

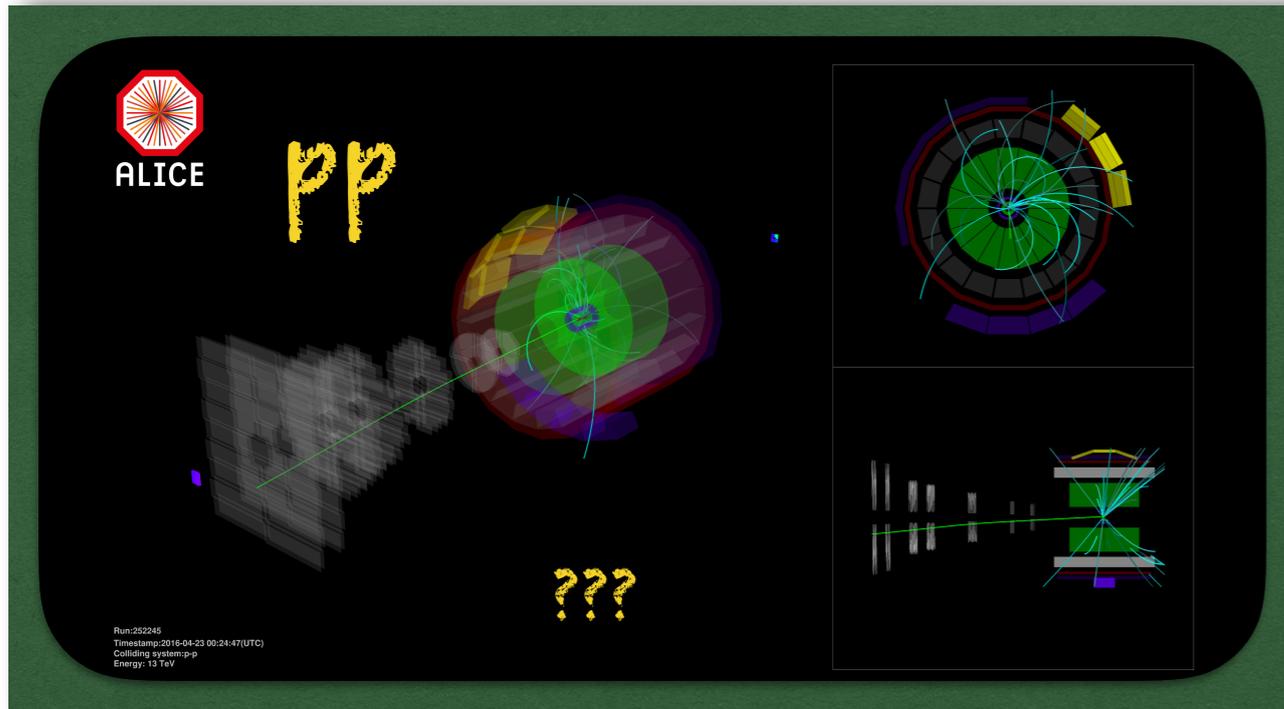
Investigating collectivity in small systems

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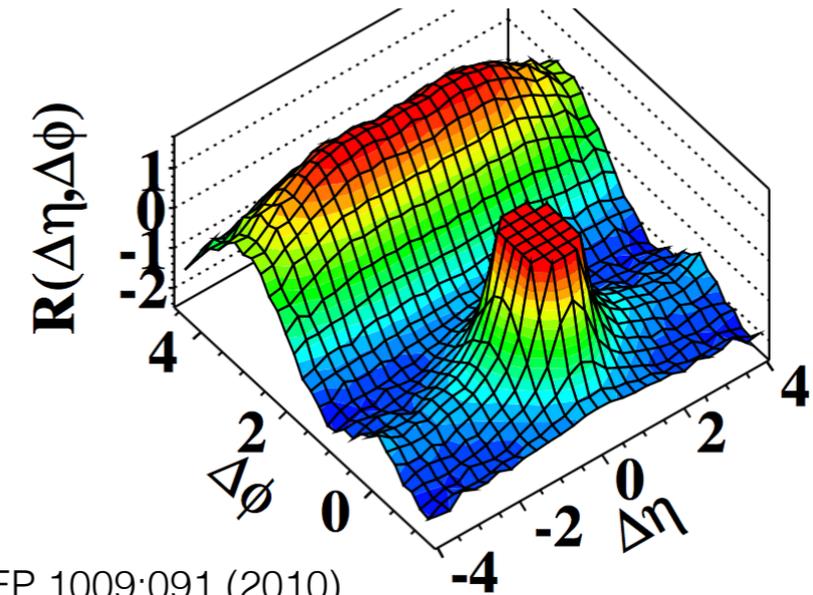
Workshop on Collectivity in Small Collision Systems

May 9, 2017

Is flow produced in pp collisions?



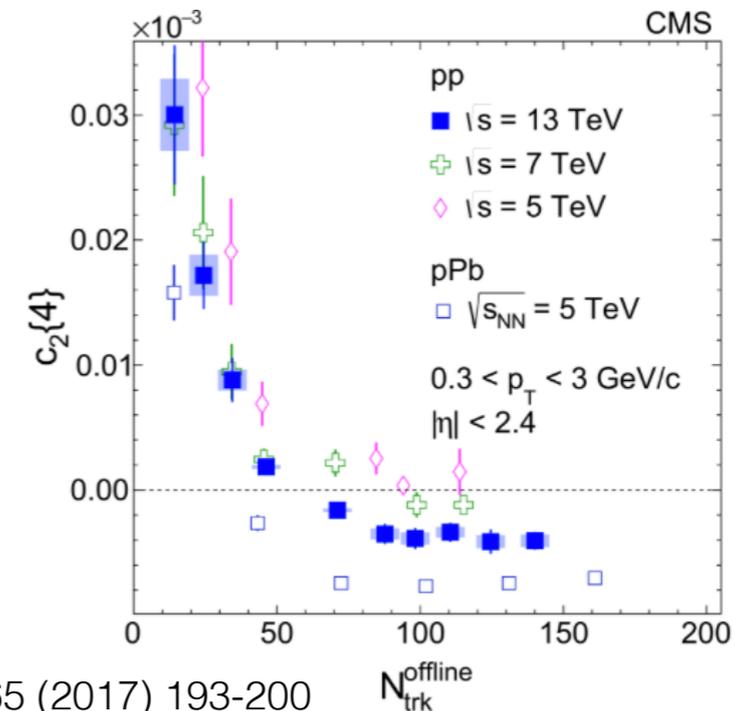
(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



CMS, JHEP 1009:091 (2010)

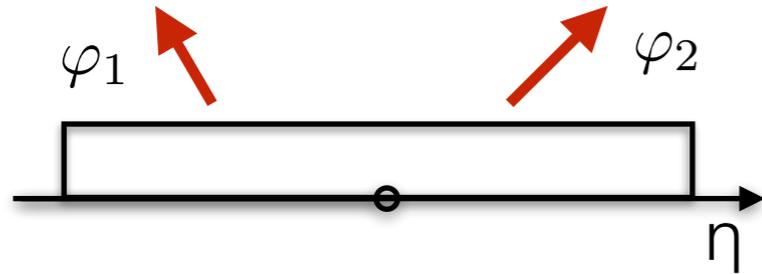
- Measurements using azimuthal correlations have shown many (surprising) results:

- **Ridge** structure in 2-particle correlations at high multiplicity pp collisions
- **Negative $c_2\{4\}$** at high multiplicity pp collisions seen by CMS



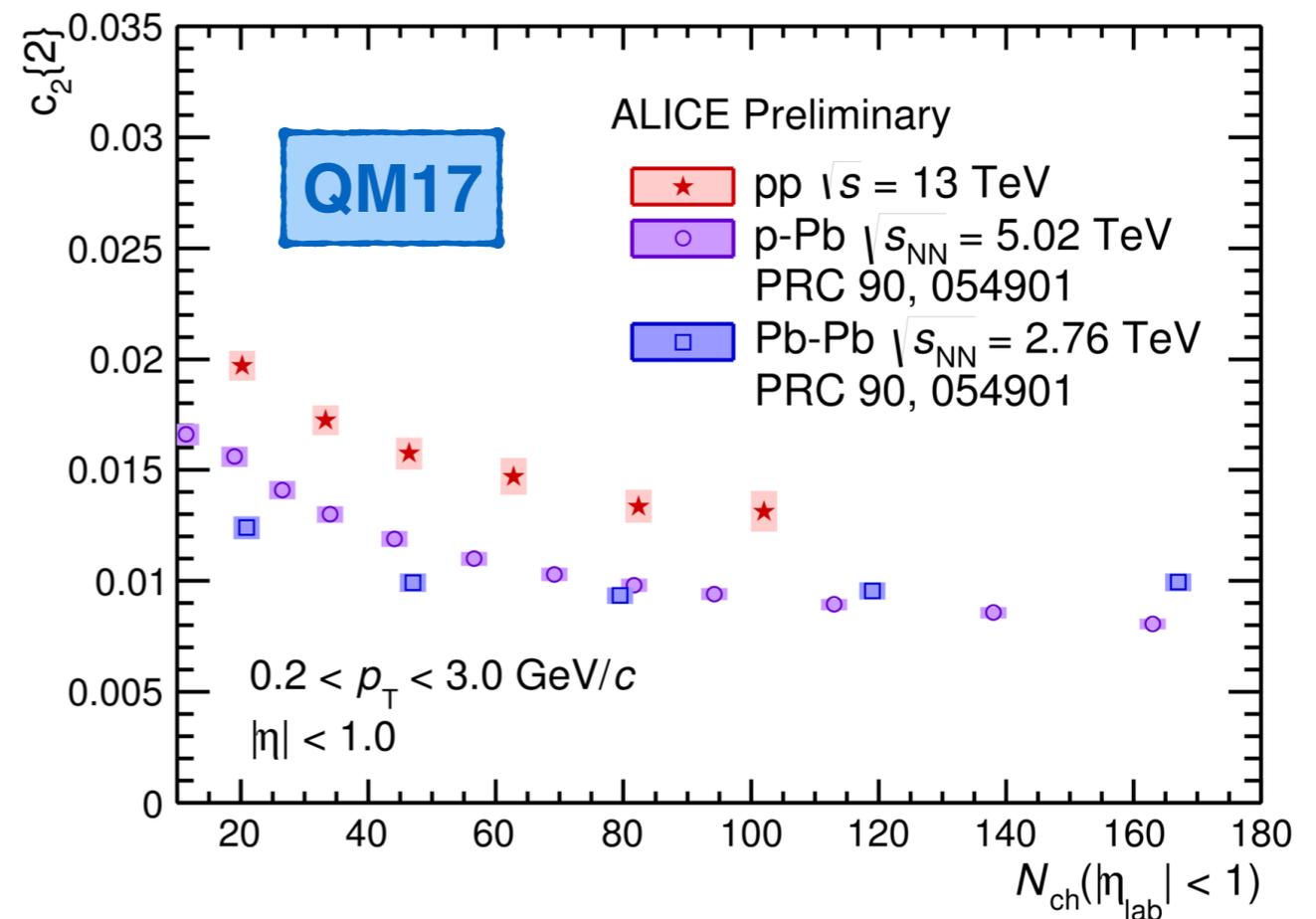
CMS, PLB 765 (2017) 193-200

2-particle cumulant



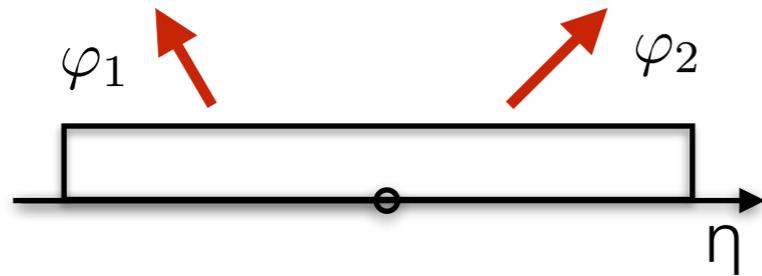
$$c_n\{2\} = \langle\langle \cos n(\varphi_1 - \varphi_2) \rangle\rangle$$

- Cumulants are an important tool in the investigation of collectivity in small systems
- $c_2\{2\}$ is clearly higher in pp collisions than in p-Pb or Pb-Pb



ALI-PREL-119544

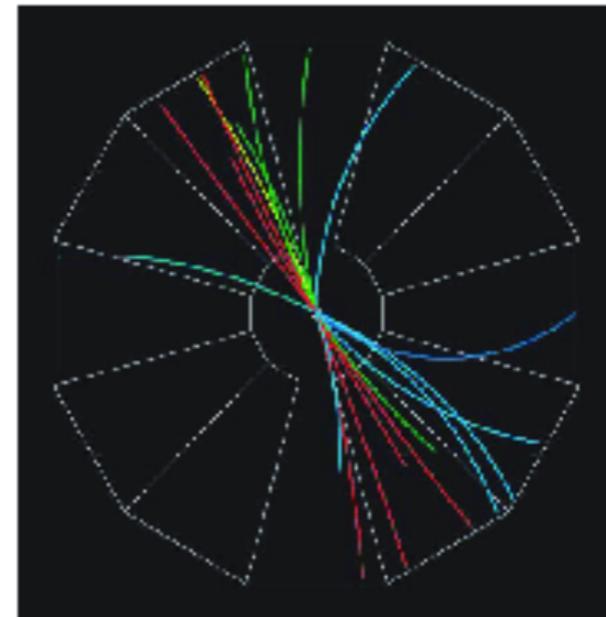
2-particle cumulant



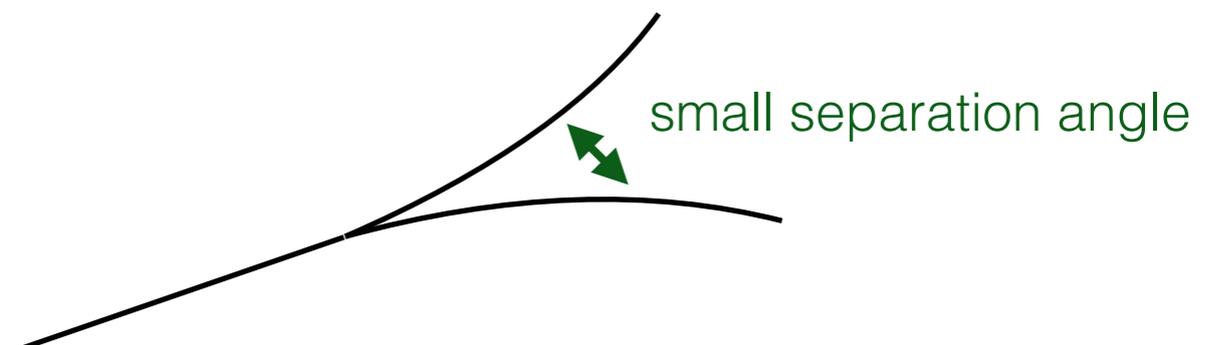
$$c_n\{2\} = \langle\langle \cos n(\varphi_1 - \varphi_2) \rangle\rangle$$

- Cumulants are an important tool in the investigation of collectivity in small systems
- Measurements are influenced by non-flow effects, especially in small systems

