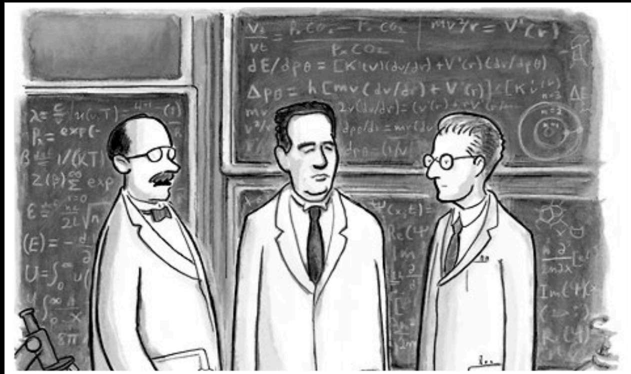
"Flow" in small systems: current and future developments



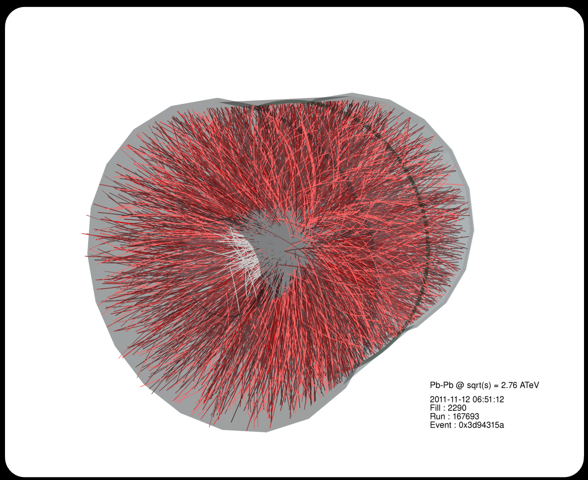
Is there collectivity in small systems? Copenhagen interpretation @ ***Workshop on Collectivity in Small Collision Systems***

You Zhou

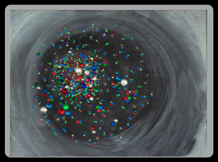
Niels Bohr Institute

Pb-Pb -> p-Pb -> pp

Pb-Pb collisions

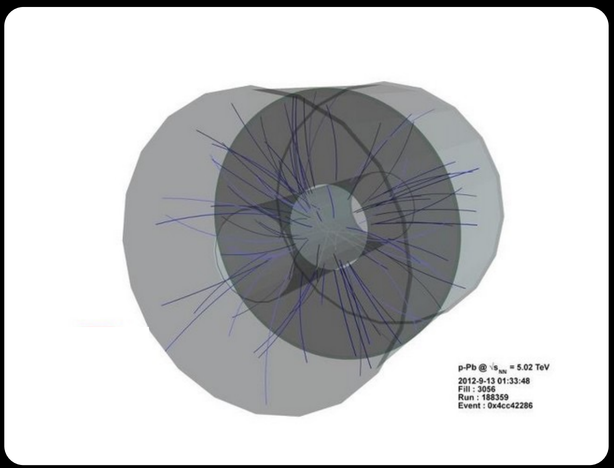


- 2.76 TeV
- 5.02 TeV



Hot QGP

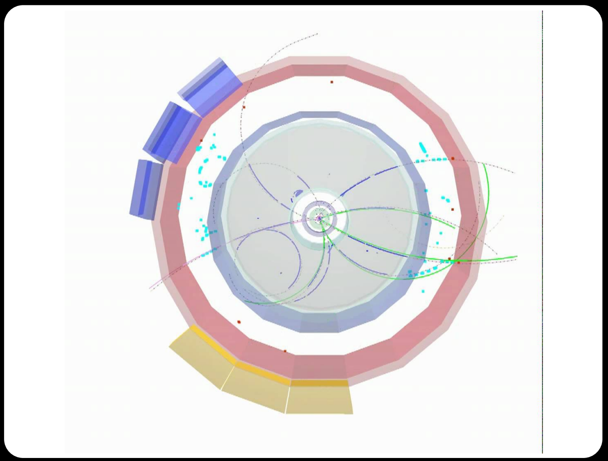
p-Pb collisions



- 5.02 TeV
- 8.16 TeV

Cold or hot ?

pp collisions



- 900 GeV
- 2.76 TeV
- 5.02 TeV
- 7 TeV
- 8 TeV
- 13 TeV

Anisotropic collectivity in AA

Credits: 

⇨ **hydro-dynamics:**

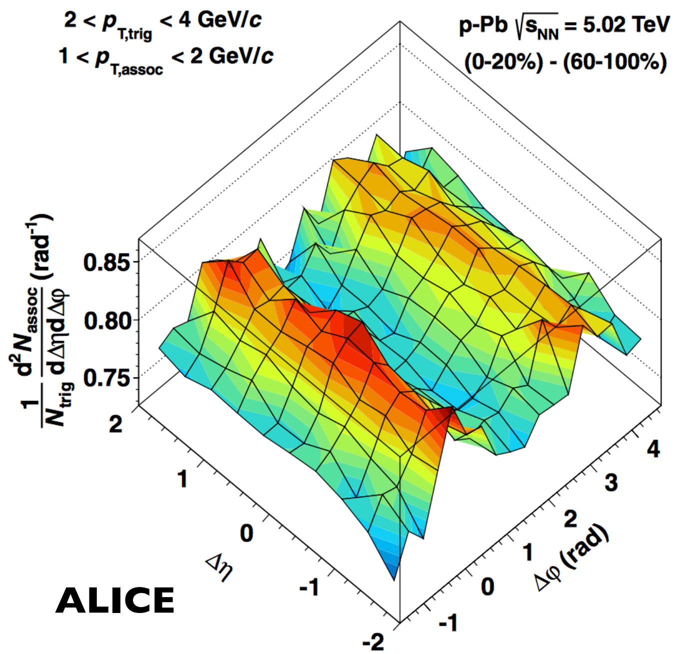
- LO : elliptic (v_2) flow for > 95% of all particles (p_t < few GeV)
- NLO : higher harmonics v_n, PID (*m* dependence) of v_n
- NNLO : non-linear mode mixing ($v_n \neq \varepsilon_n$), factorization violation $r(p_T)$, EbE $P(v_n)$, ..

- Long-range structure (ridge) LO NLO
- Anisotropic coefficient v_n LO NLO
- Mass ordering of PID v_n NLO
- Multi-particle cumulants LO
- Factorization broken NNLO
- Correlations between flow NNLO

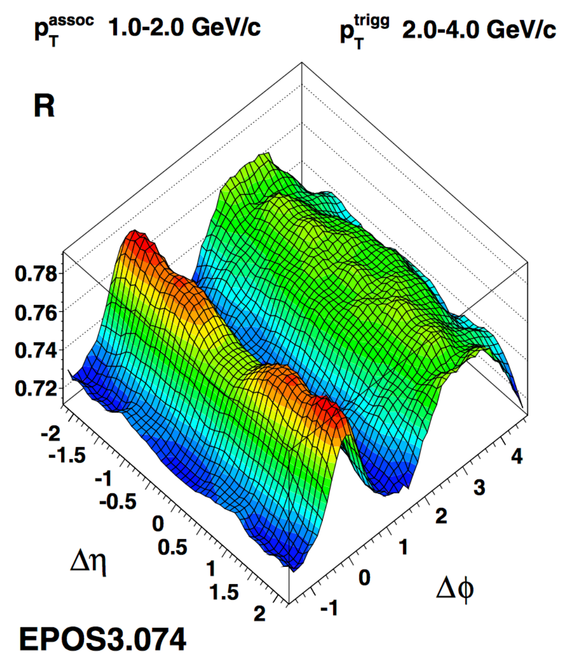
Two-particle correlations (ridge)

LO NLO

ALICE,
PLB 719, 29 (2013).



K. Werner, et. al.,
PRL. 112, 232301 (2014)

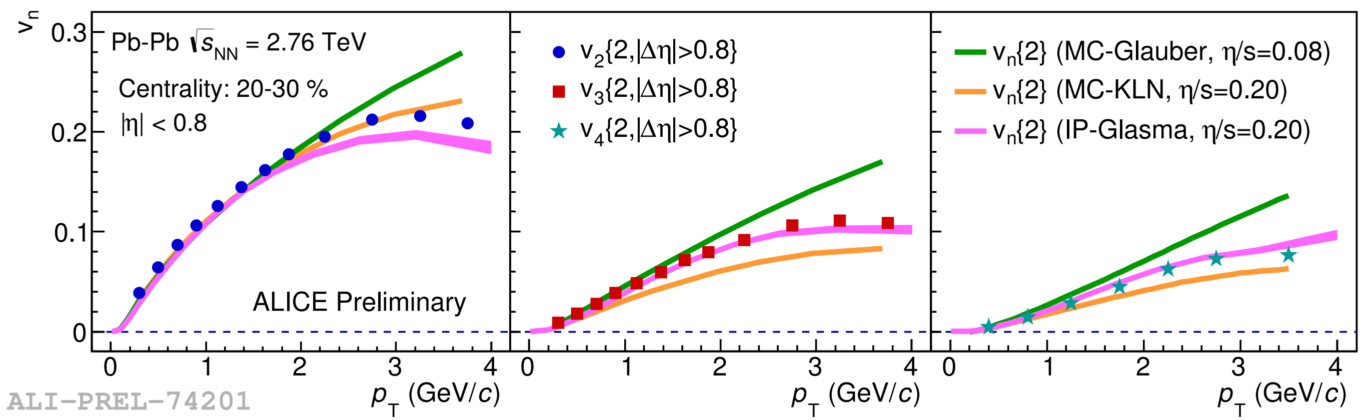


❖ Long-range correlations observed in small systems

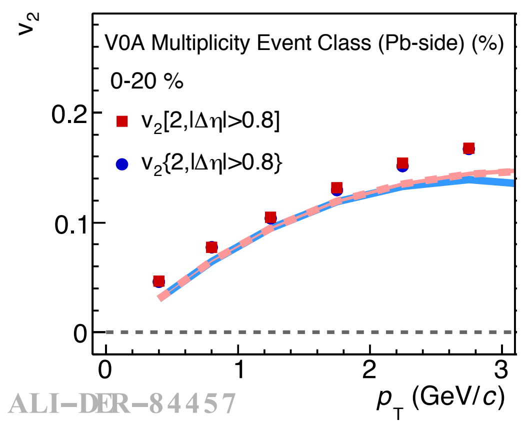
- similar correlation structure could be reproduced by hydrodynamic calculations
- collectivity?

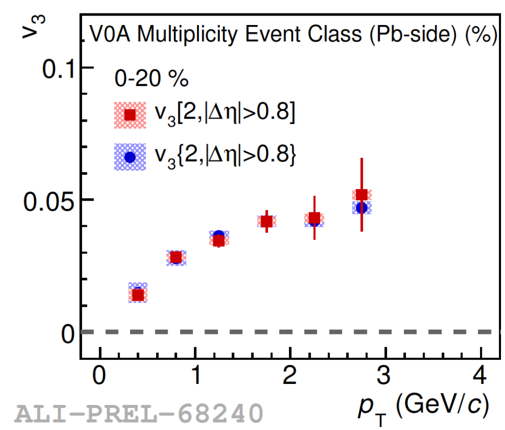
v_n shape

Pb-Pb



p-Pb





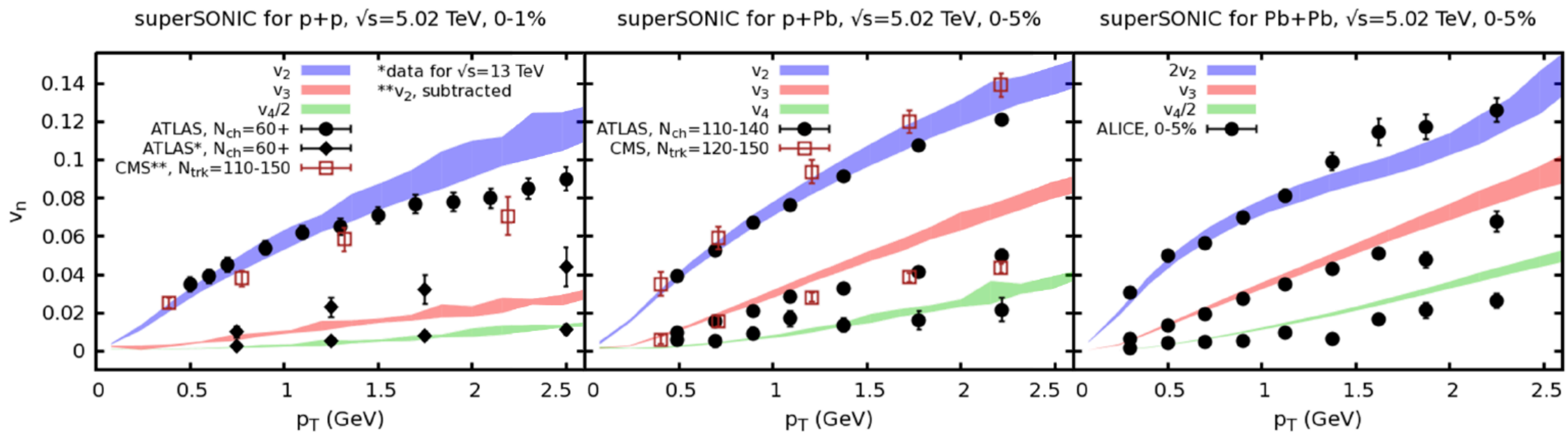
LO NLO

- The measured $v_n(p_T)$ in Pb-Pb and high multiplicity p-Pb collisions could be reproduced by hydrodynamic calculations

