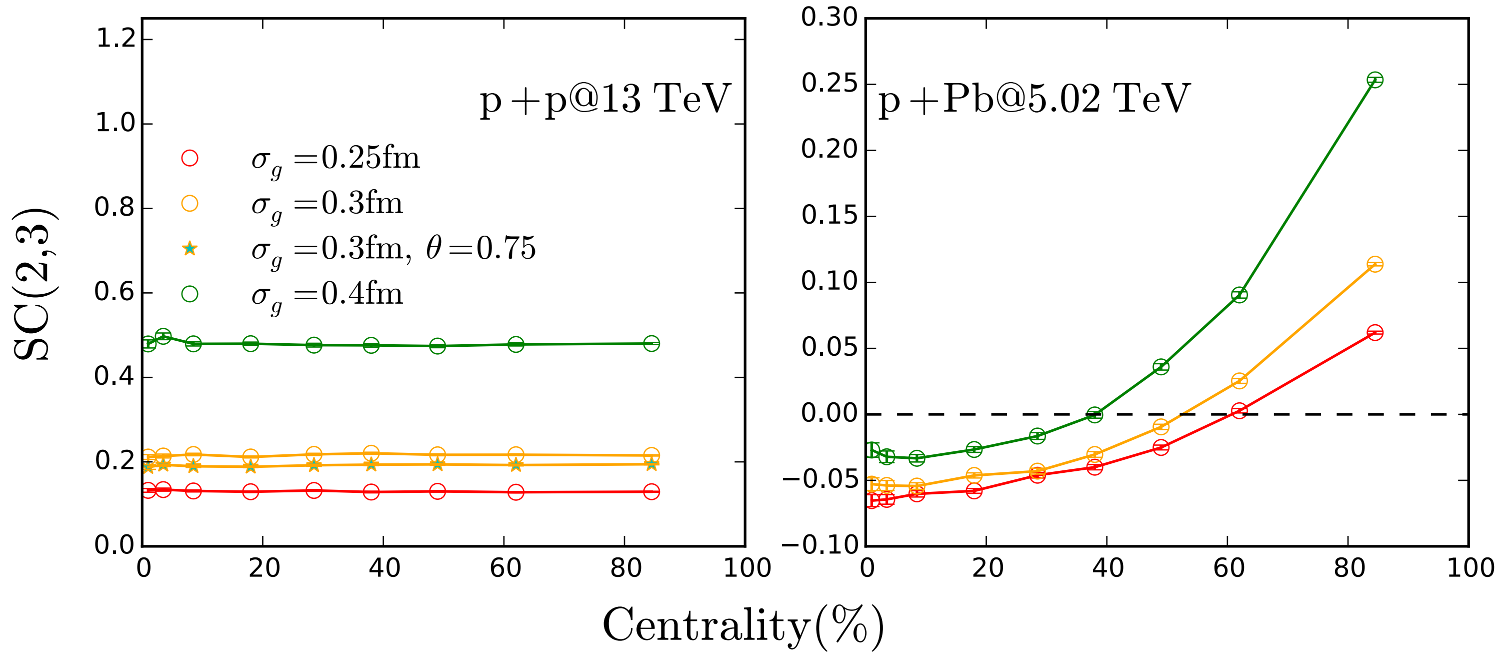

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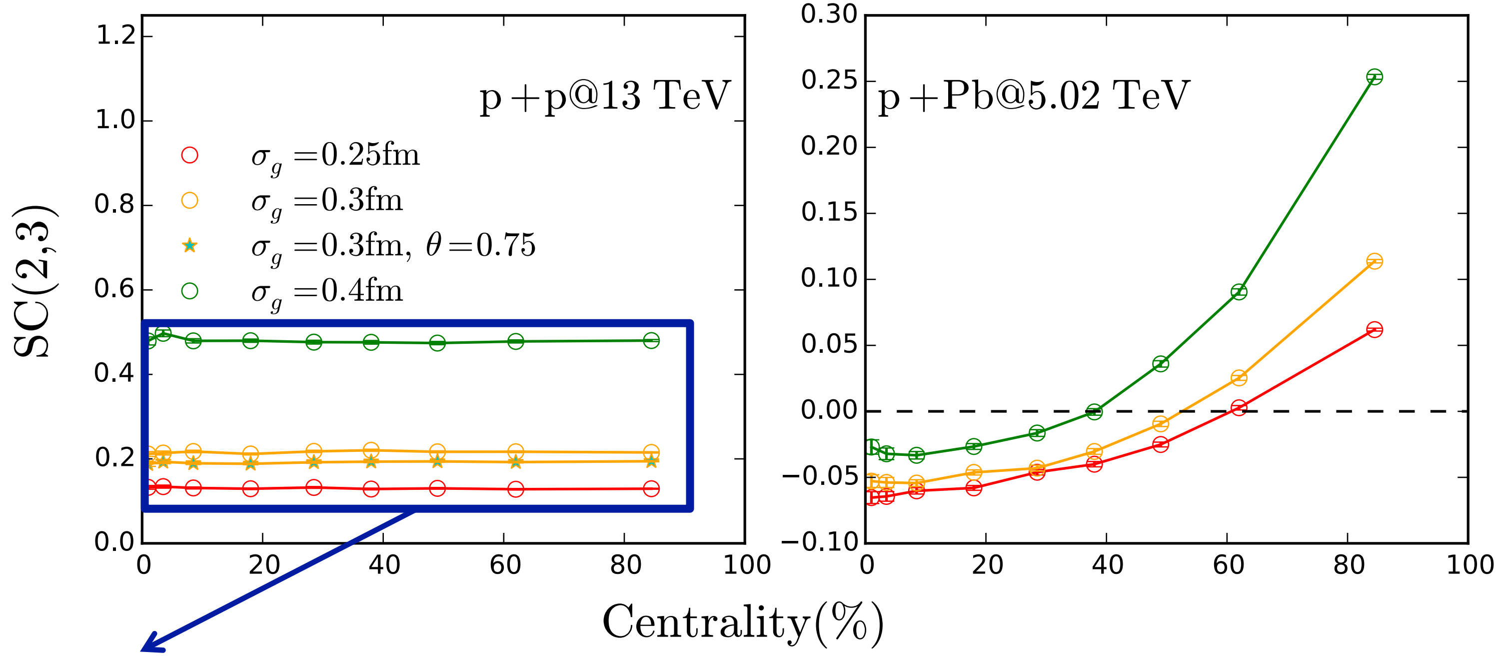
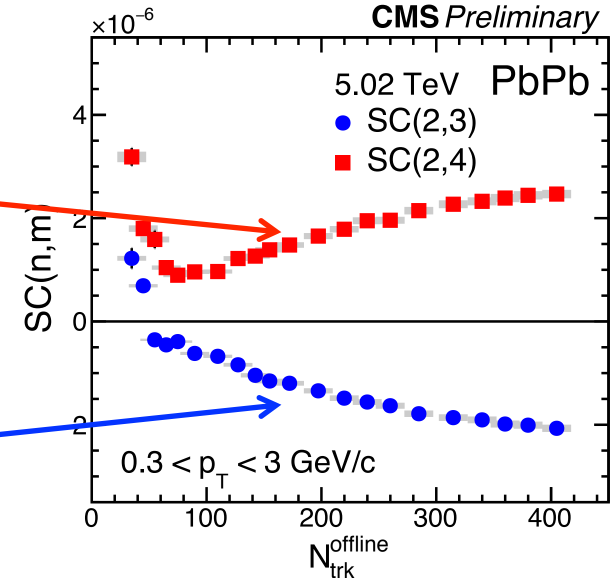


Correlation predicted in p-p → **Might be related to the number of fluctuating sources**

SC as a function of multiplicity

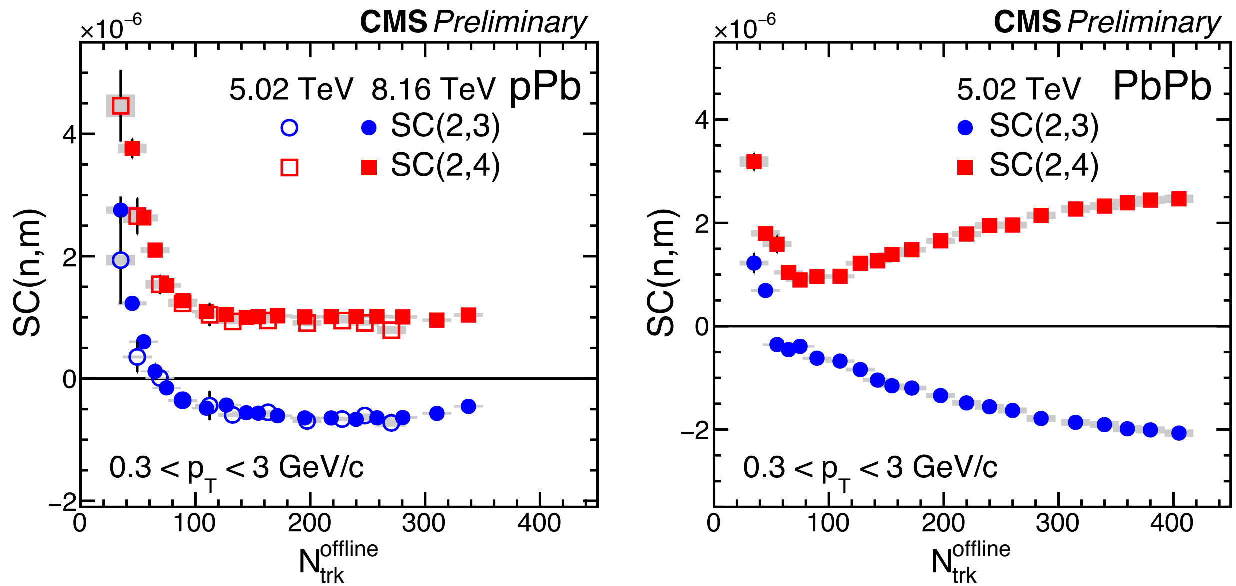
Correlation between v_2 and v_4

Anti-correlation between v_2 and v_3



CMS-PAS-HIN-16-022

SC as a function of multiplicity



CMS-PAS-HIN-16-022

- Very small energy dependence observed for p-Pb results
- pPb similar to PbPb:
  - Naturally explained by initial geometry!
  - A new challenge to initial interaction models?!