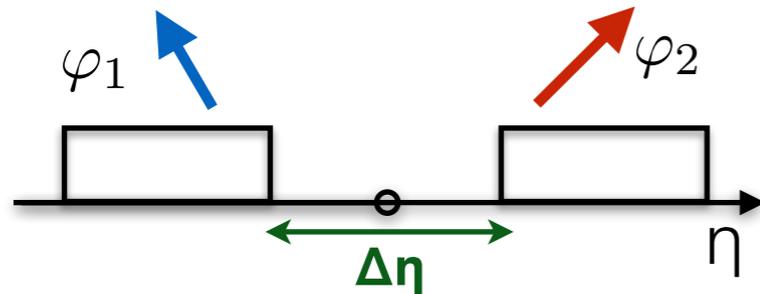
jets



resonance decays

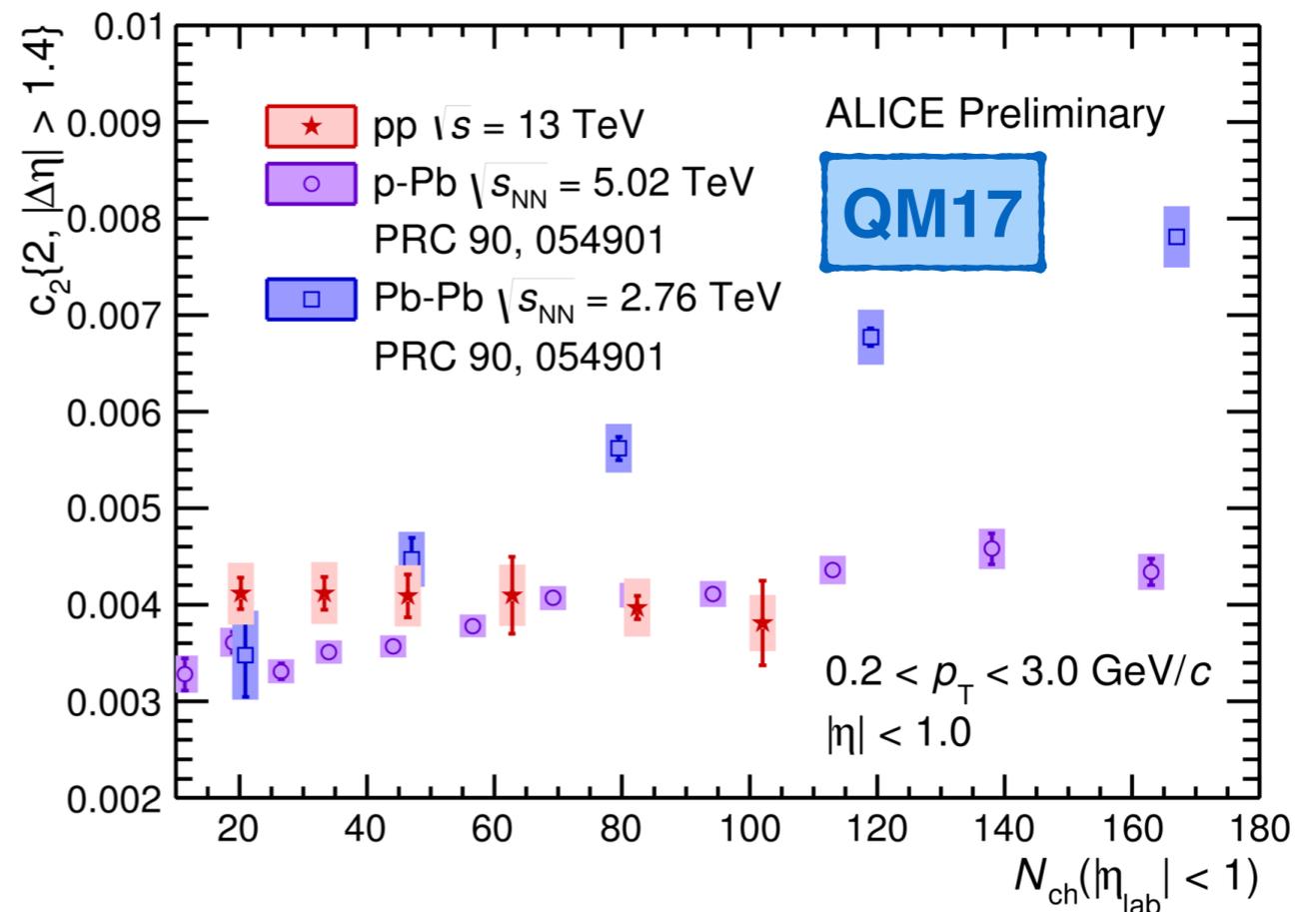


2-particle cumulant



$$c_n\{2, |\Delta\eta|\} = \langle\langle \cos n(\varphi_1 - \varphi_2) \rangle\rangle$$

- Cumulants are an important tool in the investigation of collectivity in small systems
- Measurements are influenced by non-flow effects, especially in small systems
- Splitting the acceptance into different η regions helps to avoid particles from few-body correlations, aka non-flow effects
- Overall decrease of $c_2\{2, |\Delta\eta|\}$
- Shows weak or no dependence on multiplicity in pp collisions

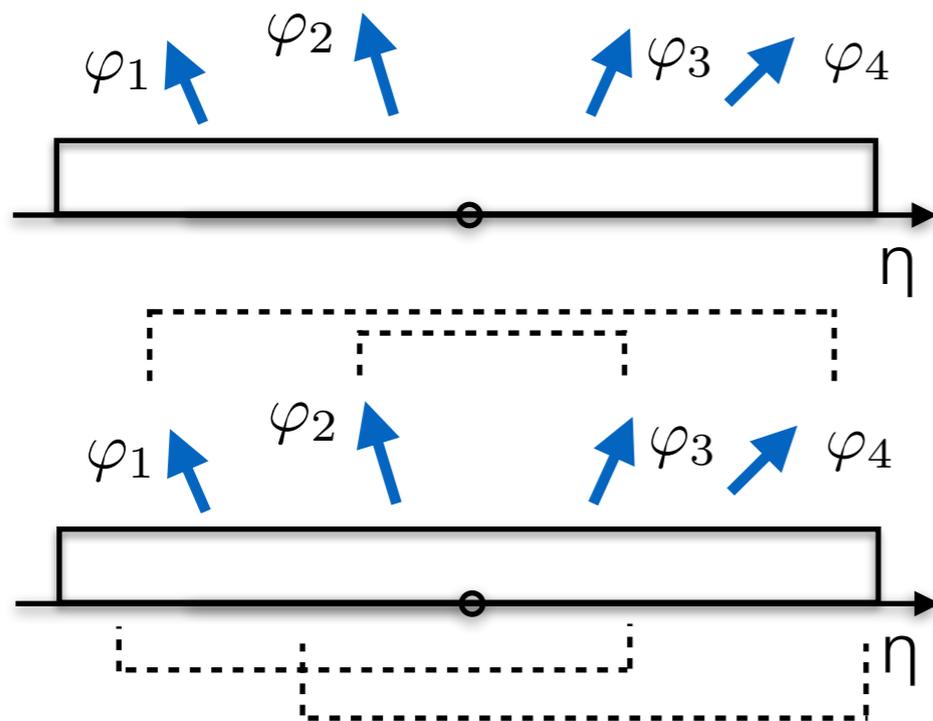


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4-particle cumulant



- Lower order correlation (including non-flow) are removed from multi-particle cumulants



$$\langle\langle 4 \rangle\rangle = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle^2 = \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle$$

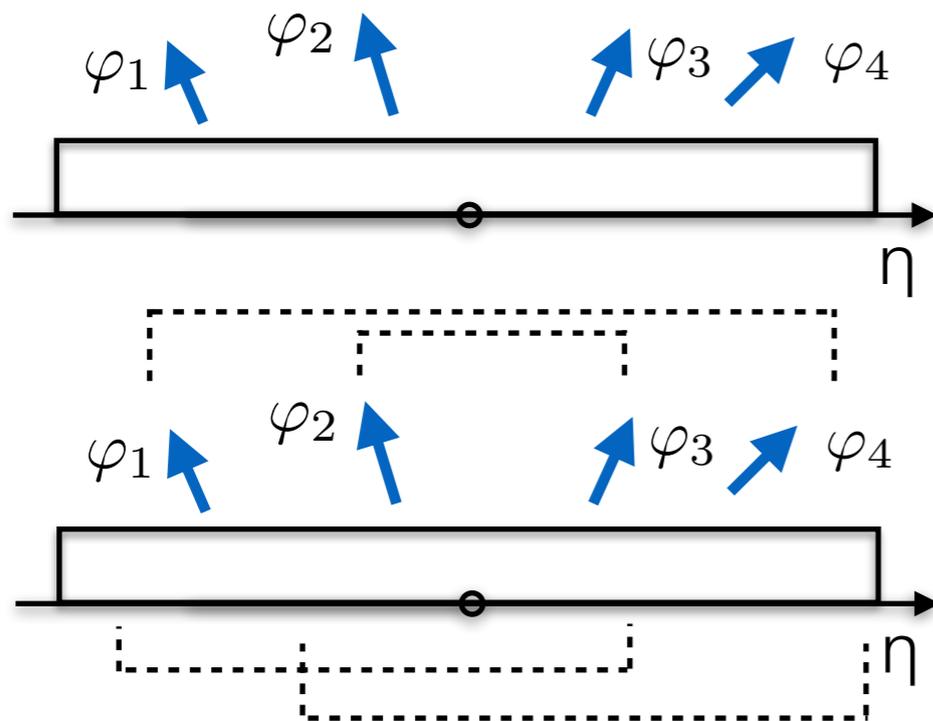
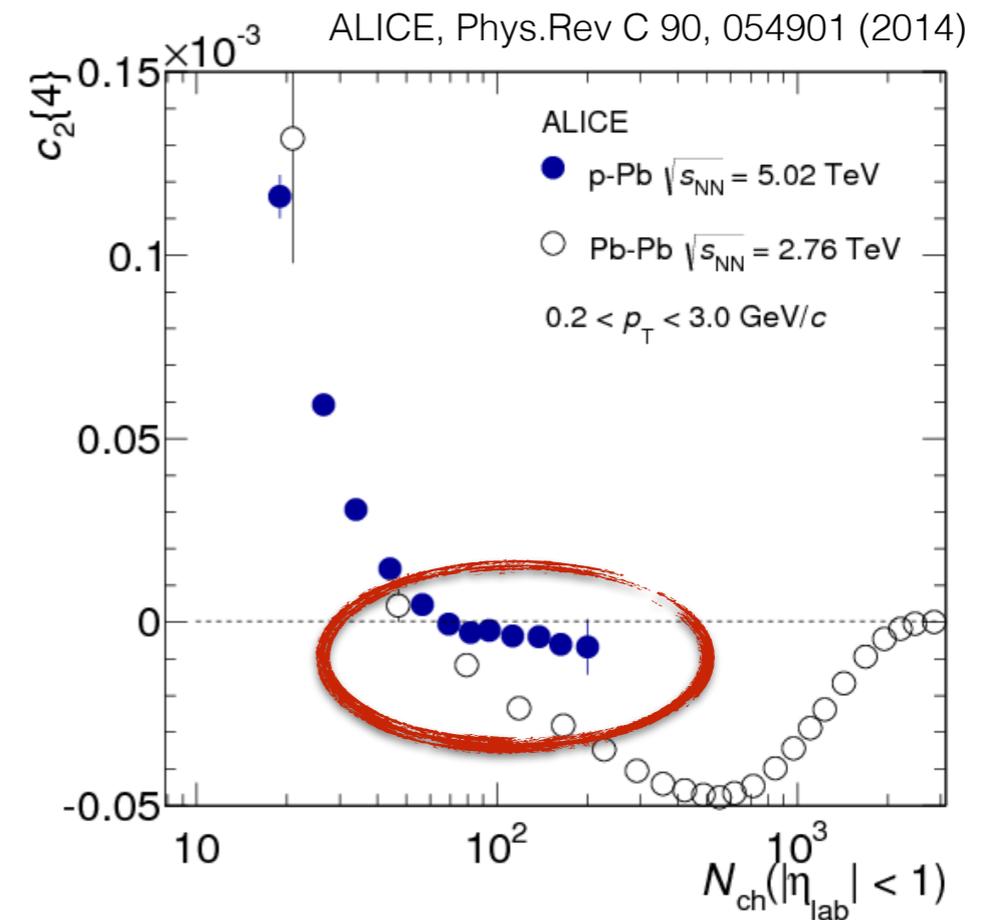
$$\langle\langle 2 \rangle\rangle^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2 \cdot \langle\langle 2 \rangle\rangle^2$$

4-particle cumulant



- Lower order correlation (including non-flow) are removed from multi-particle cumulants
- Negative sign of $c_n\{4\}$ might indicate collectivity



$$\langle\langle 4 \rangle\rangle = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle^2 = \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle$$

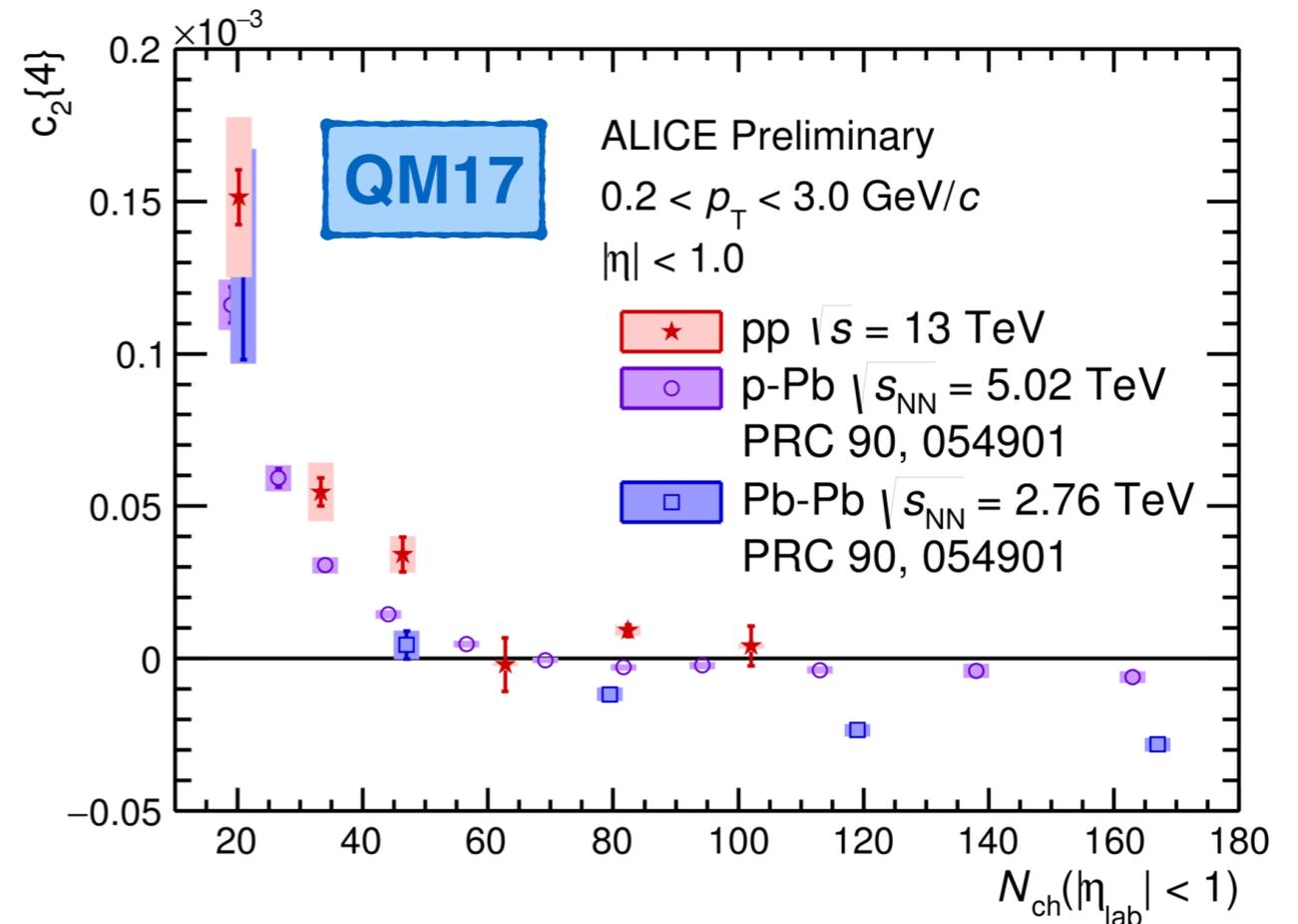
$$\langle\langle 2 \rangle\rangle^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2 \cdot \langle\langle 2 \rangle\rangle^2$$