v_n shape

LO NLO

arXiv:1701.07145

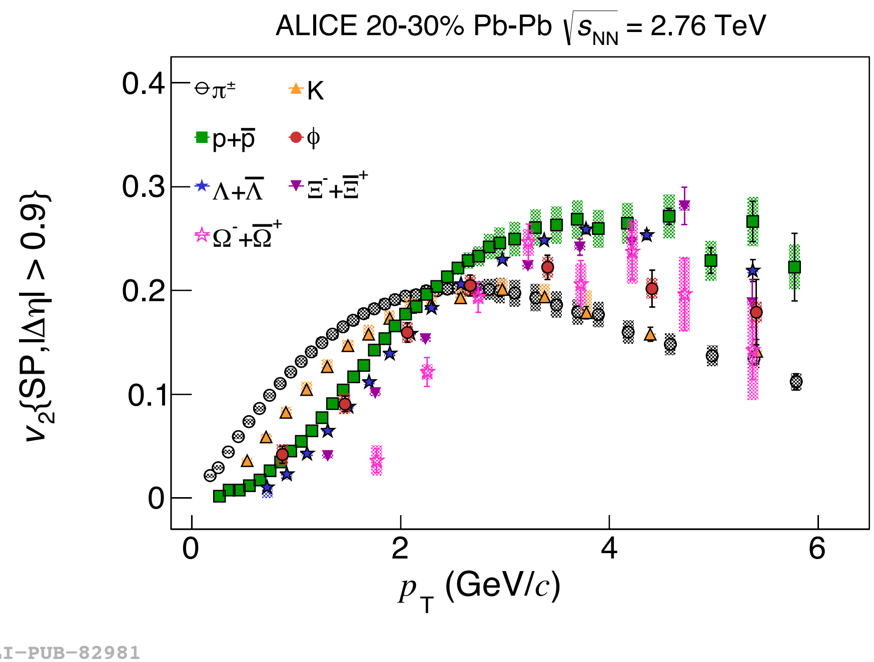


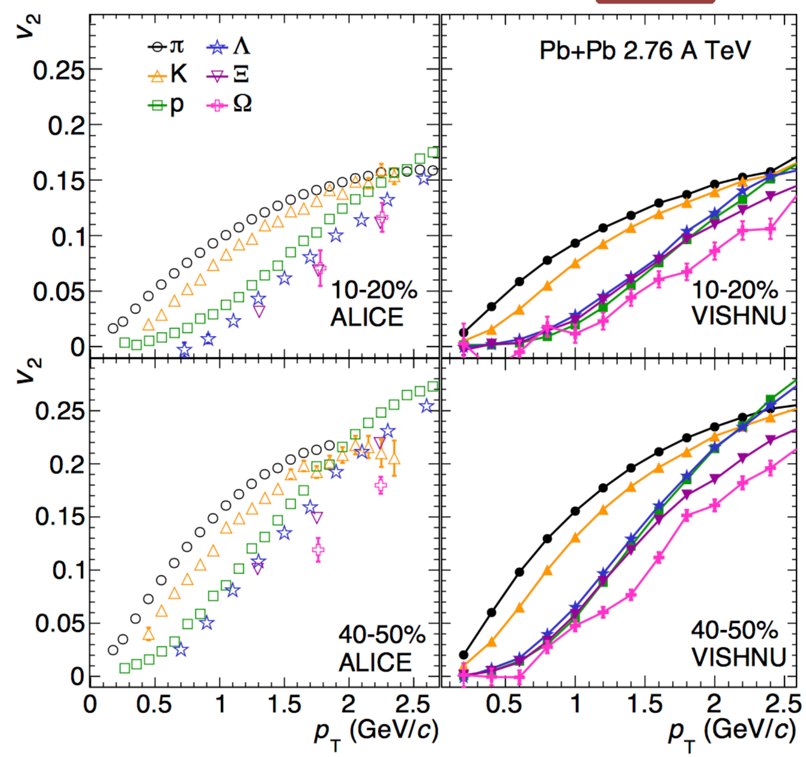
❖ One hydro model describes $v_n(p_T)$ in every system, or "***one fluid to rule them all***".

Identified particle v_2 in Pb-Pb

ALICE, JHEP 06 (2015) 190 VISHNU: PRC91 (2015), 034904

NLO



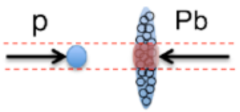


❖ v_2 of identified particles

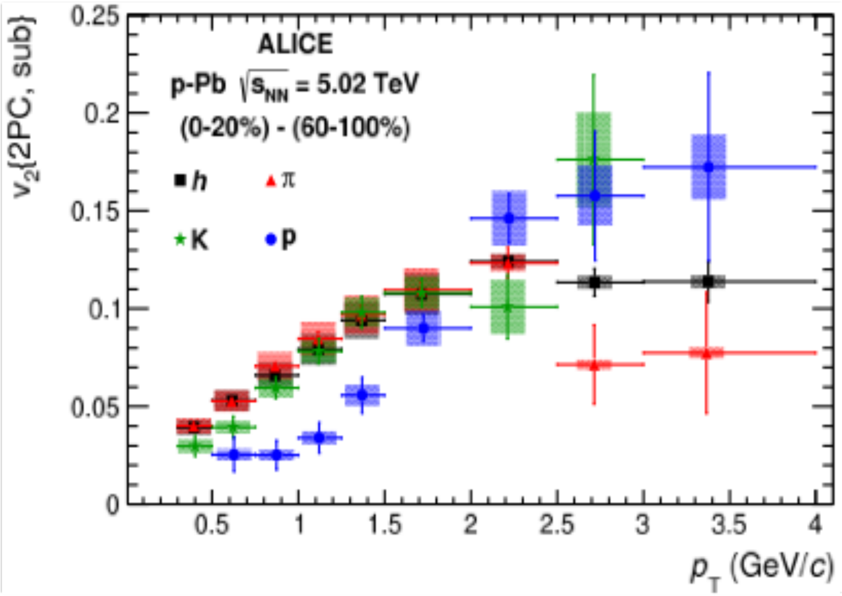
- at low p_T: mass ordering, described by VISHNU (hydro) calculations
- at immediate p_T: baryon/meson grouping and rough NCQ scaling

Identified particle v_2 in p-Pb

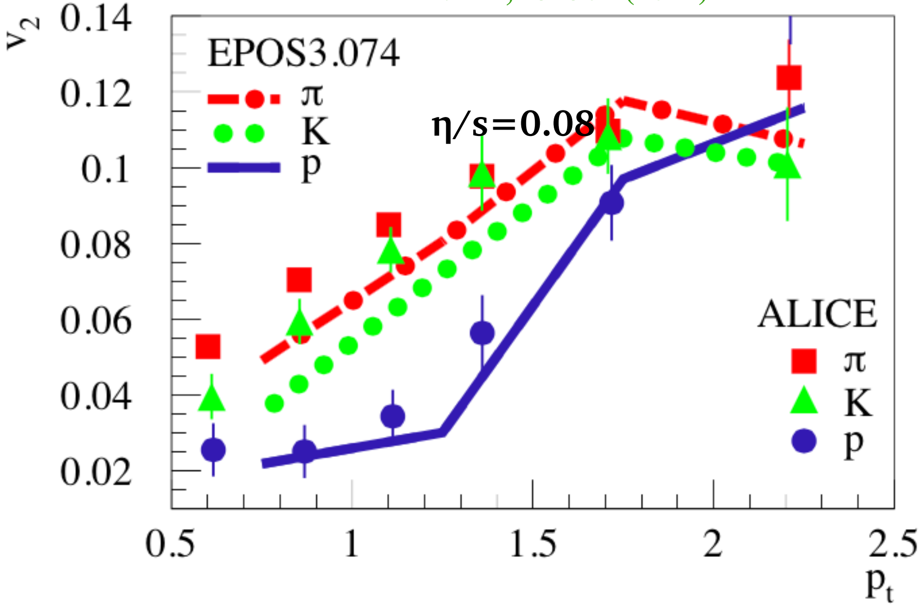
NLO



ALICE Collaboration,
Phys. Lett. B 726 (2013) 164



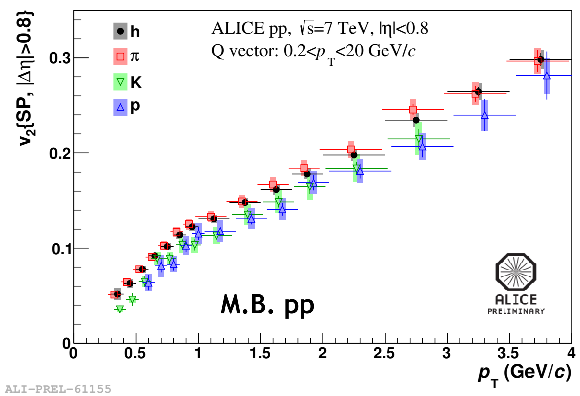
K. Werner, et. al.,
PRL. 112, 232301 (2014)

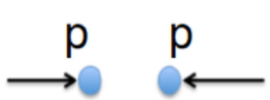


- ❖ Similar behaviors in high multiplicity p-Pb collisions
  - mass ordering, reproduced by (hybrid-)hydrodynamic calculations (e.g. EPOS)
    - indication of anisotropic flow (?)
  - baryon/meson crossing (if not clear as grouping)
    - indication of partonic collectivity (?), any calculation describes the result?

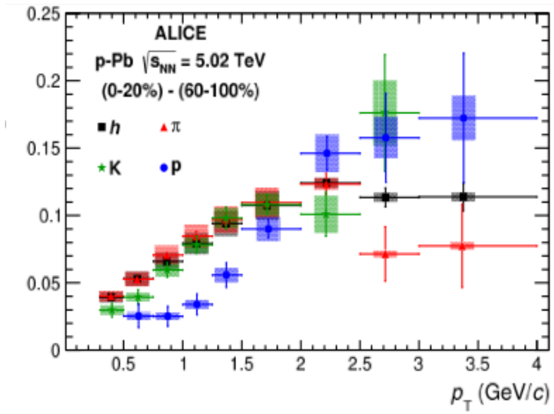
Identified particle v_2 in pp, pA, AA

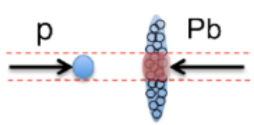
NLO



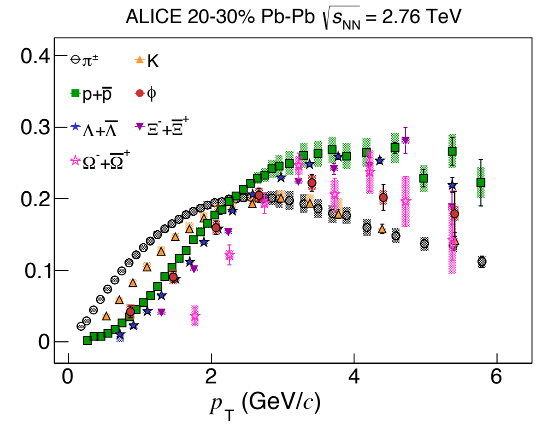


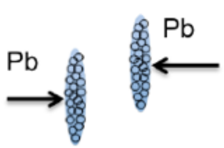
ALICE, PLB 726 (2013) 164





ALICE, JHEP 06 (2015) 190



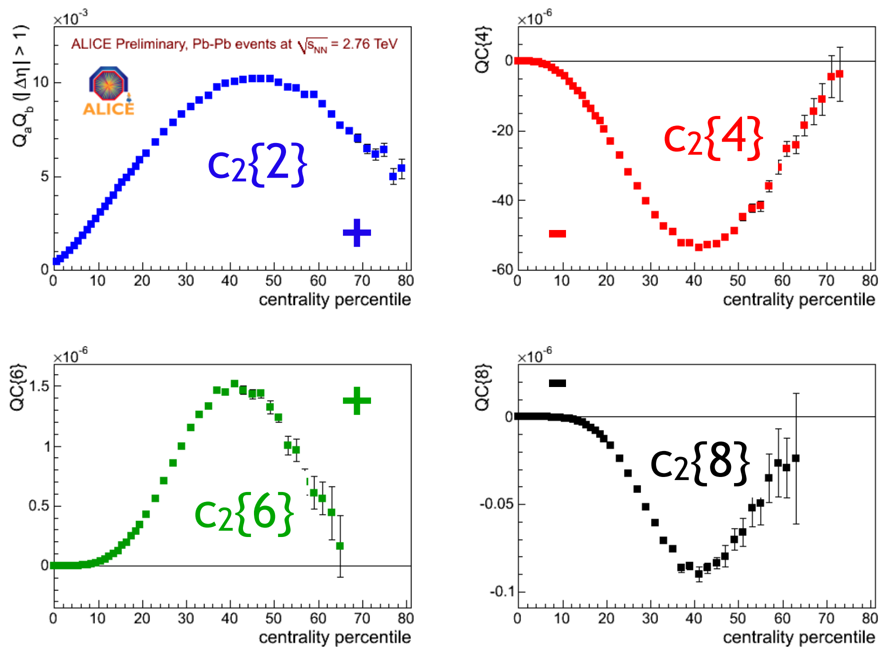


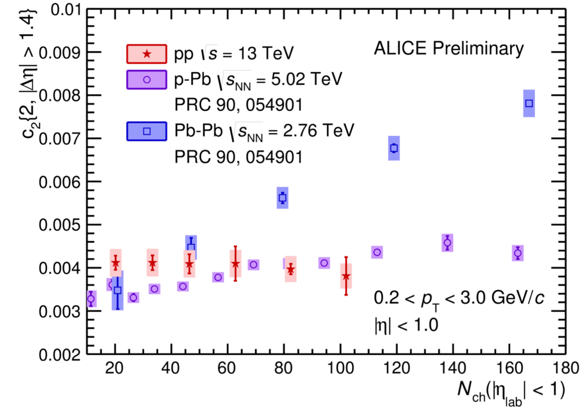
❖ v_2 of identified particles

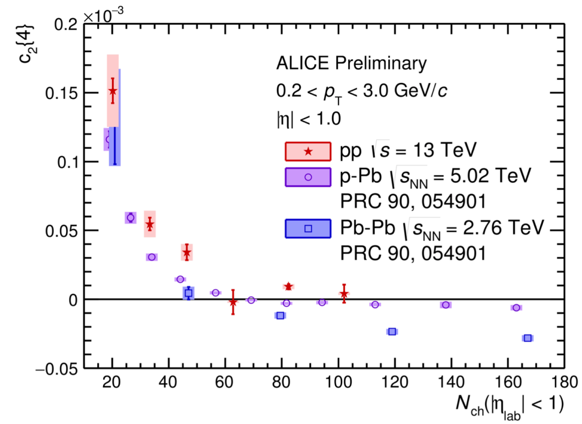
- not clear yet what does it look like in high multiplicity pp & p-Pb collisions with most particle species
- results based on high statistics RUN2 data are necessary