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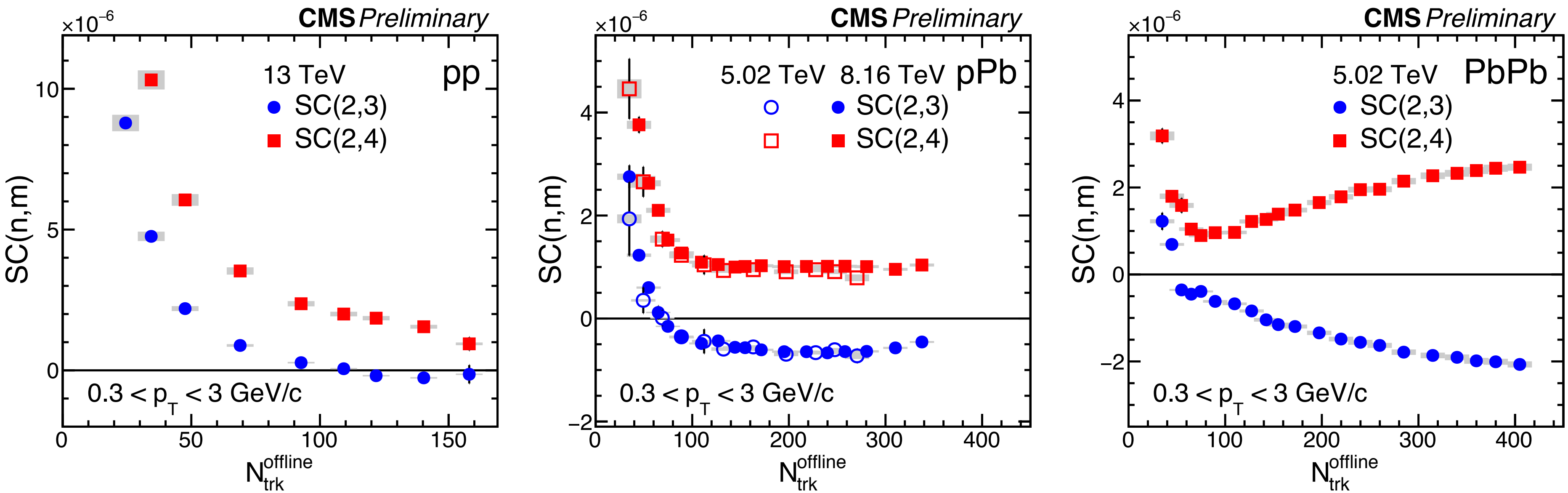

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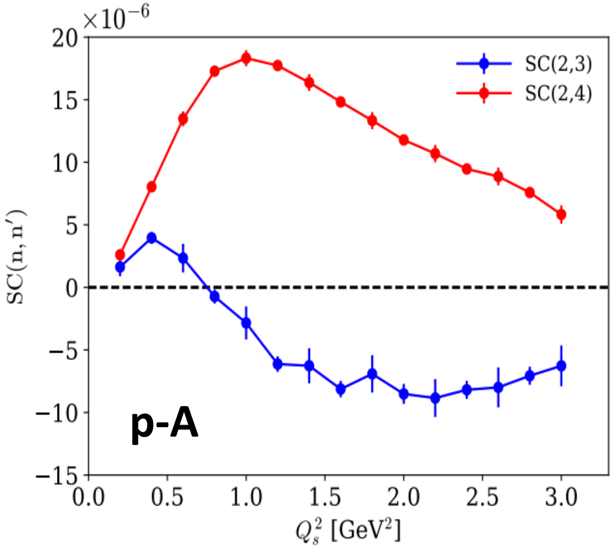
SC as a function of multiplicity



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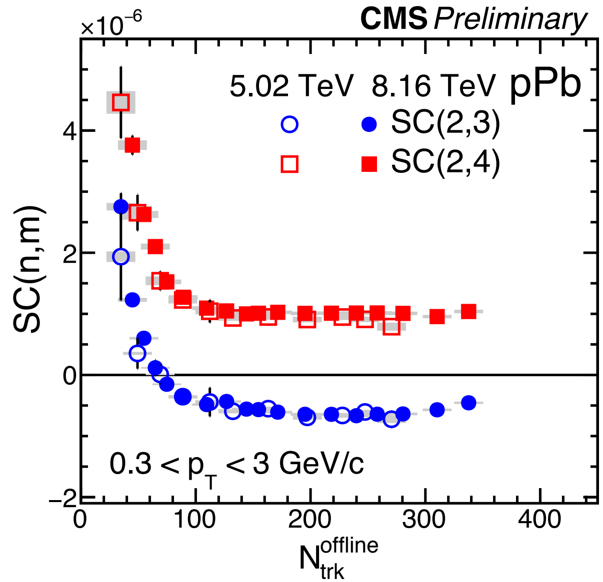
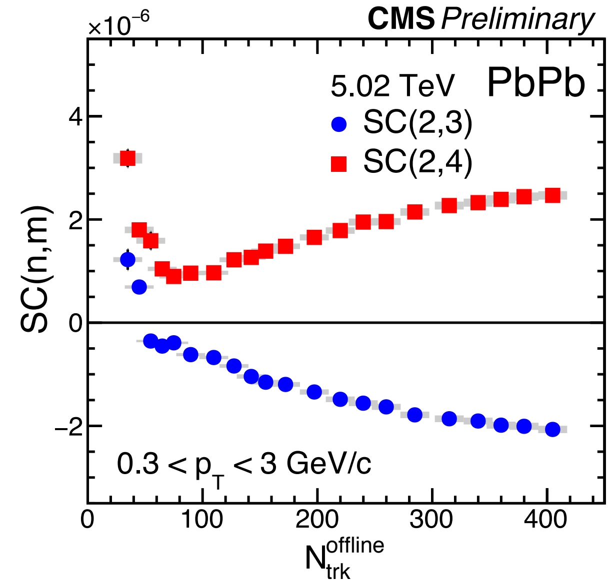


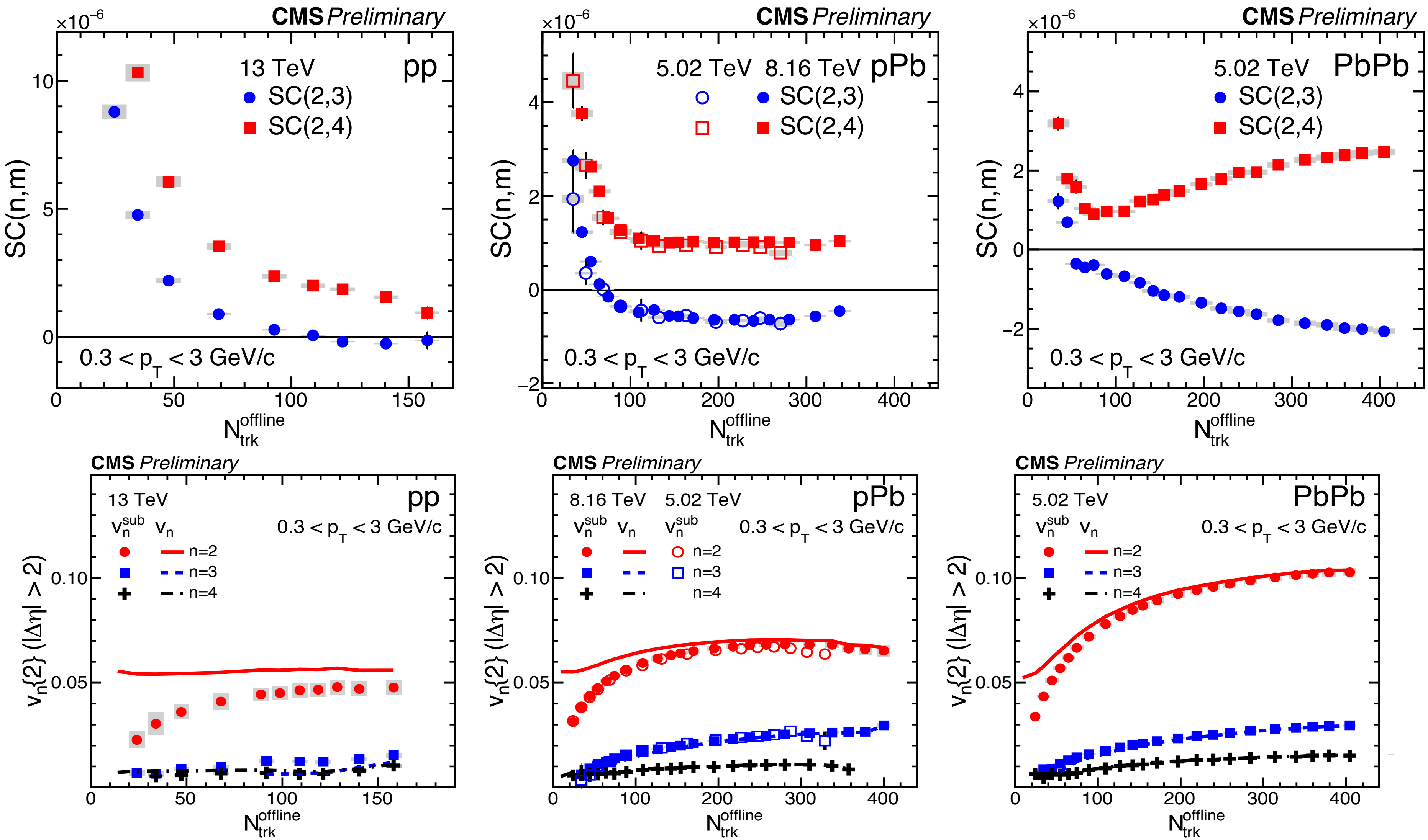


CMS-PAS-HIN-16-022

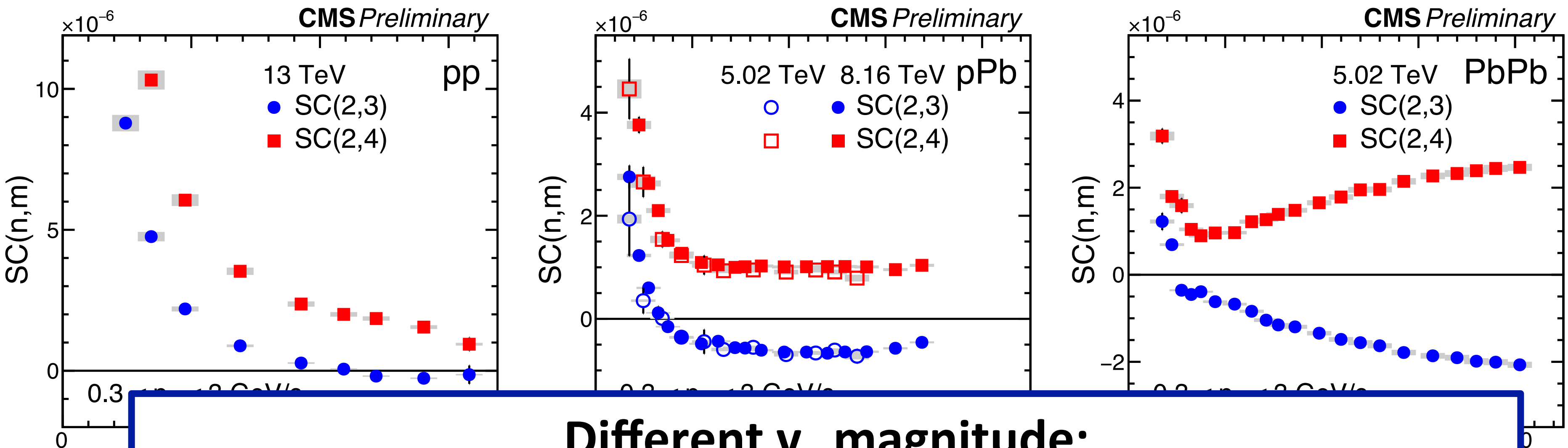
- Very small energy dependence observed for p-Pb results
- pPb and pp? similar to PbPb:
  - Naturally explained by initial geometry!
  - A new challenge to initial interaction models?! **… not really!**

How does it compare across system anyway? Universality!