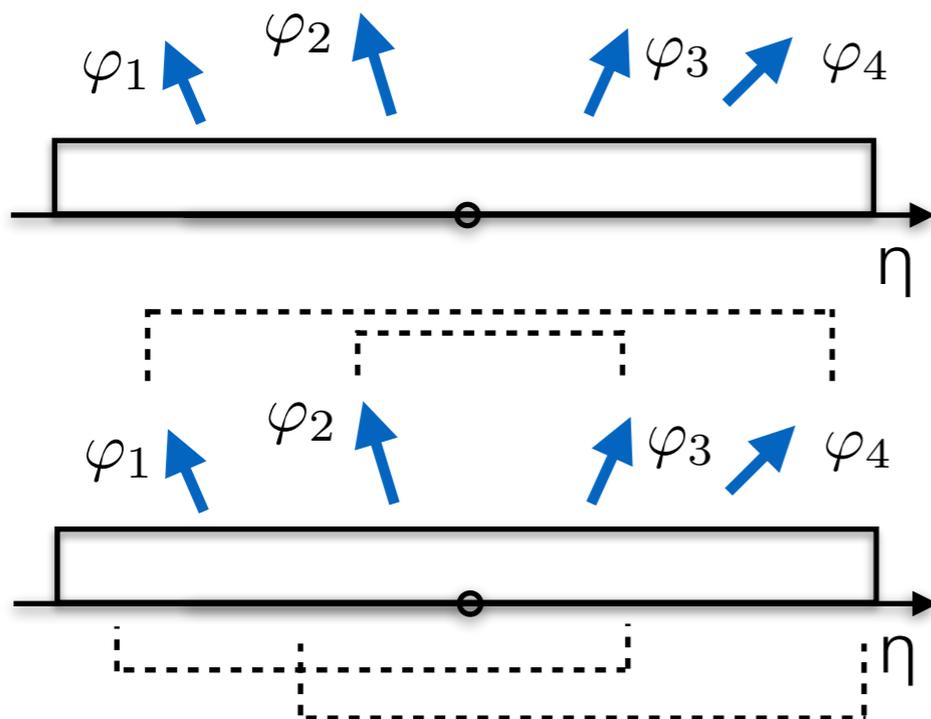
4-particle cumulant



- Lower order correlation (including non-flow) are removed from multi-particle cumulants
- Negative sign of $c_2\{4\}$ might indicate collectivity
 - Clear negative $c_2\{4\}$ observed in Pb-Pb and p-Pb collisions
 - No hint of collectivity in pp collisions



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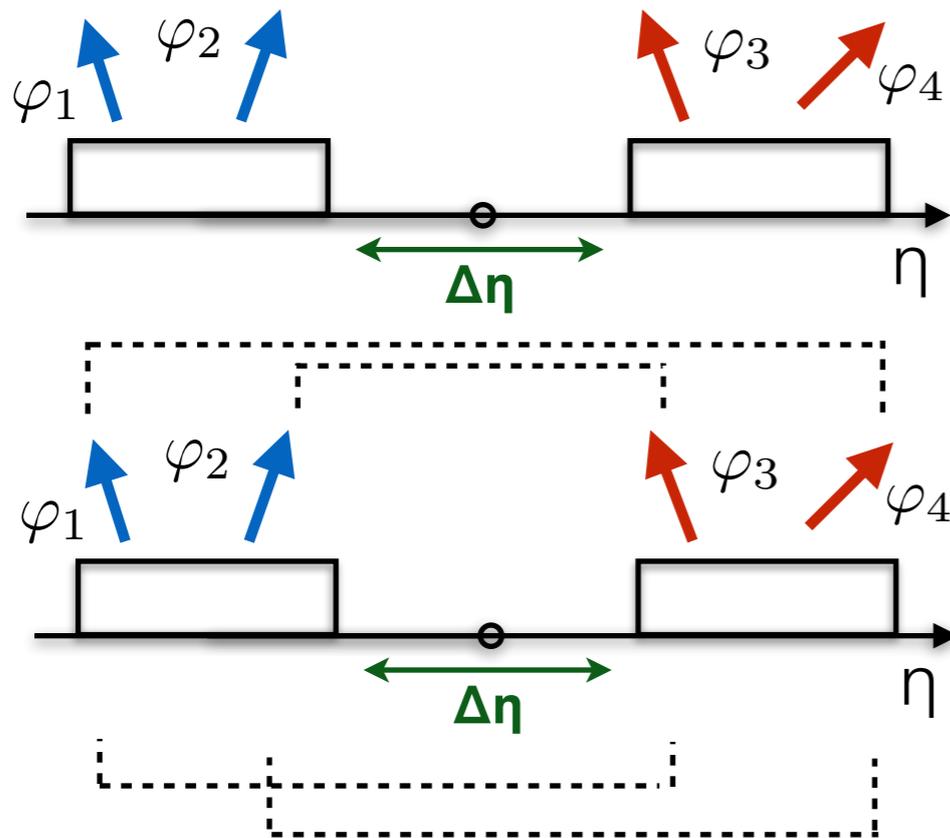
$$\langle\langle 4 \rangle\rangle = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle^2 = \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2 \cdot \langle\langle 2 \rangle\rangle^2$$

4-particle cumulant with $|\Delta\eta|$ gap



- We can **further suppress non-flow** in multi-particle cumulants using $|\Delta\eta|$ gap

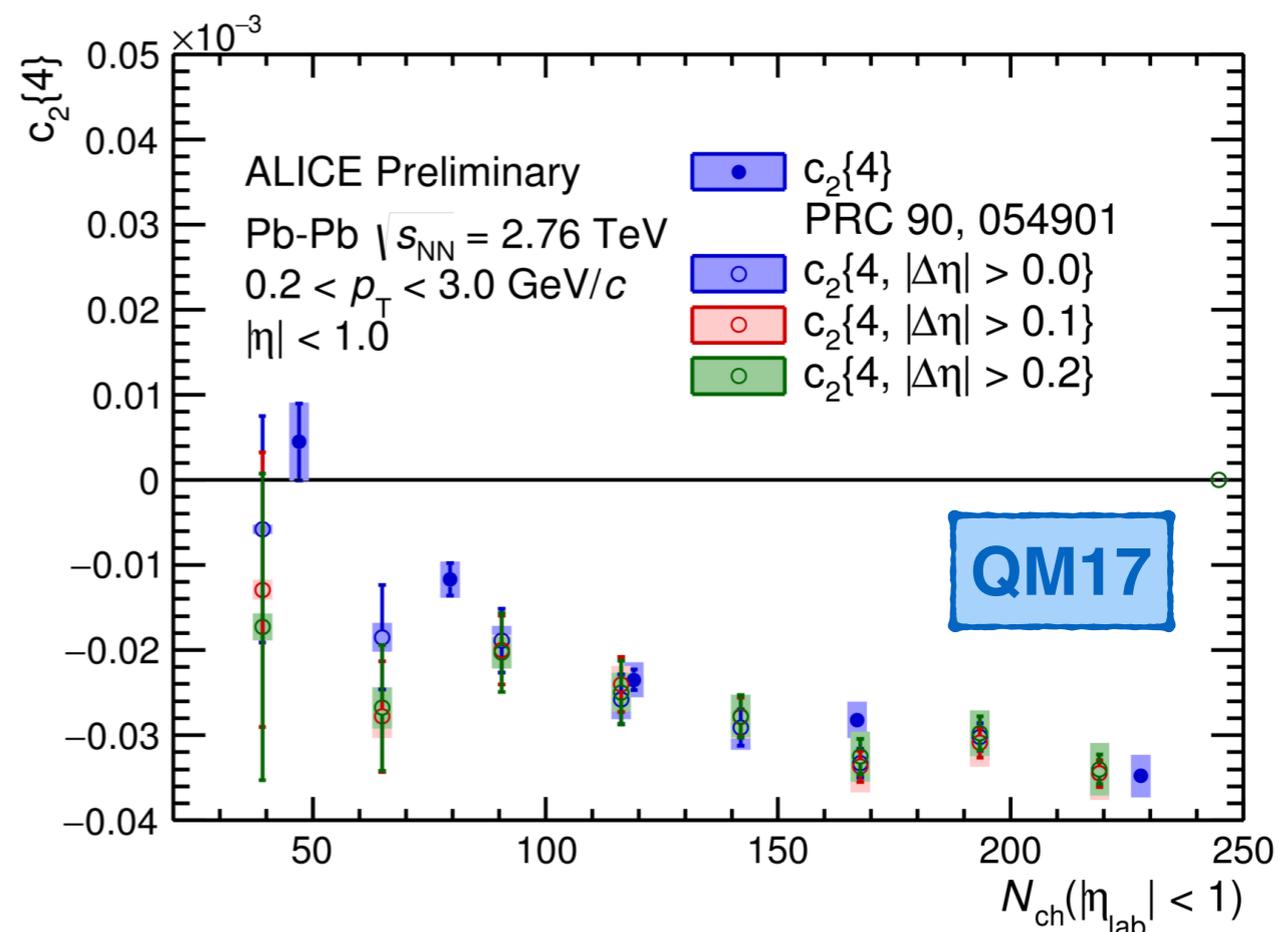
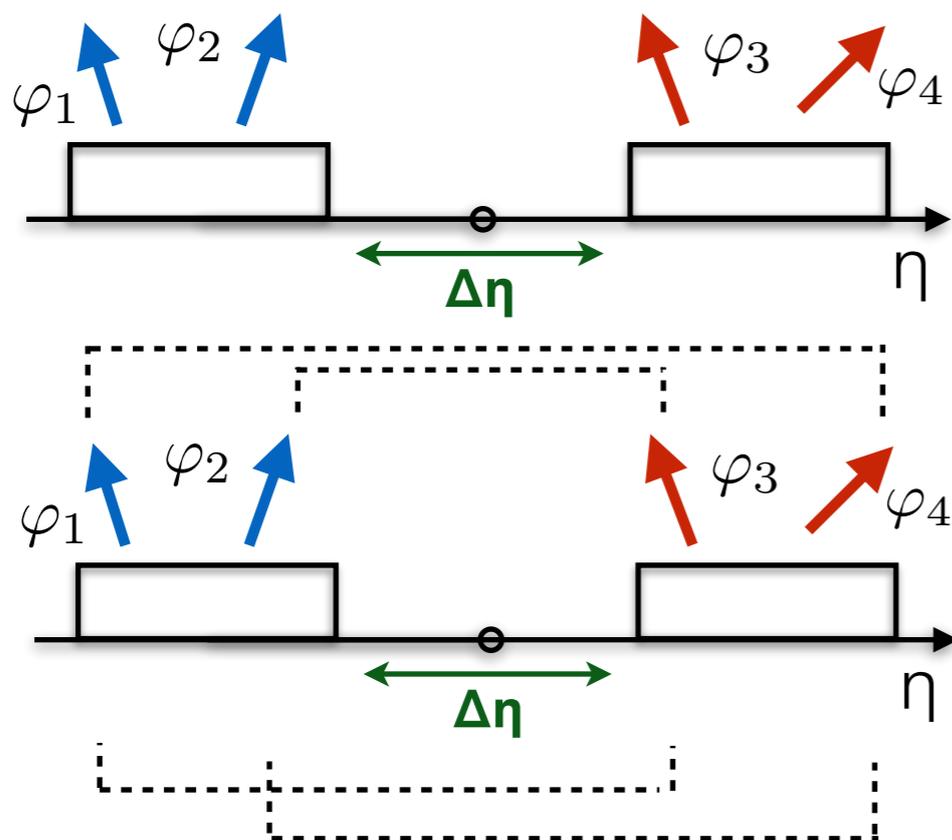
$$\langle\langle 4 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{|\Delta\eta|} = \langle\langle 4 \rangle\rangle_{|\Delta\eta|} - 2 \cdot \langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2$$

4-particle cumulant with $|\Delta\eta|$ gap



ALI-PREL-119539

- We can **further suppress non-flow** in multi-particle cumulants using $|\Delta\eta|$ gap
- 4-particle cumulant with $|\Delta\eta|$ gap in **Pb-Pb**
 - Measurements are compatible
 - Flow dominated system

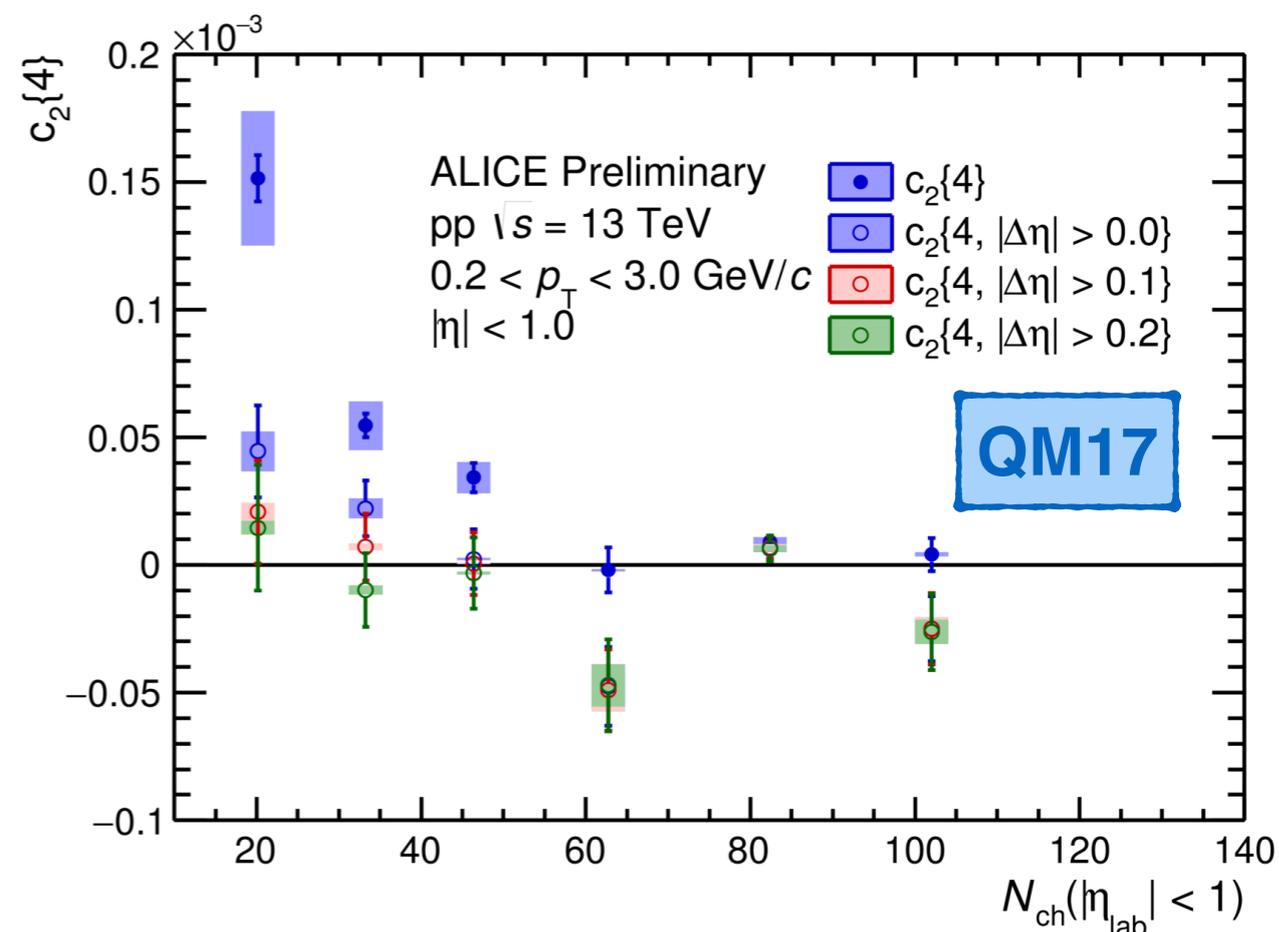
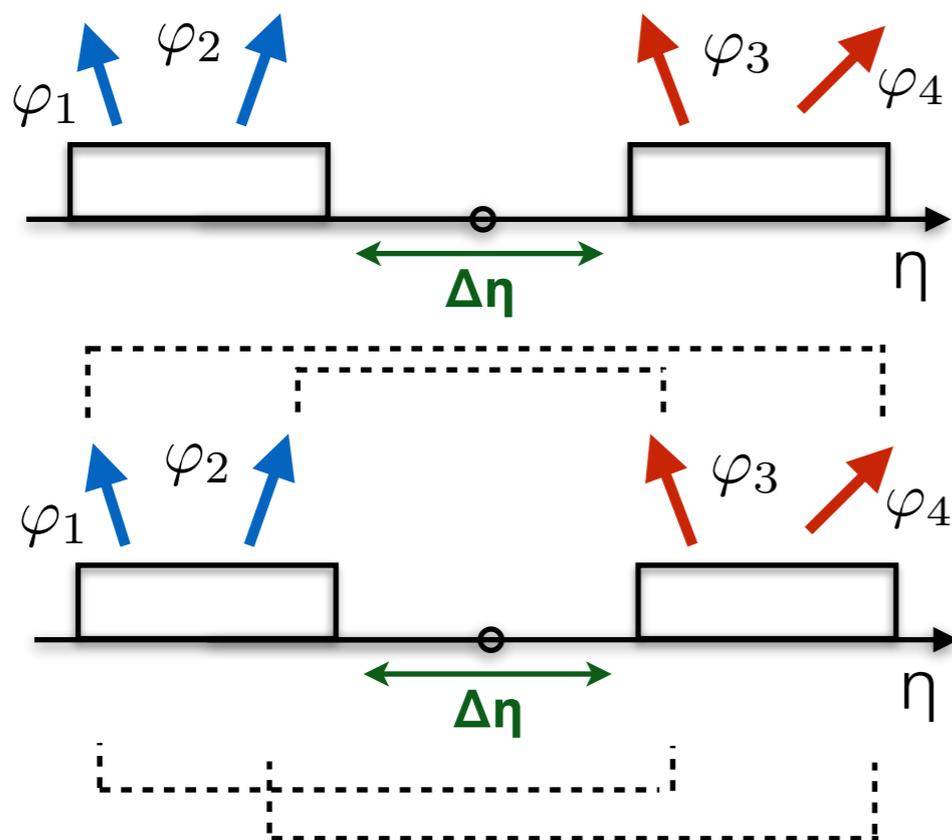
$$\langle\langle 4 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{|\Delta\eta|} = \langle\langle 4 \rangle\rangle_{|\Delta\eta|} - 2 \cdot \langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2$$