Multi-particle correlations: signs

LO







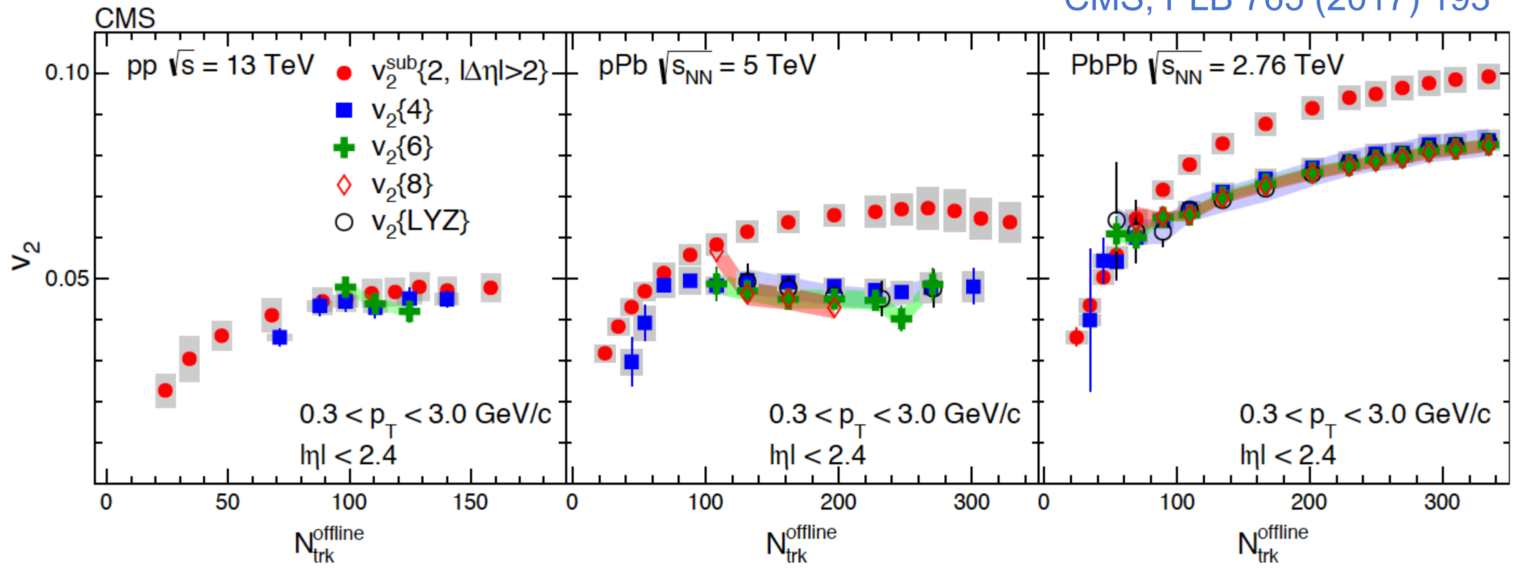
- 2- and multi-particle cumulants show +, -, +, - signs in Pb-Pb collisions
  - typical feature of collective behavior
- Similar results observed in small systems in high multiplicity regions
  - positive $c_2\{2\}$ and negative $c_2\{4\}$

Discovery

Multi-particle correlations: magnitudes

CMS, PLB 765 (2017) 193

LO



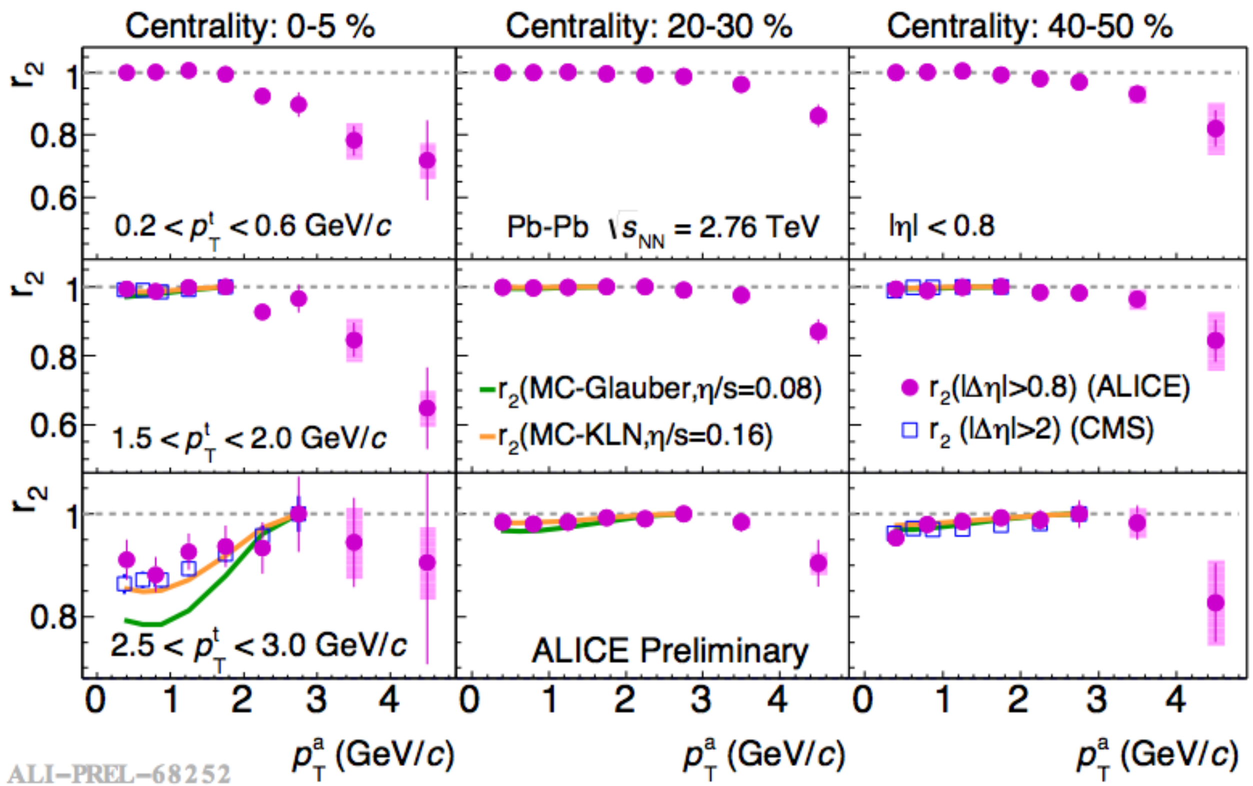
- $v_n^{sub}\{2\} \geq v_n\{4\} \approx v_n\{6\} \approx v_n\{8\} \approx v_n\{LYZ\}$
  - long-range correlations involve all particles?
  - global collectivity as Pb-Pb collisions?
- how to understand $v_2^{sub}\{2\} \approx v_n\{4\}$ in pp and $N_{trk} < 100$ for all systems?
  - because of small number of sources?
  - or over-subtraction of $v_2^{sub}\{2\}$ (using low multiplicity)?
  - <u>keep in mind that after applying sub-event method, $v_n\{4\}$ should be even higher</u>

Factorization broken in Pb-Pb & p-Pb

$$r_n = \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) \cdot V_{n\Delta}(p_T^b, p_T^b)}}$$

- r_n probes < a, b > ➡ < a, a > & < b, b >
- r_n < 1, Factorization broken

NNLO



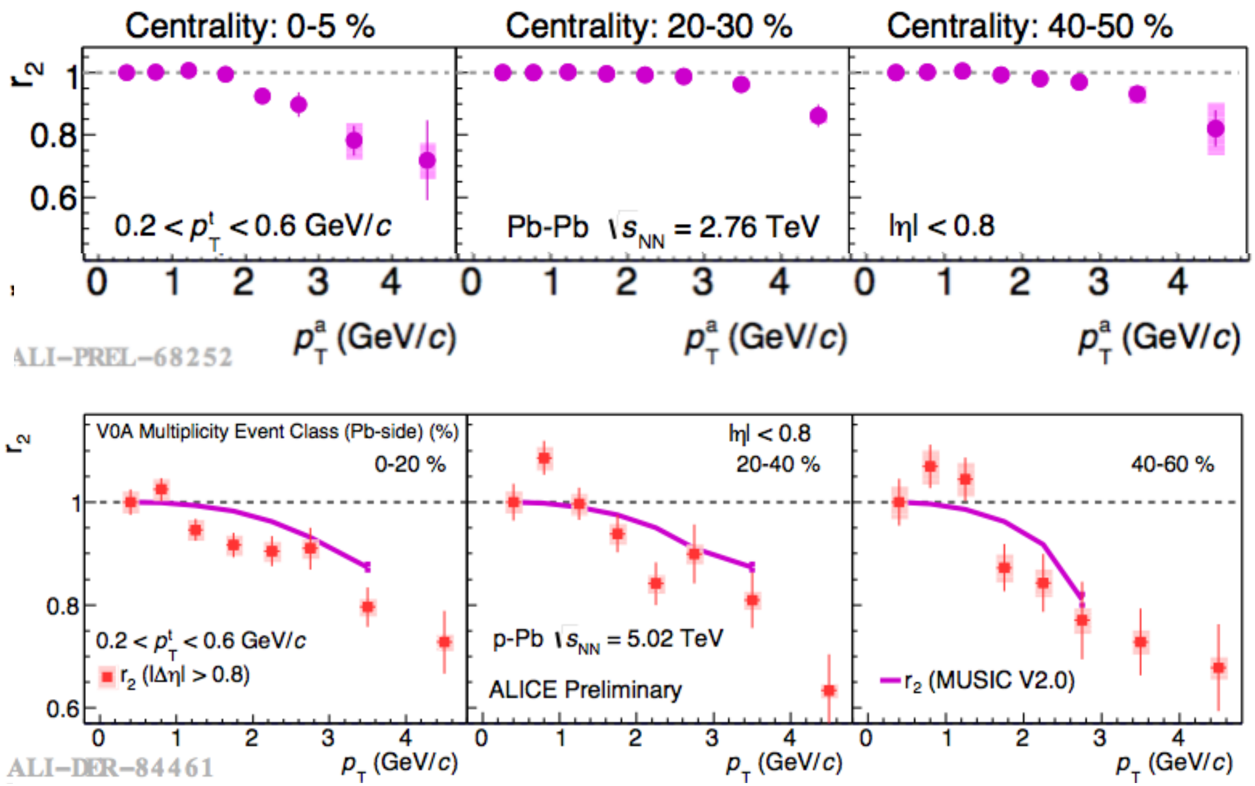
❖ Factorization broken in Pb-Pb, results can be described by hydrodynamic calculations

Factorization broken in Pb-Pb & p-Pb

$$r_n = \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) \cdot V_{n\Delta}(p_T^b, p_T^b)}}$$

- r_n probes < a, b > ➡ < a, a > & < b, b >
- r_n < 1, Factorization broken

NNLO



❖ Similar factorization broken also in p-Pb, and it can be described by hydrodynamic calculations -> similar mechanism behind?

v_m, v_n correlations with SC

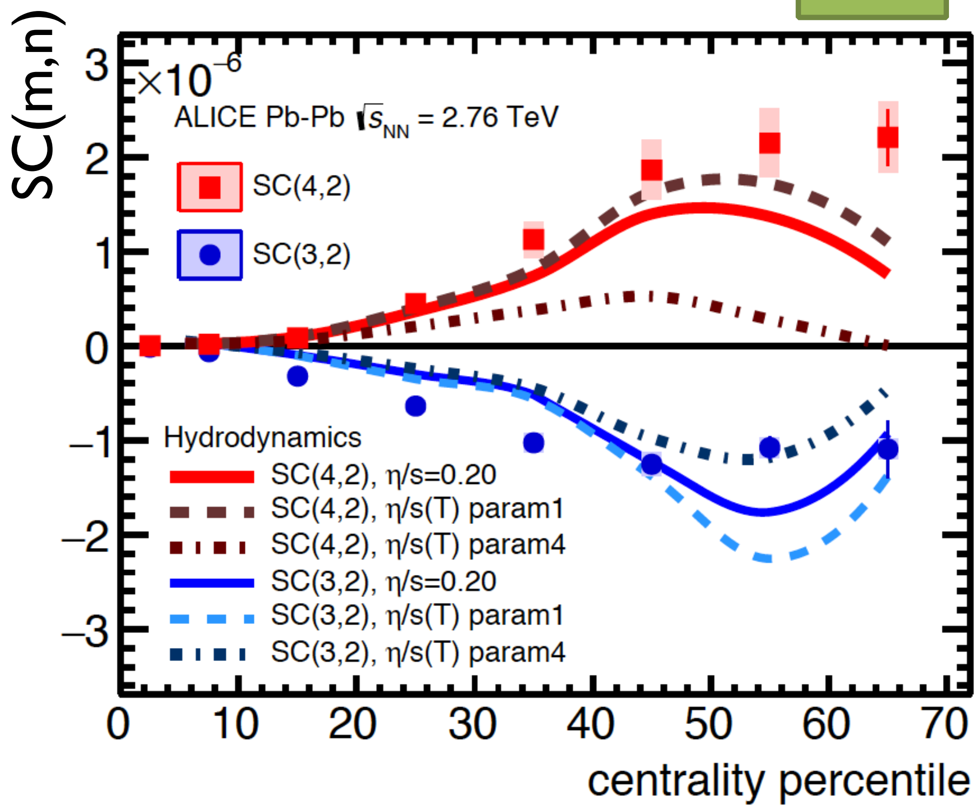
NNLO

$$SC(m,n) = \langle v_m^2 \, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

SC, proposed in
PRC 89, 064904 (2014)
(Not the PRL paper)