Comparison across systems?

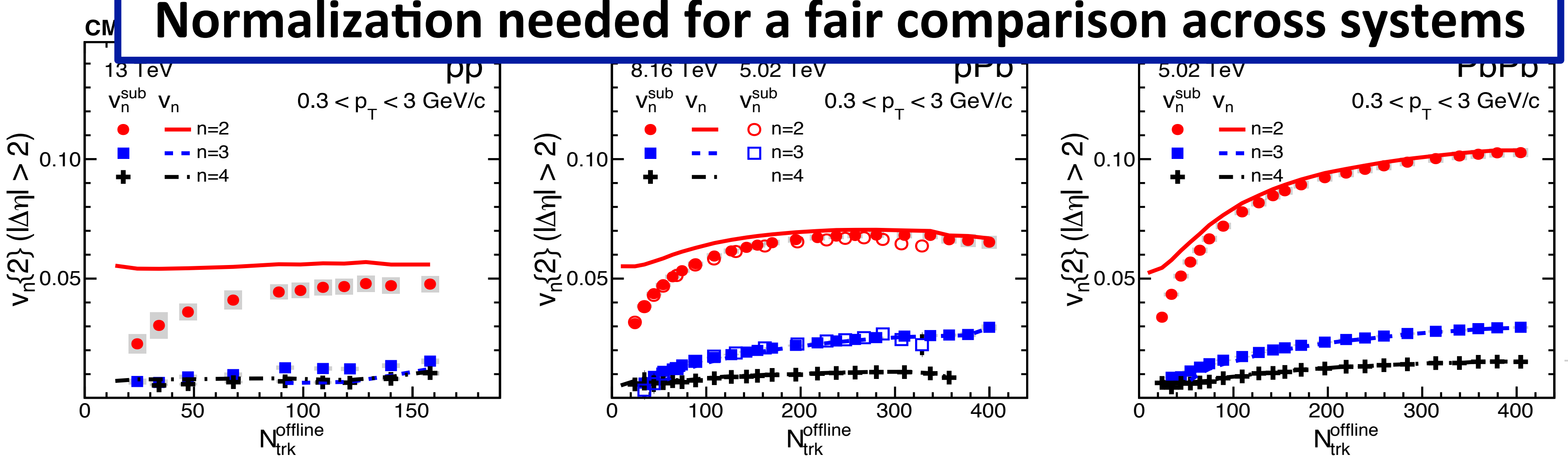


Comparison across systems?



Different v_n magnitude:

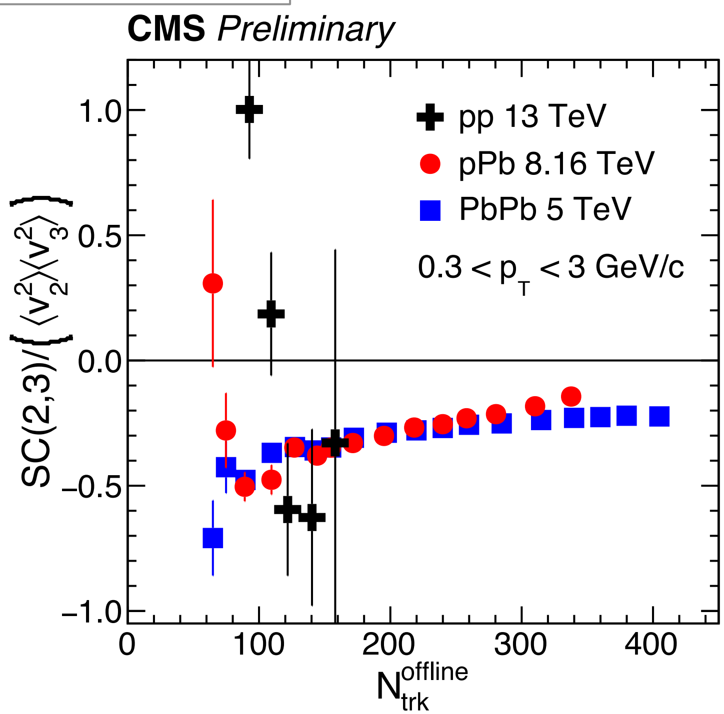
Normalization needed for a fair comparison across systems



NSC in all systems

CMS-PAS-HIN-16-022

SC normalized by <v$_n^2$>.<v$_m^2$>



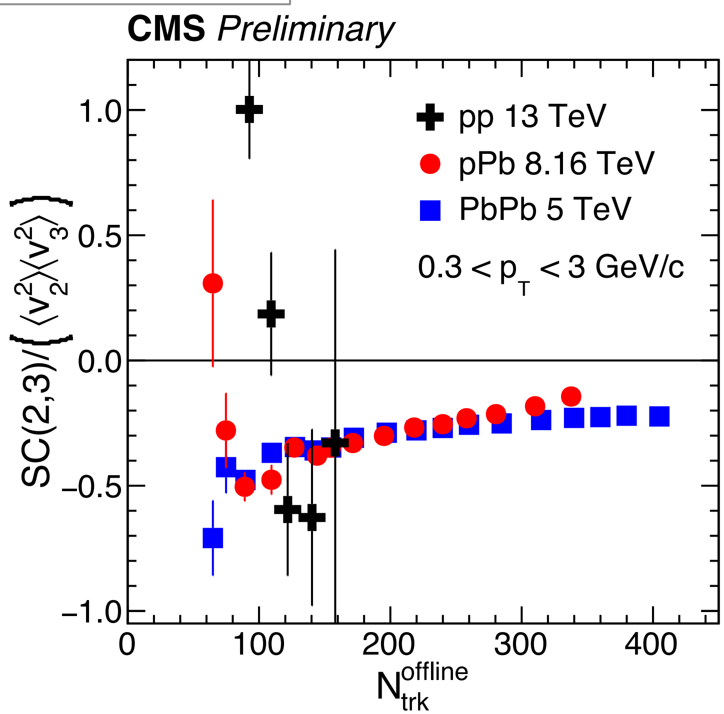
- Similar behavior in p-Pb and PbPb
- Points to similar IS fluctuations

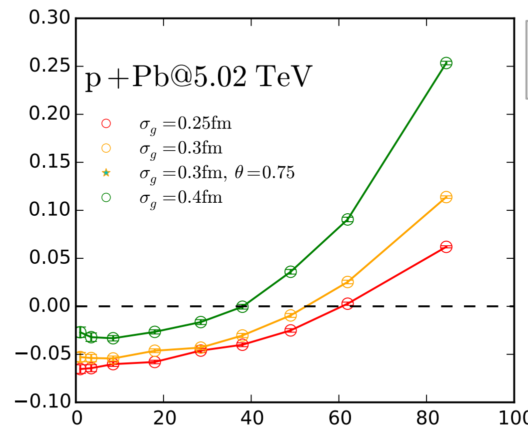
Universality???

NSC in all systems

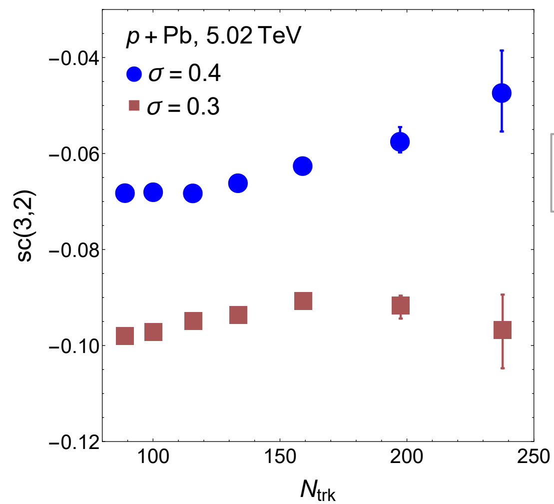
CMS-PAS-HIN-16-022

SC normalized by $\langle v_n^2\rangle.\langle v_m^2\rangle$





arXiv 1605.09418



Wounded nucleon model

arXiv 1609.05171

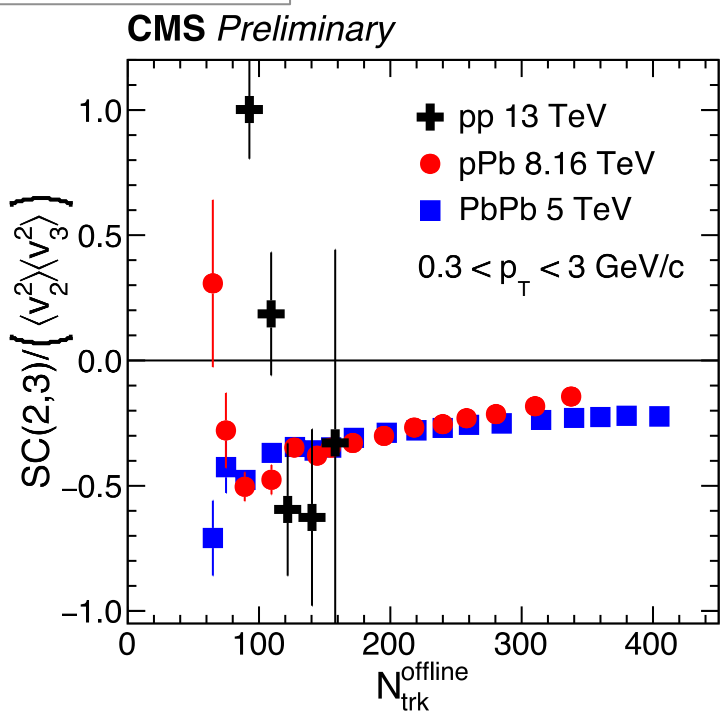
- Similar behavior in p-Pb and PbPb
- Points to similar IS fluctuations
- First calculations (ε_n correlations only)
  - Right sign
  - Magnitude is off