4-particle cumulant with $|\Delta\eta|$ gap



ALI-PREL-119434

- We can **further suppress non-flow** in multi-particle cumulants using $|\Delta\eta|$ gap
- 4-particle cumulant with $|\Delta\eta|$ gap in **pp**

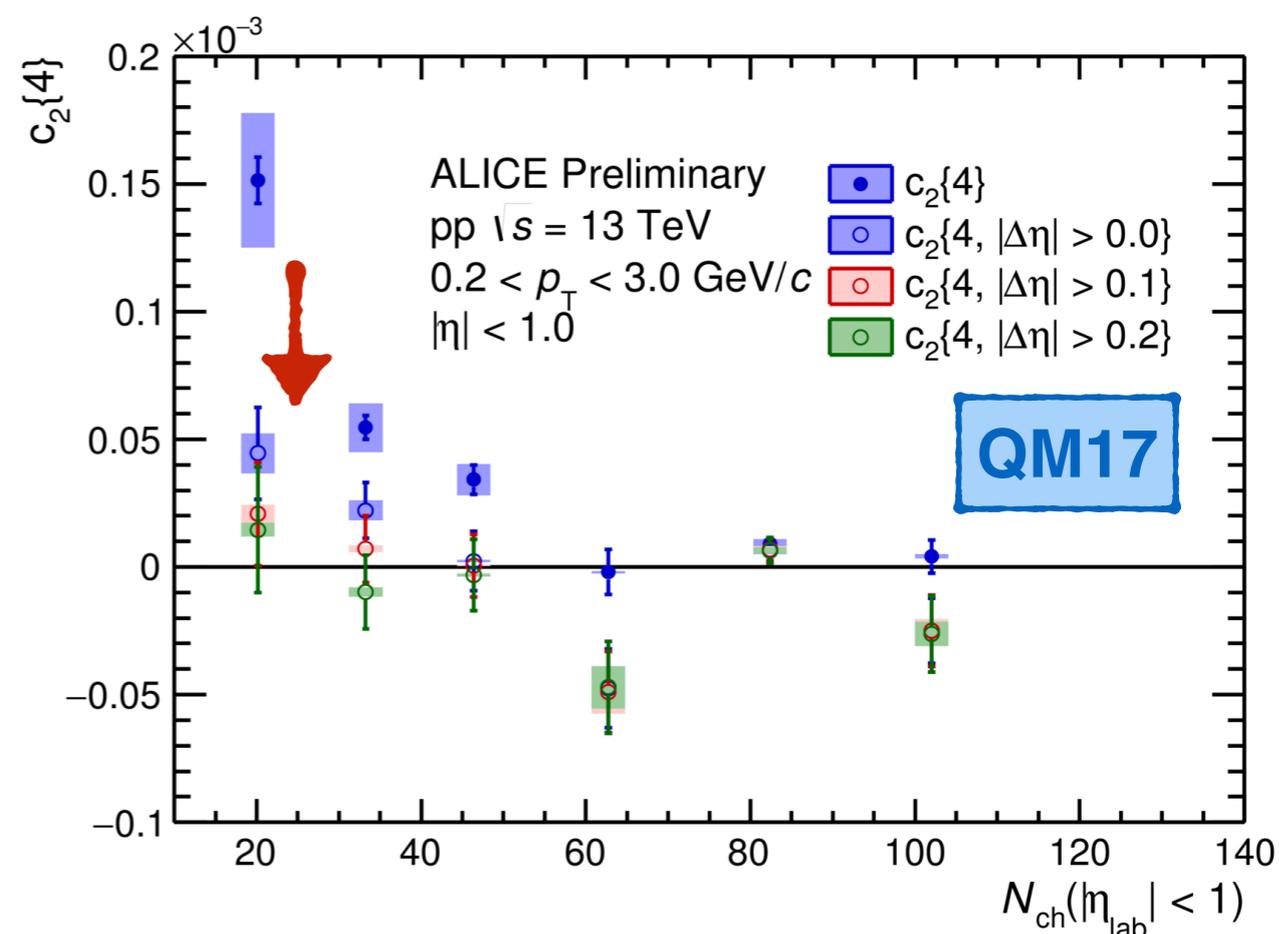
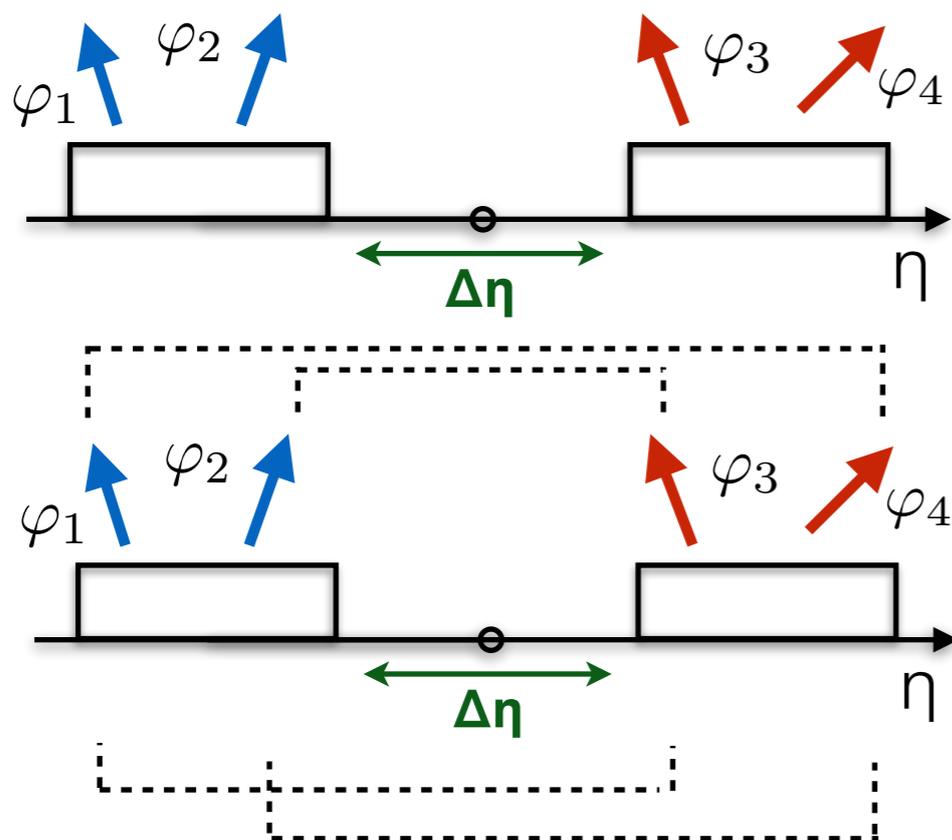
$$\langle\langle 4 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{|\Delta\eta|} = \langle\langle 4 \rangle\rangle_{|\Delta\eta|} - 2 \cdot \langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2$$

4-particle cumulant with $|\Delta\eta|$ gap



ALI-PREL-119434

- We can **further suppress non-flow** in multi-particle cumulants using $|\Delta\eta|$ gap
- 4-particle cumulant with $|\Delta\eta|$ gap in **pp**
 - $c_2\{4, |\Delta\eta|\}$ is lower than $c_2\{4\}$

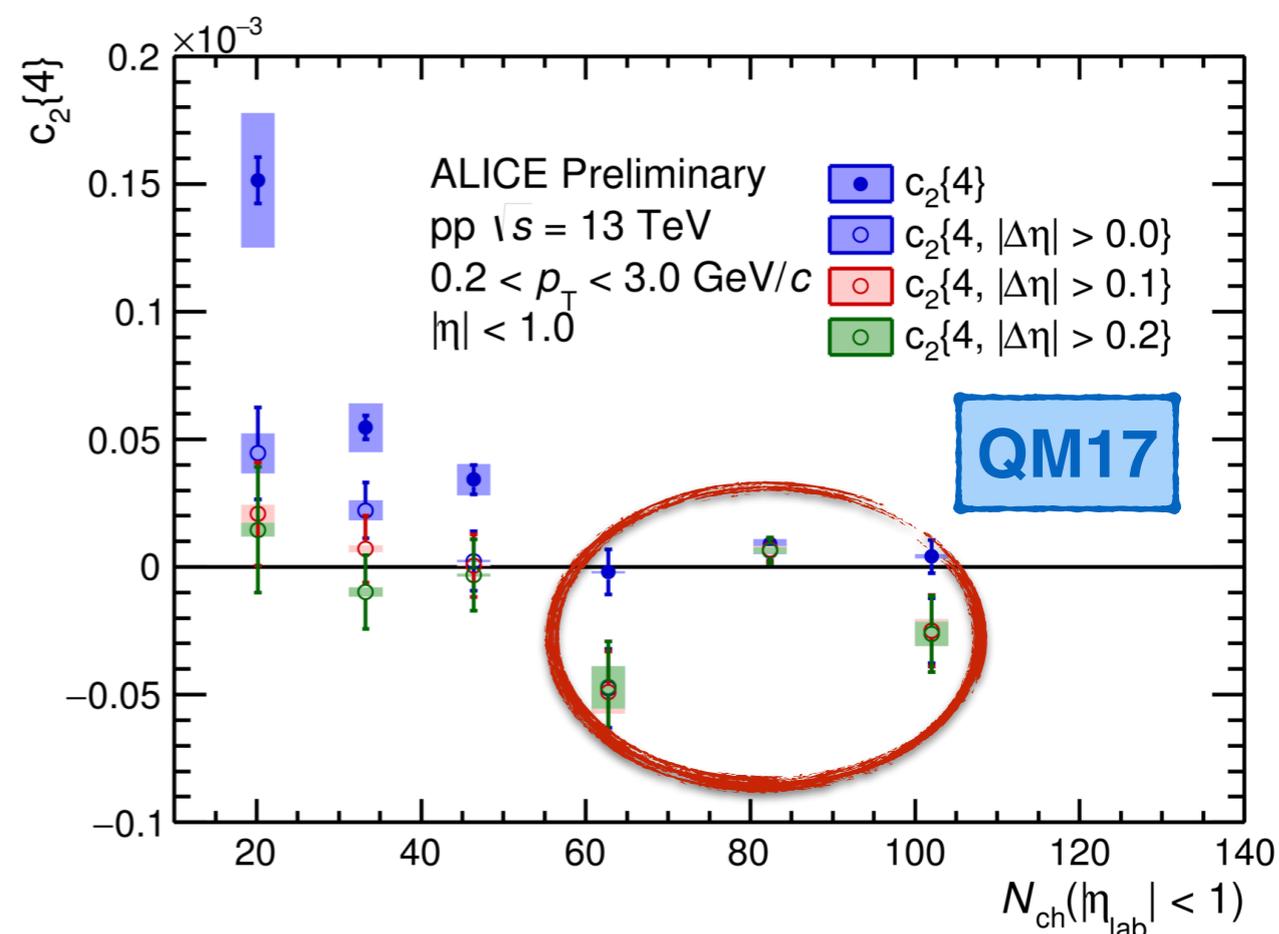
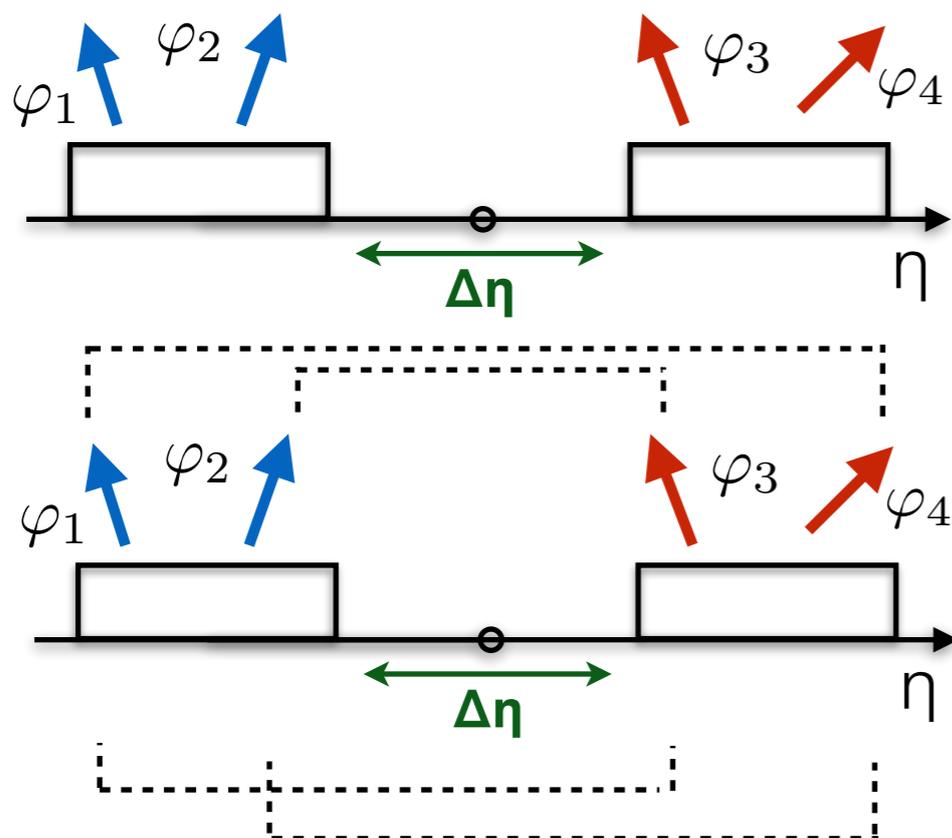
$$\langle\langle 4 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{|\Delta\eta|} = \langle\langle 4 \rangle\rangle_{|\Delta\eta|} - 2 \cdot \langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2$$

4-particle cumulant with $|\Delta\eta|$ gap



ALI-PREL-119434

- We can **further suppress non-flow** in multi-particle cumulants using $|\Delta\eta|$ gap
- 4-particle cumulant with $|\Delta\eta|$ gap in **pp**
 - $c_2\{4, |\Delta\eta|\}$ is lower than $c_2\{4\}$
 - Still no significant negative sign observed in pp collisions

$$\langle\langle 4 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{|\Delta\eta|} = \langle\langle 4 \rangle\rangle_{|\Delta\eta|} - 2 \cdot \langle\langle 2 \rangle\rangle_{|\Delta\eta|}^2$$

4-particle cumulant (3-subevent method)