ALICE,
PRL117, 182301 (2016)

EKRT: H. Niemi et. al,
PRC 93, 024907 (2016)

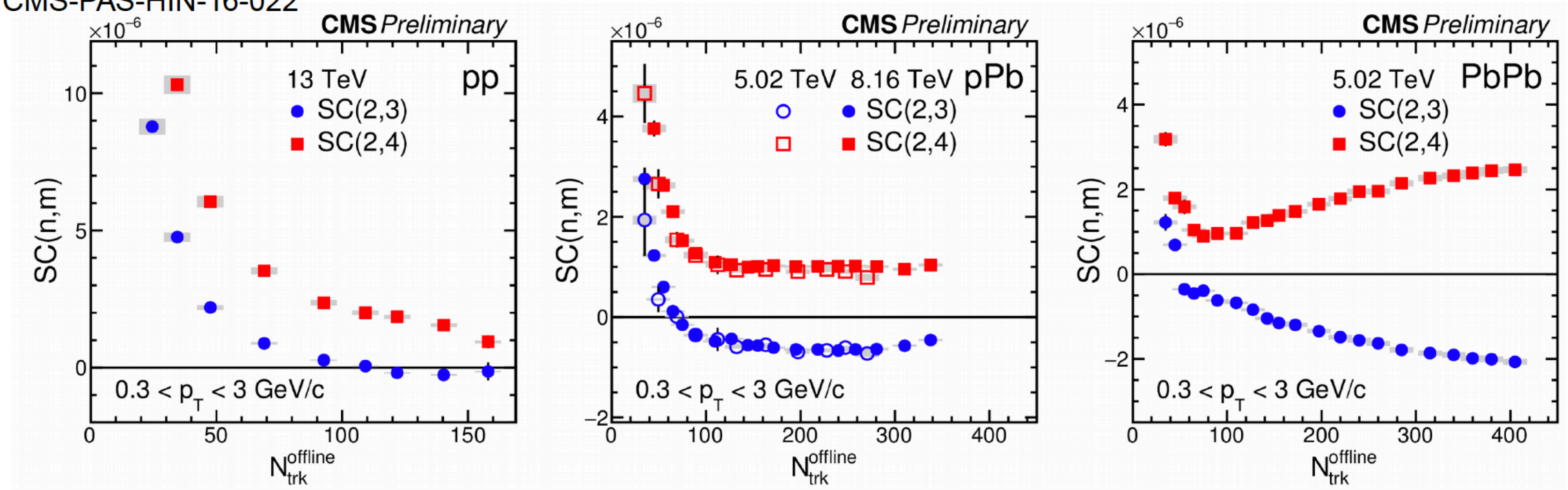


- The positive values of SC(4,2) and negative SC(3,2) are observed for all centralities.
  - suggests a correlation between v_2 and v_4, and an anti-correlations between v_2 and v_3.
  - hydro calculations qualitatively capture the trends

SC in small systems

NNLO

CMS-PAS-HIN-16-022

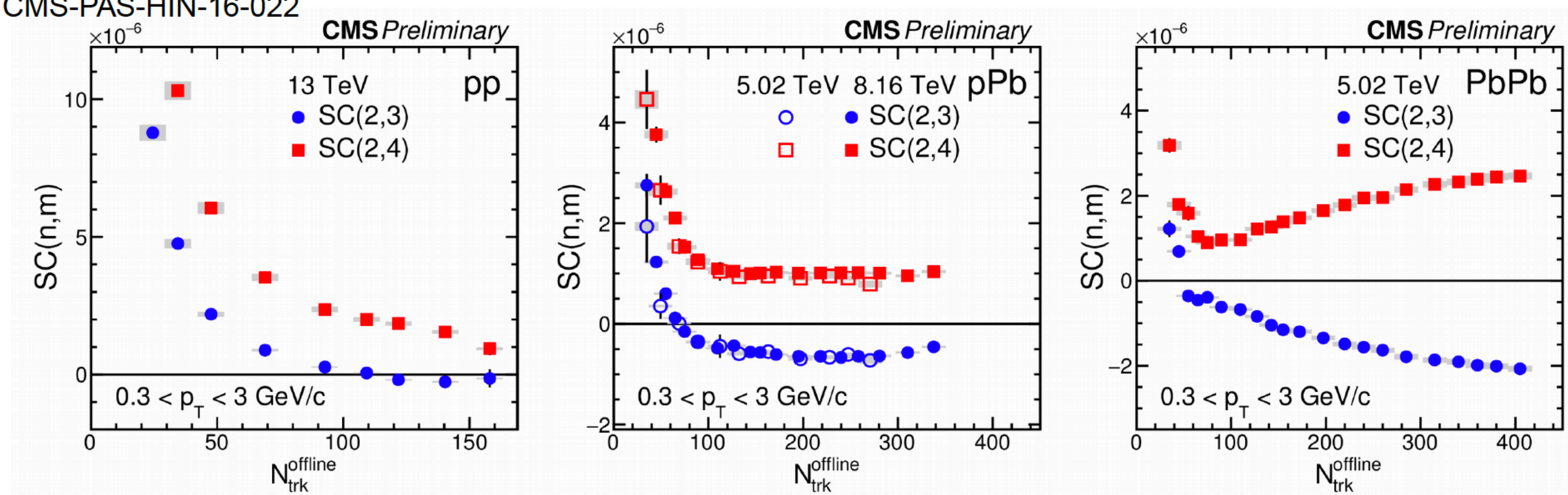


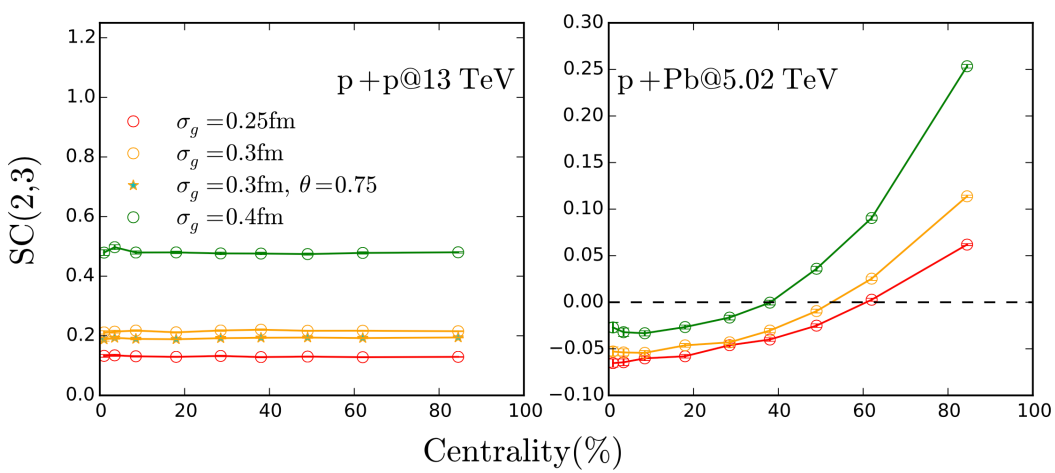
- "Similar pattern observed across systems for SC" (CMS@QM)
  - Correlations between v_2 and v_4
  - Anti-correlations between v_2 and v_3

SC in small systems

NNLO

CMS-PAS-HIN-16-022





❖ calculations based on ε_n:

- agree with data qualitatively

I : ***there is anisotropic collectivity in small systems!***

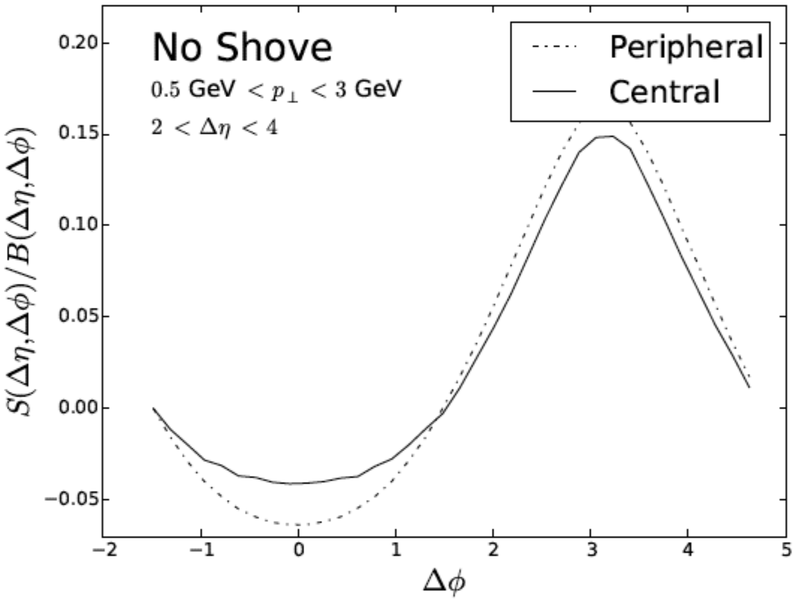
• Long-range correlations structure	flow & hydro ✓
• Anisotropic coefficient v_n	flow & hydro ✓
• PID v_n (Mass ordering & crossing)	flow & hydro ✓
• Multi-particle cumulants	flow ✓
• Factorization broken	flow & hydro ✓
• Correlations between flow	flow & hydro ✓

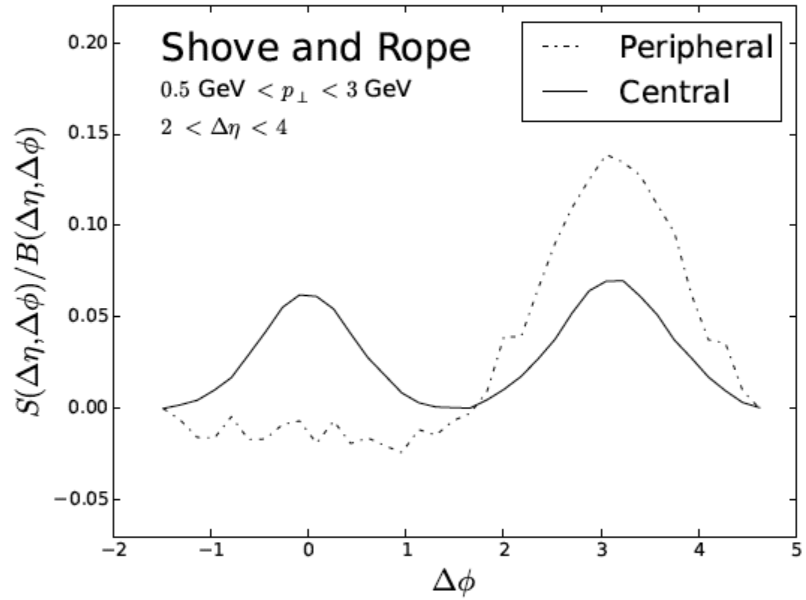
Two-particle correlations (ridge)

LO NLO

C. Bierlich et al., arXiv:1612.05132

also see talks: C. Bierlich

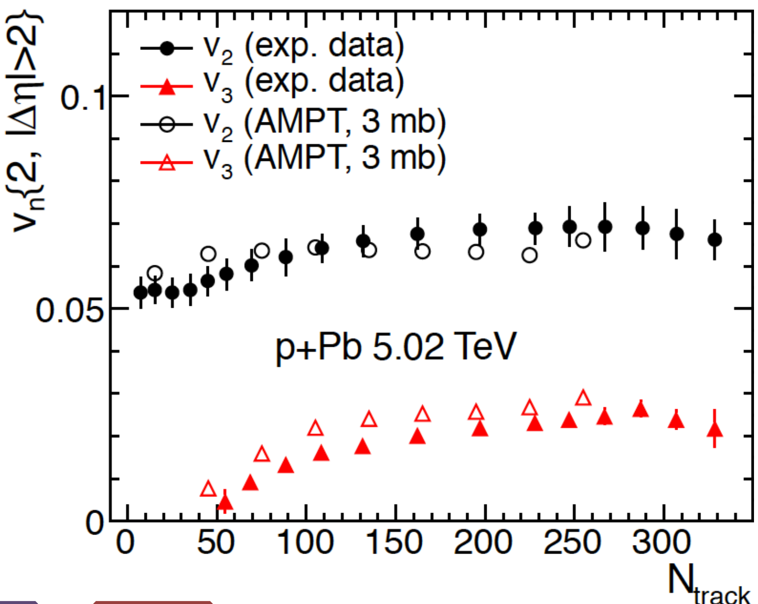




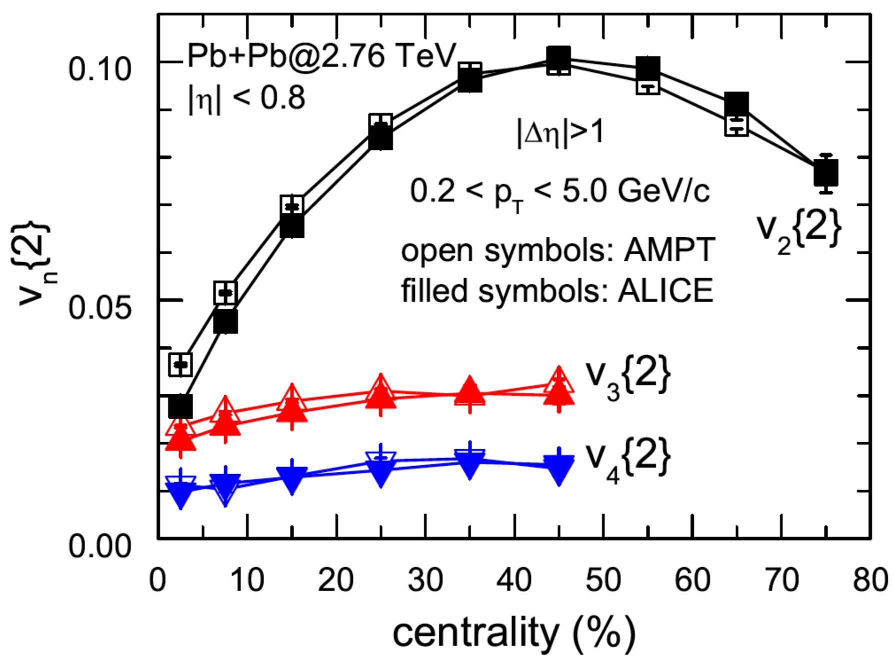
- Long-range correlations reproduced in DIPSY using rope hadronization approach
  - more see talks from Christian Bierlich (Wednesday)