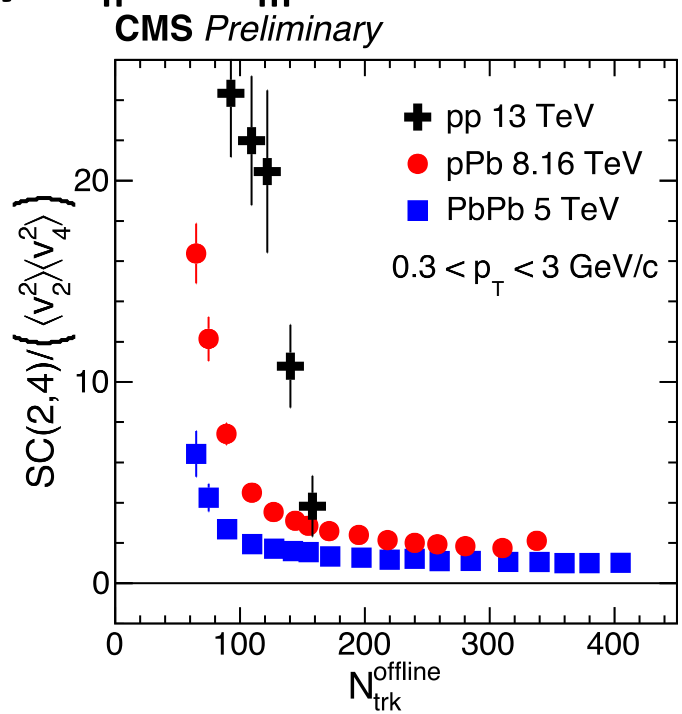
How does CGC like NSC look like?

NSC in all systems

CMS-PAS-HIN-16-022

SC normalized by <v_n^2>.<v_m^2>





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- Points to similar IS fluctuations
- First calculations (ε_n correlations only)
  - Right sign
  - Magnitude is off

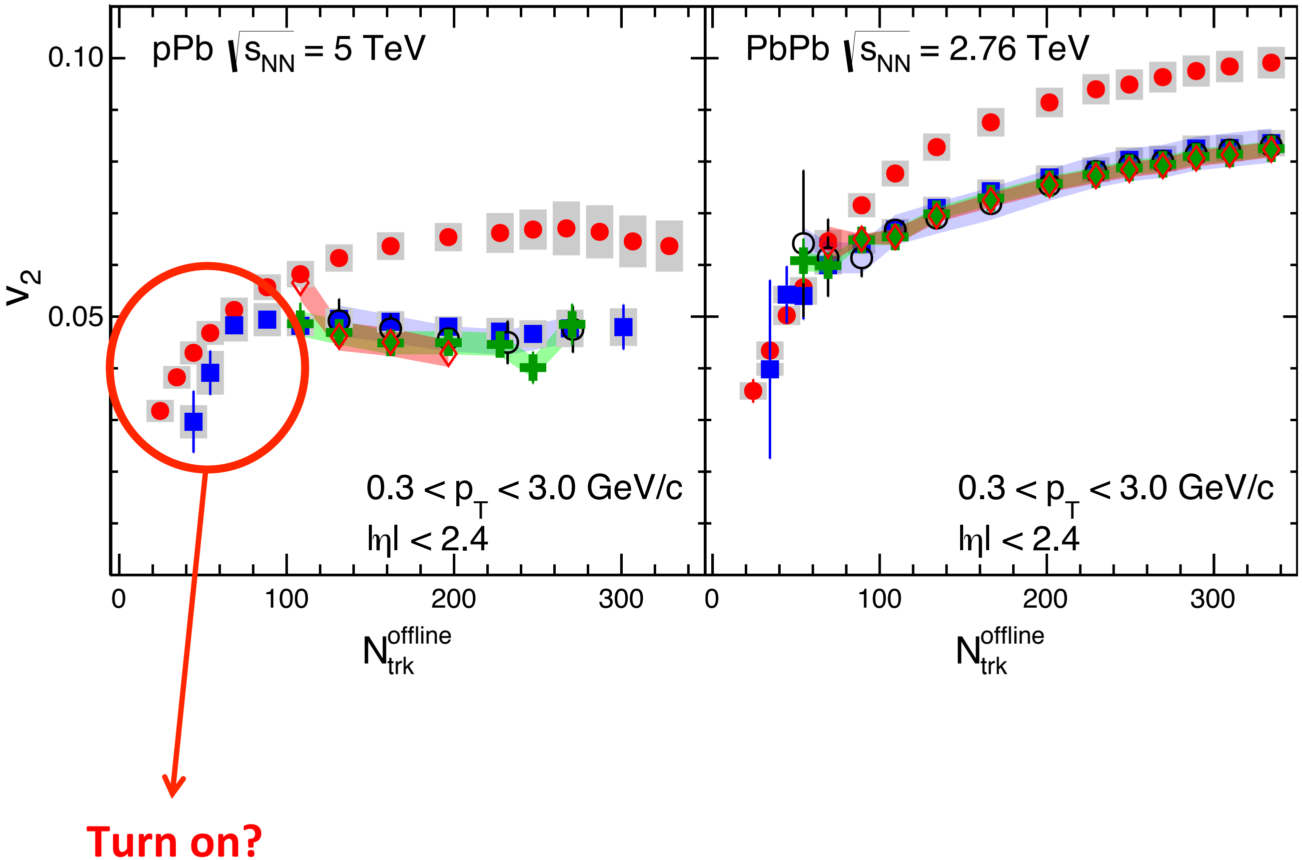
- Ordering observed:

  p-p > p-Pb > Pb-Pb

- May point to different transport properties

How to disentangle further different scenarii?

When (at which N) does collectivity turn on?



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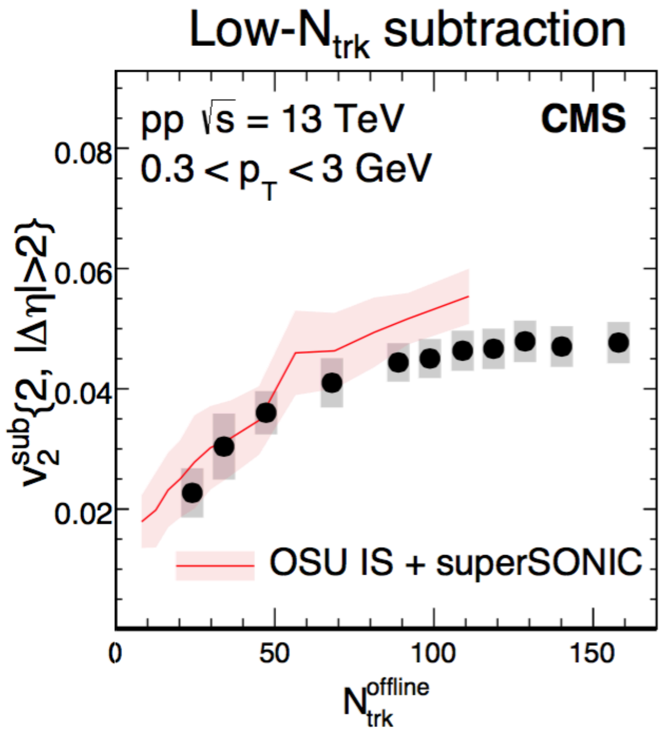
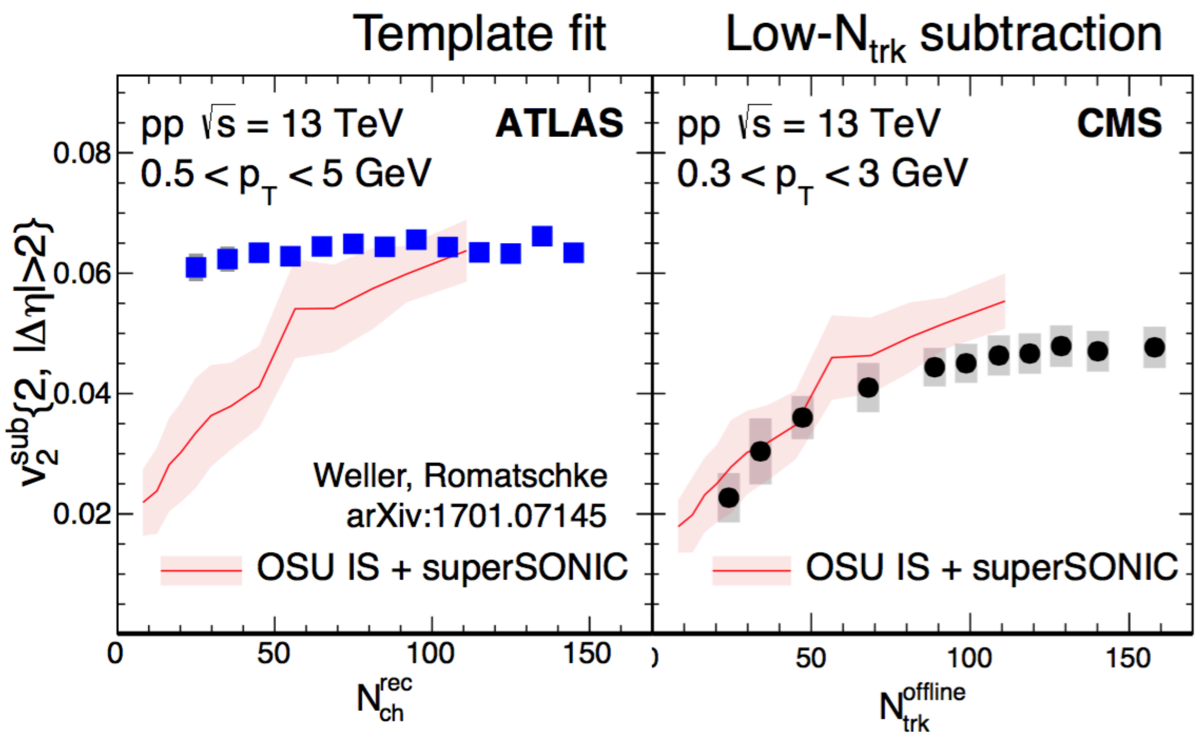
When (at which N) does collectivity turn on?



- ➢ If hydro is the explanation, v_2 should go down!

How to disentangle further different scenarii?

Does collectivity turn off?