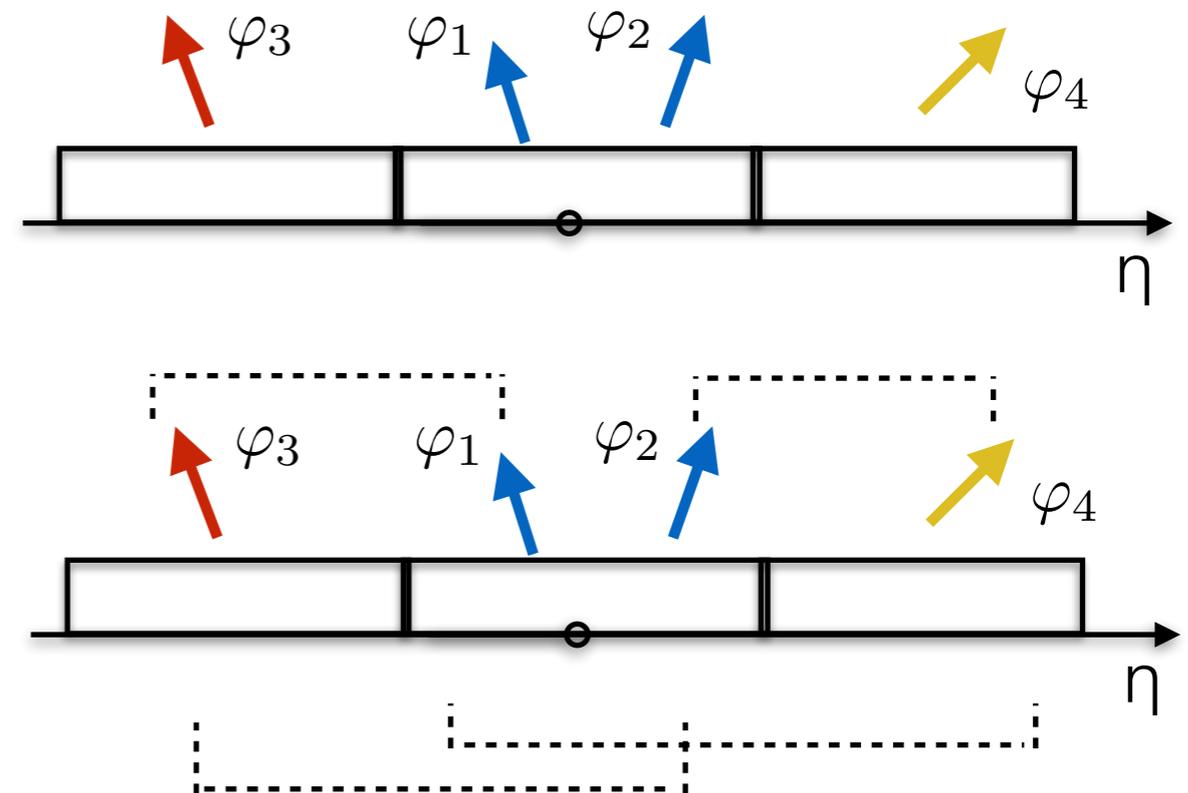
$$\langle\langle 4 \rangle\rangle_{3\text{sub}} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{3\text{sub}}^2 = \langle\langle \cos n(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

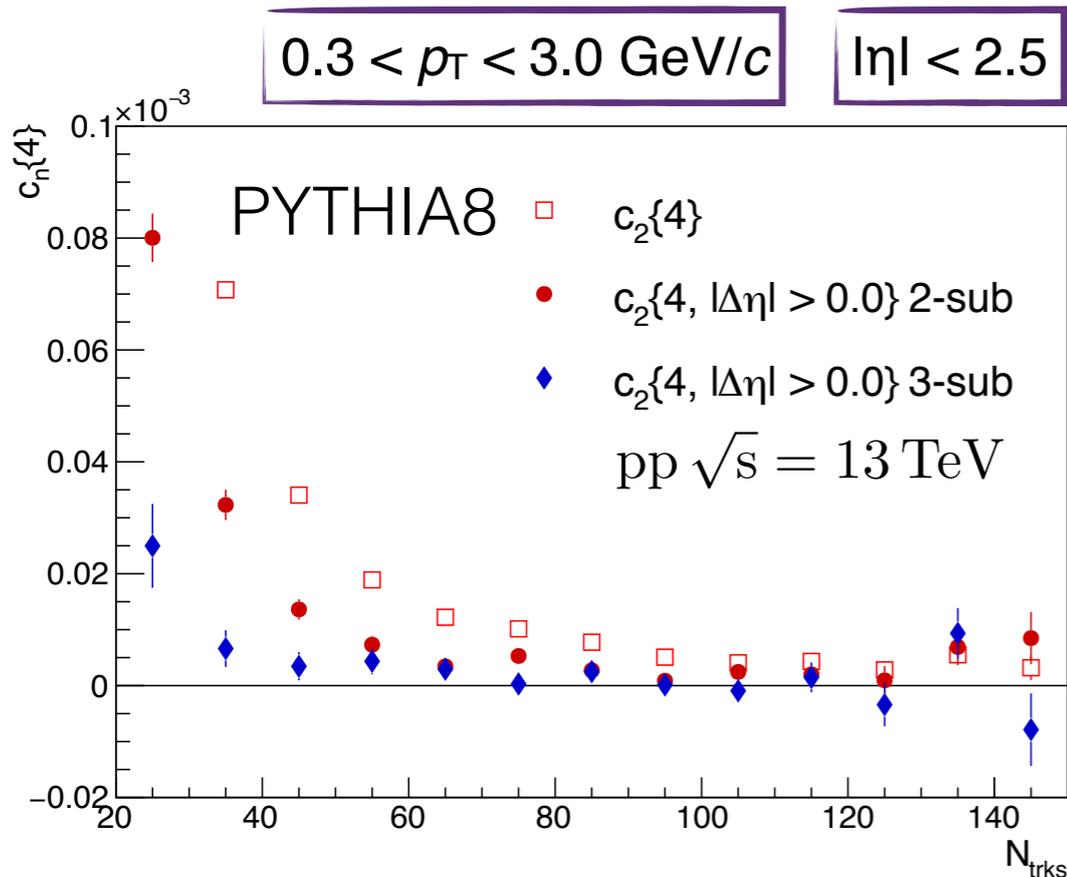
$$\langle\langle 2 \rangle\rangle_{3\text{sub}}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{3\text{sub}} = \langle\langle 4 \rangle\rangle_{3\text{sub}} - 2 \cdot \langle\langle 2 \rangle\rangle_{3\text{sub}}^2$$

- Method inspired by *arXiv: 1701.038301 [nucl-th]*, is implemented in Generic Framework and tested with PYTHIA8 simulations



4-particle cumulant (3-subevent method)



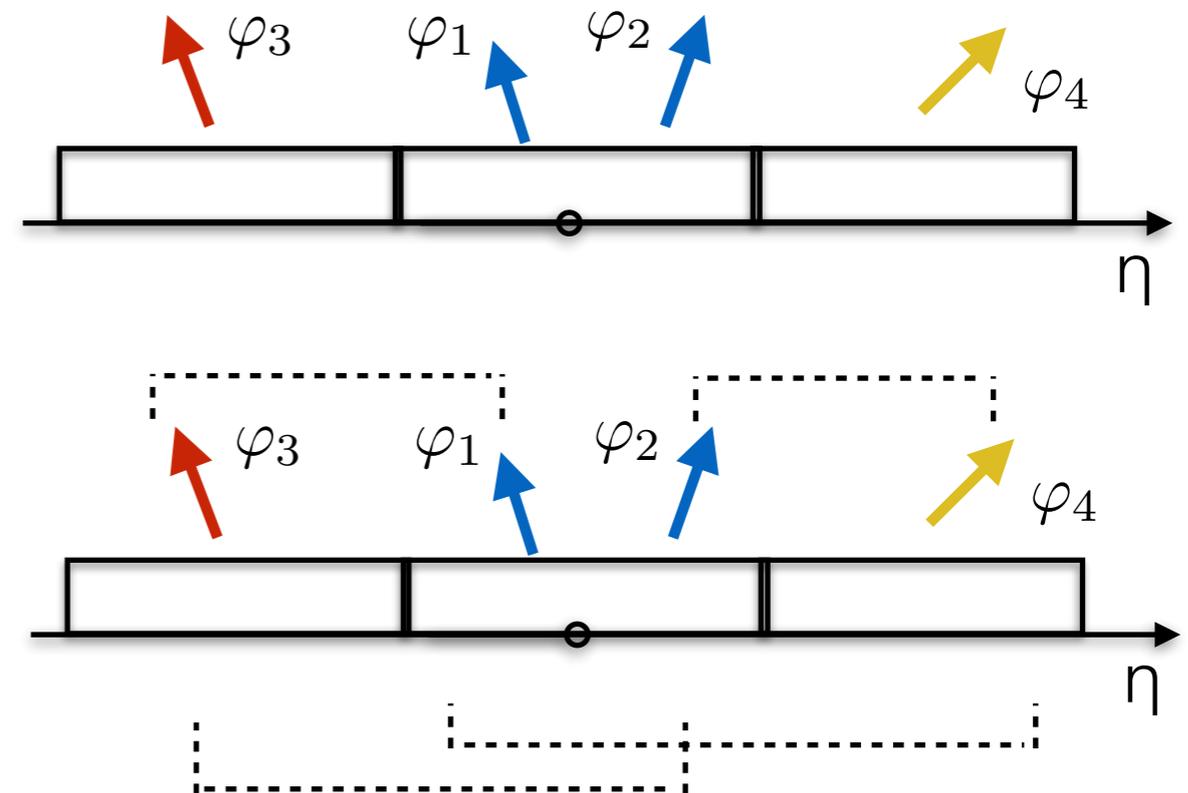
$$\langle\langle 4 \rangle\rangle_{3\text{sub}} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

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$$\langle\langle 2 \rangle\rangle_{3\text{sub}}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{3\text{sub}} = \langle\langle 4 \rangle\rangle_{3\text{sub}} - 2 \cdot \langle\langle 2 \rangle\rangle_{3\text{sub}}^2$$

- Method inspired by *arXiv: 1701.038301 [nucl-th]*, is implemented in Generic Framework and tested with PYTHIA8 simulations
- Splitting the acceptance into 3 subevents shows the ability to further suppress non-flow

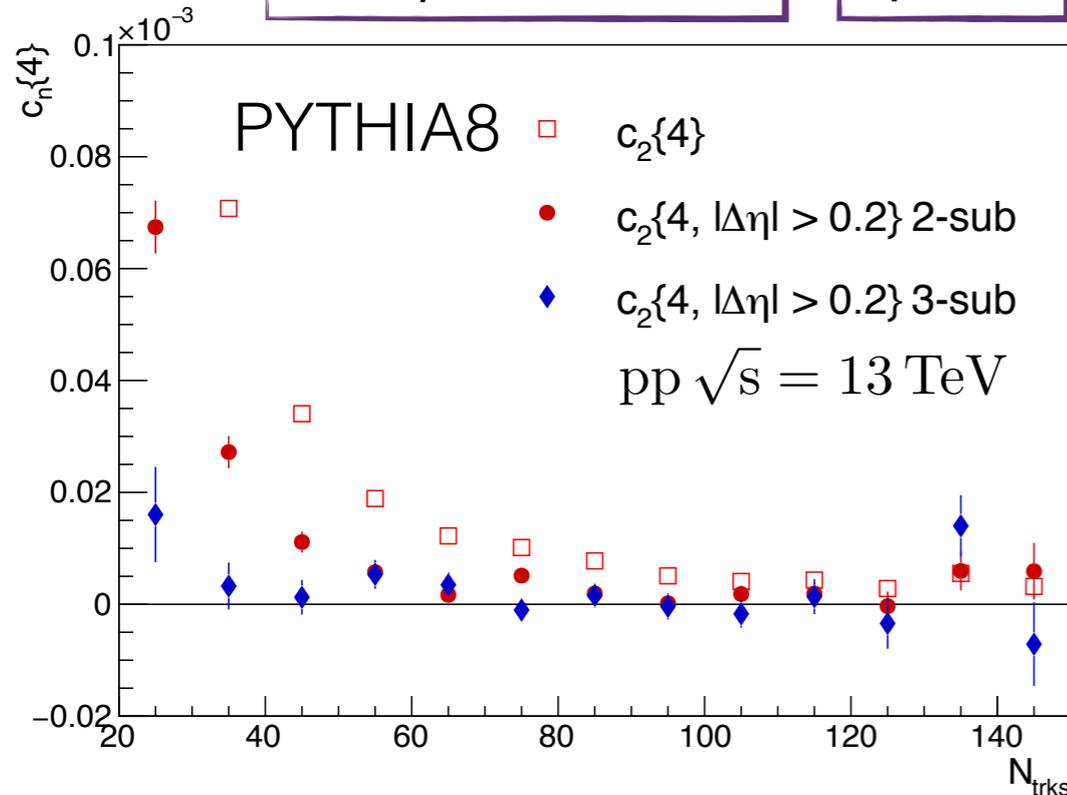


4-particle cumulant (3-subevent method)



$0.3 < p_T < 3.0 \text{ GeV}/c$

$|\eta| < 2.5$



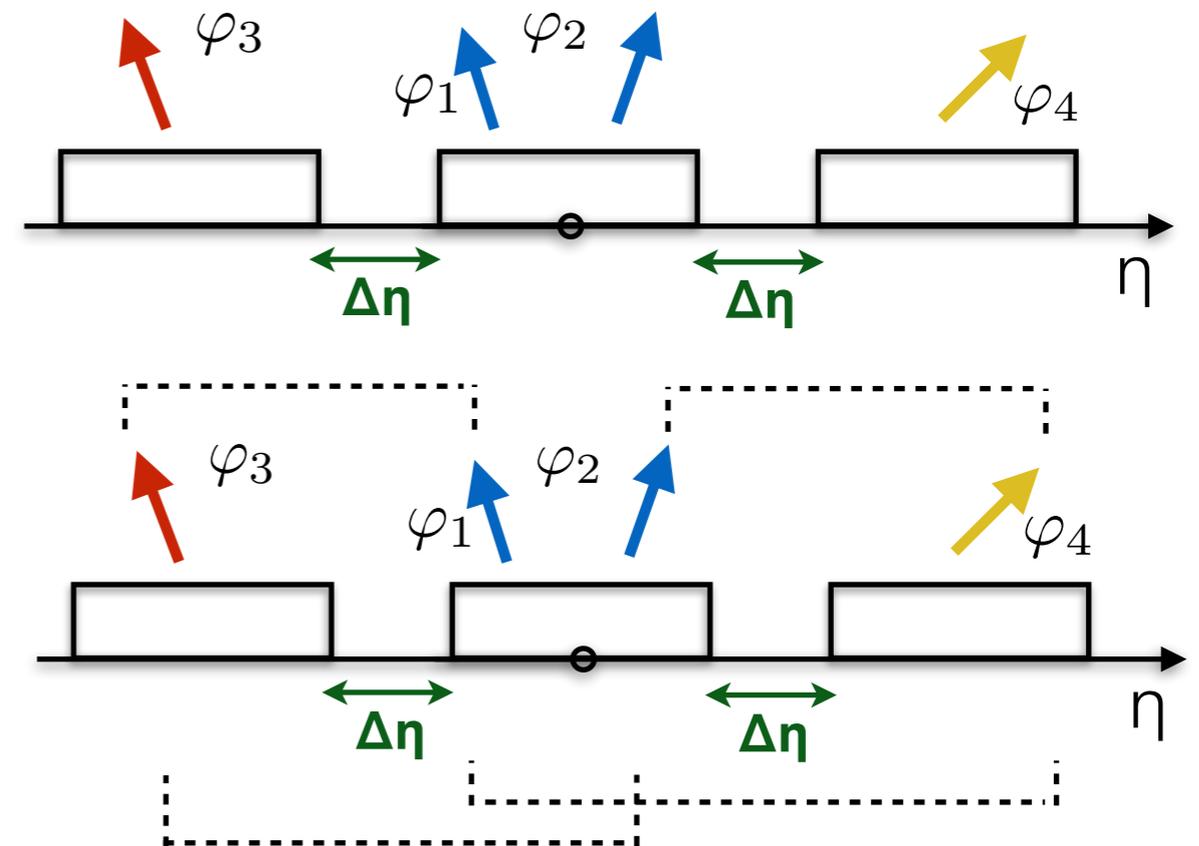
$$\langle\langle 4 \rangle\rangle_{3\text{sub}} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