v_n in AMPT

A. Bzdak et al., PRL 113, 252301 (2014)



J. Xu et al., PRC84, 044907 (2011)

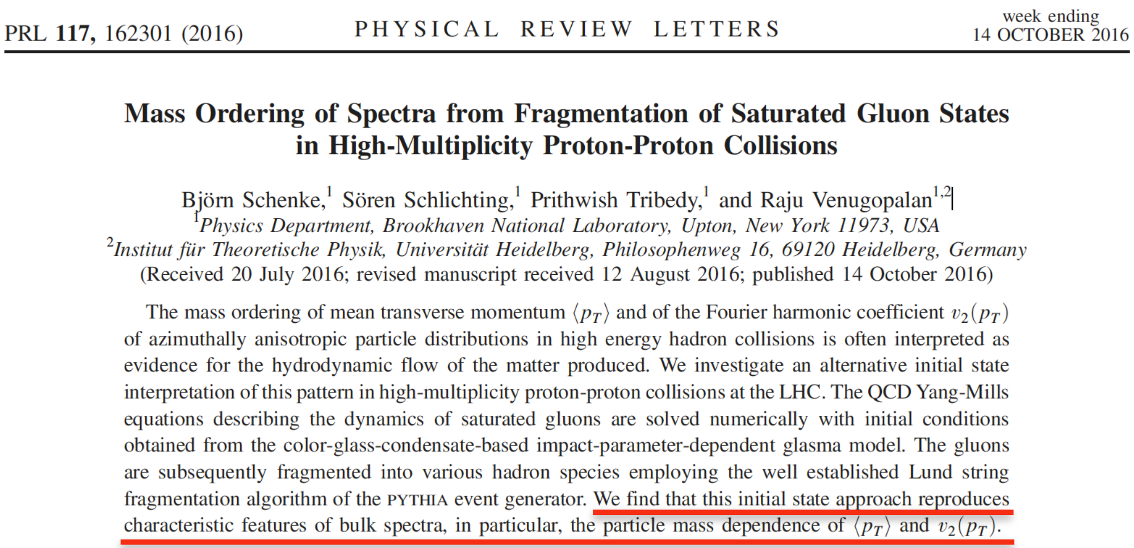


LO NLO

- AMPT (partons cascade) works very well on describing v_n shape in both Pb-Pb and p-Pb collisions
  - someone complains that "*AMPT works too good, there must be a reason behind*"

Initial stage effect: CGC+Lund

NLO



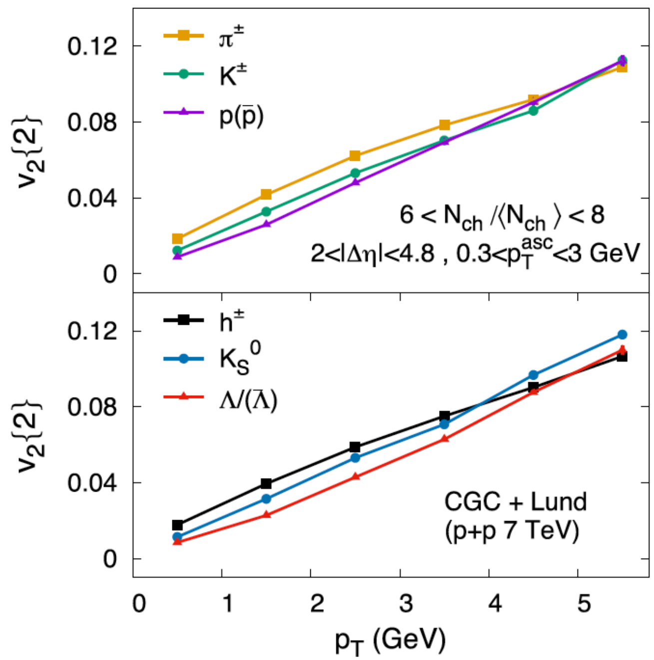
PRL **117,** 162301 (2016) PHYSICAL REVIEW LETTERS week ending 14 OCTOBER 2016

Mass Ordering of Spectra from Fragmentation of Saturated Gluon States in High-Multiplicity Proton-Proton Collisions

Björn Schenke,[1] Sören Schlichting,[1] Prithwish Tribedy,[1] and Raju Venugopalan[1,2]
[1]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[2]*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

(Received 20 July 2016; revised manuscript received 12 August 2016; published 14 October 2016)

The mass ordering of mean transverse momentum $\langle p_T \rangle$ and of the Fourier harmonic coefficient $v_2(p_T)$ of azimuthally anisotropic particle distributions in high energy hadron collisions is often interpreted as evidence for the hydrodynamic flow of the matter produced. We investigate an alternative initial state interpretation of this pattern in high-multiplicity proton-proton collisions at the LHC. The QCD Yang-Mills equations describing the dynamics of saturated gluons are solved numerically with initial conditions obtained from the color-glass-condensate-based impact-parameter-dependent glasma model. The gluons are subsequently fragmented into various hadron species employing the well established Lund string fragmentation algorithm of the PYTHIA event generator. We find that this initial state approach reproduces characteristic features of bulk spectra, in particular, the particle mass dependence of $\langle p_T \rangle$ and $v_2(p_T)$.

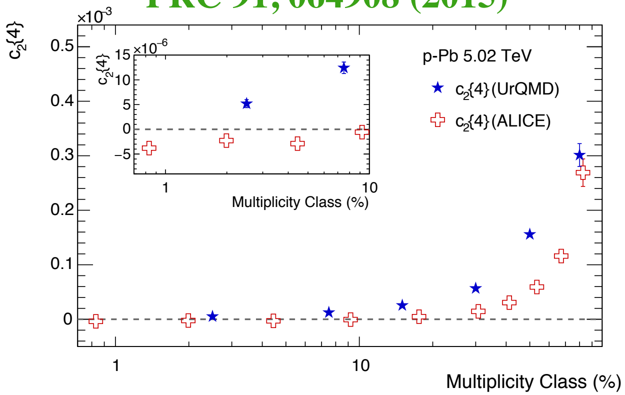


❖mass ordering from initial stage effects

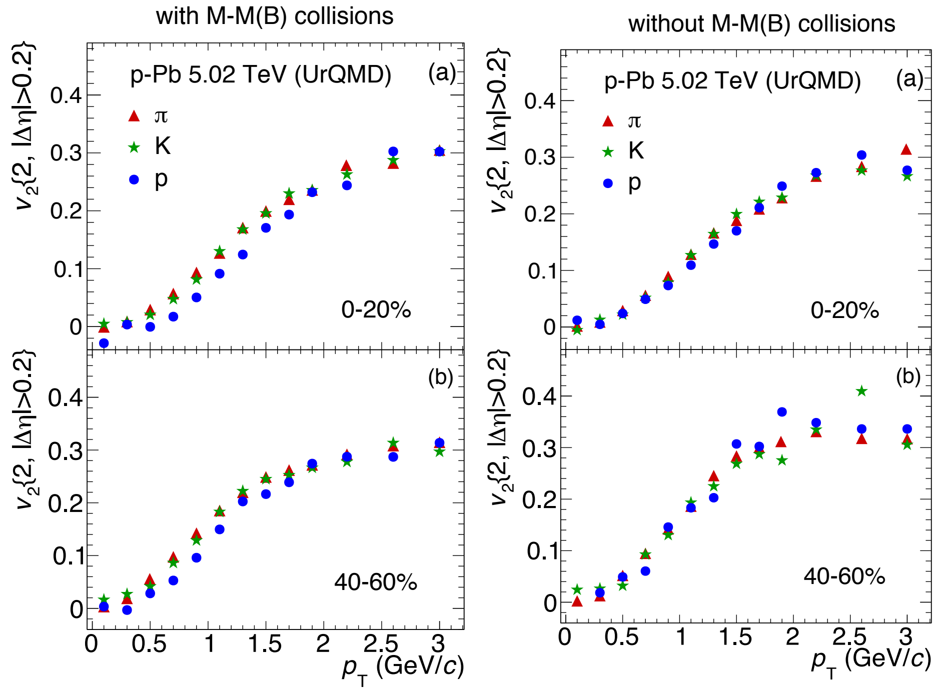
- CGC & gluons fragmented into hadrons via Lund string

Model calculations: UrQMD

Y. Zhou et al.,
PRC 91, 064908 (2015)



NLO



- ❖ Positive QC{4} observed in UrQMD
  - indication of non-flow dominant system
- ❖ The characteristic $v_2(p_T)$ mass-ordering of pions, kaons and protons is observed in UrQMD
  - the consequence of hadronic interactions
  - not necessarily associated with strong fluid-like expansions.
- ❖ No baryon-meson grouping

Collective multi-particle correlations!

(From Bjoern Schenke @ QM)

Glasma graphs lead to positive $c_2\{4\}$ for all multiplicities

Additional non-linear and non-Gaussian effects can lead to a negative contribution

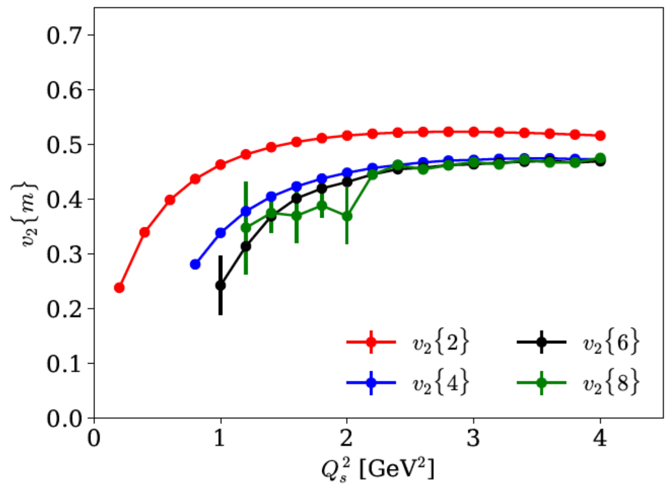
$$c_2\{4\} = \frac{1}{N_D^3}\left[\frac{1}{4(N_c^2-1)^3} - \mathcal{A}^4\right]$$

color domain model

A. Dumitru, L. McLerran, V. Skokov, Phys.Lett. B743 (2015) 134-137

LO

K. Dusling et al, arXiv:1705.00745

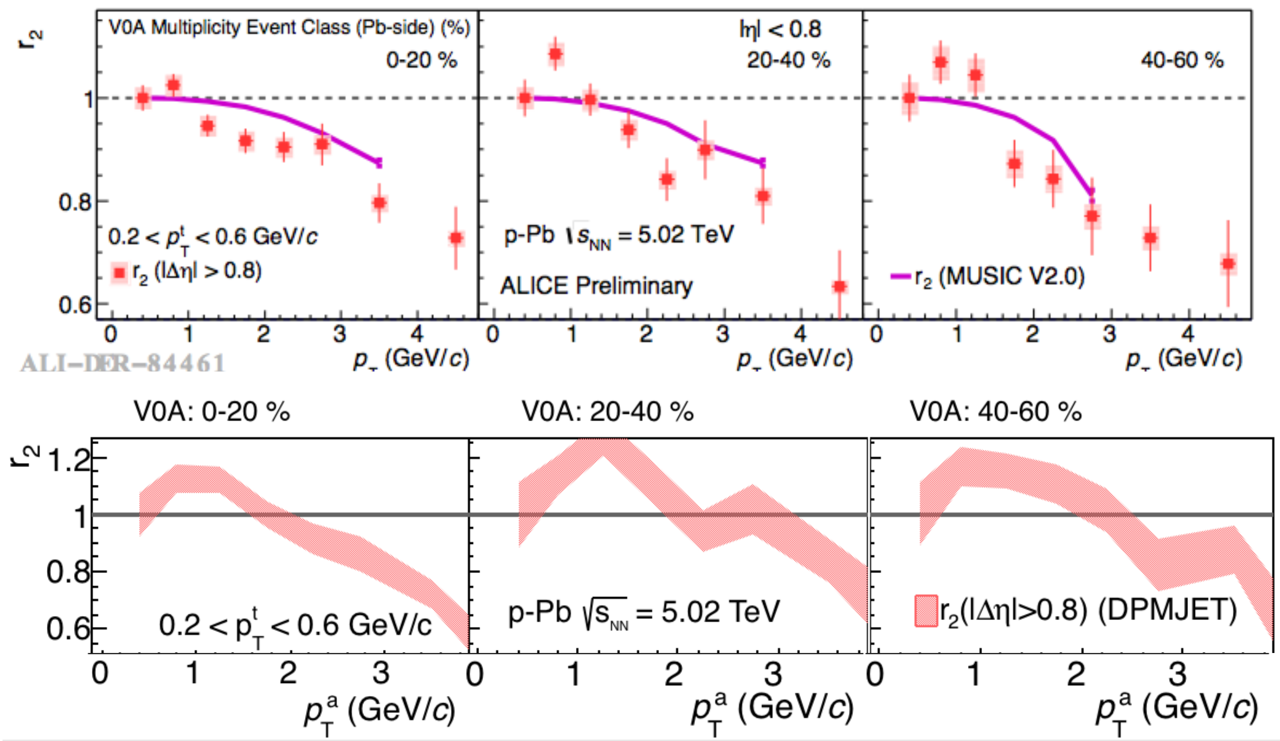


- 4-particle cumulant has a balance between glasma graphs & non-linear and non-Gaussian effects?
- $v_2\{2\} > v_2\{4\} = v_2\{6\} = v_2\{8\}$, often interpreted as a signature of collectivity, can be reproduced by plasma graphs.