- If hydro is the explanation, v_2 should go down! (see Wei's talk)
- Need measurement down to low N_{trk} w/o subtraction assumption

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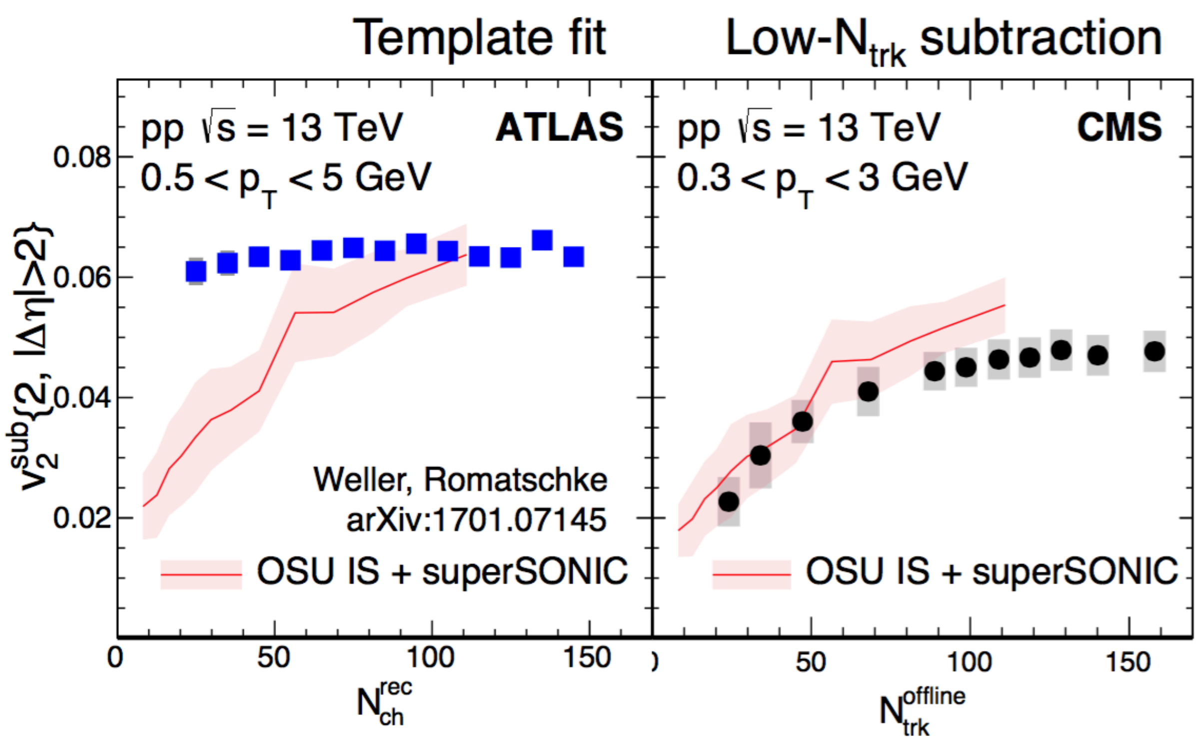
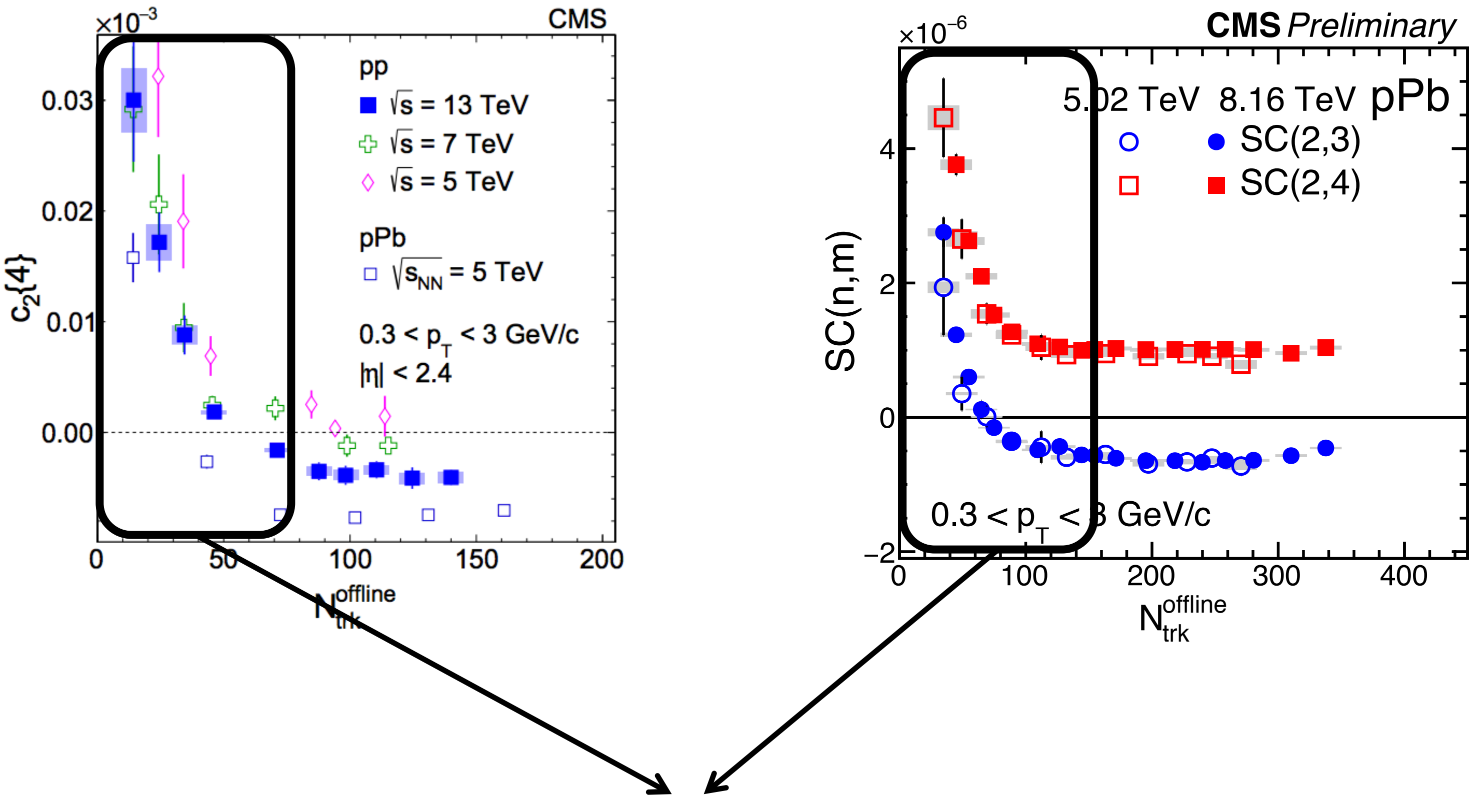


- If hydro is the explanation, v_2 should go down! (see Wei's talk)
- Need measurement down to low N_{trk} w/o subtraction assumption

Multi-particle correlation???

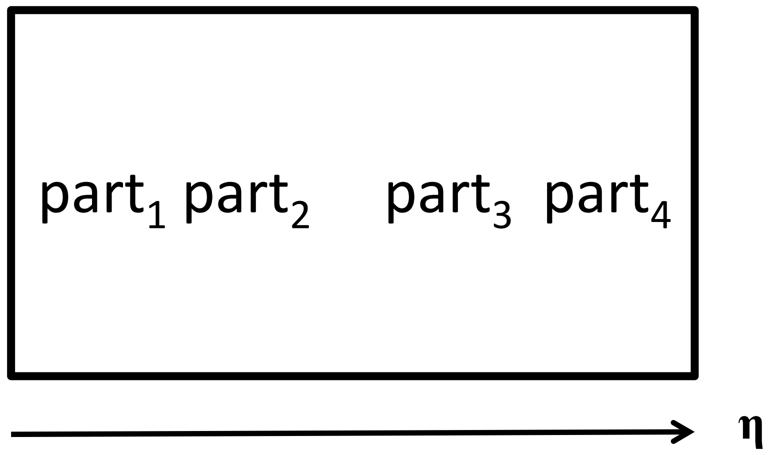
Non-flow returns

- Need to measure down to low N_{trk} but…



At low N, multi-particle technique also affected by non flow!!!

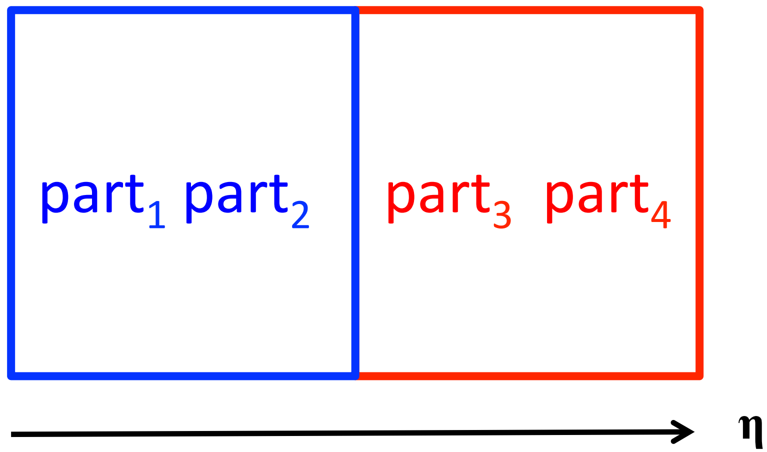
Sub-event method



- Standard 4-particle cumulant method
  - Affected by non-flow (jet induced correlation)
  - Especially at low N where jets dominate

Sub-event method

Francesco, Guilbaud, Luzum, Ollitrault
Phys. Rev. C 95, 044911 (2017)

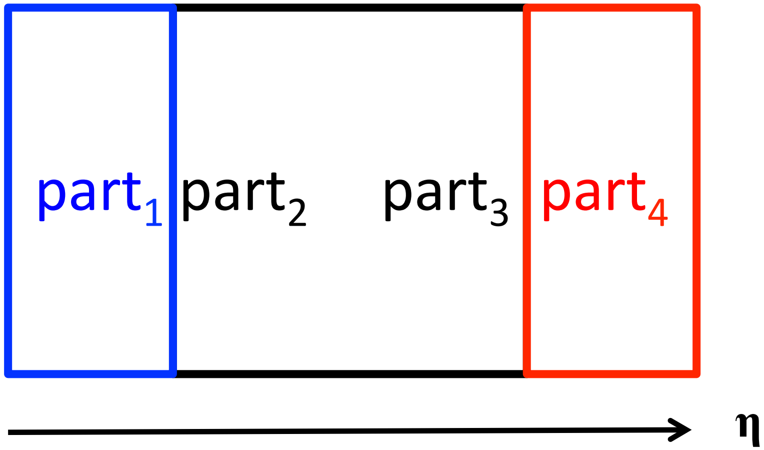


Jia, Zhou, Trzupek
arXiV 1701.03830

- 2 sub event 4-particle cumulant method
  - Suppress non flow contribution from jets