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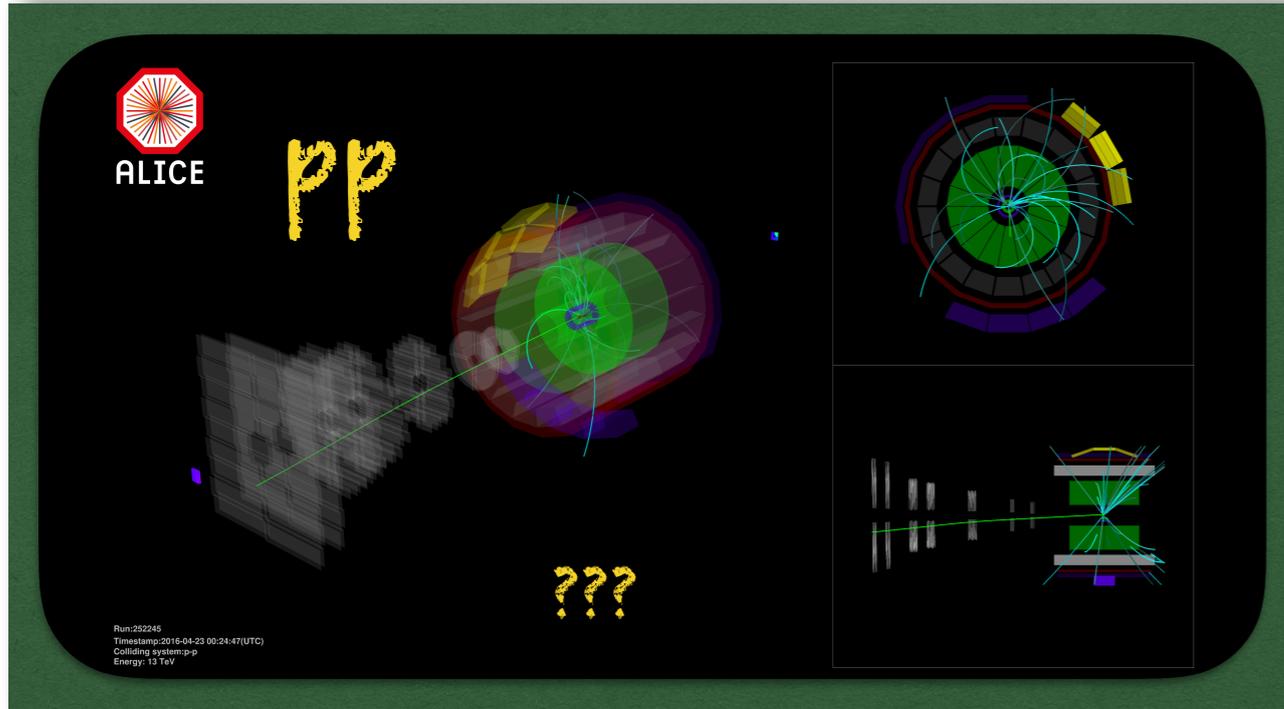
$$\langle\langle 2 \rangle\rangle_{3\text{sub}}^2 = \langle\langle \cos n(\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_3) \rangle\rangle$$

$$c_n\{4\}_{3\text{sub}} = \langle\langle 4 \rangle\rangle_{3\text{sub}} - 2 \cdot \langle\langle 2 \rangle\rangle_{3\text{sub}}^2$$

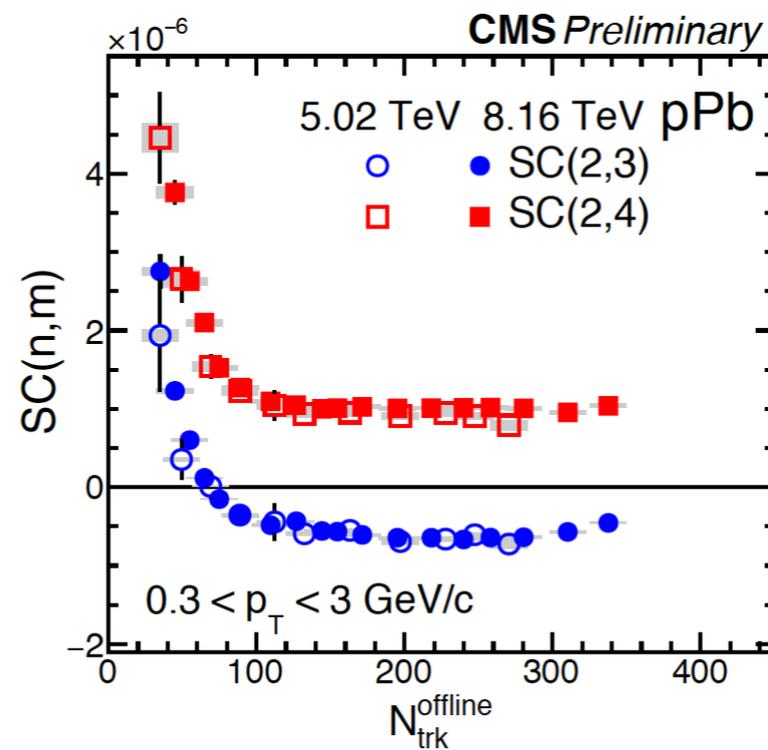
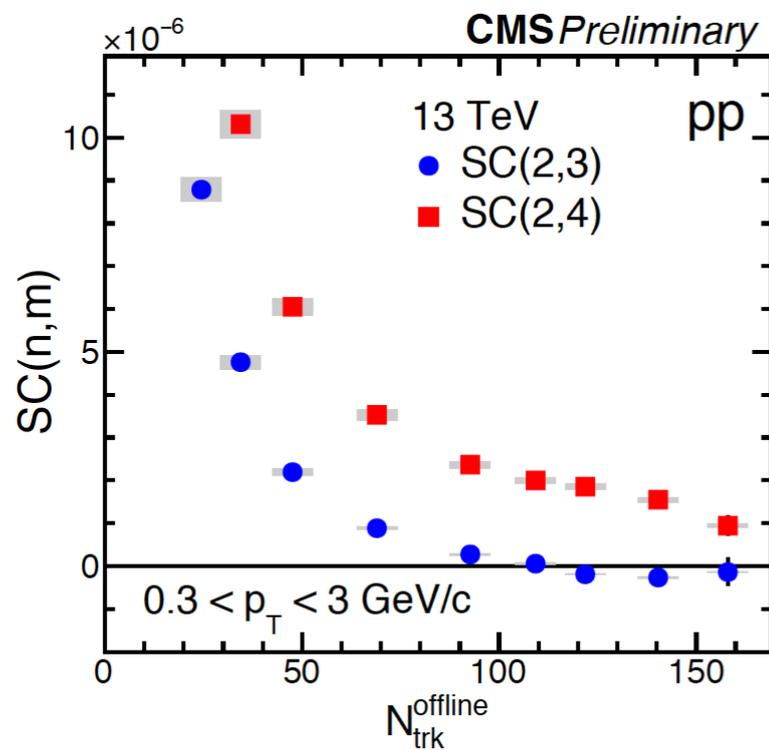
- Method inspired by *arXiv: 1701.038301 [nucl-th]*, is implemented in Generic Framework and tested with PYTHIA8 simulations
- Splitting the acceptance into 3 subevents shows the ability to further suppress non-flow
- Further decrease of $c_2\{4, |\Delta\eta| > 0.2\}$ is seen w.r.t. $c_2\{4, |\Delta\eta| > 0.0\}$



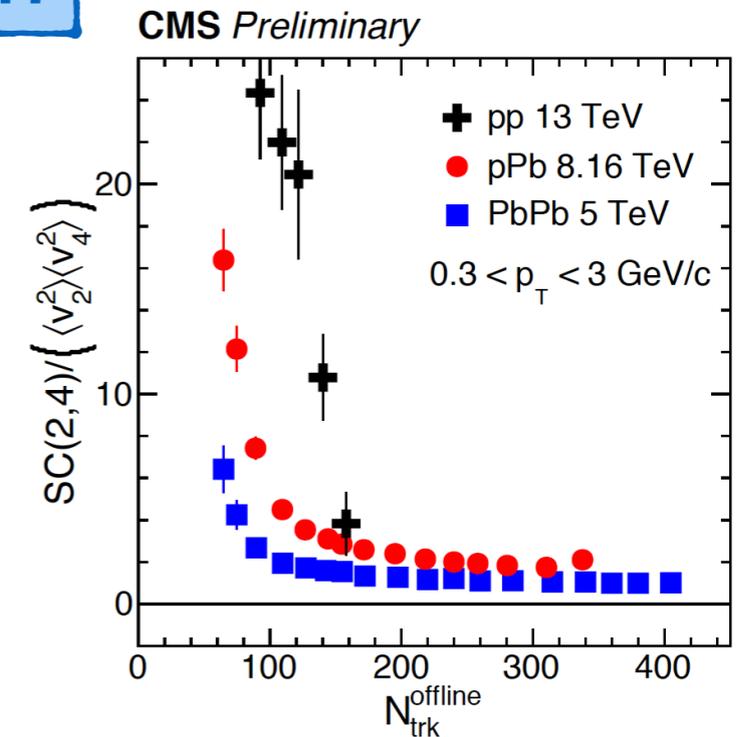
Symmetric Cumulants in small systems



- First measurements of $SC(m,n)$ in small systems were reported at QM17 by CMS
- However, **are $SC(m,n)$ measurements free of non-flow contamination, especially in small systems?**



QM17



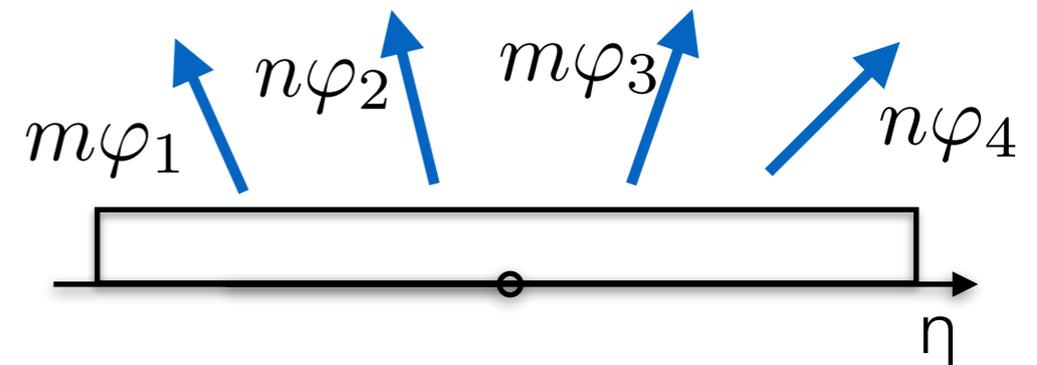
Standard $SC(3,2)$, $SC(4,2)$



$$\langle\langle 4 \rangle\rangle = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle \langle\langle 2 \rangle\rangle = \langle\langle \cos m(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$SC(m, n) = \langle\langle 4 \rangle\rangle - \langle\langle 2 \rangle\rangle \langle\langle 2 \rangle\rangle$$



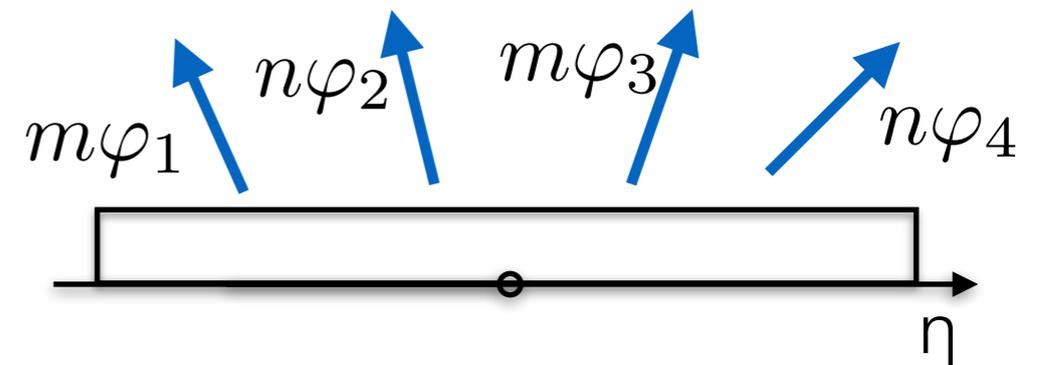
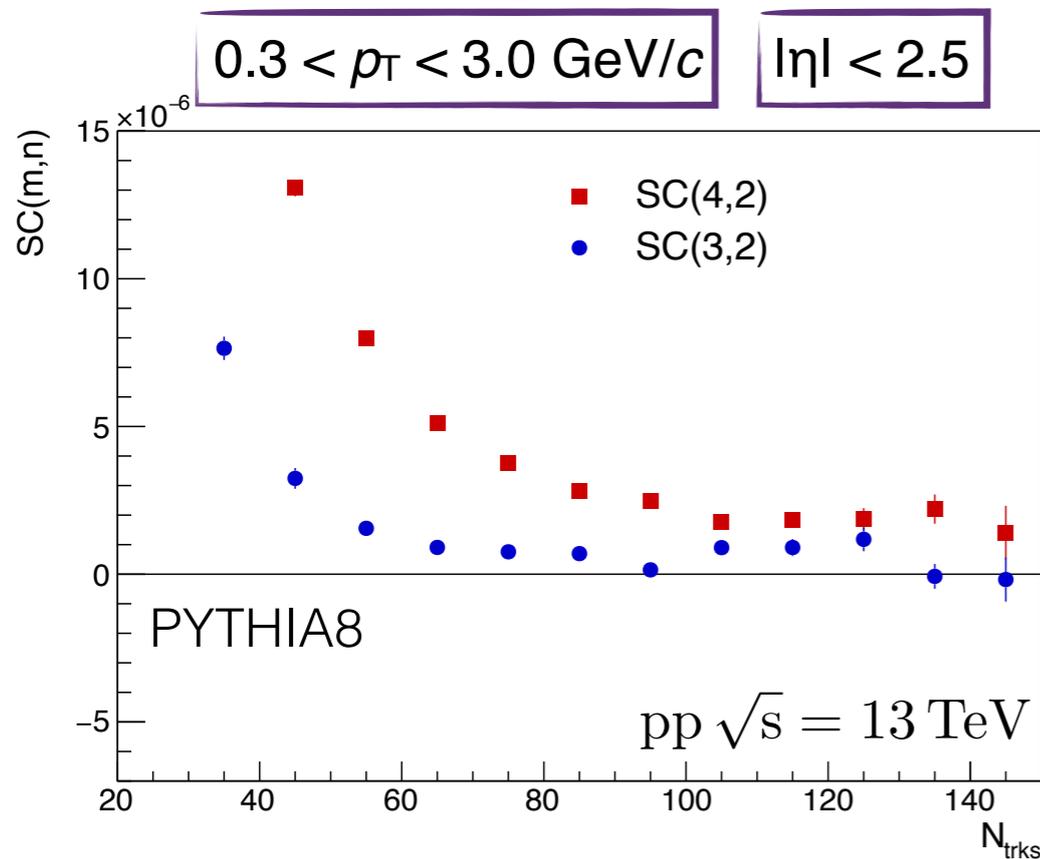
Standard SC(3,2), SC(4,2)



$$\langle\langle 4 \rangle\rangle = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle \langle\langle 2 \rangle\rangle = \langle\langle \cos m(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$SC(m, n) = \langle\langle 4 \rangle\rangle - \langle\langle 2 \rangle\rangle \langle\langle 2 \rangle\rangle$$



- Positive correlation is seen in PYTHIA between v_2 and v_3 and v_2 and v_4

Standard SC(3,2), SC(4,2)