Factorization broken in p-Pb

$$r_n = \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) \cdot V_{n\Delta}(p_T^b, p_T^b)}}$$

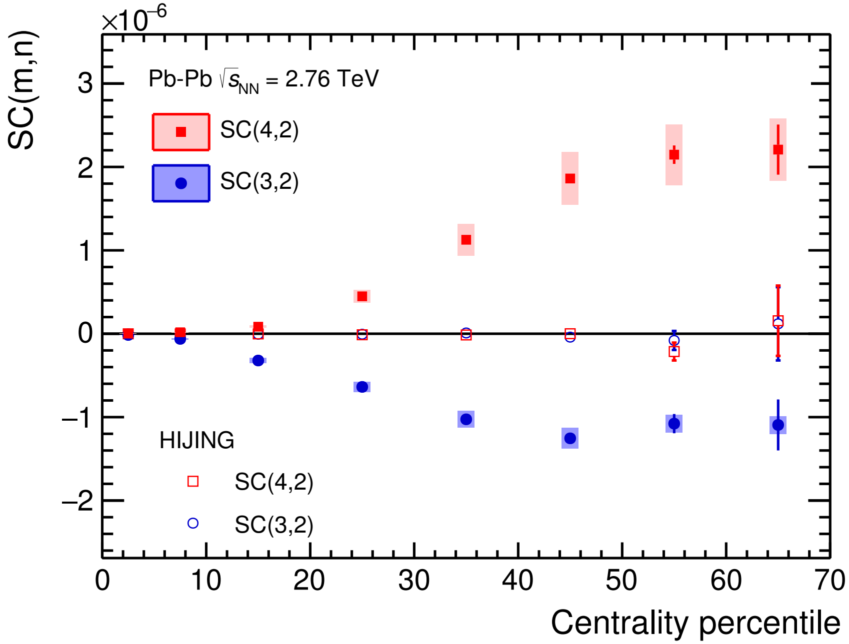
NNLO

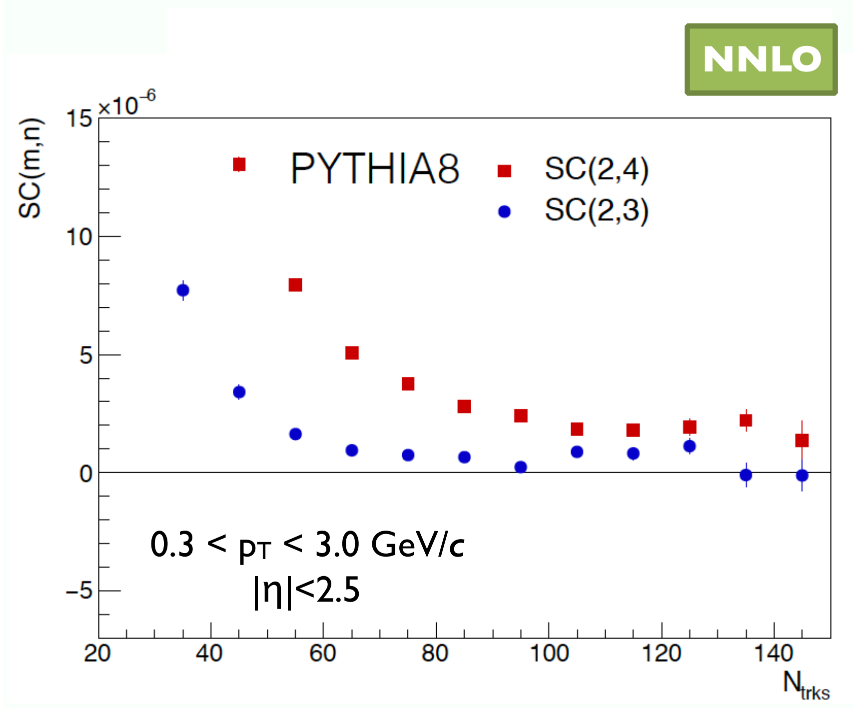


❖ factorization broken in p-Pb collisions can be produced by DPMJET (which does not generate flow)

Non-zero SC from PYTHIA

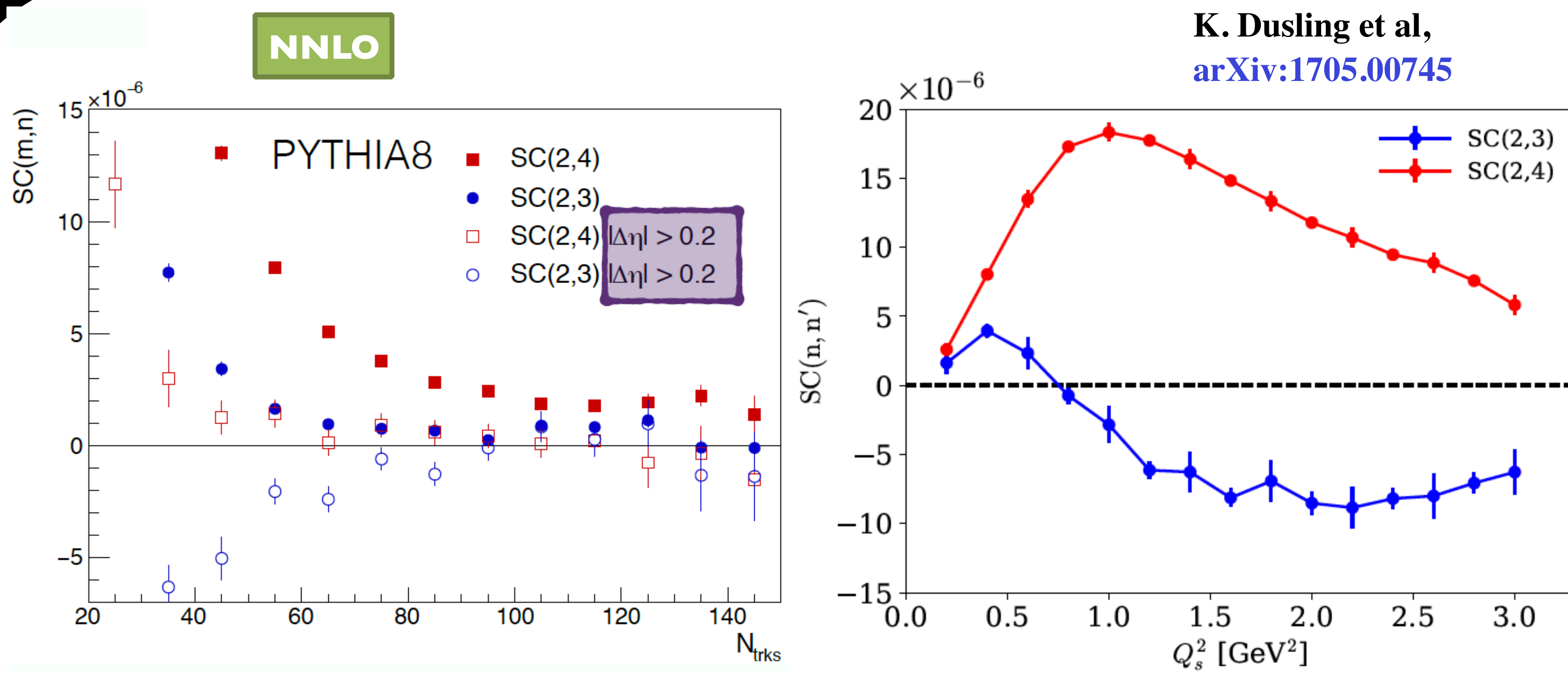
See talk: Katarína Gajdošová





- The study stops in 70% Pb-Pb collisions, because of non-flow effects (can't claim HIJING gives 0 value in ultra-peripheral collisions)
- PYTHIA calculation (do not generate flow) shows that both SC(4,2) and SC(3,2) are strongly influenced by non-flow in small system.

SC(m,n)$_{gap}$ from PYTHIA



❖ Using SC$_{gap}$ (4-particle cumulant from 2-subevent with eta gap)

- show completely different results w.r.t. standard SC

❖ Also glasma graph scenario (initial state model) demonstrates clearly that such patterns are not unique to an interpretation requiring hydrodynamic flow.

"You" : ***there might be no flow in small systems!***

• Long-range correlation structure	rope hadronization ✓
• Anisotropic coefficient v_n	cascade ✓
• PID v_n (Mass ordering & crossing)	initial stage effect / cascade (parton/hadron) ✓
• Multi-particle cumulants	glasma graphs ✓
• Factorization broken	DPMJET ✓
• Correlations between flow	Non-flow, initial stage effects ✓

"I" against "You"

Long-range correlation structure

Anisotropic coefficient v_n

PID v_n (Mass ordering & crossing)

Multi-particle cumulants

Factorization broken

Correlations between flow

Flow or not flow, it is still a question

Future improvements

- ❖ Improved methods
  - keep issue: non-flow
- ❖ New observables

MPC from Generic framework

PHYSICAL REVIEW C **89**, 064904 (2014)

Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations

Ante Bilandzic,[1] Christian Holm Christensen,[1] Kristjan Gulbrandsen,[1] Alexander Hansen,[1] and You Zhou[2,3]
[1]*Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark*
[2]*Nikhef, Science Park 105, 1098 XG Amsterdam, The Netherlands*
[3]*Utrecht University, P.O. Box 80000, 3508 TA Utrecht, The Netherlands*
(Received 20 December 2013; revised manuscript received 6 May 2014; published 9 June 2014)

We present a new generic framework which enables exact and efficient evaluation of all multiparticle azimuthal correlations. The framework can be readily used along with a correction framework for systematic biases in anisotropic flow analyses owing to various detector inefficiencies. A new recursive algorithm has been developed for higher-order correlators for the cases where their direct implementation is not feasible. We propose and discuss new azimuthal observables for anisotropic flow analyses which can be measured for the first time with our new framework. The effect of finite detector granularity on multiparticle correlations is quantified and discussed in detail. We point out the existence of a systematic bias in traditional differential flow analyses which stems solely from the applied selection criteria on particles used in the analyses and is also present in the ideal case when only flow correlations are present. Finally, we extend the applicability of our generic framework to the case of differential multiparticle correlations.

Step1 → Step2 → Step3

$$Q_{n,p} \equiv \sum_{k=1}^{M} w_k^p \, e^{in\varphi_k}$$

$$\mathrm{N}\langle m\rangle_{n_1,n_2,\ldots,n_m} \equiv \sum_{\substack{k_1,k_2,\ldots,k_m=1\\ k_1\neq k_2\neq\cdots\neq k_m}}^{M} w_{k_1} w_{k_2}\cdots w_{k_m} \times e^{i(n_1\varphi_{k_1}+n_2\varphi_{k_2}+\cdots+n_m\varphi_{k_m})},$$

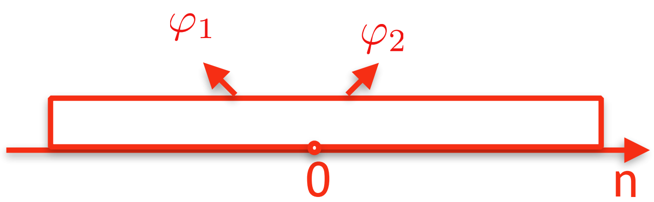
$$\mathrm{D}\langle m\rangle_{n_1,n_2,\ldots,n_m} \equiv \sum_{\substack{k_1,k_2,\ldots,k_m=1\\ k_1\neq k_2\neq\cdots\neq k_m}}^{M} w_{k_1} w_{k_2}\cdots w_{k_m} = \mathrm{N}\langle m\rangle_{0,0,\ldots,0}.$$

$$\langle m\rangle_{n_1,n_2,\ldots,n_m} \equiv \left\langle e^{i(n_1\varphi_{k_1}+n_2\varphi_{k_2}+\cdots+n_m\varphi_{k_m})}\right\rangle \equiv \frac{\sum\limits_{\substack{k_1,k_2,\ldots,k_m=1\\ k_1\neq k_2\neq\cdots\neq k_m}}^{M} w_{k_1} w_{k_2}\cdots w_{k_m}\, e^{i(n_1\varphi_{k_1}+n_2\varphi_{k_2}+\cdots+n_m\varphi_{k_m})}}{\sum\limits_{\substack{k_1,k_2,\ldots,k_m=1\\ k_1\neq k_2\neq\cdots\neq k_m}}^{M} w_{k_1} w_{k_2}\cdots w_{k_m}}.$$

Generic framework

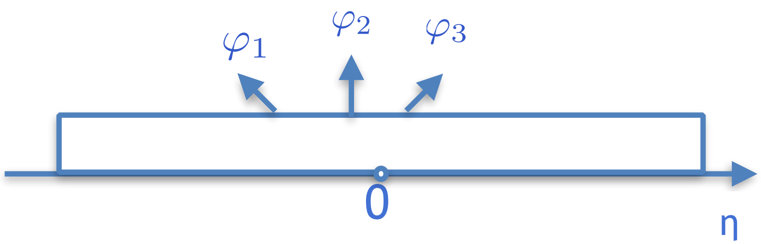
- 2-particle correlation

Details see:
PRC 89, 064904 (2014)



$$N\langle 2\rangle_{n_1,n_2} = Q_{n_1,1}Q_{n_2,1} - Q_{n_1+n_2,2},$$
$$D\langle 2\rangle_{n_1,n_2} = N\langle 2\rangle_{0,0} = Q_{0,1}^2 - Q_{0,2}.$$

- 3-particle correlation

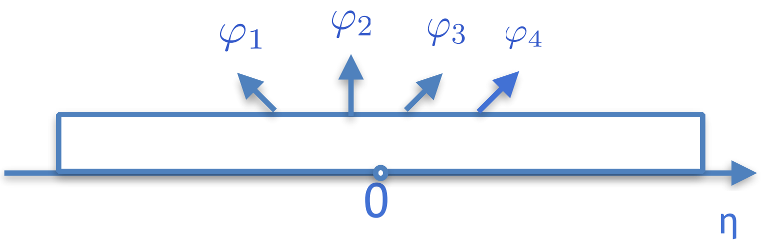


$$N\langle 3\rangle_{n_1,n_2,n_3} = Q_{n_1,1}Q_{n_2,1}Q_{n_3,1} - Q_{n_1+n_2,2}Q_{n_3,1} - Q_{n_2,1}Q_{n_1+n_3,2} - Q_{n_1,1}Q_{n_2+n_3,2} + 2Q_{n_1+n_2+n_3,3},$$
$$D\langle 3\rangle_{n_1,n_2,n_3} = N\langle 3\rangle_{0,0,0}$$
$$= Q_{0,1}^3 - 3Q_{0,2}Q_{0,1} + 2Q_{0,3}.$$