Sub-event method

Francesco, Guilbaud, Luzum, Ollitrault
Phys. Rev. C 95, 044911 (2017)



Jia, Zhou, Trzupek
arXiV 1701.03830

- 3 sub event 4-particle cumulant method
  - Suppress non flow contribution from di-jets

Sub-event method

Francesco, Guilbaud, Luzum, Ollitrault
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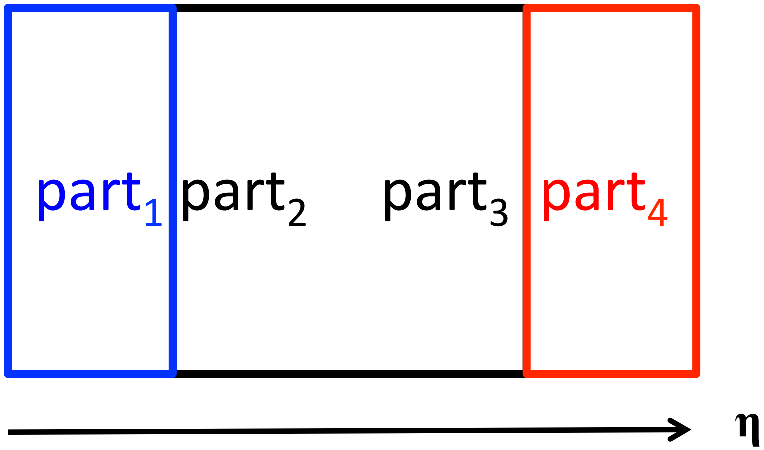
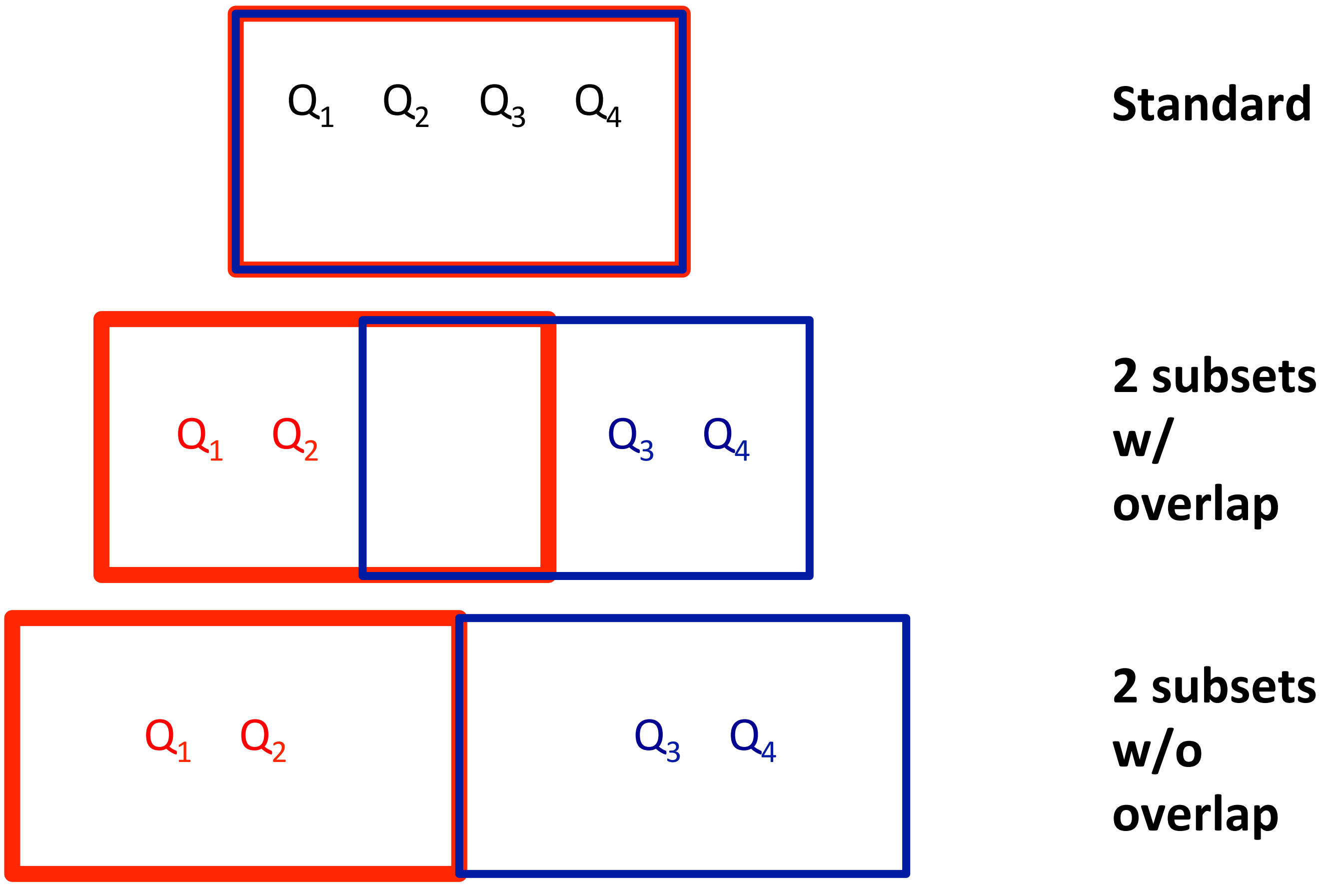


Jia, Zhou, Trzupek
arXiV 1701.03830

- 3 sub event 4-particle cumulant method
  - Suppress non flow contribution from di-jets

Understand the non-flow contribution



- Vary the amount of non flow to understand its contribution

Decomposition into moments

- Generic formula to compute cumulant up to arbitrary order with defined number of subset
  - N_{subset_max} = cumulant order
- Can be used easily to compute p_T or η dependent cumulant

$e^{in\phi_j}$

$$Q(A_i) \equiv \sum_{j_i \in A_i} q_i(j_i),$$

$$Q(A_1, \ldots, A_n) \equiv \sum_{j_1 \in A_1, \ldots, j_n \in A_n} q_1(j_1) \ldots q_n(j_n),$$

$$Q(A_1 \cap A_2) \equiv \sum_{j \in A_1 \cap A_2} q_1(j) q_2(j),$$

<4>

$$\begin{aligned} Q(A_1, A_2, A_3, A_4) = & \; Q(A_1)Q(A_2)Q(A_3)Q(A_4) \\ & - Q(A_1 \cap A_2)Q(A_3)Q(A_4) \\ & - Q(A_1 \cap A_3)Q(A_2)Q(A_4) \\ & - Q(A_2 \cap A_3)Q(A_1)Q(A_4) \\ & - Q(A_1 \cap A_4)Q(A_2)Q(A_3) \\ & - Q(A_2 \cap A_4)Q(A_1)Q(A_3) \\ & - Q(A_3 \cap A_4)Q(A_1)Q(A_2) \\ & + Q(A_1 \cap A_2)Q(A_3 \cap A_4) \\ & + Q(A_1 \cap A_3)Q(A_2 \cap A_4) \\ & + Q(A_1 \cap A_4)Q(A_2 \cap A_3) \\ & + 2Q(A_1 \cap A_2 \cap A_3)Q(A_4) \\ & + 2Q(A_2 \cap A_3 \cap A_4)Q(A_1) \\ & + 2Q(A_1 \cap A_3 \cap A_4)Q(A_2) \\ & + 2Q(A_1 \cap A_2 \cap A_4)Q(A_3) \\ & - 6Q(A_1 \cap A_2 \cap A_3 \cap A_4), \end{aligned}$$

Auto-correlation terms

Generalized cumulant calculations

- Naturally correct for detector acceptance

$c_2\{4\}$

$$
\begin{aligned}
\langle Q_1 Q_2 Q_3 Q_4 \rangle_c = \langle Q_1 Q_2 Q_3 Q_4 \rangle
& - \langle Q_1 Q_2 Q_3 \rangle \langle Q_4 \rangle \\
& - \langle Q_2 Q_3 Q_4 \rangle \langle Q_1 \rangle \\
& - \langle Q_1 Q_3 Q_4 \rangle \langle Q_2 \rangle \\
& - \langle Q_1 Q_2 Q_4 \rangle \langle Q_3 \rangle \\
& - \langle Q_1 Q_2 \rangle \langle Q_3 Q_4 \rangle \\
& - \langle Q_1 Q_3 \rangle \langle Q_2 Q_4 \rangle \\
& - \langle Q_1 Q_4 \rangle \langle Q_2 Q_3 \rangle \\
& + 2 \langle Q_1 Q_2 \rangle \langle Q_3 \rangle \langle Q_4 \rangle \\
& + 2 \langle Q_1 Q_3 \rangle \langle Q_2 \rangle \langle Q_4 \rangle \\
& + 2 \langle Q_2 Q_3 \rangle \langle Q_1 \rangle \langle Q_4 \rangle \\
& + 2 \langle Q_1 Q_4 \rangle \langle Q_2 \rangle \langle Q_3 \rangle \\
& + 2 \langle Q_2 Q_4 \rangle \langle Q_1 \rangle \langle Q_3 \rangle \\
& + 2 \langle Q_3 Q_4 \rangle \langle Q_1 \rangle \langle Q_2 \rangle \\
& - 6 \langle Q_1 \rangle \langle Q_2 \rangle \langle Q_3 \rangle \langle Q_4 \rangle .
\end{aligned}
$$

Generalized cumulant calculations

- Naturally correct for detector acceptance
- In case of perfect detector:

$c_2\{4\}$

$$
\begin{aligned}
\langle Q_1Q_2Q_3Q_4\rangle_c = &\langle Q_1Q_2Q_3Q_4\rangle \\
&-\langle Q_1Q_2Q_3\rangle\langle Q_4\rangle \\
&-\langle Q_2Q_3Q_4\rangle\langle Q_1\rangle \\
&-\langle Q_1Q_3Q_4\rangle\langle Q_2\rangle \\
&-\langle Q_1Q_2Q_4\rangle\langle Q_3\rangle \\
&-\langle Q_1Q_2\rangle\langle Q_3Q_4\rangle \\
&-\langle Q_1Q_3\rangle\langle Q_2Q_4\rangle \\
&-\langle Q_1Q_4\rangle\langle Q_2Q_3\rangle \\
&+2\langle Q_1Q_2\rangle\langle Q_3\rangle\langle Q_4\rangle \\
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&+2\langle Q_1Q_4\rangle\langle Q_2\rangle\langle Q_3\rangle \\
&+2\langle Q_2Q_4\rangle\langle Q_1\rangle\langle Q_3\rangle \\
&+2\langle Q_3Q_4\rangle\langle Q_1\rangle\langle Q_2\rangle \\
&-6\langle Q_1\rangle\langle Q_2\rangle\langle Q_3\rangle\langle Q_4\rangle.
\end{aligned}
$$