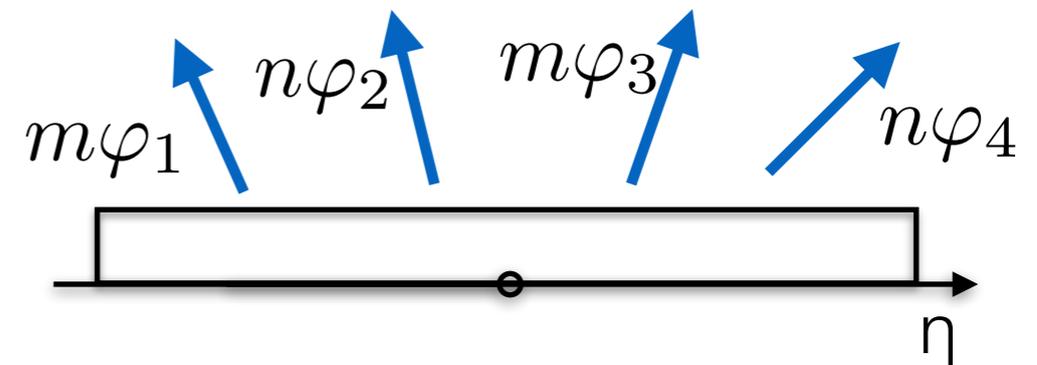
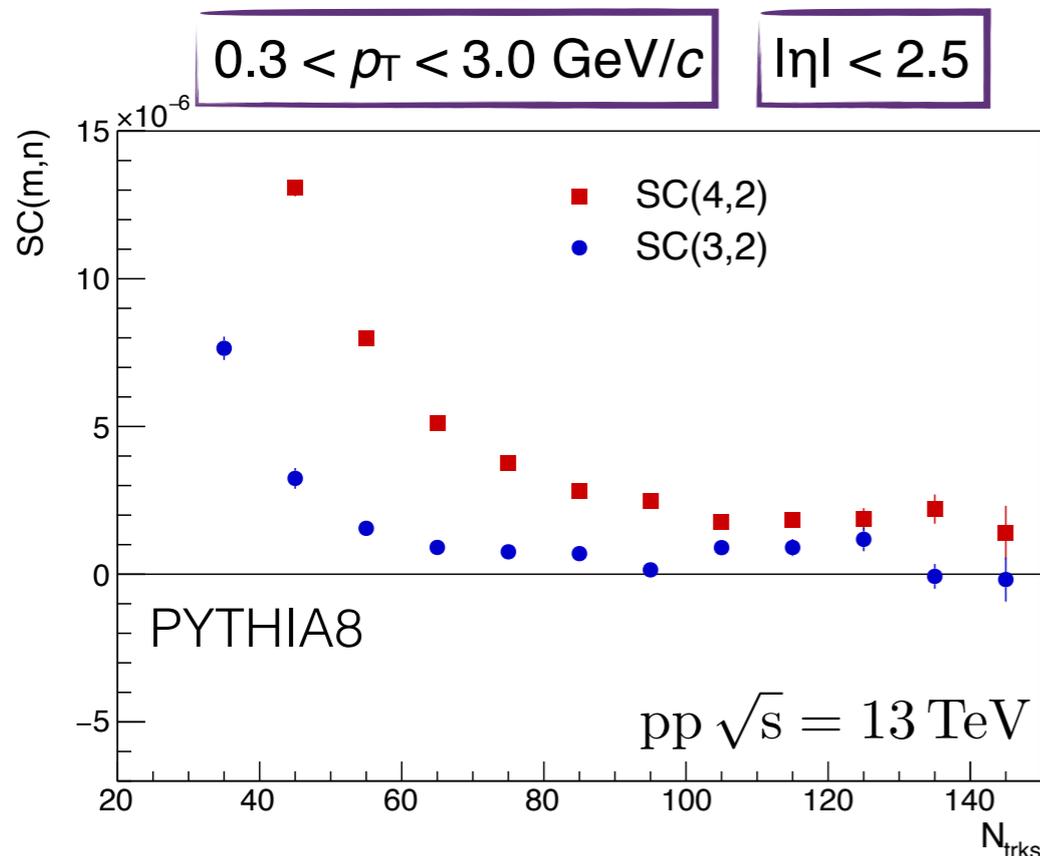


- We have seen that 4-particle cumulant still suffered from non-flow effects in pp collisions
- SC(m,n)** works on **similar principle**, therefore it **might be contaminated by non-flow**

$$\langle\langle 4 \rangle\rangle = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle \langle\langle 2 \rangle\rangle = \langle\langle \cos m(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$SC(m, n) = \langle\langle 4 \rangle\rangle - \langle\langle 2 \rangle\rangle \langle\langle 2 \rangle\rangle$$



- Positive correlation is seen in PYTHIA between v₂ and v₃ and v₂ and v₄

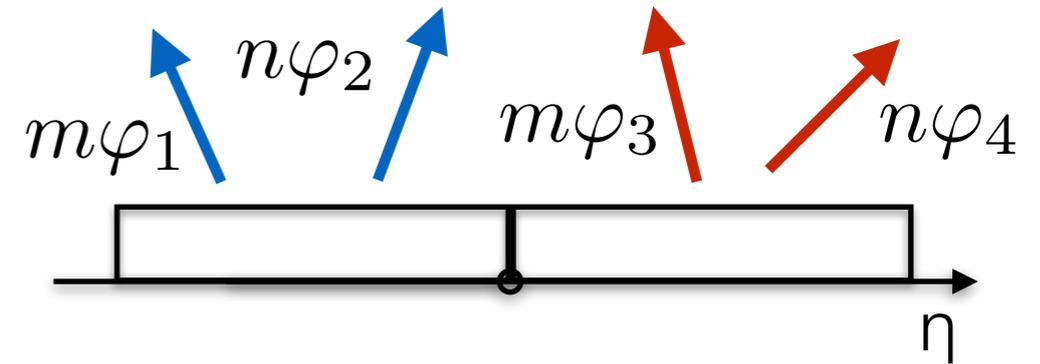
SC(3,2), SC(4,2) with $|\Delta\eta|$ gap



$$\langle\langle 4 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|} \langle\langle 2 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos m(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

$$SC(m, n)_{|\Delta\eta|} = \langle\langle 4 \rangle\rangle_{|\Delta\eta|} - \langle\langle 2 \rangle\rangle_{|\Delta\eta|} \langle\langle 2 \rangle\rangle_{|\Delta\eta|}$$



- We apply $|\Delta\eta|$ gap (or 2-subevent method) to 2- and 4-particle correlations in the calculation of SC(m,n)

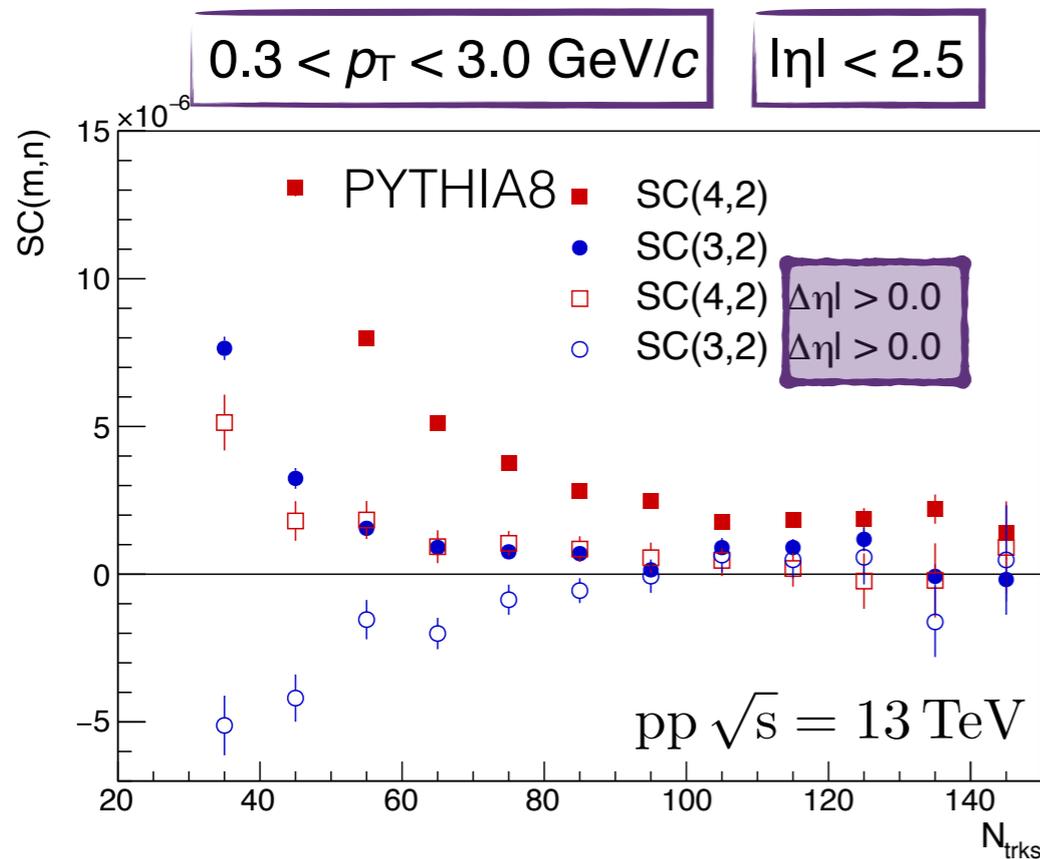
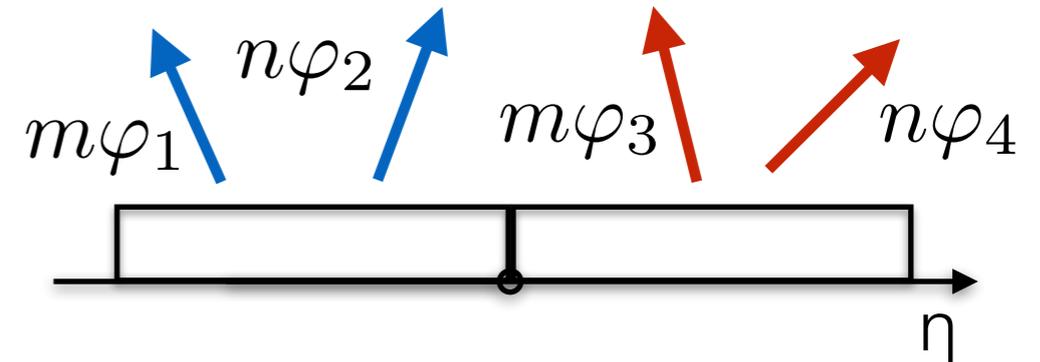
SC(3,2), SC(4,2) with $|\Delta\eta|$ gap



$$\langle\langle 4 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{|\Delta\eta|} \langle\langle 2 \rangle\rangle_{|\Delta\eta|} = \langle\langle \cos m(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

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- We apply $|\Delta\eta|$ gap (or 2-subevent method) to 2- and 4-particle correlations in the calculation of $SC(m,n)$
- Magnitudes of both $SC(4,2)$ and $SC(3,2)$ significantly decrease
- Moreover, $SC(3,2)$ turns to be negative at low multiplicity ?

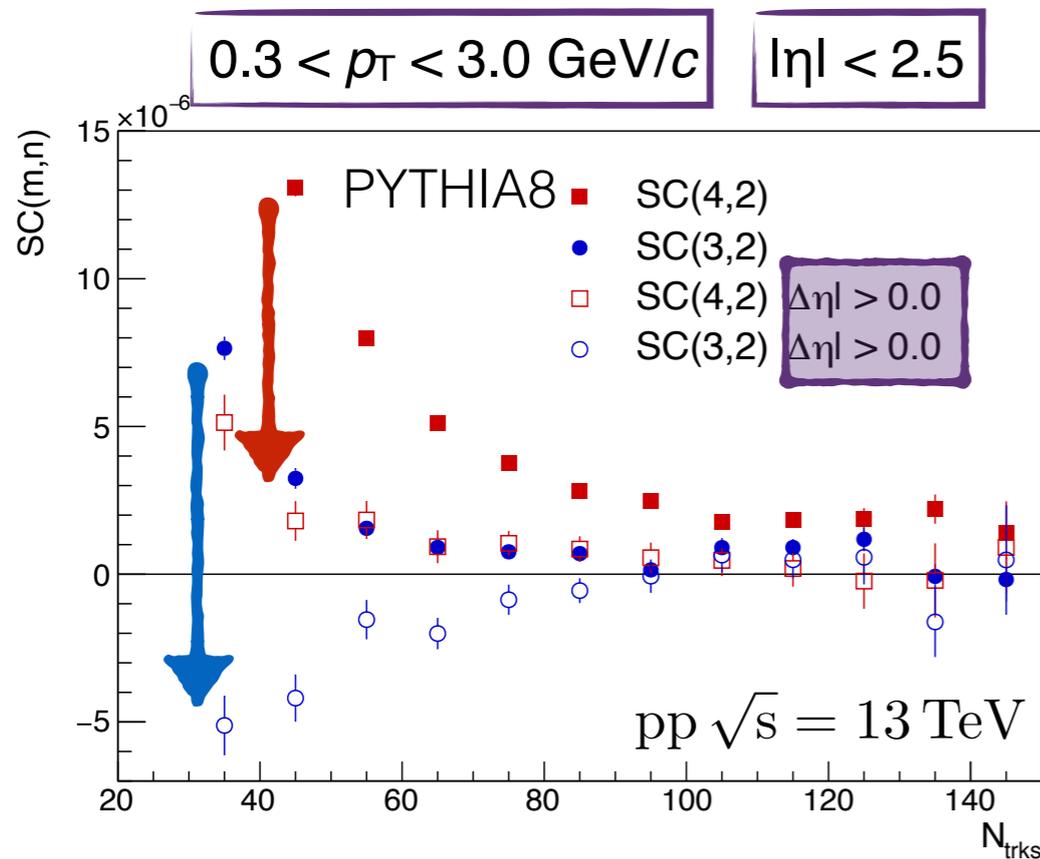
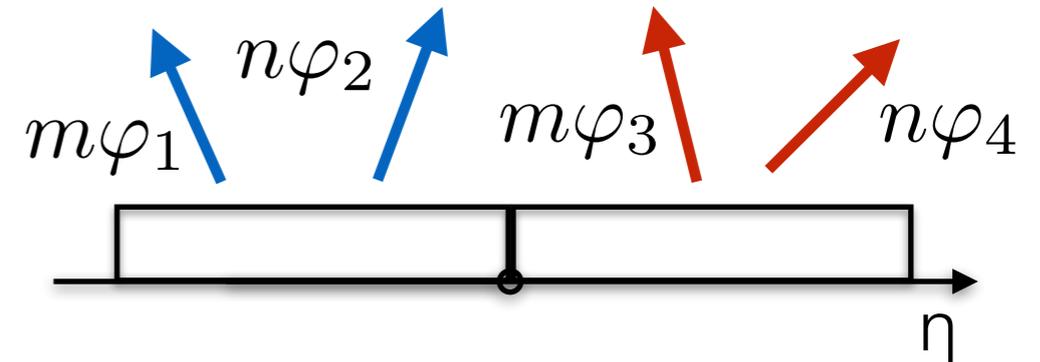
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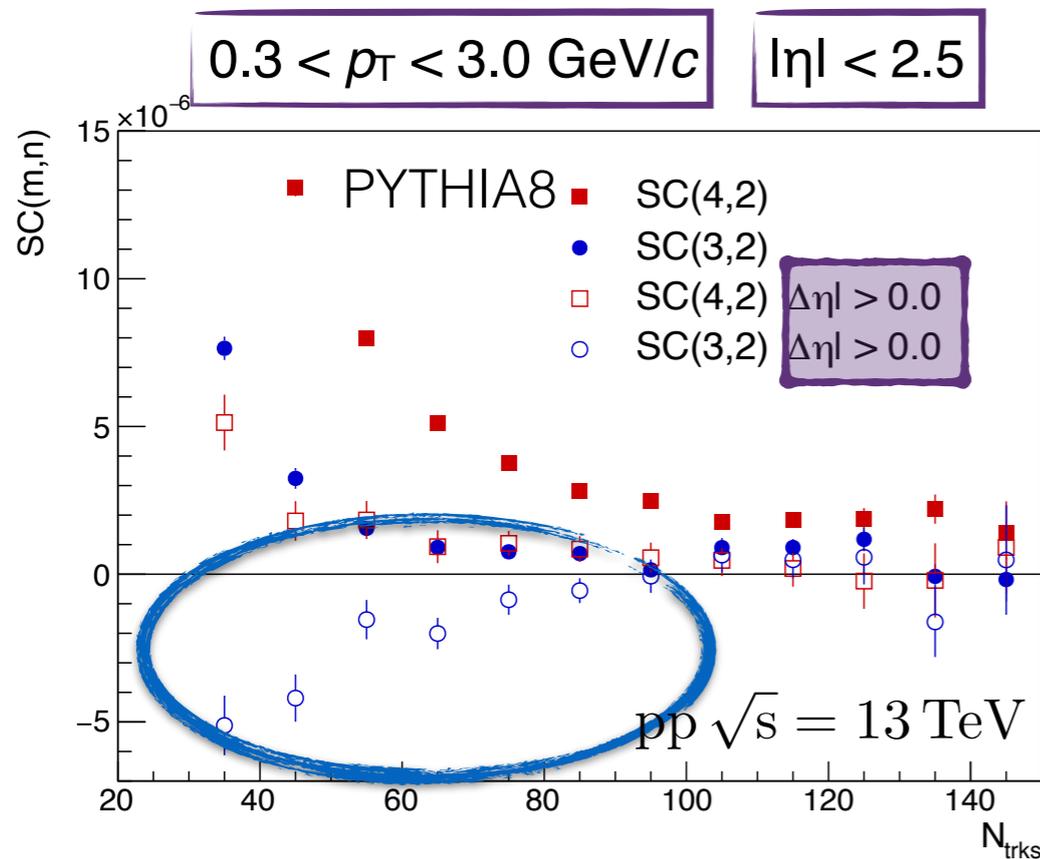
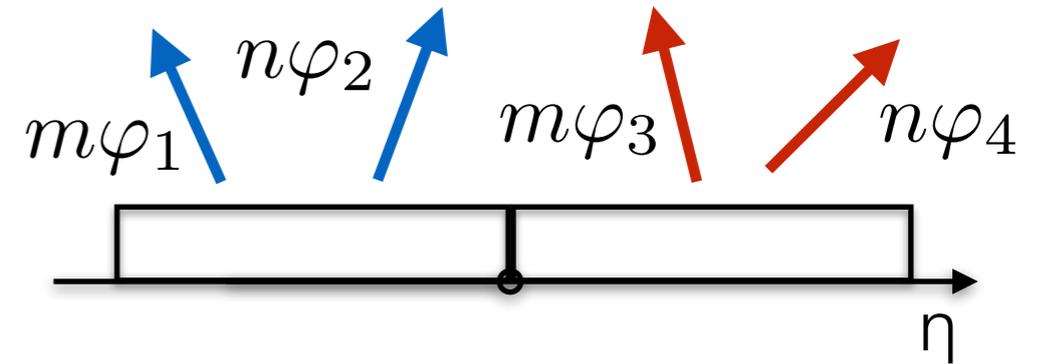
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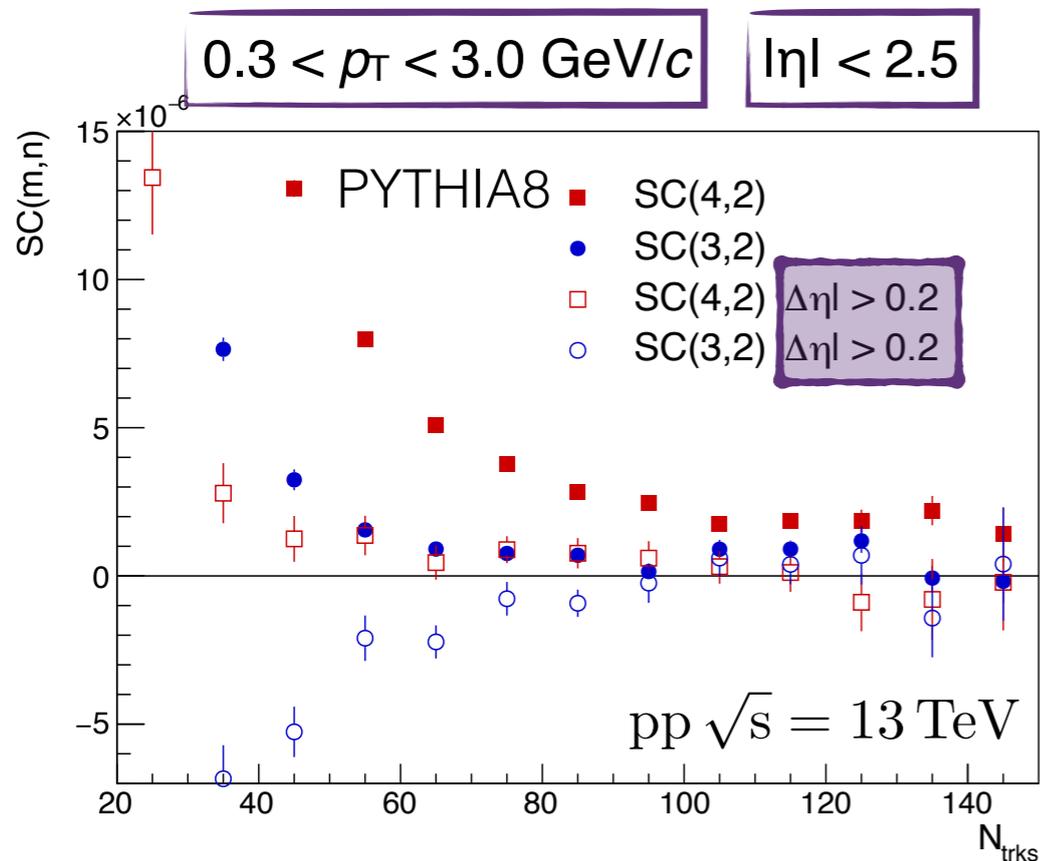
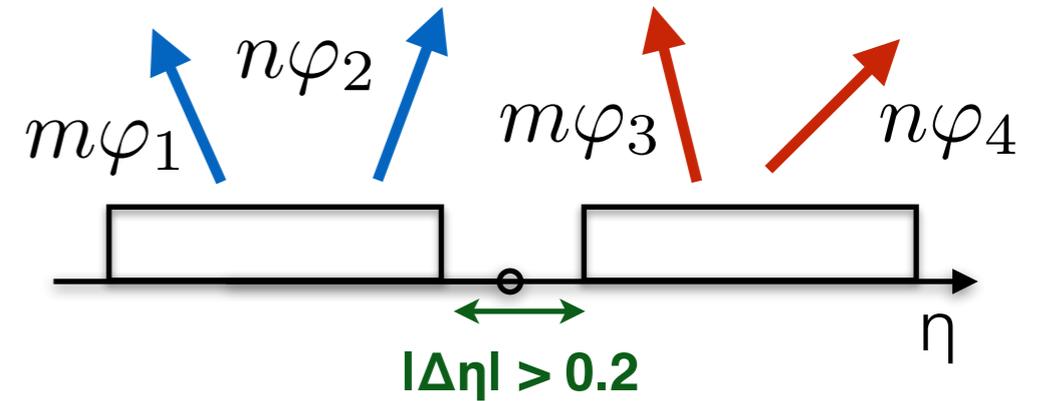
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SC(3,2), SC(4,2) with 3-subevent