Generic framework

- 4-particle correlation

Details see:
PRC 89, 064904 (2014)

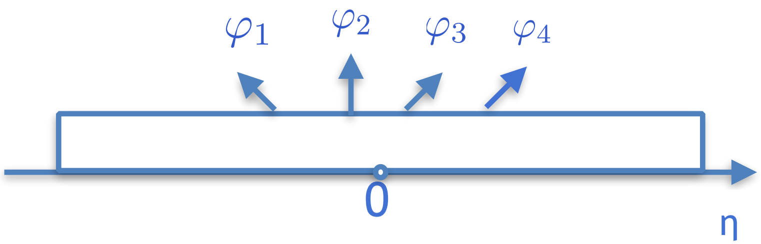


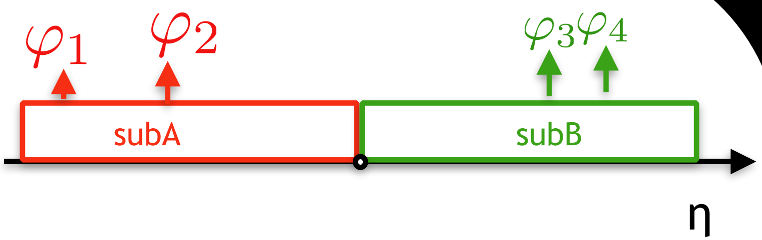
$$
\begin{aligned}
\mathrm{N}\langle 4\rangle_{n_1,n_2,n_3,n_4} = {} & Q_{n_1,1}Q_{n_2,1}Q_{n_3,1}Q_{n_4,1} - Q_{n_1+n_2,2}Q_{n_3,1}Q_{n_4,1} - Q_{n_2,1}Q_{n_1+n_3,2}Q_{n_4,1} \\
& - Q_{n_1,1}Q_{n_2+n_3,2}Q_{n_4,1} + 2Q_{n_1+n_2+n_3,3}Q_{n_4,1} - Q_{n_2,1}Q_{n_3,1}Q_{n_1+n_4,2} \\
& + Q_{n_2+n_3,2}Q_{n_1+n_4,2} - Q_{n_1,1}Q_{n_3,1}Q_{n_2+n_4,2} + Q_{n_1+n_3,2}Q_{n_2+n_4,2} \\
& + 2Q_{n_3,1}Q_{n_1+n_2+n_4,3} - Q_{n_1,1}Q_{n_2,1}Q_{n_3+n_4,2} + Q_{n_1+n_2,2}Q_{n_3+n_4,2} \\
& + 2Q_{n_2,1}Q_{n_1+n_3+n_4,3} + 2Q_{n_1,1}Q_{n_2+n_3+n_4,3} - 6Q_{n_1+n_2+n_3+n_4,4}, \\
\mathrm{D}\langle 4\rangle_{n_1,n_2,n_3,n_4} = {} & \mathrm{N}\langle 4\rangle_{0,0,0,0} = Q_{0,1}^4 - 6Q_{0,1}^2 Q_{0,2} + 3Q_{0,2}^2 + 8Q_{0,1}Q_{0,3} - 6Q_{0,4}.
\end{aligned}
$$

❖ The code is available for **any** multi-particle correlations, used by ALICE/CMS/STAR/Theory

- example: for $c_n\{4\}$ by $N\langle 4\rangle_{n,n,-n,-n}$; SC(m,n) by $N\langle 4\rangle_{m,n,-m,-n}$; or flow symmetry plane correlation e.g. $\rho_{6,222}$ by $N\langle 4\rangle_{6,-2,-2,-2}$

Improved generic framework (2-sub)





4-particle correlations

$$\langle\langle \cos n (\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\begin{aligned} N\langle 4\rangle_{n_1,n_2,n_3,n_4} = \; & Q_{n_1,1}Q_{n_2,1}Q_{n_3,1}Q_{n_4,1} - Q_{n_1+n_2,2}Q_{n_3,1}Q_{n_4,1} - Q_{n_2,1}Q_{n_1+n_3,2}Q_{n_4,1} \\ & - Q_{n_1,1}Q_{n_2+n_3,2}Q_{n_4,1} + 2Q_{n_1+n_2+n_3,3}Q_{n_4,1} - Q_{n_2,1}Q_{n_3,1}Q_{n_1+n_4,2} \\ & + Q_{n_2+n_3,2}Q_{n_1+n_4,2} - Q_{n_1,1}Q_{n_3,1}Q_{n_2+n_4,2} + Q_{n_1+n_3,2}Q_{n_2+n_4,2} \\ & + 2Q_{n_3,1}Q_{n_1+n_2+n_4,3} - Q_{n_1,1}Q_{n_2,1}Q_{n_3+n_4,2} + Q_{n_1+n_2,2}Q_{n_3+n_4,2} \\ & + 2Q_{n_2,1}Q_{n_1+n_3+n_4,3} + 2Q_{n_1,1}Q_{n_2+n_3+n_4,3} - 6Q_{n_1+n_2+n_3+n_4,4} \end{aligned}$$

$$D\langle 4\rangle_{n_1,n_2,n_3,n_4} = N\langle 4\rangle_{0,0,0,0}$$

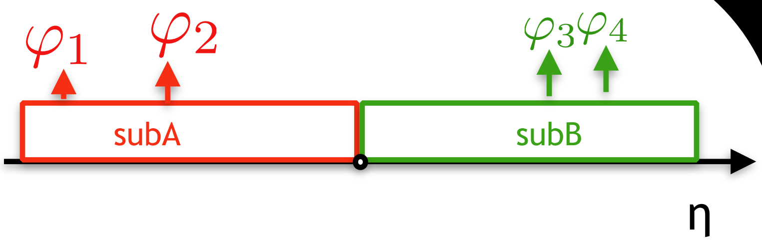
$$\begin{aligned} N\langle 4\rangle_{n_1,n_2,n_3,n_4} = \; & Q_{n_1,1}\,Q_{n_2,1}\,Q_{n_3,1}\,Q_{n_4,1} - Q_{n_1+n_2,2}\,Q_{n_3,1}\,Q_{n_4,1} \\ & - Q_{n_1,1}\,Q_{n_2,1}\,Q_{n_3+n_4,2} + Q_{n_1+n_2,2}\,Q_{n_3+n_4,2} \end{aligned}$$

$$D\langle 4\rangle_{n_1,n_2,n_3,n_4} = N\langle 4\rangle_{0,0,0,0}$$

$$\langle\langle 4\rangle\rangle_{2\text{-sub}} = \langle\langle 2\rangle\rangle_{subA} * \langle\langle 2\rangle\rangle_{subB}$$

- $\langle\langle 2\rangle\rangle_{subA}$ and $\langle\langle 2\rangle\rangle_{subB}$ could be easily calculated with Q_n-vector built in subA and subB

Improved generic framework (2-sub)



4-particle correlations

<< cos n ($\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4$) >>

<<2>>$_{\text{subA}}$

$$N\langle 2\rangle_{n_1,n_2} = Q_{n_1,1}\,Q_{n_2,1} - Q_{n_1+n_2,2}$$
$$D\langle 2\rangle_{n_1,n_2} = N\langle 2\rangle_{0,0} = Q^2_{,1} - Q_{0,2}$$

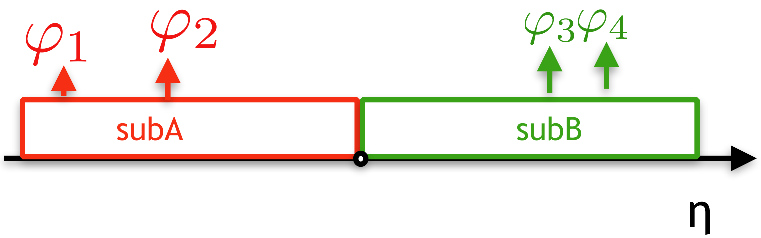
<<2>>$_{\text{subB}}$

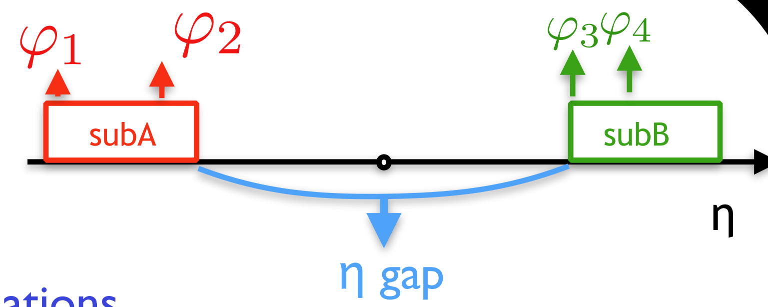
$$N\langle 2\rangle_{n_3,n_4} = Q_{n_3,1}\,Q_{n_4,1} - Q_{n_3+n_4,2}$$
$$D\langle 2\rangle_{n_3,n_4} = N\langle 2\rangle_{0,0} = Q^2_{,1} - Q_{0,2}$$

<<4>>$_{\text{2-sub}}$ = <<2>>$_{\text{subA}}$ * <<2>>$_{\text{subB}}$

$$N\langle 4\rangle_{n_1,n_2,n_3,n_4} = \left(Q_{n_1,1}\,Q_{n_2,1} - Q_{n_1+n_2,2}\right)\left(Q_{n_3,1}\,Q_{n_4,1} - Q_{n_3+n_4,2}\right)$$
$$D\langle 4\rangle_{n_1,n_2,n_3,n_4} = N\langle 4\rangle_{0,0,0,0}$$

2-sub with eta gap





4-particle correlations

$$\langle\langle \cos n (\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$N\langle 4\rangle_{n_1,n_2,n_3,n_4} = Q_{n_1,1}\,Q_{n_2,1}\,Q_{n_3,1}\,Q_{n_4,1} - Q_{n_1+n_2,2}\,Q_{n_3,1}\,Q_{n_4,1} - Q_{n_1,1}\,Q_{n_2,1}\,Q_{n_3+n_4,2} + Q_{n_1+n_2,2}\,Q_{n_3+n_4,2}$$

$$D\langle 4\rangle_{n_1,n_2,n_3,n_4} = N\langle 4\rangle_{0,0,0,0}$$

2-particle correlations

$\langle\langle \cos n (\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n (\varphi_2 - \varphi_4) \rangle\rangle$ & $\langle\langle \cos n (\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n (\varphi_2 - \varphi_3) \rangle\rangle$

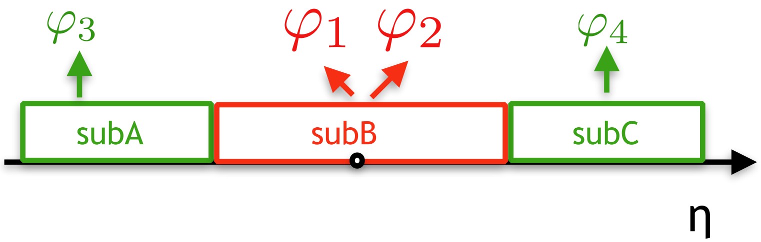
$$\langle\langle \cos n (\varphi_1 - \varphi_3) \rangle\rangle = \mathrm{N}\langle 2\rangle_{n_1,n_3} = Q_{n_1,1}\,Q_{n_3,1}$$

$$\langle\langle \cos n (\varphi_2 - \varphi_4) \rangle\rangle = \mathrm{N}\langle 2\rangle_{n_2,n_4} = Q_{n_2,1}\,Q_{n_4,1}$$

same for (1,-4)(2,-3)

- ***A simple code with all needed NUA&NUE corrections***

Improved generic framework (3-sub)



4-particle correlations

$<< \cos n\,(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) >>$

$$<<4>>_{3\text{-sub}} = Q_{n3,1} * <<2>>_{subB} * Q_{n4,1}$$

$$\mathrm{N}\langle 4\rangle_{n_1,n_2,n_3,n_4} = Q_{n_1,1}\,Q_{n_2,1}\,Q_{n_3,1}\,Q_{n_4,1} - Q_{n_1+n_2,2}\,Q_{n_3,1}\,Q_{n_4,1}$$

$$\mathrm{D}\langle 4\rangle_{n_1,n_2,n_3,n_4} = \mathrm{N}\langle 4\rangle_{0,0,0,0}$$

2-particle correlations

$<< \cos n\,(\varphi_1 - \varphi_3) >>\ << \cos n\,(\varphi_2 - \varphi_4) >>$ & $<< \cos n\,(\varphi_1 - \varphi_4) >>\ << \cos n\,(\varphi_2 - \varphi_3) >>$

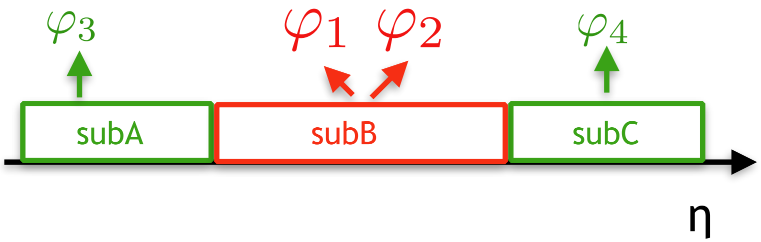
$$<< \cos n\,(\varphi_1 - \varphi_3) >> = \mathrm{N}\langle 2\rangle_{n_1,n_3} = Q_{n_1,1}\,Q_{n_3,1}$$