$c_2\{4\} = \langle\langle 4\rangle\rangle - 2\langle\langle 2\rangle\rangle^2$

Generalized cumulant calculations

- Naturally correct for detector acceptance
- In case of perfect detector:

$c_2\{4\}$

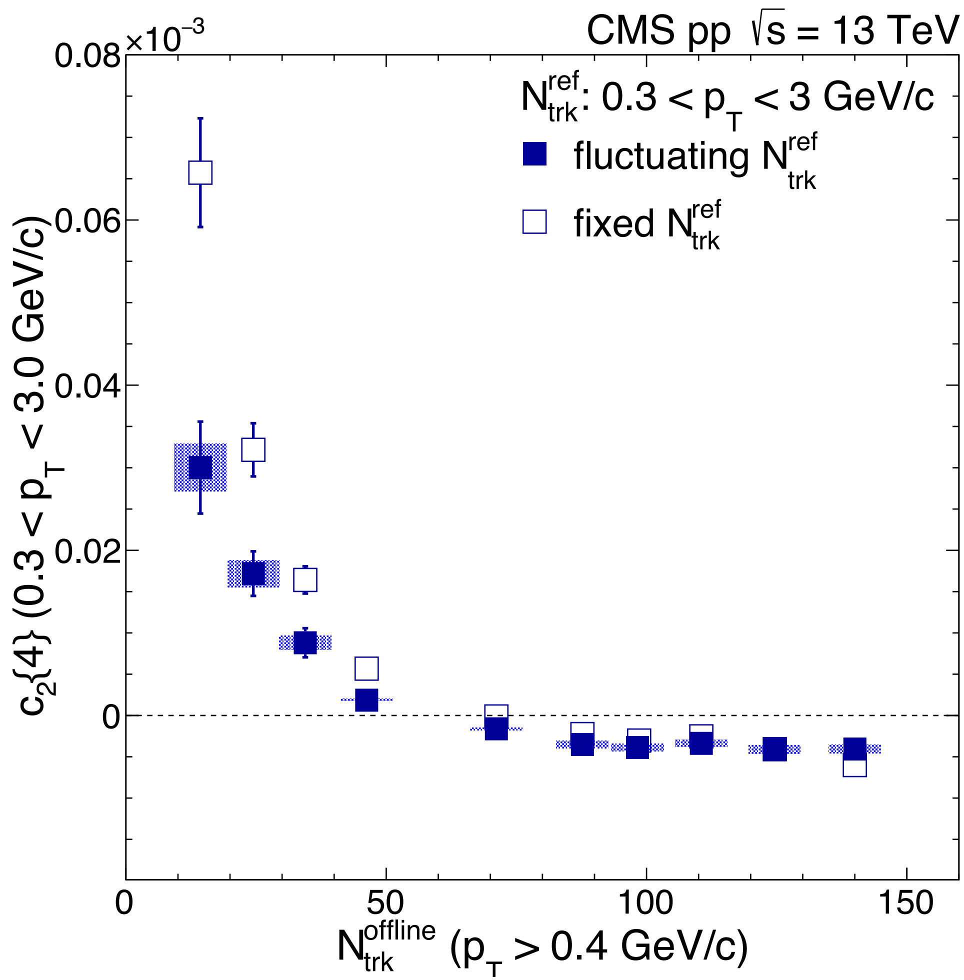
$$\begin{aligned}\langle Q_1Q_2Q_3Q_4\rangle_c = \langle Q_1Q_2Q_3Q_4\rangle \\ -\langle Q_1Q_2Q_3\rangle\langle Q_4\rangle \\ -\langle Q_2Q_3Q_4\rangle\langle Q_1\rangle \\ -\langle Q_1Q_3Q_4\rangle\langle Q_2\rangle \\ -\langle Q_1Q_2Q_4\rangle\langle Q_3\rangle \\ -\langle Q_1Q_2\rangle\langle Q_3Q_4\rangle \\ -\langle Q_1Q_3\rangle\langle Q_2Q_4\rangle \\ -\langle Q_1Q_4\rangle\langle Q_2Q_3\rangle \\ +2\langle Q_1Q_2\rangle\langle Q_3\rangle\langle Q_4\rangle \\ +2\langle Q_1Q_3\rangle\langle Q_2\rangle\langle Q_4\rangle \\ +2\langle Q_2Q_3\rangle\langle Q_1\rangle\langle Q_4\rangle \\ +2\langle Q_1Q_4\rangle\langle Q_2\rangle\langle Q_3\rangle \\ +2\langle Q_2Q_4\rangle\langle Q_1\rangle\langle Q_3\rangle \\ +2\langle Q_3Q_4\rangle\langle Q_1\rangle\langle Q_2\rangle \\ -6\langle Q_1\rangle\langle Q_2\rangle\langle Q_3\rangle\langle Q_4\rangle.\end{aligned}$$

New method! But will it fix our problems and fulfill our goals???

1. Multiplicity fluctuations
2. Measurement down to low N_{trk}
3. Method sensitivity

Multiplicity fluctuations

- Multiplicity fluctuations have significant effect at low N in the standard method
- That can be overcome by binning carefully the measurement



Multiplicity fluctuations

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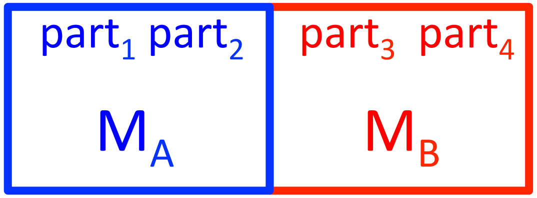
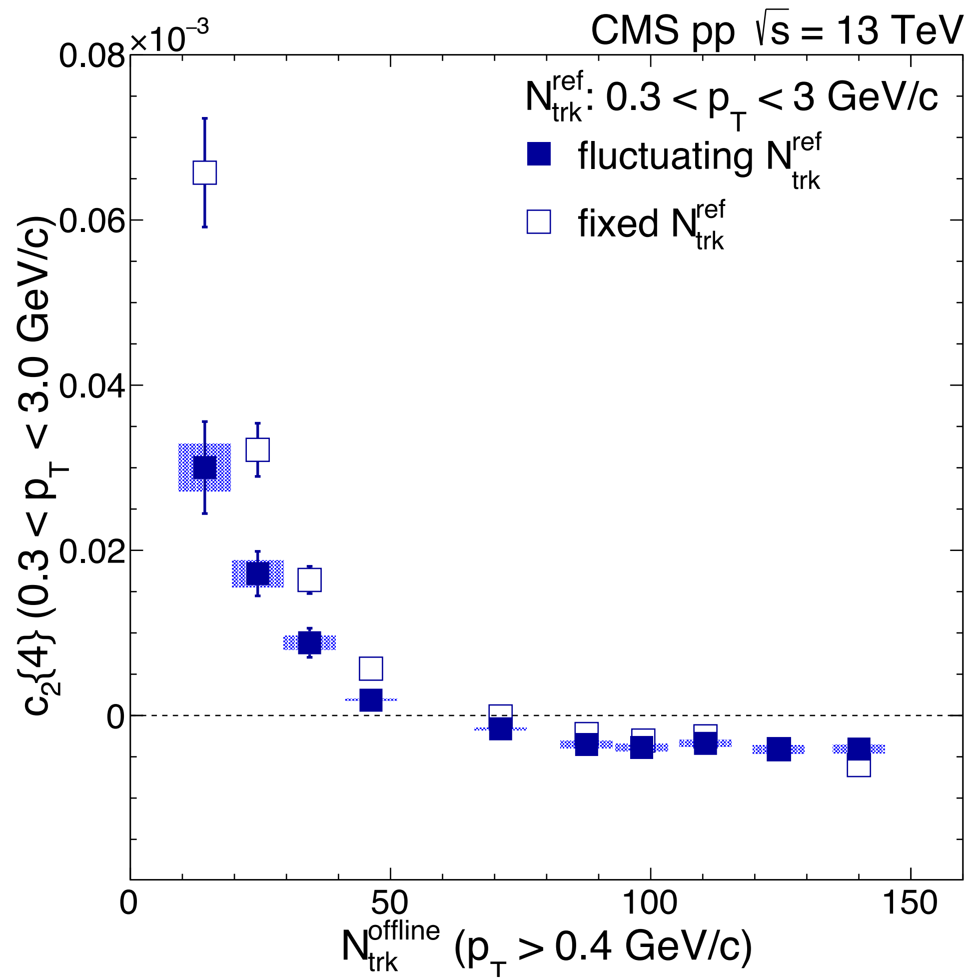
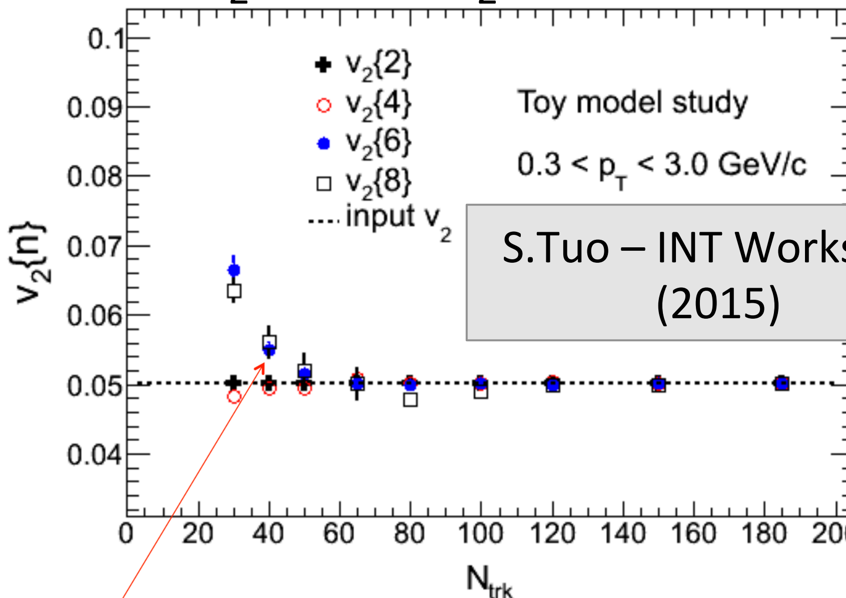
- In case of sub-events, what about multiplicity fluctuations in sub-events?