$$\langle\langle 4 \rangle\rangle_{m,n,-m,-n} = \langle\langle \cos (m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

$$\langle\langle 2 \rangle\rangle_{m,-m} \langle\langle 2 \rangle\rangle_{n,-n} = \langle\langle \cos m(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

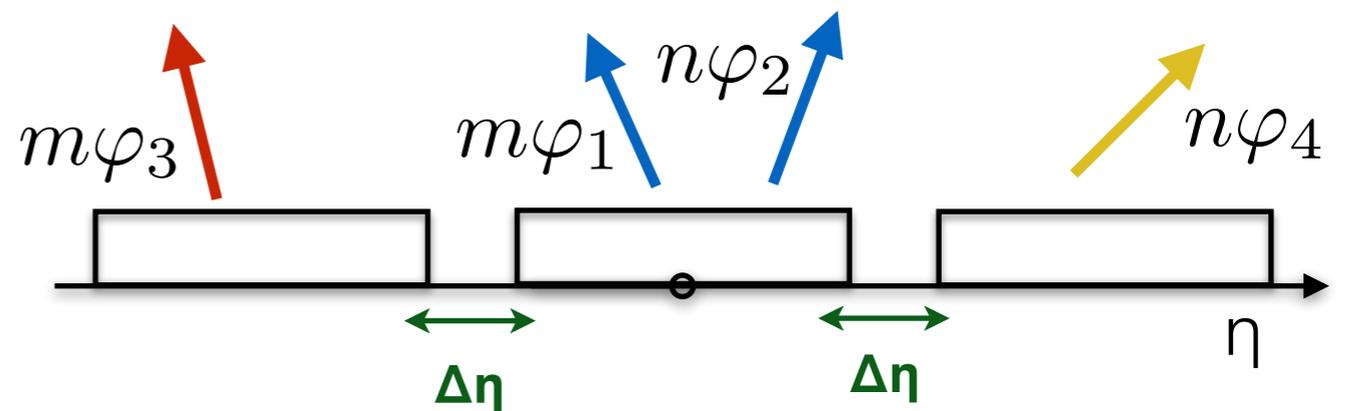


$$\langle\langle 4 \rangle\rangle_{m,n,-n,-m} = \langle\langle \cos (m\varphi_1 + n\varphi_2 - n\varphi_3 - m\varphi_4) \rangle\rangle$$

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$$SC(m, n)_A = \langle\langle 4 \rangle\rangle_{m,n,-m,-n} - 2 \cdot \langle\langle 2 \rangle\rangle_{m,-m} \langle\langle 2 \rangle\rangle_{n,-n}$$

$$SC(m, n)_B = \langle\langle 4 \rangle\rangle_{m,n,-n,-m} - 2 \cdot \langle\langle 2 \rangle\rangle_{n,-n} \langle\langle 2 \rangle\rangle_{m,-m}$$



SC(3,2), SC(4,2) with 3-subevent



$$\langle\langle 4 \rangle\rangle_{m,n,-m,-n} = \langle\langle \cos (m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

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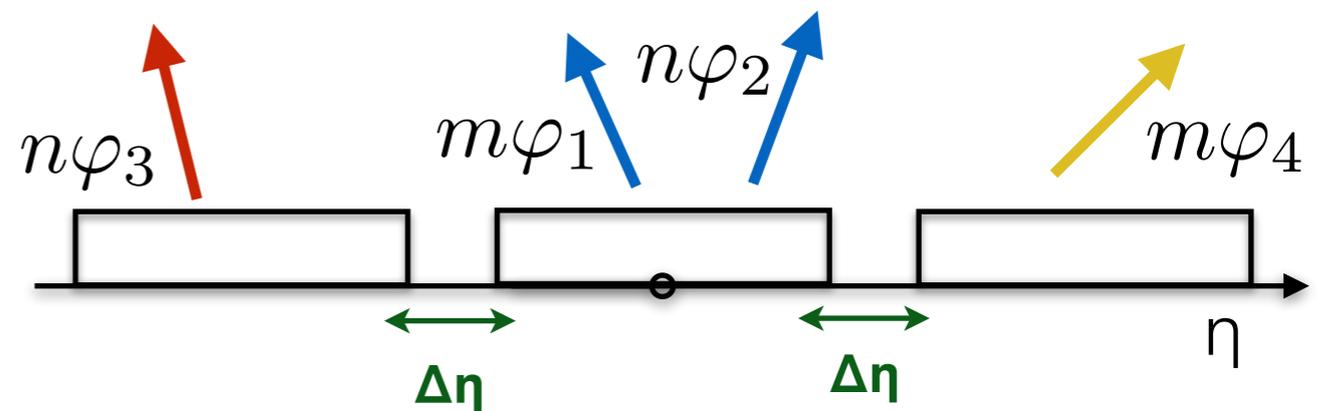


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SC(3,2), SC(4,2) with 3-subevent



$$\langle\langle 4 \rangle\rangle_{m,n,-m,-n} = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle$$

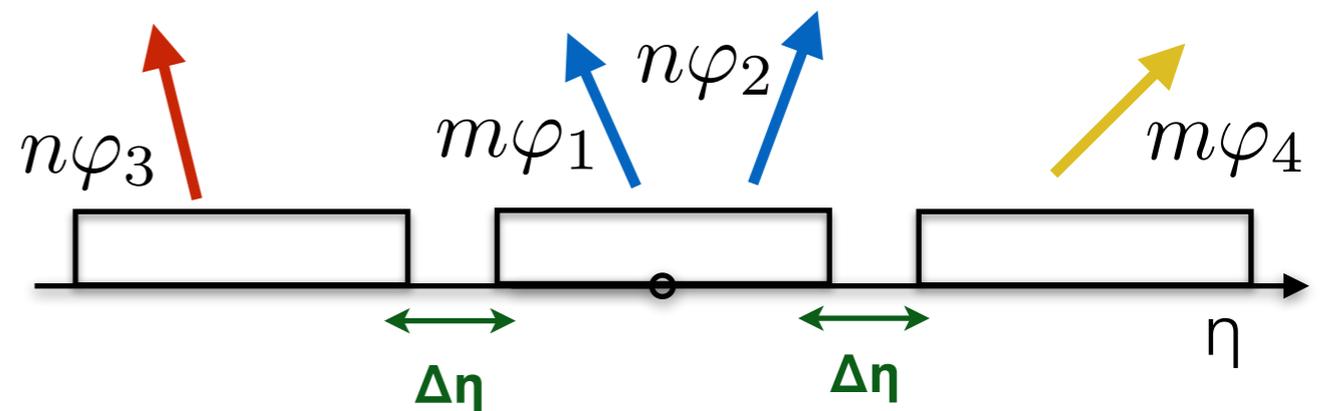
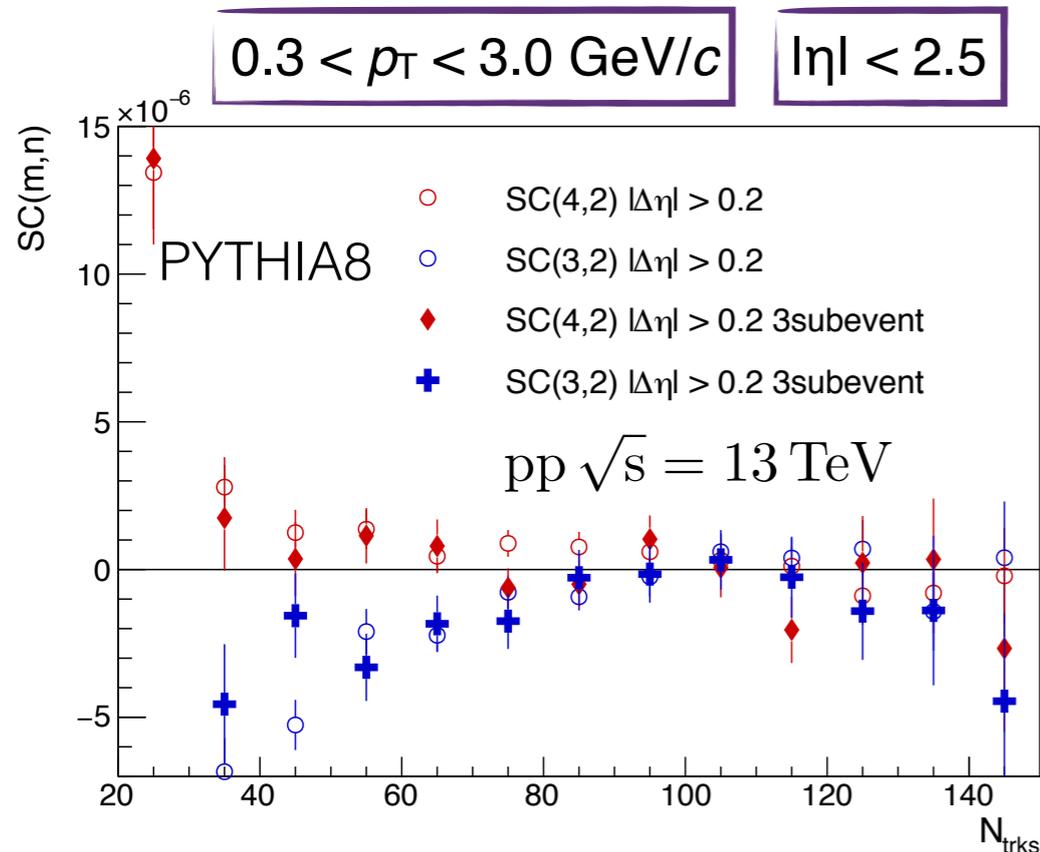
$$\langle\langle 2 \rangle\rangle_{m,-m} \langle\langle 2 \rangle\rangle_{n,-n} = \langle\langle \cos m(\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle$$

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- We calculate SC(m,n) with 3-subevent method
- There is no significant difference between 2- or 3-subevent method

Summary

- Measurements of 2- and 4-particle cumulants in pp, p-Pb and Pb-Pb collisions were presented
 - While negative sign of $c_2\{4\}$ was observed in Pb-Pb and p-Pb collisions for $N_{\text{trk}} > 60$, **no definitive flow-like signature** was observed in pp collisions
 - **Further suppression of non-flow did not reveal** a definitive **negative sign** in $c_2\{4, |\Delta\eta|\}$ measurements
- Simulation study of Symmetric Cumulants with 2- and 3-subevent method was shown
 - Clear **decrease of magnitude** of the measurements when using 2- or 3-subevent method
 - **SC(3,2) changed sign** after the suppression of non-flow contribution
- It is important to perform the measurements of $c_n\{4\}$ and $\text{SC}(m,n)$ with subevent method to avoid large non-flow contamination