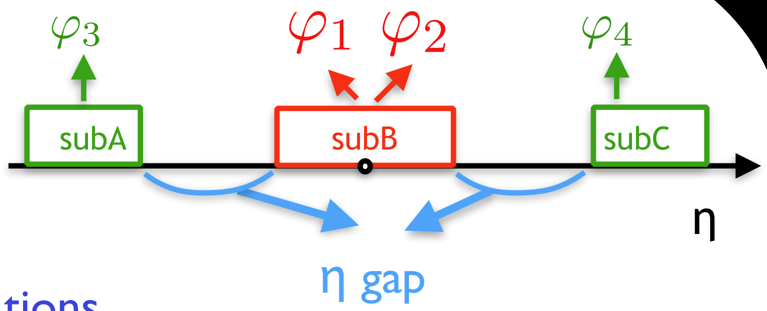
$$<< \cos n\,(\varphi_2 - \varphi_4) >> = \mathrm{N}\langle 2\rangle_{n_2,n_4} = Q_{n_2,1}\,Q_{n_4,1}$$

same for (1,-4)(2,-3)

- ***A simple code with all needed NUA&NUE corrections***

3-sub with eta gap





4-particle correlations

$$\langle\langle \cos n (\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle\rangle$$

$$\mathrm{N}\langle 4\rangle_{n_1,n_2,n_3,n_4} = Q_{n_1,1}\, Q_{n_2,1}\, Q_{n_3,1}\, Q_{n_4,1} - Q_{n_1+n_2,2}\, Q_{n_3,1}\, Q_{n_4,1}$$

$$\mathrm{D}\langle 4\rangle_{n_1,n_2,n_3,n_4} = \mathrm{N}\langle 4\rangle_{0,0,0,0}$$

2-particle correlations

$\langle\langle \cos n (\varphi_1 - \varphi_3) \rangle\rangle \langle\langle \cos n (\varphi_2 - \varphi_4) \rangle\rangle$ & $\langle\langle \cos n (\varphi_1 - \varphi_4) \rangle\rangle \langle\langle \cos n (\varphi_2 - \varphi_3) \rangle\rangle$

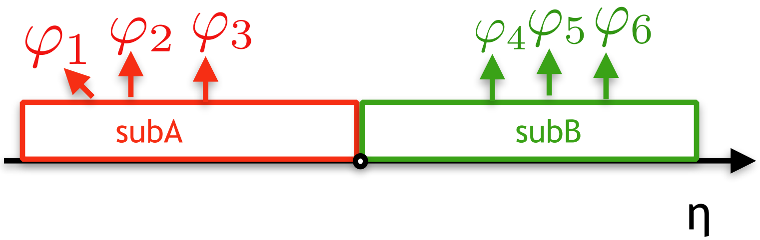
$\langle\langle \cos n (\varphi_1 - \varphi_3) \rangle\rangle = \mathrm{N}\langle 2\rangle_{n_1,n_3} = Q_{n_1,1}\, Q_{n_3,1}$

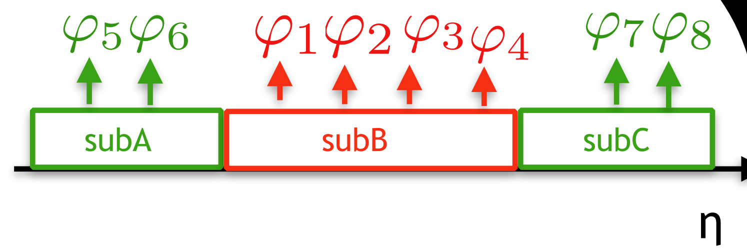
$\langle\langle \cos n (\varphi_2 - \varphi_4) \rangle\rangle = \mathrm{N}\langle 2\rangle_{n_2,n_4} = Q_{n_2,1}\, Q_{n_4,1}$

same for (1,-4)(2,-3)

- ***A simple code with all needed NUA&NUE corrections***

6- and 8-particle with sub-event





$\langle\langle \cos n\,(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6) \rangle\rangle_{2\text{-sub}}$

$\langle\langle 6 \rangle\rangle_{2\text{-sub}} = \langle\langle 3 \rangle\rangle_{subA} * \langle\langle 3 \rangle\rangle_{subB}$

$\langle\langle \cos n\,(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6 - \varphi_7 - \varphi_8) \rangle\rangle_{2\text{-sub}}$

$\langle\langle 8 \rangle\rangle_{2\text{-sub}} = \langle\langle 4 \rangle\rangle_{subA} * \langle\langle 4 \rangle\rangle_{subB}$

$\langle\langle \cos n\,(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6 - \varphi_7 - \varphi_8) \rangle\rangle_{3\text{-sub}}$

$\langle\langle 8 \rangle\rangle_{2\text{-sub}} = \langle\langle 2 \rangle\rangle_{subA} * \langle\langle 4 \rangle\rangle_{subB} * \langle\langle 2 \rangle\rangle_{subC}$

- ❖ This is not a joke, but is useful to suppress the possible non-flow effects in $v_n\{6\}$ and $v_n\{8\}$, when using them to probe the underlying *p.d.f.*
  - idea: see talk from Jean-Yves Ollitrault

Generic multi-particle correlations

1701.03830

Revealing long-range multi-particle collectivity in small collision systems via subevent cumulants

Jiangyong Jia,[1,2,*] Mingliang Zhou,[1,†] and Adam Trzupek[3]

- ❖ Similar idea of subevent method proposed earlier by Jiangyong et al.
- ❖ Our method is more general for any multi-particle correlations including mixed harmonic with unique formulas.
  - e.g. $<4>^{2\text{-sub}}_{m,n,-m,-n}$ (for SC), $c_n\{4\}^{2\text{-sub}}$ sharing the same equation
  - can be used easily in mixed harmonic correlations with little (or without) further modifications

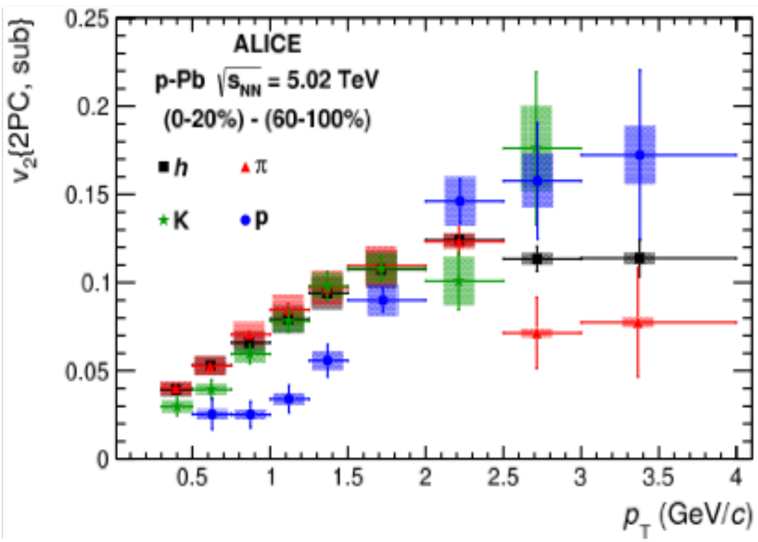
$$N\langle 4\rangle_{n_1,n_2,n_3,n_4} = Q_{n_1,1}\,Q_{n_2,1}\,Q_{n_3,1}\,Q_{n_4,1} - Q_{n_1+n_2,2}\,Q_{n_3,1}\,Q_{n_4,1} - Q_{n_1,1}\,Q_{n_2,1}\,Q_{n_3+n_4,2} + Q_{n_1+n_2,2}\,Q_{n_3+n_4,2}$$

$$D\langle 4\rangle_{n_1,n_2,n_3,n_4} = N\langle 4\rangle_{0,0,0,0}$$

PID v_2 at intermediated p_T

What on the list

- LO : elliptic (v_2) flow for > 95% of all particles (p_t < few GeV)
- NLO : higher harmonics v_n, PID (*m* dependence) of v_n, v_n at intermediated p_T
- NNLO : non-linear mode mixing ($v_n \neq \varepsilon_n$) and the possible symmetry plane correlations, factorization violation $r(p_T)$, EbE $P(v_n)$, …



- more particle species and better precision would be very crucial
- looking forward to results from Run2

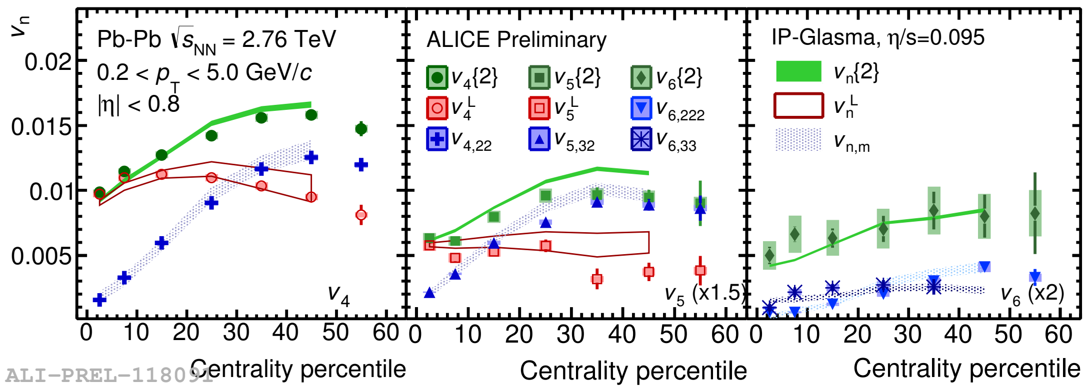
❖ How do we understand v_2 of baryon/meson crossing or grouping at intermediated p_T in high multiplicity p-Pb collisions? any recent calculation?

❖ What should we expect in pp collisions with most particle species?

Linear and non-linear response

⇨ **What on the list**

- LO : elliptic (v_2) flow for > 95% of all particles (p_t < few GeV)
- NLO : higher harmonics v_n, PID (*m* dependence) of v_n
- NNLO : non-linear mode mixing ($v_n \neq \varepsilon_n$) and the possible symmetry plane correlations, factorization violation $r(p_T)$, EbE $P(v_n)$, …



NNLO

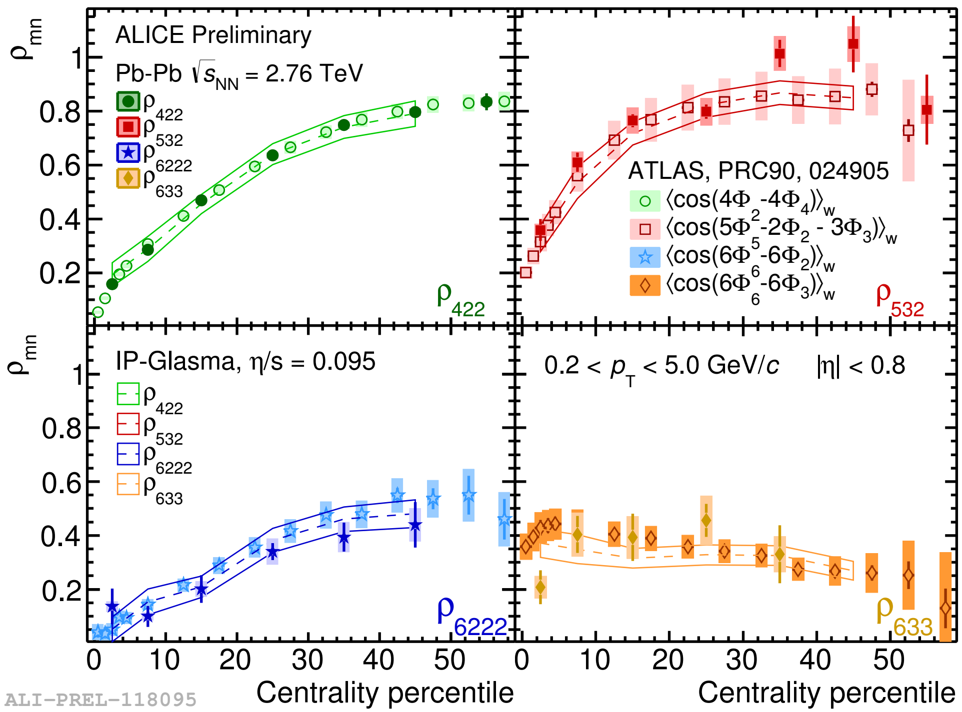
❖ non-linear component $v_{n,m}$ in Pb-Pb collisions
  - increase with increasing centrality, becomes dominant in peripheral collisions
  - non-linear response are mostly related to v_2 modes (due to geometry) to some power.

❖ What will happen in p-Pb collisions (no elliptic shape)

Linear and non-linear response

⇨ **What on the list**

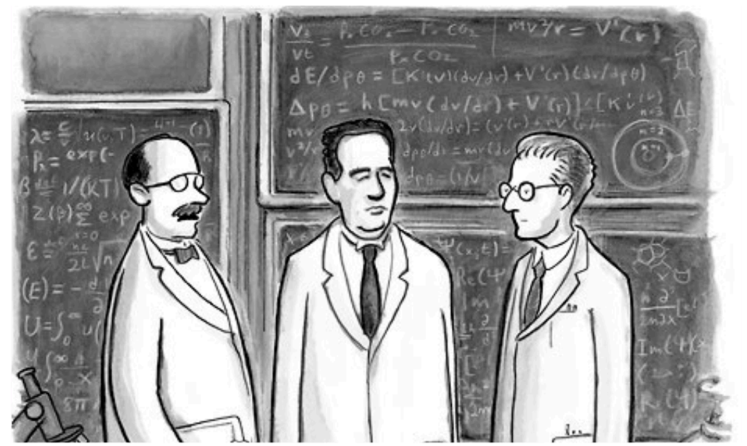
- **LO** : elliptic (v_2) flow for > 95% of all particles (p_t < few GeV)
- **NLO** : higher harmonics v_n, PID (*m* dependence) of v_n at intermediated p_T
- **NNLO** : non-linear mode mixing ($v_n \neq \varepsilon_n$) and the possible symmetry plane correlations, factorization violation $r(p_T)$, EbE $P(v_n)$, …



NNLO

- characterized correlation pattern
- reproduced by hydro

instead of a new Copenhagen Interpretation,
one step closer to a bright future



Is there collectivity in small systems? Copenhagen interpretation @
Workshop on Collectivity in Small Collision Systems