Measurement down to low N_{trk}

- It has been already shown that at low N $c_2\{6\}$ and $c_2\{8\}$ cannot return the signal value

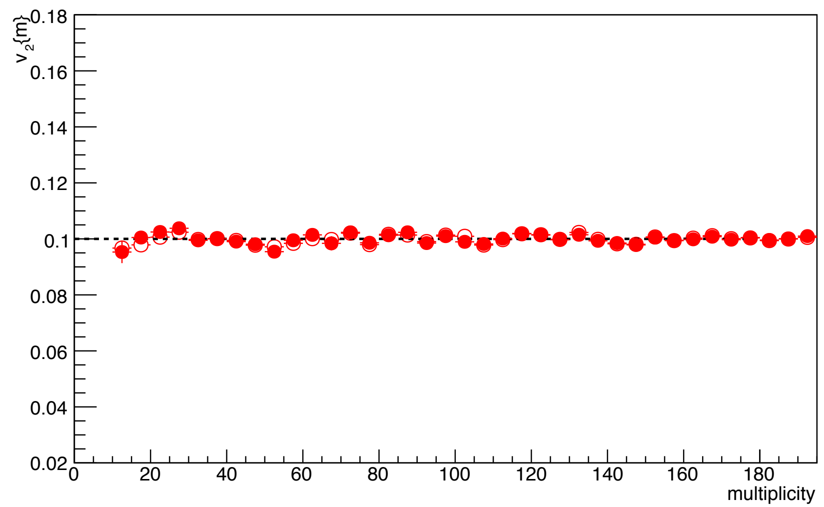


S.Tuo – INT Workshop (2015)

$v_2\{6\}$ and $v_2\{8\}$ over estimate the signal in low multiplicity events

Measurement down to low N_{trk}

- It has been already shown that at low N $c_2\{6\}$ and $c_2\{8\}$ cannot return the signal value
- What about gap method?
  - Recover the truth down to N = 10



Simple toy MC: no v_n fluc, no mult fluct

········ Truth
○ v2{2} std
● v2{2} sub evt

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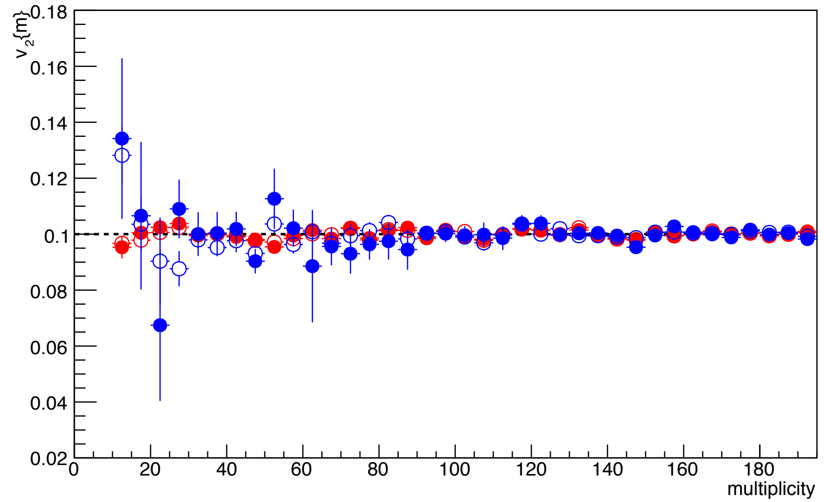

Simple toy MC: no v_n fluc, no mult fluct

Truth

$v_2\{4\}$ std

$v_2\{2\}$ std

$v_2\{4\}$ sub evt

$v_2\{2\}$ sub evt

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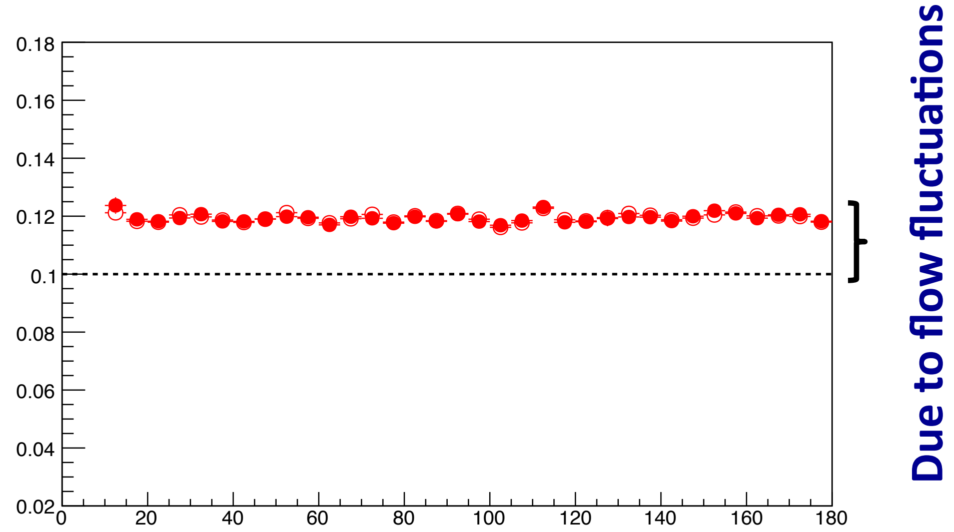
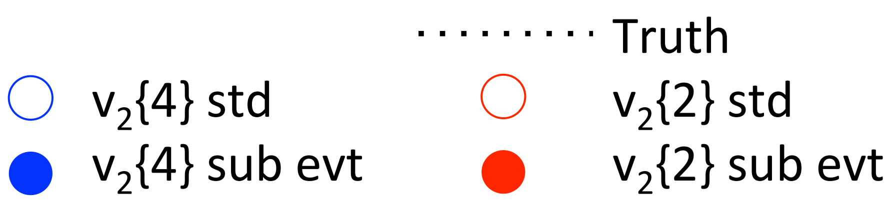


Simple toy MC: $\mathbf{v_n}$ **fluc**, no mult fluct



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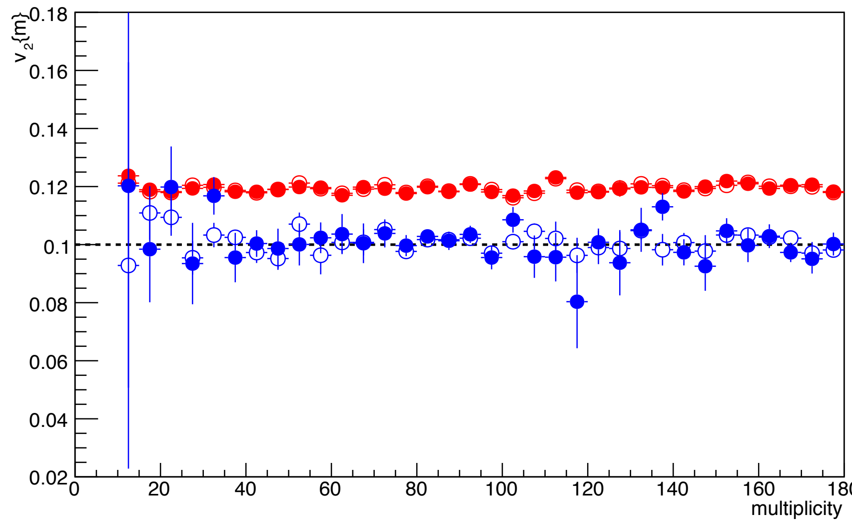


Simple toy MC: $\mathbf{v_n}$ **fluc**, no mult fluct

.... Truth

$v_2\{4\}$ std

$v_2\{4\}$ sub evt

$v_2\{2\}$ std

$v_2\{2\}$ sub evt

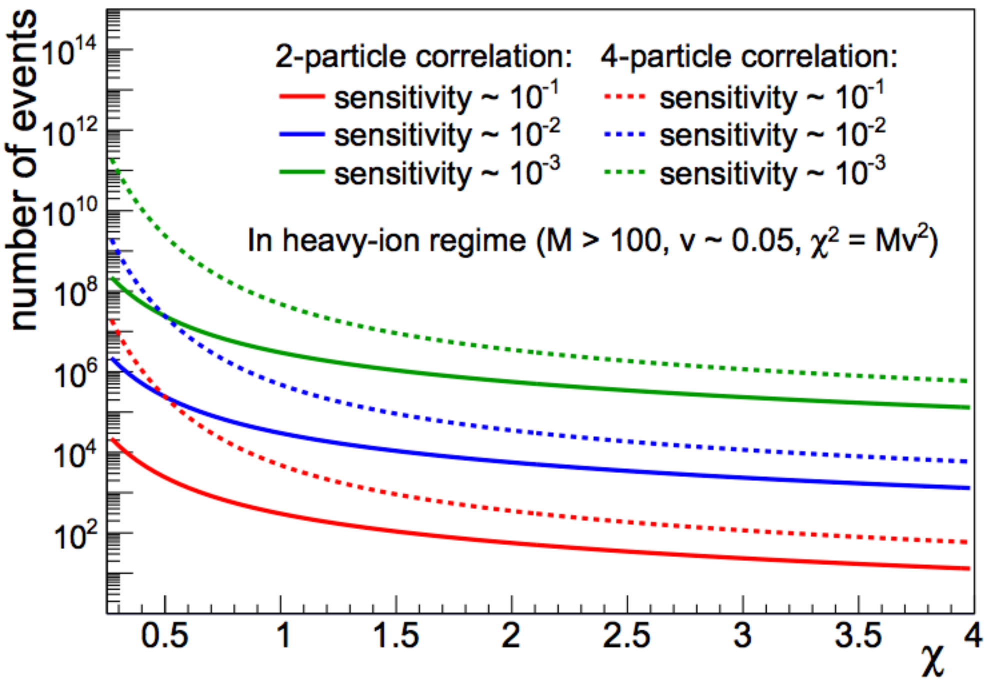
Method sensitivity

Default cumulant method

- Number of event needed increase drastically when M decrease at fix v_n
- At fix M sensitivity goes with v_n^2

What about the sub-event method?

- Probably worse and has to be studied…



Summary

- Clear evidence of collectivity in small systems observed thanks to multi-particle cumulant measurement

- Distinguishing between different theoretical scenarii is not trivial at the moment
  - Better understanding of the initial-state fluctuation is needed first
    - In particular, testing eccentricity driven universality seems to be crucial
  - Nevertheless, if a coherent picture can be found to describe all hadronic system that would probably be the most convincing scenario

- New experimental development are in progress to look for "flow turn-on"
  - New sub-event method to be studied in details
  - Few caveat to solve like multiplicity fluctuations in sub-event, sensitivity of the method and applicability down to low N

Summary

- In general, we should find a way to express our results that does not depend on the experiment
- It is also, most of the time, non trivial to compare to theoretical calculation.
  - Ideas about common way of plotting observable would be convenient
- In any cases, we might encounter some limitations in the multi-particle cumulant method that has to be define and understood
- Anyway, if we are able to remove non-flow using sub-event method, the fine splitting between cumulant order might be the key to discriminate among models